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Extending Asterism decoding to QAM and its complexity in Rician fading MIMO systems

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***Abstract*— The area of Multiple Input Multiple Output (MIMO) communications systems has received enormous attention recently as they can provide a roughly linear increase in data rate by using Multiple Transmit and Receive antennas. The optimal detection strategy for a MIMO receiver is to perform a Maximum-Likelihood (ML) search over all possible transmitted symbol combinations has an exponential complexity when the constellation size of number of transmit antennas increase. A number of sub-optimal decoders, such as VBLAST, provide linear decoding only where the number of receive antennas is at least equal to the number of transmit antennas .**

Asterism decoding proposed in [7] [8], described a scheme that achieved ML performance for PSK MIMO systems. By considering the larger complex constellation created by a multiple transmit antennas and a single receive antenna. The decoder was then extended to achieve ML like performance for any number of receive antennas.

This paper investigates the extension of Asterism decoding for QAM modulation schemes. It further shows that Asterism decoding has an approximate order of magnitude reduction in computational complexity when compared to ML decoding, and shows that this reduction is possible not only for Rayleigh fading channels, but also most Rician flat fading channels.

I. Introduction

In recent years the employment of multi-element antenna arrays at both transmit and receive sites has received much interest because it is capable of enormous theoretical capacity over wireless communications systems, in particular the area of Multiple-In-Multiple-Out(MIMO) systems [1]. However, the design of practical signaling and signal processing schemes capable of supporting data rates close to the capacity limit remains a major challenge.

A number of sub-optimal decoding schemes such as Zero Forcing (ZF), the Bell Labs Layered Space-Time (BLAST)[1] [3] and the QR Decomposition [4] perform best when the number of receive antennas is greater than the number of transmit antennas, while performance is less-optimal when antenna numbers are equal and cannot be used for systems where there are less receive antennas than transmit. Whereas the optimal detection strategy for a MIMO receiver is to perform a Maximum-Likelihood (ML) decoding has a computational complexity exponential with the number of transmit antennas and constellation size.

It was shown in [5], that using such a ML decoder with less receive than transmit antennas could still provide sufficient increase in data rate, hence removing the need for additional receive antennas on a receiving device, such as a mobile terminal. Reducing the size and cost of mobile terminals by reducing the number receive antennas leads to the need for a lower complexity decoder that achieves ML like performance for systems where the number of receive antennas is less than the number of transmit antennas.

Asterism decoding proposed in [7] [8], described a scheme that achieved ML performance for PSK MIMO systems. By considering the larger complex constellation created by a multiple transmit antennas and a single receive antenna. The decoder was then extended to achieve ML like performance for any number of receive antennas. This paper investigates the extension of Asterism decoding for modulation schemes with symbols that have varying amplitudes, such as 16QAM.

The paper is ordered as follows. Section 2 gives a brief system description of MIMO systems and decoders such as VBLAST and Maximum Likelihood. Section 3 describes Asterism decoding for MIMO systems with 16QAM. While Section 4 gives a simple comparison the complexity of Asterism decoding and ML decoding and shows that mean complexity only slightly increases for Rician channels.

II. Multiple In Multiple Out Systems

The Multiple In Multiple out approach was first introduced by Lucent's Bell Labs, with their BLAST family of Space Time Code structures [2]. An uncoded Vertical Bell Laboratories Layered Space-Time (VBLAST) scheme, where the input bit stream is de-multiplexed into n_t substreams, is considered in this paper. Let n_t be the number of transmit and n_r be the number of receive antennas, and $s = (s_1, s_2, ...s_{n_t})^T$ denote the vector of symbols of constellation size C, transmitted in one symbol period. The received vector $R = (R_1, R_2...R_{n_t})^T$ is:

$$R = Hs + n \tag{1}$$

where $n = (n_1, n_2, ...n_{n_t})^T$ is the noise vector of additive white Gaussian noise of variance σ^2 equal to $\frac{1}{2}$ per dimension. The $n_r \times n_t$ channel matrix:

$$H = \begin{pmatrix} h_{1,1} & . & . & h_{1,n_t} \\ . & . & & . \\ h_{n_r,1} & . & . & h_{n_r,n_t} \end{pmatrix} \tag{2}$$

contains independent identical distribution (i.i.d.) complex fading gains $h_{i,j}$ from the j^{th} transmit antenna to the i^{th}

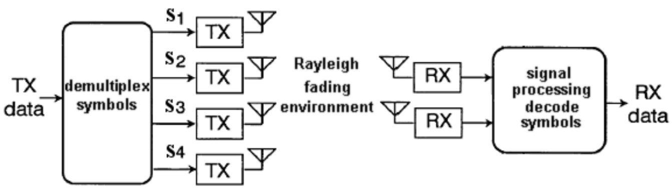

Fig. 1. Block diagram of a Layered Space-Time system where $n_t = 4$ transmit and $n_r = 2$ receive antennas.

receive antenna. We assume Rayleigh flat fading where the magnitude of the elements of H have a Rayleigh distribution. A block diagram of Layered Space-Time system is shown in Figure 1.

A. VBLAST decoding

The sub-optimal but less complex V-BLAST detector was proposed [1] [3] as a reduced complexity method to decode Layered Space-Time systems. A nulling (ZF) process was first introduced, which uses a pseudo inverse of H to produce estimates, $\widetilde{s}$, of the individual symbols, which are then passed to individual decoders. Conceptually, each transmitted symbol is considered in turn to be the desired symbol and the remaining symbols are treated as interferers.

$$\widetilde{s} = H^{\dagger} Y \tag{3}$$

where †is the Moore-Penrose pseudo inverse [2].Another method of nulling with better performance is to modify the receiver antenna pre-processing to carry out Minimum Mean Squared Error (MMSE) rather than ZF.

$$\widetilde{s} = \left(\left(\left(H^H H + \left(\sigma^2 I \right) \right)^{-1} \right) H^H \right) Y \tag{4}$$

where σ^2 is the noise variance.

MMSE and ZF nulling have the disadvantage that some of the diversity potential of the receiver antenna array is lost in the decoding process. To take advantage of the diversity potential, nonlinear techniques, such as Ordered Successive Interference Cancelation (OSIC) have been introduced [3] and shown to have superior performance.

The OSIC decoding (as called V-BLAST decoding) algorithm uses the detected symbol $\widetilde{s_i}$, obtained by the zero forcing, to produce a modified received vector with $\widetilde{s_i}$ canceled out. This modified received vector has fewer interferers and better performance due to a higher level of diversity. This process is continued until all n_t symbols have been detected. Obviously an incorrect symbol selection in the early stages will create errors in the following stages. Therefore the order in which the components are detected becomes important to the overall system performance.

While Zero Forcing, MMSE, OSIC (and others) provide a low complexity and reasonable performance when $n_r > n_t$, their performance is substantially reduce when $n_r = nt$ and cannot decode when $n_r < n_t$. This could potentially restrict future mobile terminals by requiring a larger number of receive antennas.

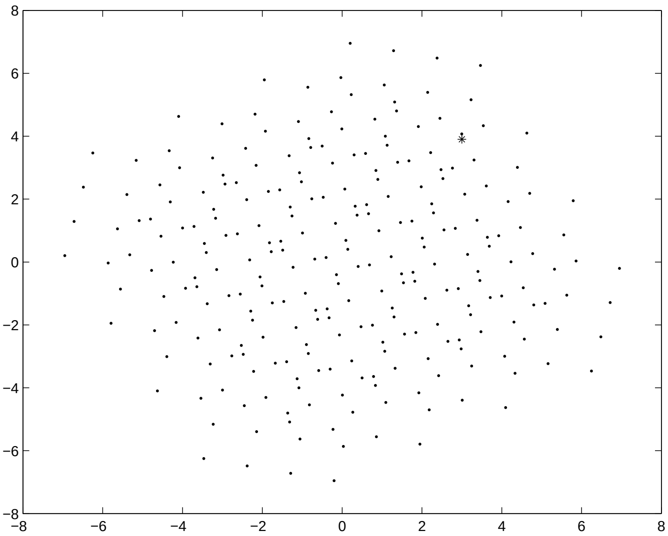

Fig. 2. Large complex constellation Y with 2 transmit 1 receive antennas, 16QAM, and receive vector R represented by *.

B. Maximum Likelihood Detector

Optimal Maximum Likelihood decoding is achieved by minimising

$$\parallel Hs - R \parallel^2 \tag{5}$$

for all elements of s, which are symbols of constellation of size C. This would produce a search of length C^{n_t}, which for a system using 4 transmit antennas and 16QAM gives 65536 possibilities. This leads to the development of ML performance decoding methods with a reduced computational complexity.

If we plot each of these possibilities, as shown in 2, for the simple MIMO system with $n_t = 2$ and $n_r = 1$ and 16QAM with two random complex channel coefficients, a larger more complex constellation of size C^{n_t}=256 is created. The ML solution is found by distant measurement from each point of the complex constellation to the received vector R represented by an Asterisk.

III. Asterism Decoding for QAM

Asterism decoding, proposed in [7] [8], was created to reduce the computational complexity of Maximum Likelihood decoding and yet retain the performance and flexibility of reducing the number of receive antenna. By considering the larger complex constellation created by a multiple transmit antennas and a single receive antenna.

It can be seen that the complex constellation of Figure 2 can be divided into C (in this case 16) smaller groups or Asterisms [6], as shown in Figure 3. Each of these Asterisms can in turn be divided into C smaller Asterisms, and so on for n_t.

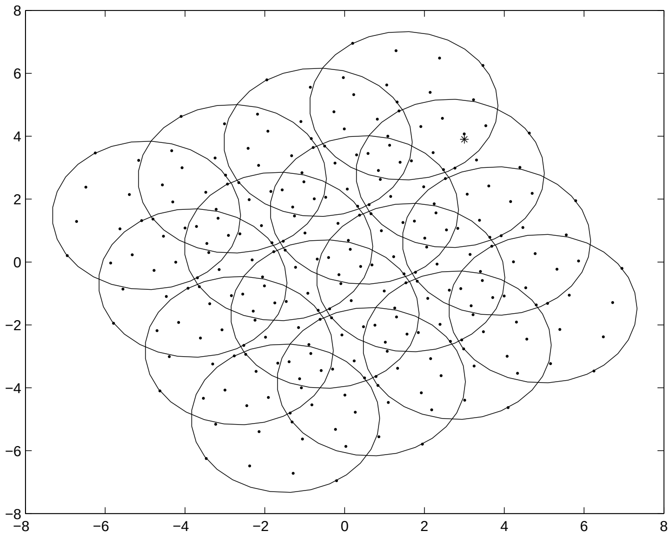

Fig. 3. Complex constellation Y grouped into 16 Asterisms.

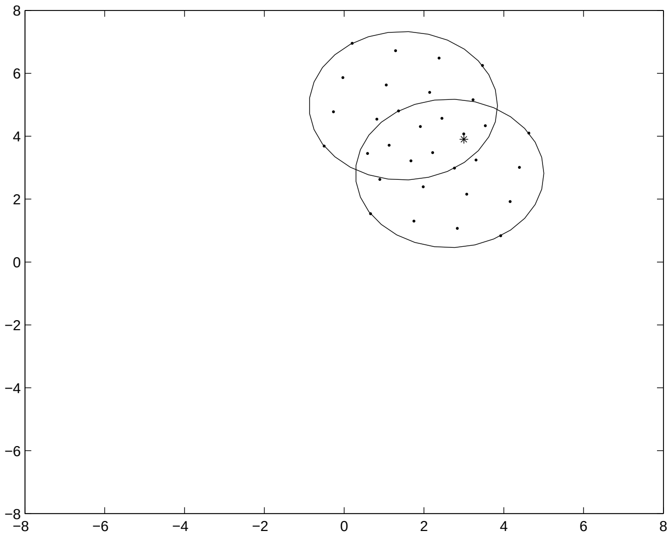

Fig. 4. 2 remainding Asterism from the complex constellation Y grouped into 2 Asterisms.

Finding the ML solution without having to test every point by grouping the complex constellation into Asterisms is the main concept behind the Asterism decoding.

For ease of explanation, we make the assumption that the magnitude of H in (3) is decreasing i.e. $|h_1|$ is the largest and $|h_3|$ is the smallest. The radius of the Asterism radius at detection stage k is:

$$Radius_{(k)} = \beta \times \sum_{j=k+1}^{n_t} |h(j)| \tag{6}$$

where $\beta =$ largest symbol magnitude, which for 16QAM is $\sqrt{18}$ the magnitude for the symbols $[3+3i, -3+3i, -3-3i, 3-3i]$. These Asterisms at the first detection stage are centered at $h_1 \times s_i$. Every possible combination is covered by these 16 Asterism circles. The size and the amount of overlap of these circles is determined by the number of transmit antennas, the magnitude of the elements of H and the Hamming distance of the constellation.

Simply put, if the received vector R is inside the one or more circles it is possibly the ML solution. The algorithm then subtracts this possible solution from R and determines whether modified R is in one of the new Asterism circles centered at $h_2 \times s_i$and of radius $|h_3|$. This recursive process continues until all n_t symbols are found. If there is more than one combination found, the combination with the lowest complex distance measurement is chosen to be the ML solution.

When an instantaneous noise vector places the received vector outside any of the Asterisms the algorithm will find no symbol combination and fail. This can happen not only at the first stage of decoding, where noise puts R outside the area covered by the largest Asterisms, but also at any later stage of decoding where R may be part of a larger Asterism but not part of a small Asterism of later stages of decoding.

To overcome this the decoding algorithm must not only find with Asterism the received vector is inside, but also allow for the case where it is inside none of the Asterisms. In this case the decoding chooses the Asterism to which it is closest to and continues the process to find the ML solution.

A. Extending to multiple receive antennas

While using Asterism decoding to multiple transmit and a single receiver antenna system produces the ML performance, the performance of this type of system is poor as shown later in Section IV. To overcome this the use of multiple antennas at the receive is now considered. The received vector where $n_t = 3$ and $n_r = 2$ becomes:

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} \\ h_{1,2} & h_{2,2} & h_{2,3} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \tag{7}$$

To take advantage of the information provided by the additional antenna a number of traditional receive diversity combining schemes, such as equal gain and Maximum Ratio combining, were tested and found to have sub-ML performance. This loss in performance was overcome by applying a modified Maximum Ratio combining at each stage of decoding and was found to have near ML performance. For the first detected symbol the modified Received vectors becomes:

$$R_{m_{1st}} = h^*_{11} R_1 + h^*_{21} R_2 \tag{8}$$

After the first stage of decoding the modified received vector becomes:

$$R_{m_{2nd}} = h^*_{12} R_1 + h^*_{22} R_2 \tag{9}$$

this process is continued to detect all n_t symbols are found. Because the order is which the symbols are detected, the detection order needs to be tested to ensure that the modified Maximum ratio combining does not change the detection ordering and that the symbol with the largest channel magnitude

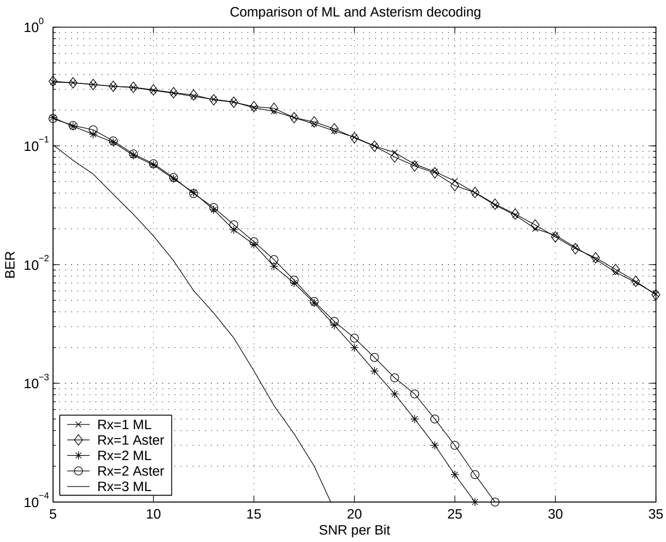

Fig. 5. Performance comparison of Asterism and ML decoding for $n_t = 2$, 16QAM and $n_r = 1$, 2 and 3 antennas.

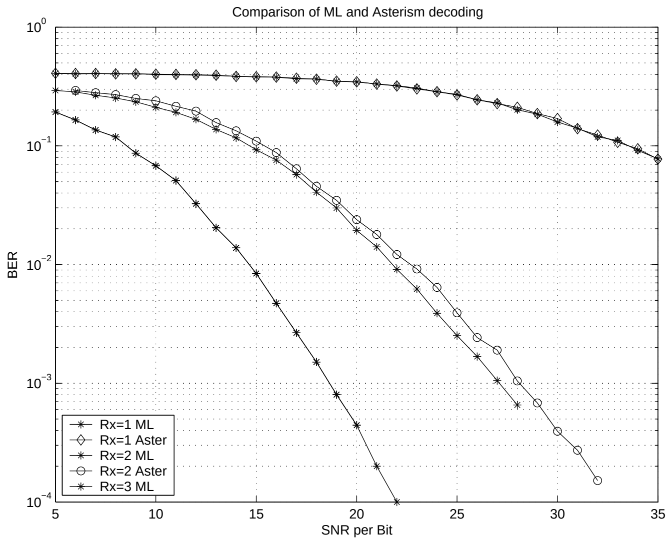

Fig. 6. Performance comparison of Asterism and ML decoding for $n_t = 3$, 16QAM and $n_r = 1$, 2 and 3 antennas.

is detected first while the symbol with the smallest channel magnitude is detected last.

Because the order is which the symbols are detected, the detection order needs to be tested to ensure that the modified Maximum ratio combining does not change the detection ordering and that the symbol with the largest channel magnitude is detected first while the symbol with the smallest channel magnitude is detected last.

IV. Simulation Results

In this section, we provide simulation results, by Monte Carlo method, to illustrate the performance and complexity of Asterism decoding. The channels are assumed quasistatic flat-fading, i.e., they are the same over a data burst and changes from burst to burst, with L the burst length equal to 100. The fading coefficients are generated according to

Figure 5 and Figure 6 show the performance of Asterism and ML decoding 16QAM MIMO systems with $n_t = 2$ and 3 respectively and $n_r = 1, 2, 3$ for Rayleigh fading channels. While Figure 7 shows the distribution of Complex distance calculations for Asterism decoding for $n_t = 3$ and Figure 8 displays Boxplots for the complexity of Asterism decoding for various values of the Rician factor K.

A. Performance Results

It can be seen from Figure 5, that the performance of Asterism decoding is similar to that of ML decoding for $n_r = 1$. For the symmetric system where $n_t = 2 = n_r$ Asterism decoding is less than $1dB$ worse than ML at a BER of 10^{-3}. Also, the gain between $n_r = 2$ is over $15dB$ gain over the system with one receive antenna. Whereas the increase of a third receive antenna produces only a gain of approximately $5db$.

Similarly Figure 6 shows that Asterism decoding matches ML decoding for n_t, $n_r = 1$, and when $n_t = 3 n_r = 2 and 3$ Asterism decoding is less than $1dB$ at a BER of 10^{-3}. It can also be seen that there is over $15dB$ gain over the system with one receive antenna.

As the number of transmit antennas increases the lower the performance of the system with one receive antenna, irrespective of the decoder. The simplest was to increase performance is by adding receive antennas, as shown in figure 5 and Figure 6. It can be seen that the greatest improvement is when the number of receive antenna is increased from 1 to 2, with less dramatic increase with the addition of further receive antennas.

B. Complexity in Rician fading environment

The ML decoding worse case scenario is when all channels have the same magnitude and computation will be equal to C^{n_t}(4096 for described system, or $512 \times C$) complex distance calculations. Whereas the best case is when each channel magnitude is equal to or greater than the sum of the remaining channel magnitudes, equal to $n_t \times C$ complex distance measurements.

Figure 7 shows the distribution of Complex distance calculations for Asterism decoding for $n_t = 3$. By using Monte Carlo simulations for the Rayleigh fading system of $n_t = 3$, 16QAM fading and by counting the number of C complex distance operations to decode the system, it was found that computational complexity is reduced to an approximate mean of 33 with a standard deviation of 25. This suggests an approximate order of magnitude reduction in the complexity when compared to ML decoding.

Until now we have considered the fading environment to be Rayleigh, that is purely Non Line of Site (NLOS). We now consider the case where the channel coefficients are a combination of Line Of Site (LOS) and Non Line of Site coefficients. The channel coefficients of Rician fading become:

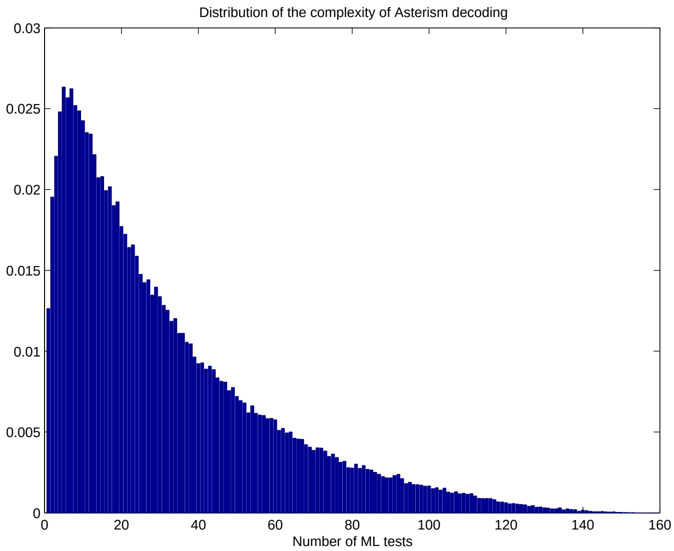

Fig. 7. Distribution of Complex distance calculations for Asterism decoding for $n_t = 3$ antennas, Rayleigh fading and $16QAM$.

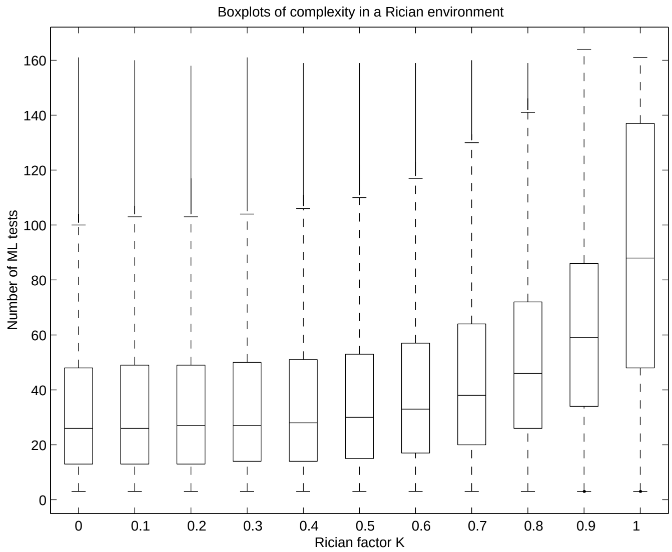

Fig. 8. Boxplots for the complexity of Asterism decoding for various values of the Rician factor K.

$$H_{Rician} = K \times LOS + \sqrt{(1-K)} \times NLOS \qquad (10)$$

where K is the Rician factor.

The boxplots of Figure 8 show the distribution of the complexity of Asterism decoding for a Rician environment in quartiles. The rectangle represents the data between the first and the third quartile with the horizontal line showing the median. As to allow reference the distribution of Figure 7 uses the same data as the Boxplot in Figure 8 when $K = 0$.

When $K = 0$, Rayleigh fading, the boxplot not only shows that the Median value is Approximately 25 but also that the third quartile is less than 50, ie 75% of the simulation had less than 50 ML tests, substantially less than the ML decoding described in Section II. It can also been sheen from Figure 8 that as K increase the median and third quartile only slightly increasing a median of 50 and third quartile of 60 at $K = 0.7$, before more significant increases as $K \Rightarrow 1$. It should also be noted that the performance of Asterism decoding in a purely LOS ($K = 1$) environment is extremely poor making it unacceptable for this type of system.

V. Conclusion

This paper has extended the previously described method of decoding uncoded Multiple-In-Multiple-Out systems called Asterism decoding. Asterism decoding looks for a more efficient way of finding the ML solution by first considering the case of multiple transmit antennas and a single receive antenna. By taking into consideration the magnitude of the symbols of the modulation scheme, Asterism decoding can be used for not PSK but also QAM. It was then shown to has an approximate order of magnitude reduction in computational complexity when compared to ML decoding and that the decoding complexity only slightly increases as the Rician K increases.

Further areas of the development of Asterism decoding may include a smart searching/ordering search to reduce complexity without affecting performance, multi-dimensional approach to Asterism search rather than proposed combining for multiple receive antennas systems and applying to coded systems such as Turbo coding and decoding separately or embedding into the Asterism decoding algorithm.

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