12-1-2005

# Using Asterisms to Decode Multi-User MIMO Systems

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***Abstract*— The area of Multiple Input Multiple Output (MIMO) communications systems has received enormous attention recently as they can provide a roughly linear increase in data rate by using multiple transmit and receive antennas. MIMO combined with an Turbo coding has been shown as a promising way to achieve near capacity for wireless channels.**

**This paper proposes applying an Asterism based decoder to produce an additional multi user access scheme on top of the primary access method for Ad Hoc networks. After reviewing Sphere and Asterism decoding for MIMO systems with a larger number of transmit than receive antennas, this paper then shows how multiple users transmitting simultaneously can be represented as a MIMO system with more transmit than receive antennas.**

## I. Introduction

The area of Multiple Input Multiple Output(MIMO) communications systems has received enormous attention recently as they can provide a roughly linear increase [1] in data rate by using multiple transmit and receive antennas. Incorporating a multiple antenna system with error correcting code, such as Turbo coding, is a popular approach to achieving increased data rates [2].

At the same time as increasing data rates, future mobile communications are promised to be small, lightweight mobile devices which present problems to the implementation of multiple antenna systems. Thus reducing the size and cost of mobile terminals by reducing the number receive antennas will be factor in the implementation of MIMO especially for larger number of transmit antennas.

Of the previously described MIMO decoding schemes such as Maximum Likelihood (ML), Zero Forcing (ZF), Bell Labs Layered Space-Time (BLAST) [3] and Sphere decoding [2], only Maximum Likelihood and Sphere decoding can successfully decode MIMO systems with more transmit than receive antennas. This is at the detriment of computational complexity. Asterism decoding [4] is a scheme that achieved Maximum Likelihood performance for MIMO systems for any number of receive antennas, by considering the larger complex constellation created by a multiple transmit antennas and a single receive antenna. It was then extended to operate in combination with Sphere decoding to reduce the computational complexity of MIMO decoding for any number of transmit and receive antennas.

Another area of recent interest is the proposal of mobile networks with distributed transmitters and receivers, typically referred to as mobile ad hoc networks (MANET). A large volume of research in MANET utilize the IEEE 802.11 standards [5], where the number of concurrent users communicating severely limits the performance, and results in reduced throughput and efficiency. This paper proposes applying an Asterism based decoder to effectively produce an additional multi user access scheme on top of the primary access method for MANET.

The paper is ordered as follows: Section 2 describes Multiple In Multiple Out systems and previous decoders such as Maximum Likelihood and Sphere decoder for systems with a larger number of transmit than receive antennas ($n_t > n_r$). Section 3 reviews Asterism based decoders for such MIMO systems with $n_t > n_r$, while Section 4 gives a brief system description of a Multiple User Access scheme that uses multi path fading to distinguish between users and can be represented as a MIMO system with $n_t > n_r$. Finally Section 5 shows the performance of uncoded and Turbo coded Multi-User MIMO systems decoding using an Asterism based decoder.

## II. Multiple In Multiple Out Systems

The Multiple In Multiple Out approach was first introduced by Lucent's Bell Labs, with their BLAST family of Space Time Code structures [3]. A Vertical Bell Laboratories Layered Space-Time (VBLAST) scheme with Turbo error correction is considered in this paper. A VBLAST (also known as MIMO) system is one where the Turbo encoded bit stream, is interleaved and encoded into complex symbols before demultiplexed into $n_t$ substreams. Let $n_t$ be the number of transmit and $n_r$ be the number of receive antennas, and $s = (s_1, s_2, ...s_{n_t})^T$ denote the vector of symbols from the constellation size $C$, transmitted in one symbol period. The received vector $R = (R_1, R_2...R_{n_r})^T$ becomes:

$$R = Hs + n \tag{1}$$

where $n = (n_1, n_2, ...n_{n_r})^T$ is the noise vector of additive white Gaussian noise (AWGN) of variance $\sigma^2$ equal to $\frac{1}{2}$ per dimension.

The $n_r \times n_t$ channel matrix:

$$H = \begin{pmatrix} h_{1,1} & . & . & h_{1,n_t} \\ . & . & & . \\ h_{n_r,1} & . & . & h_{n_r,n_t} \end{pmatrix} \tag{2}$$

contains independent identical distribution (i.i.d.) complex fading gains $h_{i,j}$ from the $j^{th}$ transmit antenna to the $i^{th}$ receive antenna. We assume flat fading where the magnitude of the elements of $H$ have a Rayleigh distribution.

### A. *Previous MIMO decoders for $n_t > n_r$*

There has been considerable research for decoding MIMO systems, almost all scheme concentrate on systems with the same or more receive antennas than transmit antennas.

Optimal Maximum Likelihood decoding is achieved by minimising

$$\| Hs - R \|^2 \tag{3}$$

for all elements of $s$, which are symbols of constellation of size $C$. This would produce a search of length $C^{n_t}$. A system using $n_t = 4$ and $16QAM$ would have to test each of the $65536$ possibilities, far beyond being easily implemented on low cost and low power hardware [6].

The principle of Sphere decoding is to search the closest lattice point to the received vector within a sphere of radius $d$ centered at zero forcing estimate of the symbols $\widetilde{s}$. The Sphere decoder described in [2] decomposes the channel matrix and received vector from complex numbers into its real and imaginary parts to produce a channel matrix of twice the size of the original. The multi-dimensional Sphere decoder now becomes an interval centered around $\widetilde{s}$ and a Cholesky factorisation of $H$ is also used to determine the interval size based on the starting radius and any previously decoded symbols.

The Sphere decoder is essentially the same for $n_t > n_r$ but uses the $Padded$ channel matrix $H_{pad}$ to not only determine $\widetilde{s}$ but also the Cholesky factorisation. While the $padding$ of $H$ is satisfactory for the calculation of $\widetilde{s}$, when used for the Cholesky factorisation, the padded $H$ produces very small numbers for the lowest $n_t - n_r$ levels of the triangular matrix. This means that the Sphere decoder has a complexity of the order of $C^{n_t - n_r}$ i.e. Sphere decoder decomposes to the complexity of ML detection when $n_r = 1$.

## III. Asterism Decoding

Asterism decoding, proposed in [4], was created to reduce the computational complexity of Maximum Likelihood decoding and yet retain the performance and flexibility of reducing the number of receive antenna. This is achieved by considering the larger complex constellation created by a multiple transmit antennas and a single receive antenna.

The large complex constellation generated by equation (3) for all values of $C$ substituted into $s$, plotted in Figure 1, can be divided into $C$ smaller groups or Asterisms. Each of these Asterisms can in turn be divided into $C$ smaller Asterisms, and so on for all $n_t$ symbols.

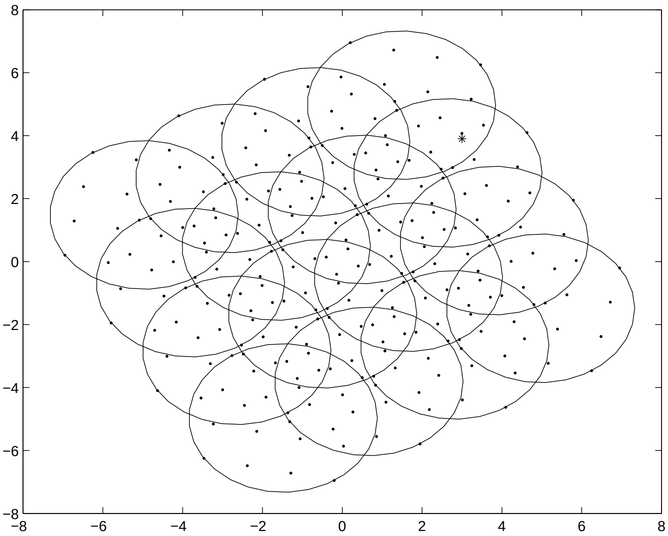


Fig. 1. Complex constellation R grouped into 16 Asterisms.

Finding the ML solution without having to test every point by grouping the complex constellation into Asterisms is the main concept behind the Asterism decoding.

For ease of explanation, we make the assumption that the magnitude of $H$ in (3) is decreasing i.e. $|h_1|$ is the largest and $|h_3|$ is the smallest. The radius of the Asterism radius at detection stage $k$ is:

$$Radius_{(k)} = \beta \times \sum_{j=k+1}^{n_t} |h(j)| \tag{4}$$

where $\beta =$ largest symbol magnitude, which for 16QAM is $\sqrt{18}$ the magnitude for the symbols $[3{+}3i, -3{+}3i, -3{-}3i, 3{-}3i]$. These Asterisms at the first detection stage are centered at $h_1 \times s_i$. Every possible combination is covered by these Asterism circles. The size and the amount of overlap of these circles is determined by the number of transmit antennas, the magnitude of the elements of $H$ and the Hamming distance of the constellation.

If the received vector $R$ is inside the one or more circles it is possibly the ML solution. The algorithm then subtracts this possible solution from $R$ and determines whether modified $R$ is in one of the new Asterism circles centered at $h_2 \times s_i$and of radius $|h_3|$. This recursive process continues until all $n_t$ symbols are found. If there is more than one combination found, the combination with the lowest complex distance measurement is chosen to be the ML solution.

When noise places the received vector outside any of the Asterisms the algorithm will find no symbol combination and fail. This can happen not only at the first stage of decoding, where noise puts $R$ outside the area covered by the largest Asterisms, but also at any later stage of decoding where $R$ may be part of a larger Asterism but not part of a small Asterism of later stages of decoding.

To overcome this the decoding the algorithm must not only find which Asterism the received vector is inside, but also

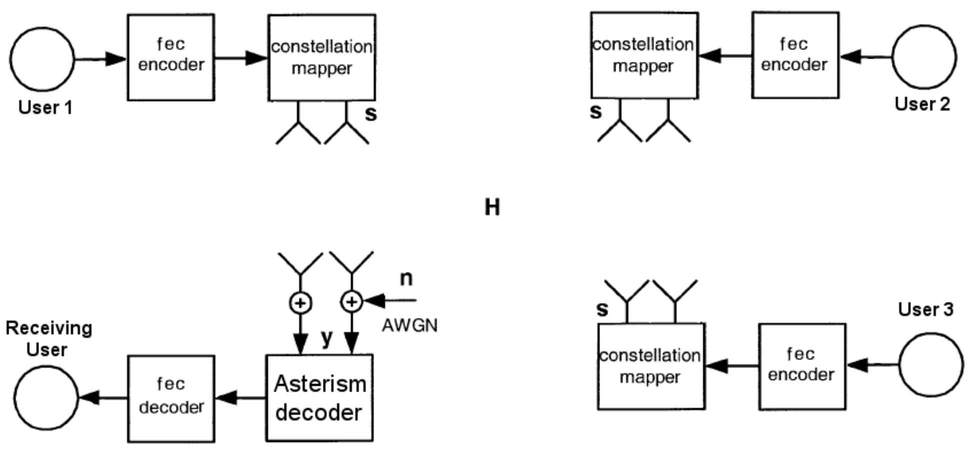


Fig. 2. Simplified block diagram of the Multiuser Access scheme, where each user has Multiple Transmit and Receive antennas.

allow for the case where it is inside none of the Asterisms. In this case the decoding chooses the Asterism to which it is closest to and continues the process to find the ML solution.

While using Asterism decoding to multiple transmit and a single receiver antenna system produces the ML performance, the performance of this type of system is relatively poor. To overcome this the use of multiple antennas at the receive was considered. The received vector where $n_t = 4$ and $n_r = 2$ becomes:

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\ h_{2,1} & h_{2,2} & h_{2,3} & h_{2,4} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{5}$$

To take advantage of the information provided by additional antennas Maximum Ratio combining is used at each stage of decoding and was found to have near Maximum Likelihood performance.

To further reduce the complexity of Asterism decoding with multiple receive antennas a combined Sphere-Asterism decoder was proposed [7] that used Asterism decoding to decode the first $n_t - n_r$ detected symbols. The Sphere decoder algorithm described in Section 2 applied to a real system of equations and chooses symbols from a real lattice, as opposed to complex symbols for Asterism decoding. To allow a combined Asterism and Sphere decoder we need to use complex Sphere decoder similar to [2], but make the adjustment of calculating a complex distance test to determine the points inside the sphere rather than the cosine algorithm.

## IV. MIMO as a Multiuser Access scheme

A simplified block diagram of a very simple MANET system using MIMO described in this paper is shown in Figure 2. Each of the users has $n_t = n_r = 2$ MIMO antennas and including error encoding/decoding. In a traditional Time Division Multiple Access system (TDMA), such as IEEE 802.11, when a user (say User 1) transmits its message to the receiver, the remaining users (Users 2 and 3) not only cannot transmit to the receiver, but also cannot transmit a message to any other user. Normally if more than one user transmits simultaneously the the signals will interfere with each other causing an error in transmission. If we assume that the users are separated enough so that their fading coefficients can be considered uncorrelated (similar to the main assumption of MIMO), then the channel matrix of $n$ users transmitting simultaneously becomes:

$$H = \begin{pmatrix} H_{user1} & H_{user2} & . & H_{user_n} \end{pmatrix} \tag{6}$$

where $H_{user}$ is a $n_t \times n_r$ channel matrix (in this example a $2 \times 2$ matrix)between each of the users' transmit antennas to the receiver's antennas.

A multiuser MIMO system with 2 users transmitting simultaneously is equivalent to the MIMO system of $n_t = 4$ and $n_r = 2$ describe in Section 2. Equation (5) now becomes:

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} h_{User1_{1,1}} & h_{User1_{2,1}} \\ h_{User1_{1,2}} & h_{User1_{2,2}} \\ h_{User2_{1,1}} & h_{User2_{2,1}} \\ h_{User2_{1,2}} & h_{User2_{2,2}} \end{bmatrix}^T \begin{bmatrix} s_{User1_1} \\ s_{User1_2} \\ s_{User2_1} \\ s_{User2_2} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{7}$$

Similarly, a system with 3 users transmitting simultaneously would be equal to a MIMO system with 6 transmit and 2 receive antennas and with 4 users would be equivalent to a MIMO with 8 transmit and 2 receive.

As described in Section 3, an Asterism based decoder can successfully decode such a MIMO system with more transmit than receive antennas, hence applying such a decoder to such a system with multiple transmitting users, creates a Multi-User access scheme in addition to the TDMA scheme often used for Ad-Hoc networks.

## V. Simulation Results

In this section we provide simulation results, by Monte Carlo methods, to illustrate the performance of an Asterism

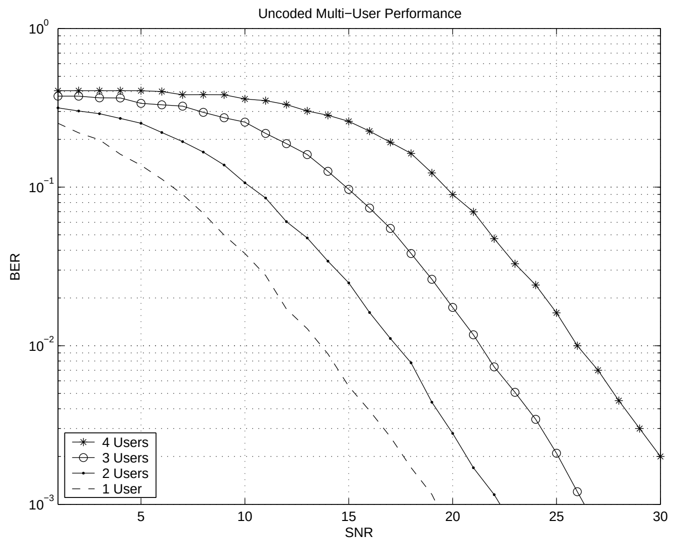


Fig. 3. Performance comparison of Asterism decoding for $n_t = 4$, $n_r = 1, 2, 3$ and $4$ using QPSK without error coding.

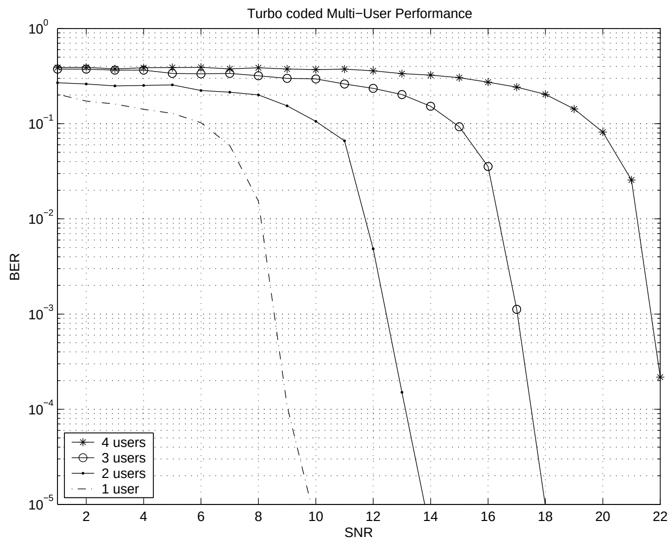


Fig. 4. Performance comparison of Turbo Asterism decoding for $n_t = 4$, $n_r = 1, 2, 3$ and $4$ using QPSK.

decoded Multi-User Access scheme describe in Section IV. The channels are assumed fast flat-fading, i.e. they are the constant over a single data burst and change from burst to burst, this is due to the movement of node, and the fading coefficients are generated according to a Rayleigh distribution. The assumption is made that the users' transmissions are synchronized but no other implementation MAC or high layers are made at this stage.

Figure 3 shows the performance of an Asterism decoded Multi-User MIMO system with 1, 2, 3 and four simultaneous transmitting users and that the performance of the system reduce by approximately $4dB$ for each additional user transmitting simultaneously. The loss between 4 transmitting users and a single user is approximately $12dB$. This may seem quite poor at first, but considering that without an Asterism based decoder at the receiver, only one user could transmit at a time producing congestion in the network and producing a higher BER when multiple users transmit simultaneously.

Figure 4 shows results of the same systems as Figure 3 with the addition of Turbo coding. The Turbo decoder used in our simulations consisted of two Log-MAP decoders [8] permuted by a deinterleaver. For our simulation we used a frame of length 9216 bits, two (7,5) convolution coders with puncturing giving a total rate of $1/2$ and all results are for $5$ iterations of the Turbo decoder. The error correction produced by the Turbo decoder has a consistent reduction of BER of approximately $8dB$. Which produces a BER of $10^{-5}$ at a SNR of $10dB$, again for each additional user the SNR increases by approximately $4dB$ to produce the same BER, with 2 users $14dB$, 3 users $18dB$ and approximately $22dB$.

## VI. Conclusion

This paper proposed applying an Asterism based decoder to produce an additional multi user access scheme on top of the primary access method that could be used for Mobile Ad Hoc networks. By applying an Asterism based decoder and hence the possibility of an additional multi user access scheme (on top of a primary access method, such as TDMA), multiple transmissions from different users can be decoded successfully, potentially increasing the efficiency and throughput of MANETs.

Further avenues of research include investigating methods that take advantage of the Multi-User access scheme and increasing the performance beyond what has been shown in this paper. If increasing the computational complexity were possible, further research could include reducing the loss in performance by each additional user by possibly a combination of more powerful error correction codes and joint error detection of the different coding schemes.

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