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A Physically Based Two-Dimensional Infiltration Model for Furrow Irrigation

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Summary: A two-dimensional physically based infiltration model was developed for furrow irrigation. Infiltration was simulated using the Green-Ampt infiltration method. The Green-Ampt infiltration parameters are available from numerous sources, unlike the Kostiaikov infiltration parameters. Simulation tests showed the two-dimensional model capable of estimating cumulative infiltration volume within 8% compared to simulated infiltration using the finite element model, Hydrus-2D. Application of the two-dimensional model in a surface irrigation advance model allows irrigation performance parameters to be predicted without extensive soil experiments.

Keywords: Furrow irrigation, infiltration, simulation

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A Physically Based Two-Dimensional Infiltration Model for Furrow Irrigation

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Introduction and Objectives

Fresh water is an important natural resource and has become the subject of increasing public concern as discussions have focused on water right and quality issues. The public has demanded more of the available water resources for environmental issues and economic growth which has increased competition for the limited amount of fresh water. Irrigation is a major consumer of water resources and public attention has increasingly focused on the agricultural sector.

The Nebraska Agricultural Statistics Service reported that approximately 46.4 million acres (Hamlin and Groskurth, 2000) were farmed in Nebraska 1999. Of the 19.4 million cultivated acres, approximately 8.1 million were irrigated. Approximately half (Yonts et. al., 1994) of the irrigated area is irrigated with center pivot systems while the remaining use surface irrigation. Furrow irrigated fields typically require a large amount of water to adequately irrigate the bottom end of a field. As demand for water resources increases, focus is shifting to improve the efficiency of these systems while improving or maintaining water quality.

Researchers have spent considerable time and resources studying methods to improve the efficiency of surface irrigated fields. Field experiments are required to adequately prove the proposed irrigation methods significantly impact the performance of an irrigation system; however, field experiments are expensive and time consuming. Simulation models have been developed to aid in the identification of promising technologies and reduce the time and expense of field experiments by focusing resources on these most promising technologies. A key component of these models is the ability to adequately describe the infiltration occurring during the irrigation. There are three basic types of infiltration equations which are used in these models (Walker and Skogerboe, 1987) and are described as either empirically or physically based. The physically based models are generally derived from either Richards equation or the Green-Ampt infiltration equation. Solutions based on Richards equation are generally solved numerically as there is often no closed-form solution to the differential equation without imposing severe restrictions or limitations. Conversely, the Green-Ampt equation is solved algebraically and offers a simpler solution to the problem.

The empirically based Kostiakov (1932) equation as modified by Lewis (1937) is the most predominantly used equation in surface irrigation models due to its simplicity of relating cumulative infiltration to the infiltration opportunity time (Eddebbah and Podmore, 1988). This equation works well when the fitting parameters have been established for an irrigation event. However, due to the empirical nature of the Kostiakov method, it becomes increasingly difficult to estimate the fitting parameters in the absence of water advance or infiltration studies. This does not allow for apriori establishment of the parameters which are only determined after the irrigation event has occurred. The Kostiakov parameters effectively lump initial soil moisture content, soil properties and dimensionality into the infiltration estimates which limits the effectiveness of this equation in modeling an irrigation event where limited irrigation data are available (Bautista and Wallender, 1993). Furthermore, the Kostiakov parameters, once determined, are not consistent within furrow

sets and between successive irrigation events which compounds the problem of modeling a field throughout the growing season. This limits the use of surface irrigation models that use the Kostiakov equation.

To overcome the limitations of empirically based infiltration equations, a two-dimensional, physically-based infiltration equation will be developed based on the Green-Ampt infiltration model. The objective of this paper is to develop a two-dimensional physically based model to predict the volume of infiltrated water across the furrow boundary.

Two-Dimensional Furrow Infiltration Models

Water infiltration is the process of water penetrating from the ground surface into the soil (Chow et. al, 1988). Infiltration is a key component of the hydrologic cycle and has been described using many different equations. Infiltration models based on the Richards equation are generally more useful where complex boundary conditions and/or non-uniform soils are encountered. Richards equation is limited to soils where air pressure effects (i.e. two-phase flow systems) are insignificant (Stephens, 1995) and also requires knowledge of the unsaturated hydraulic properties of the soil which is generally determined through the Brooks-Corey (1964) or van Genuchten (1980) models. Determining the fitting parameters of either the Brooks-Corey or van Genuchten models requires laboratory testing and often results in considerable variation across a field (House et. al., 1999).

The Richards equation can be solved analytically considering the simple case of vertical or horizontal, one-dimensional flow. However, extending the model to two or three dimensions greatly complicates the solution and no general closed form analytic solution exists without imposing severe restrictions on the model. Further difficulties arise when considering complex initial and boundary conditions of the modeled domain. There are several models which implement the Richards equation to solve water flow through variably saturated soils; however, they can be time consuming depending on the complexity of the modeled domain. It is also difficult to develop a model based on the Richards equation capable of handling all cases of initial and boundary conditions without imposing simplifying assumptions which limit the model functionality.

Other physically based models are generally derived from the Green-Ampt equation. Three infiltration parameters are required in the Green-Ampt model which considers a single homogeneous soil. The infiltration parameters allow for a wide variety of soil conditions. Numerous sources are available to describe the range of these parameters for a given soil classification. Solutions to these equations are much simpler than those based on Richards equation. A major assumption in the Green-Ampt infiltration model requires a sharp wetting front to develop which distinguishes saturated soil conditions behind the wetting front while initial soil moisture conditions remain ahead of the advancing front (Chow et. al., 1988).

Infiltration in a furrow is a two-dimensional process and requires a two-dimensional solution. Fok and Chiang (1984) envisioned a two-dimensional model, based on the Green-Ampt model, consisting of three separate infiltration zones (vertical, horizontal and sides) which all contribute to total infiltration. A rectangular furrow profile was assumed where the infiltration was calculated using the one-dimensional solution of Green-Ampt in the horizontal and vertical directions. The side zone boundary follows an assumed elliptical shape extending from the edge of the horizontal zone to the bottom of the vertical zone. A significant portion of the total infiltrated water is contained in the side zone and significantly influences the shape of the cumulative infiltration curve

which rapidly becomes linear, or even concave upwards, with increasing infiltration (Skonard et. al., 1999).

Singh et. al. (1987) also recognized the need to estimate infiltration in two dimensions when designing furrow irrigation systems. They developed a generalized infiltration model based on the streamtube concept using the continuity equation, energy equation and Darcy's law and extending the model to 1-D, 2-D and 3-D cases. A unique development in the model used Darcy's law alternatively expressed as a function of hydraulic resistance which allowed for computation of converging and diverging flows.

The two-dimensional model by Singh et. al. (1987) was verified using the experimental data from Fok and Chiang (1984) but with a significant difference in the shape representation of the furrow profile. While Fok and Chiang (1984) assumed a rectangular profile, the 2-dimensional model of Singh et. al. (1987) used a semi-circular furrow profile and assumed the wetted perimeters between the two profiles were equal. Results of the two-dimensional model compared well with that of Fok and Chiang (1984). Maximum relative errors using the cumulative infiltration were within 8 percent for a wet soil and generally less than 6 percent for a dry soil. Singh et. al. (1987) concede that their two-dimensional model has difficulty obtaining analytic solutions for other furrow profiles (i.e.. trapezoidal, triangular, etc).

Schmitz (1993a) treated the two-dimensional infiltration process as a sum of weighted one-dimensional processes in separate soil columns, particularly noting the influence of the shape of the furrow profile on infiltration. Infiltration is calculated using an analytic solution to a one-dimensional modified Richards equation. The modification accounts for the varying influence of gravity near the free water surface in the furrow where at this point infiltration is purely horizontal. The shape of the furrow profile is incorporated in the model using a geometry factor which is based on the radius of curvature of the profile. Within each streamtube, water is assumed to infiltrate orthogonally to the furrow perimeter and local infiltration opportunity time is determined based on the rise and fall of the water in the furrow.

In summary, an effective two-dimensional physically based model must account for the varying influence of gravity on infiltration. As water flows across a boundary it may assume any flow direction between the horizontal and vertical directions. In the purely horizontal direction, gravity has no influence on infiltration and is driven only by the soil matric potential. However, gravity gradually begins to dominate the infiltration process as the direction of infiltration approaches the vertical direction. This is a key physical process which must be incorporated into two-dimensional infiltration models.

Physical furrow profile properties must also be considered in an effective two dimensional infiltration model as curvature and wetted perimeter effects have been shown to strongly influence infiltration. Consider the case of infiltration during furrow irrigation, as water advances down the furrow, the depth of water in the furrow changes with time. As the depth increases, more of the furrow perimeter is used to infiltrate water into the soil profile. The curvature of the furrow and water flow in the furrow determine how much of the furrow perimeter is exposed to the water, thus influencing the wetted perimeter and the rate of infiltration. Also, the hydraulics in the furrow are constantly changing. The lowest portion of the furrow, generally considered the center of a uniformly shaped furrow, has the largest water pressure head on the furrow perimeter whereas at the free water surface no pressure head exists. The changing hydraulic head at any point along the

wetted furrow surface will influence the infiltration rate and must be accounted for in the infiltration model.

Infiltration Model Development

The two-dimensional model is based on a model proposed by Martin et. al. (1997). The domain of the conceptual model is illustrated in Figure 1. Since the profile of the furrow is assumed symmetric about the y axis, only one-half of the domain is simulated. The wetted perimeter of the furrow is divided into n discrete points with equal delta x increments. Infiltration is assumed to flow normal to the furrow boundary. The normal line, L_n , located at any point along the furrow boundary makes an angle, β , to the horizontal. Without knowing the true streampath of a water particle, it is assumed that water flows radially outward from the center of the furrow along L which is allowed to move vertically up and down with changing water depths in the furrow. The radially outward flow of water from the center of the furrow to the wetting front makes an angle α , to the horizontal. Green-Ampt assumptions are used in the development providing for a sharp wetting front where saturated conditions occur behind the wetting front in a homogeneous soil. Many different furrow geometries (Fok and Chiang, 1984) have been used to describe the shape of the furrow profile (i.e. trapezoidal, elliptical, etc). A power law geometry has been used in this development and takes the form:

$$y(x) = ax^b \quad (1)$$

where $y(x)$ = elevation above the bottom of the furrow (m)
 x = position along the horizontal axis perpendicular to the length of the furrow (m)
 a, b = fitting parameters

The Green-Ampt method employs Darcy's law which is applied at n points along the furrow boundary. Even though water is assumed to flow radially outward, all infiltration calculations are based along L_n , the normal line to the furrow at x to determine the flux across the furrow boundary. Applying Darcy's law at a given x yields the following equation for the flux across the furrow boundary.

$$dq = K_s \left[\frac{Y(t) - y(x) + L_n \sin \beta + h_w}{L_n} \right] \quad (2)$$

where dq = incremental infiltration flux ($m^3/m^2/hr$)
 K_s = saturated hydraulic conductivity (m/hr)
 $Y(t)$ = hydraulic head at x (m) at time t (hr)
 $y(x)$ = elevation at x (m)
 L_n = normal radial distance to the wetting front (m)
 β = angle from the horizontal (rad)
 h_w = wetting front suction head (m)

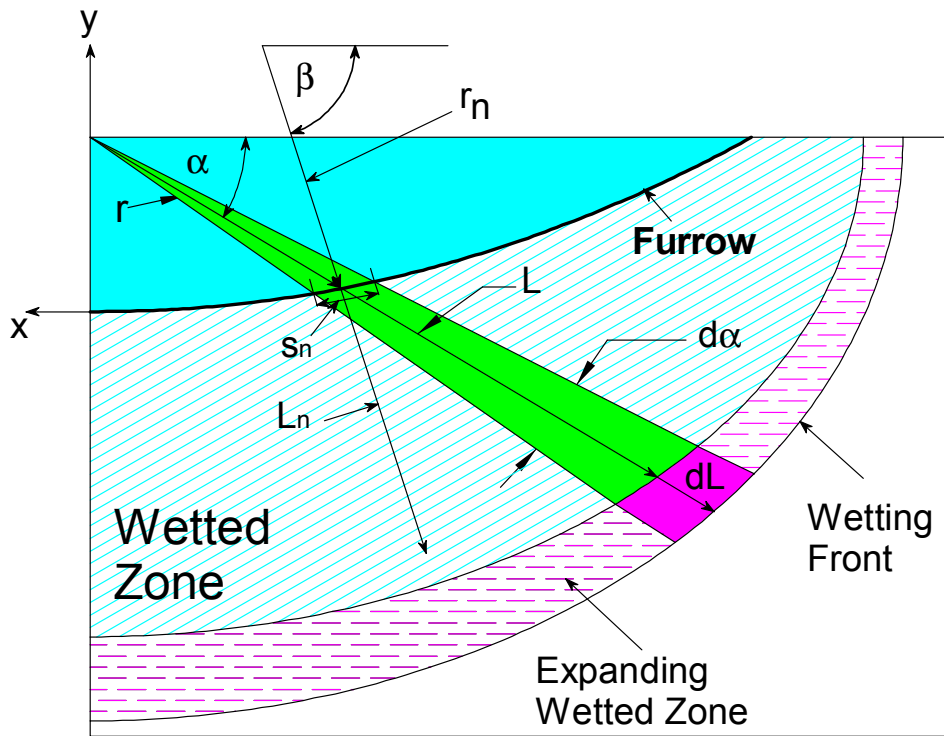


Figure 1 Diagram of components used in the Green-Ampt based two-dimensional infiltration model.

Multiplying the flux times the incremental wetted perimeter and time provides the incremental volume of water flowing across s_n :

$$dV = s_n K_s \left[\frac{Y(t) - y(x) + L_n \sin \beta + h_w}{L_n} \right] dt \quad (3)$$

where dV = incremental change in volume crossing the furrow boundary (m^3/m)
 s_n = incremental length of the wetted perimeter (m)

which must equal the incremental change in volume for the sector along L

$$dV = \Delta \theta s_\alpha \left(\frac{L+r}{r} \right) dL \quad (4)$$

where $\Delta \theta = \theta_s - \theta_i$
 θ_s = saturated moisture content (m/m)
 θ_i = initial moisture content (m/m)

s_α = elemental wetted perimeter in the α direction (m)
 L = radial distance from the furrow at x to the wetting front in the α direction (m)
 r = radial distance from the origin to the furrow boundary in the α direction (m).

Setting equations (3) and (4) equal yields the following equation:

$$s_n K_s \left[\frac{Y(t) - y(x) + L_n \sin \beta + h_w}{L_n} \right] dt = \Delta \theta s_\alpha \left(\frac{L+r}{r} \right) dL \quad (5)$$

Substituting into equation 5 using the following relationships between s_n and s_α and the flow lengths L and L_n gives:

$$s_\alpha = s_n \cos(\beta - \alpha)$$

$$L_n = L \cos(\beta - \alpha)$$

Figure 2

which yield the following differential equation in terms of L :

$$\frac{dL}{dt} = \left(\frac{K_s}{\Delta \theta} \right) \left(\frac{r}{L} \right) \left[\frac{Y(t) - Y(x) + L \cos(\beta - \alpha) \sin \beta + h_w}{(L+r) \cos^2(\beta - \alpha)} \right] \quad (6)$$

Once L has been determined in the α direction, equations (6) and (2) may be applied to calculate the flux at the specified time increment in the normal (β) direction. Integrating the flux over the entire wetted perimeter allows the determination of the incremental volume across the furrow boundary during each time step. Equation (8) determines the cumulative volume of water infiltrated over time:

$$V_t = V_{t-1} + \frac{dV}{dt} \Delta t \Big|_t \quad (7)$$

where V_t = volume of water infiltrated at the current time step (m^3/m)
 V_{t-1} = volume of water infiltrated at the previous time step (m^3/m)
 dV/dt = time rate of change of infiltrated water ($\text{m}^3/\text{m}/\text{hr}$)
 Δt = time step (h).

Testing

The two-dimensional Green-Ampt model was compared to Hydrus-2D (Simunek et. al., 1996) which is a finite element model that simulates water flow in variably saturated media. Hydrus-2D was used as the comparative model due to the difficulties of obtaining actual infiltration data at a specific point along a furrow. Also, a large number of samples are required to adequately assess the two-dimensional infiltration model during a single irrigation which is physically not possible using

many furrows. For these reasons, it was decided that testing would be conducted using a computer model capable of modeling variably saturated flow.

Simulations were conducted using three furrow profiles and three representative soils. The cumulative volumes infiltrated were compared to evaluate the capabilities of the two-dimensional model. The soils represent a broad spectrum of soil conditions typically exhibited in furrow irrigation in Nebraska. The soil hydraulic parameters used in the analysis are listed in Table 1. Hydrus-2D uses the van Genuchten soil hydraulic function to describe the unsaturated soil hydraulic properties.

Table 1. Listing of soil hydraulic properties used in the analysis.

Soil	ϕ	θ_r	n	α	K_s	h_w	$\Delta\theta$
S1	0.509	0.073	1.333	0.009012	1.28	15.58	0.250
S2	0.472	0.081	1.321	0.006244	0.53	21.44	0.203
S3	0.509	0.085	1.317	0.014062	3.11	9.37	0.259

ϕ = Saturated Porosity	K_s = Saturated Hydraulic Conductivity (cm/h)
θ_r = Residual Moisture Content	h_w = Wetting Front Suction Head (cm)
n = van Genuchten model parameter	$\Delta\theta$ = Soil Moisture Content Differential
α = van Genuchten model parameter	

Table 2. Listing of furrow properties.

Furrow	Power Law Coefficients		Wetted Perimeter at 0.05 m depth (m)
	a	b	
F1	0.641	1.402	0.351
F2	1.611	2.085	0.396
F3	6.623	3.056	0.425

Results

The two-dimensional radial infiltration model, equation (7), was compared to Hydrus-2D (Simunek et. al., 1996). Nine simulations were conducted using three furrow geometries and three soils. The soil types used in the analysis are typical of furrow irrigated fields in central Nebraska and the selected furrow geometries represent the extreme conditions as measured in field experiments. For a given depth of water, furrow F1 has the smallest wetted perimeter, conversely, furrow F3 exhibits much greater curvature and allows for a greater depth of water throughout the width of the furrow. Larger hydraulic heads at the modeled nodes are achieved throughout more of furrow F3 which combined with a longer wetted perimeter allows for a greater amount of water to infiltrate across the furrow boundary.

The cumulative infiltration volume was used to validate the physically based infiltration model. Figures 2 and 3 show the results of simulations using a constant water depth of 0.05 m. Tables 3 and 4 provide simulation results along with the calculated relative error between the two simulation methods. Generally, the relative error between the radial infiltration model and Hydrus-2D was less than 5% with a maximum error across all simulations of approximately 8%. Total predicted cumulative infiltration is reasonable for the silt loam furrows of the Central Platte Valley in Nebraska. Predicted infiltrated depths range from 4 to 9 cm across the range of soil conditions simulated using a wetted furrow spacing of 1.52 m and 12-hour cutoff time.

The radial infiltration model is sensitive to the soil hydraulic properties and wetted perimeter. The wetted perimeter for each furrow at a constant depth of 0.05 m is listed in Table 2. It is evident that the total infiltrated volume increases as the wetted perimeter increases. A similar trend occurs with the soil properties based on increasing hydraulic conductivity of the three soils.

Conclusion

A physically based two-dimensional infiltration model has been developed to estimate the volume of water which infiltrates across a furrow boundary based on the Green-Ampt infiltration procedure. Simulation analysis has shown that the two-dimensional physically based infiltration model is capable of estimating the cumulative infiltrated volume when compared to a finite element analysis of flow through a porous media. Close agreement was achieved across a range of soils and furrow profiles. The two-dimensional model has been incorporated into a surface irrigation advance model to predict irrigation performance and distribution of infiltrated water along a furrow thus providing predictive capability to surface irrigation simulation. The model was sensitive to reflect changes in soil hydraulic properties and wetted perimeter.

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Table 3 Results of the simulation the furrow F2.

Time (h)	Soil S1			Soil S2			Soil S3		
	Hydrus (m ³ /m)	Radial (m ³ /m)	%Rel Err	Hydrus (m ³ /m)	Radial (m ³ /m)	%Rel Err	Hydrus (m ³ /m)	Radial (m ³ /m)	%Rel Err
1	0.019	0.020	2.07%	0.012	0.012	-0.17%	0.029	0.031	4.42%
2	0.030	0.032	4.97%	0.018	0.019	3.09%	0.049	0.051	5.33%
3	0.040	0.042	5.75%	0.024	0.025	3.81%	0.067	0.070	4.46%
4	0.049	0.052	5.89%	0.028	0.030	4.93%	0.085	0.088	3.29%
5	0.058	0.061	5.86%	0.033	0.035	4.82%	0.103	0.105	2.14%
6	0.067	0.070	5.56%	0.038	0.040	5.32%	0.121	0.122	1.00%
7	0.075	0.079	5.20%	0.042	0.044	5.00%	0.138	0.138	-0.07%
8	0.083	0.087	4.81%	0.046	0.049	4.98%	0.156	0.154	-1.09%
9	0.091	0.095	4.38%	0.050	0.053	5.18%	0.173	0.170	-1.91%
10	0.099	0.104	4.12%	0.054	0.057	5.17%	0.191	0.186	-2.68%
11	0.107	0.111	3.63%	0.058	0.061	4.98%	0.208	0.201	-3.41%
12	0.115	0.119	3.21%	0.062	0.065	4.66%	0.226	0.216	-4.38%

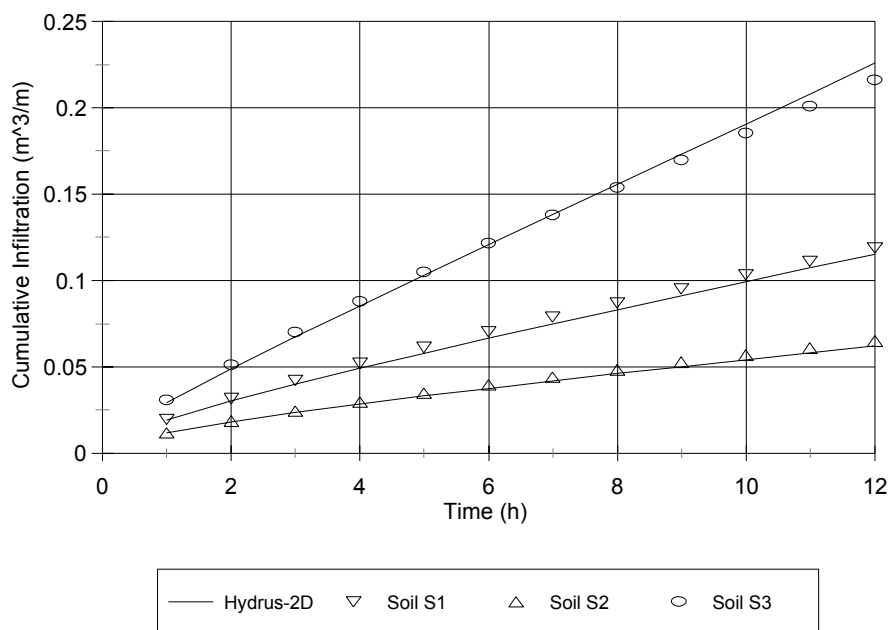
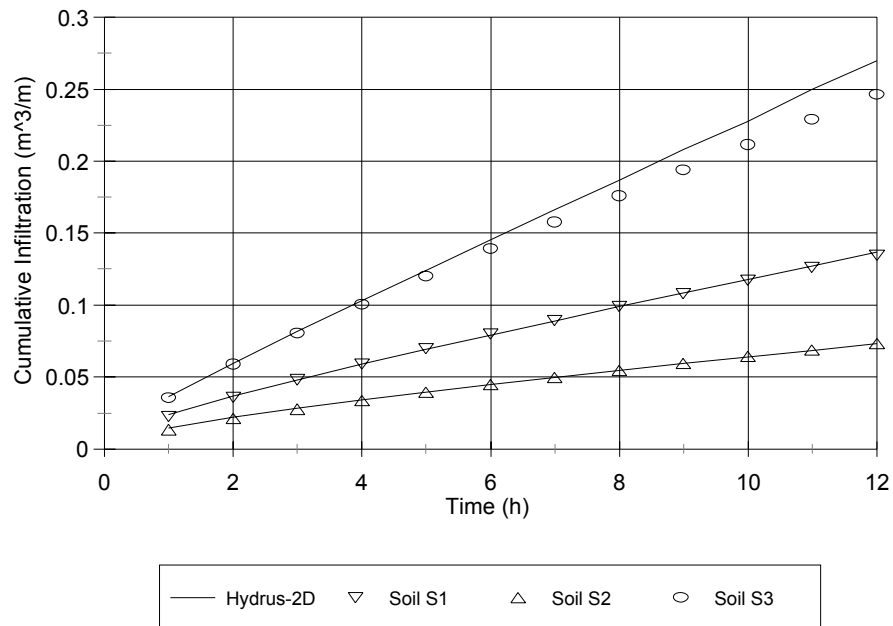
**Figure 2** Cumulative infiltration from the radial model compared to Hydrus-2D using three soils with the geometry for furrow F2.

Table 4 Results of the simulation for furrow F3.

Time (h)	Soil S1			Soil S2			Soil S3		
	Hydrus (m ³ /m)	Radial (m ³ /m)	%Rel Err	Hydrus (m ³ /m)	Radial (m ³ /m)	%Rel Err	Hydrus (m ³ /m)	Radial (m ³ /m)	%Rel Err
1	0.0238	0.0226	-5.04%	0.0148	0.0136	-7.86%	0.0364	0.0355	-2.47%
2	0.0368	0.0363	-1.36%	0.0220	0.0213	-3.18%	0.0596	0.0591	-0.84%
3	0.0482	0.0483	0.21%	0.0284	0.0279	-1.76%	0.0816	0.0806	-1.23%
4	0.0590	0.0593	0.51%	0.0342	0.0339	-0.88%	0.1030	0.1008	-2.14%
5	0.0694	0.0698	0.58%	0.0396	0.0395	-0.25%	0.1242	0.1203	-3.14%
6	0.0794	0.0799	0.63%	0.0448	0.0448	0.00%	0.1454	0.1393	-4.20%
7	0.0892	0.0896	0.45%	0.0498	0.0500	0.40%	0.1662	0.1578	-5.05%
8	0.0990	0.0991	0.10%	0.0546	0.0549	0.55%	0.1870	0.1760	-5.88%
9	0.1086	0.1083	-0.28%	0.0594	0.0597	0.51%	0.2080	0.1940	-6.73%
10	0.1180	0.1174	-0.51%	0.0640	0.0644	0.62%	0.2280	0.2117	-7.15%
11	0.1274	0.1263	-0.86%	0.0686	0.0690	0.58%	0.2500	0.2291	-8.36%
12	0.1368	0.1350	-1.32%	0.0732	0.0735	0.41%	0.2700	0.2465	-8.70%

**Figure 3** Cumulative infiltration from the radial model compared to Hydrus-2D using three soils with the geometry for furrow F3.