University of Nebraska - Lincoln
DigitalCommons@University of Nebraska - Lincoln

Faculty Publications in Computer & Electronics Engineering (to 2015) | Electrical & Computer Engineering, Department of

2003

Antenna Selection Scheme for Wireless Channels Utilizing Differential Space-Time Modulation

Le Chung Tran
University of Wollongong, lctran@uow.edu.au

Tadeusz A. Wysocki
University of Nebraska-Lincoln, wysocki@uow.edu.au

Follow this and additional works at: http://digitalcommons.unl.edu/computerelectronicfacpub

Part of the Computer Engineering Commons

Tran, Le Chung and Wysocki, Tadeusz A., "Antenna Selection Scheme for Wireless Channels Utilizing Differential Space-Time Modulation" (2003). *Faculty Publications in Computer & Electronics Engineering (to 2015)*. 52.
http://digitalcommons.unl.edu/computerelectronicfacpub/52

This Article is brought to you for free and open access by the Electrical & Computer Engineering, Department of at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Faculty Publications in Computer & Electronics Engineering (to 2015) by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.

Antenna Selection Scheme for Wireless Channels Utilizing Differential Space-Time Modulation

Le Chung Tran and Tadeusz A. Wysocki

School of Electrical, Computer and Telecommunications Engineering
Wollongong University, Northfields Avenue, NSW 2522, Australia
Phone: 61-2-4221 3413. Fax: 61-2-4227 3277
Email: {lct71,wysocki}@uow.edu.au

Abstract

In this paper, the authors prove that the differential space-time modulation techniques proposed in literature provide a full spatial diversity. Based on that, the authors propose an antenna selection scheme called general N-out-of-M antenna selection technique for wireless communications channels utilizing differential space-time modulation. The simulation shows that differential detection associated with our proposed technique provides much better bit error performance over that without antenna selection, and even over coherent detection without antenna selection at certain signal-to-noise ratios.

1. Introduction

Space-time codes have been examined intensively and various proposals for space-time codes have been mentioned in literature so far. Specifically, space-time codes can be decoded coherently when transmission gains of the channels between transmit and receive antennas are assumed to be known at the receiver. This assumption is suitable for the scenario where channels are assumed to be quasi-static flat fading, i.e. the fade changes so slowly that transmission gains are constant over a frame comprising multiple symbol periods. Therefore, the transmitter is able to transmit the training signals enabling the receiver to estimate the channels. In faster fading channels, however, this assumption is not reasonable any more. The differential space-time modulation (DSTM) [1, 2, 3, 4] is then a more practical candidate. In all existing DSTM techniques, transmission gains are not known at either transmitter or receiver. The drawback of differential detection is the worse of bit error performance.

In order to improve the performance of space-time codes in fading channels, several antenna selection techniques (ASTs) have been proposed in [5, 6] and our improved AST has been mentioned in [7] for coherent detection. The fundamental idea for selecting transmit (and/or receive) antennas in coherent detection is selecting the best channels out of available ones to transmit signals with an assumption that the transmission gains are known at the receiver. Although, this idea cannot be directly applied in differential detection as the transmission gains are not known at the receiver, it suggests us to search for an AST in case of DSTM. As mentioned in more details in Section 4, we make use of transmitting the initial code block to enable the receiver to predict semiblindly, with the maximum likelihood, the best channels out of all available ones. The receiver then informs the transmitter via a feedback loop enabling the transmitter to select these transmit antennas. It should be emphasized that, although, the transmission gains change faster than those in coherent detection, so that the transmission of training signals is impractical and, consequently, the utilization of DSTM is useful, but they are assumed not to change too fast to transmit a few feedback bits. Otherwise, no closed-loop transmit diversity technique, certainly, is applicable.

The rest of this paper is organized as follows. In Section 2, we recall some main points of DSTM techniques proposed in literature. The diversity of space-time codes utilizing DSTM is examined in the next section. In Section 4, we propose an antenna selection scheme called (general) N-out-of-M AST for wireless channels utilizing DSTM. The simulation is presented in Section 5 and Section 6 concludes the paper. The detailed mathematical proof of expressions used to prove the full spatial diversity of DSTM is given in the Appendix.

2. Differential space-time modulation

In this section, at first, we briefly outline the DSTM scheme proposed by Ganesan et. al. [1], which is based on the theory of unitary space-time block codes. Then, we shortly consider the DSTM schemes proposed by Hughes [2] and by Hochwald et. al. [3]. We consider a system with n transmit and m receive antennas. Let R_t, A, N_t be the $(m \times n)$-sized matrices of received signals at time t, transmission gains between receive and transmit antennas, and noise at the receive antennas, respectively. The $\kappa\eta^{th}$ element of A, namely $a_{\kappa\eta}$, is the gain factor of the path between the η^{th} transmit antenna and the κ^{th} receive antenna. Transmission gains are assumed to be identically independently distributed (i.i.d.) complex Gaussian random variables with the distribution $CN(0,1)$, which remain constant during every frame comprising several symbol periods and change from frame to frame. It is important to note that, by using the term "frame" here, the authors do not mean that the considered channels are very slow fading (or quasi-static fading) ones like in coherent detection. We just use this term to make it easier to explain the proposed AST in a very general case mentioned in Section 4. Up to date, in all existing DSTM techniques, the channel gains have been assumed to be constant during, at least, two consecutive code blocks. Therefore, when the Alamouti DSTM, for instance, is used, the size of frames here is four symbol periods. Again, the fade in the channels considered here is still fast enough, so that the utilization of DSTM is useful. Noises are assumed to be i.i.d. complex Gaussian random variables with the distribution $CN(0,\sigma^2)$. Let $\{s_j\}_{j=1}^{p}=\{s_j^R+is_j^I\}_{j=1}^{p}$ (where i^2=-1, s_j^R and s_j^I are the real and imaginary parts of s_j, respectively) be the set of p unitary symbols, which are derived from a signal constellation S and transmitted in the t^{th} block. Since the symbols are unitary, each symbol has the unit energy:

$$|s_j|^2=1 \tag{1}$$

We define: $Z_t=\frac{1}{\sqrt{p}}\sum_{j=1}^{p}(X_js_j^R+iY_js_j^I)$, where $\{X_j\}_{j=1}^{p}$ and $\{Y_j\}_{j=1}^{p}$ form a set of matrices of size $n\times n$, satisfying the following properties, which are linked to the theory of amicable orthogonal designs [8]:

$$\begin{aligned} X_jX_j^H &= I;\ Y_jY_j^H=I \qquad \forall j && (2)\\ X_jX_k^H &= -X_kX_j^H;\ Y_jY_k^H=-Y_kY_j^H\ \forall k\neq j && (3)\\ X_jY_k^H &= Y_kX_j^H \qquad \forall k,j && (4)\end{aligned}$$

where I is an identity $n\times n$ matrix and $(.)^H$ is the Hermitian transpose operation of the argument matrix. From (1), (2), (3) and (4), we have:

$$Z_tZ_t^H=\frac{1}{p}(\sum_{j=1}^{p}|s_j|^2)I=I$$

Hence, Z_t is a unitary matrix.

In the DSTM scheme proposed in [1], at the beginning of every frame, an initial matrix $W_0=I_{n\times n}$ is transmitted. Then, the matrix transmitted at time t $(t=1,2,3\dots)$ is given by:

$$W_t=W_{t-1}Z_t \tag{5}$$

As Z_t is a unitary matrix, the matrix W_t is also a unitary one. The model of the channel at the t^{th} transmission time $(t=0,1,2\dots)$ is as follows (the 0^{th} transmission means the initial transmission):

$$R_t=AW_t+N_t \tag{6}$$

If the transmission gain matrix A is assumed to be constant over two blocks $t-1$ and t, then the maximum likelihood (ML) detector for the symbols $\{s_j\}_{j=1}^{p}$ is calculated as follows [1], [9]:

$$\{\hat{s}_j\}_{j=1}^{p}=Arg\max_{\{s_j\},s_j\in S}Re\{tr(R_t^HR_{t-1}Z_t)\} \tag{7}$$

where $tr(.)$ is the trace operation. Hence, the ML detector for the symbol s_j is:

$$\begin{aligned}\hat{s}_j=Arg\max_{s_j\in S}[&Re\{tr(R_t^HR_{t-1}X_j)s_j^R\}+\\ &+Re\{tr(R_t^HR_{t-1}iY_j)s_j^I\}]\end{aligned} \tag{8}$$

If we denote:

$$D_j=Re\{tr(R_t^HR_{t-1}X_j)\}+iIm\{tr(R_t^HR_{t-1}Y_j)\} \tag{9}$$

where $Re\{.\}$ and $Im\{.\}$ are the real and the imaginary parts of the argument, respectively, then (8) becomes (see equation (13) in Appendix):

$$\hat{s}_j=Arg\max_{s_j\in S}Re\{D_j^*s_j\} \tag{10}$$

where D_j^* is the conjugate of D_j. Therefore, at the receiver, we form the statistic D_j to decode the symbol s_j. Expressions (9) and (10) show that the detection of the symbol s_j is carried out without the knowledge of transmission gains. Particularly, the symbol s_j can be decoded by using the received signal blocks in the two consecutive transmission times. The penalty of the differential detection mentioned above is that the signal-to-noise ratio (SNR) required for the same bit error rate as in the case of coherent detection is 3 dB higher. This is explicitly proven in [9] (equations (5.5) and (5.30)).

Independently, Hughes proposed the DSTM scheme based on the group theory in [2] and Hochwald et. al. devised a DSTM technique based on unitary space-time codes in [3]. It is interesting that the detector for the symbol s_j in these schemes (see equation (16) in [2] and Section V. C in [3]) is similar to that mentioned in (7), although these approaches are different from each other. This note is important in the sense that the consideration mentioned in the next section is true for all these DSTM schemes.

3. The spatial diversity of differentially modulated space-time codes

It is proven in Appendix (equation (12)) that the statistic D_j in (9) is calculated as follows:

$$\begin{aligned} D_j &\approx \frac{1}{\sqrt{p}}tr(A^HA)s_j+Re\{tr(W_t^HA^HN_{t-1}X_j)\}\\ &+ Re\{tr(N_t^HAW_{t-1}X_j)+iIm\{tr(W_t^HA^HN_{t-1}Y_j)\}\\ &+ iIm\{tr(N_t^HAW_{t-1}Y_j)\}\\ &\approx \frac{1}{\sqrt{p}}\sum_{\kappa=1}^{m}\sum_{\eta=1}^{n}|a_{\kappa\eta}|^2s_j+\eta_j\end{aligned}$$

where:

$$\begin{aligned}\eta_j &= Re\{tr(W_t^HA^HN_{t-1}X_j)\}+\\ &+ Re\{tr(N_t^HAW_{t-1}X_j)+\\ &+ iIm\{tr(W_t^HA^HN_{t-1}Y_j)\}+\\ &+ iIm\{tr(N_t^HAW_{t-1}Y_j)\}\end{aligned}$$

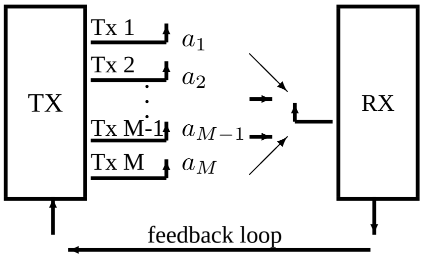

Fig. 1. The diagram of systems with the general N-out-of-M antenna selection scheme in the differential detection scenario.

The statistic D_j has a form of the received signal in the case that the symbol s_j is transmitted while the noise of the channel is η_j. The coefficient of s_j in the statistic is small only when all $m \times n$ complex modules of transmission gains are small. In other words, the DSTM schemes proposed in [1], [2] and [3] provide a full spatial diversity of $m \times n$ level. It is shown in [9] (equation (5.30)) that the SNR of the statistic D_j is approximately:

$$SNR_{diff} \approx \frac{1}{p}\frac{tr(A^H A)}{2\sigma^2} \approx \frac{\sum_{\kappa=1}^{m}\sum_{\eta=1}^{n}|a_{\kappa\eta}|^2}{2p\sigma^2} \tag{11}$$

Clearly, the larger the SNR_{diff} is, the more precise the detection is.

In addition, not only the DSTM techniques proposed in [1], [2] and [3], but also the one proposed by Tarokh et. al. [4] is proven to provide a full spatial diversity (equation (26) in [4]). Generally speaking, all existing DSTM techniques have the same property that the SNR of the decision metric is linearly proportional to $\sum_{\kappa=1}^{m}\sum_{\eta=1}^{n}|a_{\kappa\eta}|^2$. This note is important in the sense that the AST mentioned below is applicable to all existing DSTM schemes.

4. Antenna selection technique for channels utilizing DSTM

In this section, we consider a system comprising M transmit antennas and one receive antenna while the transmitted code blocks have a size of $N \times N$ $(N < M)$. The redundant transmit antennas are used to provide spatial diversity.

If the transmission gains are known at the receiver then, from the equation (11), one realizes that the optimal AST is selecting the N transmit antennas out of M available ones, from which the complex modules of the transmission gains of the paths to the receive antenna are maximal. This scheme has been well examined for coherent detection [5]. However, this principle cannot be directly applied for differential detection as the receiver has no knowledge about transmission gains. Therefore, we propose here an AST for differential detection where the transmitter selects transmit antennas based on the statistical comparison carried out at the receiver between received signals during the initial transmission time of each frame.

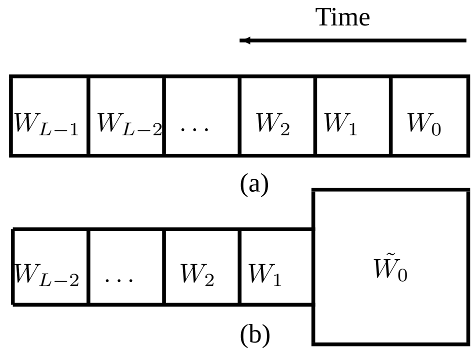

Fig. 2. Code blocks transmitted in a frame with (b) and without (a) the antenna selection technique. The delay of transmitting the feedback information is not considered.

Let us consider a system comprising $M = 4$ transmit antennas and only one receive antenna using the DSTM based on the Alamouti code ($N = 2$) as an example. The transmission gain matrix is assumed to be constant in a frame comprising $2L$ symbol periods. Again, as we emphasize in Section 2, this assumption does not mean that the channels are very slow fading (or quasi-static fading) ones like in coherent detection. The proposed AST is as follows:

At the beginning of every frame, the transmitter sends an initial block $\tilde{W}_0 = I_M$ via M transmit antennas, instead of sending an initial block $W_0 = I_N$ via N transmit antennas as in every existing DSTM technique. Because the size of the initial matrix W_0 has been changed, the size of the other matrices in (6) have also been changed. We emphasize the change in the size of matrices by using the tilde mark for matrices as below:

$$\tilde{W}_0 = I_4, \tilde{A} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \end{pmatrix}$$
$$\tilde{N}_0 = \begin{pmatrix} n_{01} & n_{02} & n_{03} & n_{04} \end{pmatrix}$$

where a_η (η=1...4) is the transmission gain of the channel from the η^{th} transmit antenna to the receive antenna, and $n_{0\eta}$ is the noise affecting this channel during the initial transmission. The received signal in the initial transmission time is:

$$\begin{aligned}\tilde{R}_0 &= \tilde{A}\tilde{W}_0 + \tilde{N}_0 \\ &= \begin{pmatrix} a_1 + n_{01} & a_2 + n_{02} & a_3 + n_{03} & a_4 + n_{04} \end{pmatrix}\end{aligned}$$

Let $\tilde{R}_0 = \begin{pmatrix} r_{01} & r_{02} & r_{03} & r_{04} \end{pmatrix}$. After determining the received matrix $\tilde{R}_0$, the receiver carries out two tasks.

First, the receiver semiblindly estimates the N best channels based on the matrix $\tilde{R}_0$ by comparing the complex modules of four received signals, namely $|r_{01}|$, $|r_{02}|$, $|r_{03}|$ and $|r_{04}|$, and then finding out the two received signals corresponding to the first and the second maximum complex modules. Without

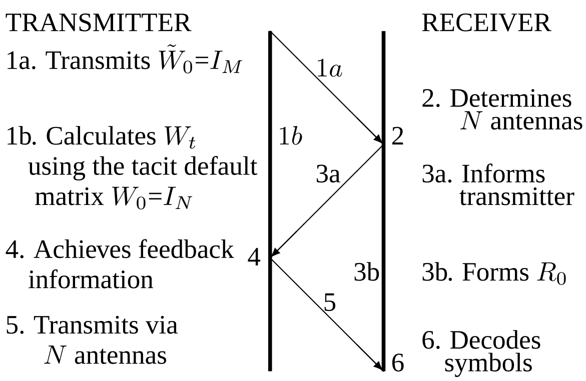

Fig. 3. The general N-out-of-M antenna selection technique for the system using DSTM.

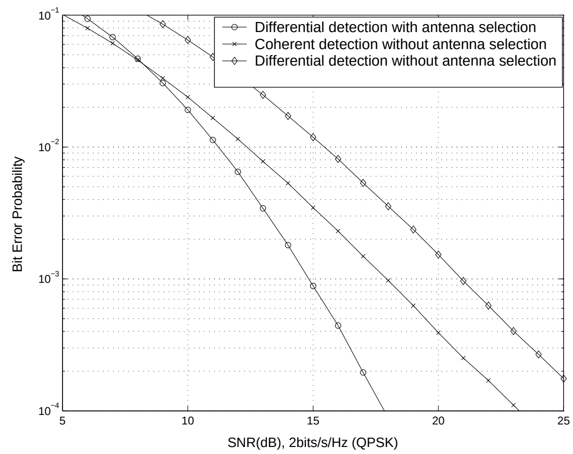

Fig. 4. Differential modulation with the proposed antenna selection techniques.

loss of generality, we assume here that they are the first and the second received signals. Then the receiver informs the transmitter via a feedback loop to select the first and the second channels (antennas) to transmit the rest of data of the considered frame (see Figure 1). Again, it is emphasized in Section 1 that, although, the transmission gains change faster than those in coherent detection, so that the transmission of training signals is impractical and, consequently, the utilization of DSTM is useful, but they are assumed not to change too fast to transmit a few feedback bits.

Second, the receiver forms the matrix R_0, which is used to decode the code blocks transmitted in the next transmission times, by taking the first and the second elements of the matrix $\tilde{R}_0$, which are corresponding to the first and the second maximum complex modules, i.e. R_0=$\begin{pmatrix} a_1 + n_{01} & a_2 + n_{02} \end{pmatrix}$. The transmission of the rest of data in the considered frame after the initial transmission is exactly the same as that in the system using the first and the second transmit antennas only. In other words, at the transmitter, the next transmitted matrices W_t (t=1, 2, 3, ...) in (5) are calculated by using the tacit default matrix W_0=I_N (in the example, the default matrix W_0=I_2). The formation of the matrices W_t does not necessarily take place after the transmitter achieves the feedback information.

It is worth to note that, in all existing DSTM techniques, the initial matrix W_0=I_N is only used to initialize the transmission (W_0 is utilized to calculate the next transmitted blocks W_t ($t \geq 1$), and to form the received matrix R_0, which is indispensable to decode the code blocks transmitted in the next transmission times). Unlike these techniques, in the proposed technique, the initial identity matrix $\tilde{W}_0$=I_M is transmitted. This matrix has two main roles. It enables the receiver to form the initially received matrix R_0 indirectly (from the received matrix $\tilde{R}_0$), which is used to decode the next code blocks. Simultaneously, in some sense, it also plays a role of training signals, i.e. it provides the receiver with the statistic of the channels. This is the main difference between the differential modulation with our AST and the one without antenna selection.

The transmission procedure of a whole frame including L code blocks is shown in Figure 2. The code block $\tilde{W}_0$ is transmitted via four transmit antennas in four symbol periods. The following blocks are transmitted via two transmit antennas in two symbol periods. We can realize that another difference between the differential modulation with our AST and the one without antenna selection is the number of code blocks transmitted in a frame. If one can transmit L-1 code blocks $W_1, \ldots, W_{L-1}$ carrying 2(L-1) symbols in a frame in a differential modulation technique without antenna selection, then the number of those in a frame in the proposed technique is L-2 (or 2(L-2) symbols are transmitted). Additionally, if the channels are required to be constant during, at least, four symbol periods in all existing DSTM techniques, then they are required to be unchanged during, at least, six symbol periods in our proposed AST. In the above consideration, the delay of transmitting the feedback information from the receiver to the transmitter is not considered.

We call the scheme mentioned above the general 2-out-of-4 AST. The scheme can be generalized to apply for other space-time block codes of a larger dimension N as well as for any number of transmit antennas M ($N < M$) without any difficulty. This scheme is called the general N-out-of-M AST (or just N-out-of-M AST whenever there is no ambiguity) and presented in Figure 3.

5. Simulation Results

In this Section, the performance of differential modulation with the proposed AST is presented. The Alamouti code and the QPSK signal constellation are considered. In this simulation, we consider the 2-out-of-4 AST, i.e. N=2 and M=4, and the receiver uses 3 feedback bits to inform the transmitter. The SNR is defined to be the ratio between the total power that the receiver receives during each symbol period and the power of noise. It can be seen from Figure 4 that differential detection without antenna selection has a 3 dB worse performance compared to coherent detection (without antenna selection). The performance of differential detection with the general 2-out-of-4 AST is worse than that of the coherent detection at low SNRs. However, at SNRs > 8 dB, the antenna selection remarkably improves the performance of differential detection. For instance, at $BER = 10^{-4}$, the performance of differential modulation is 5 dB better than that of coherent detection and, therefore, 8 dB better than that of differential detection without antenna selection.

This is interpreted as follows. The received signal $r_{0\eta}$ (η=1$\ldots M$) can be expressed: $r_{0\eta} = a_\eta + n_{0\eta}$, where a_η and $n_{0\eta}$ are defined in Section 4. Because both a_η and $n_{0\eta}$ are the i.i.d. Gaussian random variables with the distribution $CN(0,1)$ and $CN(0,\sigma^2)$ respectively, $r_{0\eta}$ is also an i.i.d. Gaussian random variable with zero expectation and its variance is given as follows:

$$Var(r_{0\eta}) = Var(a_\eta) + Var(n_{0\eta}) = 1 + \sigma^2$$

where $Var(.)$ is the variance operation. It is important to notice that, essentially, the N-out-of-M AST proposed in Section 4 is based on selecting the N best antennas, which correspond to the N received signals of the highest power (or highest variance). Therefore, if the SNR is large enough so that the variance of transmission gains is large enough, compared to the variance of noise σ^2, i.e. $1 \gg \sigma^2$, then the variance of the received signal can be roughly approximated to that of the transmission gain, i.e. $Var(r_{0\eta}) \approx Var(a_\eta)$. As a result, at high SNRs, the problem of selecting the N maximum complex modules among $|r_{0\eta}|$ (η=1$\ldots M$) may be considered as the problem of selecting the N maximum values among $|a_{0\eta}|$. Consequently, at low SNRs, selecting the maximum norms of the received signals does not always lead to selecting the optimal antennas which are corresponding to the transmission gains of the biggest norms, because the variance of noises σ^2, which is inversely proportional to the SNR, is not small enough compared to the variance of the transmission gains. In other words, the proposed AST does not always select the best antennas. However, at high SNRs, specifically at SNRs $> 8dB$, the contrast scenario usually happens. It means that, at larger SNRs, the proposed antenna selection scheme is more precise.

6. Conclusion

In this paper, we prove that all existing DSTM techniques provide a full spatial diversity of the $m \times n$ level. Then we propose an antenna selection scheme called (general) N-out-of-M AST for wireless channels utilizing differential detection. Instead of transmitting the initial matrix $W_0 = I_N$ (N is the required number of transmit antennas), we transmit the initial matrix $\tilde{W}_0$=I_M (M is the number of available transmit antennas). In all existing DSTM techniques, the signals received when the initial matrix W_0 is transmitted are used to initialize the transmission only. However, in our proposal, these signals are also utilized to enable the receiver to make the maximum likelihood decision about the best channels. The receiver then informs the transmitter via a feedback loop to select those channels. The channels are much faster fading ones compared to those in coherent detection, and consequently, the utilization of DSTM is useful (the transmission of training signals is impractical), but they are assumed not to change too fast to transmit some feedback information.

The simulation shows that differential detection with the proposed technique provides a much better bit error rate over that without antenna selection. Moreover, in comparison with the performance of coherent detection, the proposed scheme also provides a better bit error rate at any SNR >8 dB. Therefore, the proposed AST remarkably improves the performance of wireless channels utilizing DSTM.

7. Appendix

In this section, the authors mention the expression of the statistic D_j mentioned in (9). Then, we prove that the detector for the symbol s_j is given by:

$$\hat{s}_j = Arg \max_{s_j \in S} Re\{D_j^* s_j\}$$

Before proceeding further, it is important to note that:

1. $\{tr(\Psi A^H A)\}$ is real if Ψ is a Hermitian matrix, i.e. Ψ=Ψ^H. Consequently, $Im\{tr(\Psi A^H A)\}$=0.

2. $tr(\Omega\Upsilon)$=$tr(\Upsilon\Omega)$ if Ω and Υ are square matrices.

3. $Z_t^H W_{t-1}^H$=W_t^H

4. $Z_t = \frac{1}{\sqrt{p}} \sum_{k=1}^{p} (X_k s_k^R + i Y_k s_k^I)$

5. $\{X_k\}$ and $\{Y_k\}$ satisfy the following properties:

   a. $X_k X_k^H = I;\ Y_k Y_k^H = I \quad \forall k$
   b. $X_k X_l^H$=$-X_l X_k^H$; $Y_k Y_l^H$=$-Y_l Y_k^H \quad \forall k \neq l$
   c. $X_k Y_l^H = Y_l X_k^H; \quad \forall k, l$

One has:

$$\begin{aligned} R_t^H R_{t-1} &= (AW_{t-1}Z_t + N_t)^H (AW_{t-1}Z_t + N_t) \\ &= Z_t^H W_{t-1}^H A^H A W_{t-1} + \\ &+ Z_t^H W_{t-1}^H A^H N_{t-1} + \\ &+ N_t^H A W_{t-1} + N_t^H N_{t-1} \end{aligned}$$

If the noise variance is small enough, the term $N_t^H N_{t-1}$ is negligible. We have the following trans-forms:

$$\begin{aligned} D_j^R &= Re\{tr(R_t^H R_{t-1} X_j)\} \\ &\approx \frac{1}{\sqrt{p}} Re\{tr(X_j^H W_{t-1}^H A^H A W_{t-1} X_j) s_j^R\} + \\ &+ Re\{tr[\frac{1}{\sqrt{p}} \sum_{k=1}^{p} (X_k^H s_k^R - iY_k^H s_k^I) \times \\ &\qquad \times W_{t-1}^H A^H N_{t-1} X_j]\} - \\ &- Re\{tr(\frac{1}{\sqrt{p}} \sum_{k=1}^{p} iY_k^H W_{t-1}^H A^H A W_{t-1} X_j s_k^I)\} \\ &+ Re\{tr(N_t^H A W_{t-1} X_j)\} \\ &\approx \frac{1}{\sqrt{p}} tr(A^H A) s_j^R + \\ &+ Re\{tr(Z_t^H W_{t-1}^H A^H N_{t-1} X_j)\} - \\ &- Im\{tr(\frac{1}{\sqrt{p}} \sum_{k=1}^{p} X_j Y_k^H A^H A s_k^I)\} + \\ &+ Re\{tr(N_t^H A W_{t-1} X_j)\} \\ &\approx \frac{1}{\sqrt{p}} tr(A^H A) s_j^R + Re\{tr(W_t^H A^H N_{t-1} X_j)\} \\ &- Im\{tr(\frac{1}{\sqrt{p}} \sum_{k=1}^{p} X_j Y_k^H s_k^I A^H A)\} + \\ &+ Re\{tr(N_t^H A W_{t-1} X_j)\} \end{aligned}$$

Let Ξ=$\sum_{k=1}^{p} X_j Y_k^H s_k^I$. Clearly, Ξ =Ξ^H, i.e. Ξ is a Hermitian matrix. Therefore:

$Im\{tr(\frac{1}{\sqrt{p}} \sum_{k=1}^{p} X_j Y_k^H s_k^I A^H A)\}$=0 and:

$$\begin{aligned} D_j^R &\approx \frac{1}{\sqrt{p}} tr(A^H A) s_j^R + \\ &+ Re\{tr(W_t^H A^H N_{t-1} X_j)\} + \\ &+ Re\{tr(N_t^H A W_{t-1} X_j)\} \end{aligned}$$

Similarly, we have:

$$\begin{aligned} D_j^I &= Re\{tr(R_t^H R_{t-1} iY_j)\} \\ &\approx \frac{1}{\sqrt{p}} tr(A^H A) s_j^R + Re\{tr(W_t^H A^H N_{t-1} iY_j)\} \\ &+ Re\{tr(N_t^H A W_{t-1} iY_j)\} \end{aligned}$$

The statistic for decoding the symbol s_j is given below:

$$\begin{aligned} D_j &= D_j^R + iD_j^I \\ &= \frac{1}{\sqrt{p}} tr(A^H A) s_j + \\ &+ Re\{tr(W_t^H A^H N_{t-1} X_j)\} + \\ &+ Re\{tr(N_t^H A W_{t-1} X_j) + \\ &+ iIm\{tr(W_t^H A^H N_{t-1} Y_j)\} + \\ &+ iIm\{tr(N_t^H A W_{t-1} Y_j)\} \end{aligned} \tag{12}$$

Decoding the symbol s_j is equivalent to minimizing the following expression (note that $|s_j|^2$=1):

$$\begin{aligned} |D_j - \frac{1}{\sqrt{p}} tr(A^H A) s_j|^2 &= D_j^* D_j + \frac{1}{p}(tr(A^H A))^2 \\ &- \frac{2}{\sqrt{p}} tr(A^H A) Re\{D_j^* s_j\} \end{aligned}$$

Therefore, the detector of the symbol s_j is:

$$\begin{aligned} \hat{s}_j &= Arg \max_{s_j \in S} [Re\{tr(R_t^H R_{t-1} X_j) s_j^R\} + \\ &+ Re\{tr(R_t^H R_{t-1} iY_j) s_j^I\}] \\ &= Arg \max_{s_j \in S} Re\{D_j^* s_j\} \end{aligned} \tag{13}$$

Equations (12) and (13) are the aim of the proof.

References

[1] G. Ganesan and P. Stoica, “Differential modulation using space-time block codes”, *IEEE Sign. Process. Lett.*, vol. 9, no. 2, pp. 57-60, Feb. 2002.

[2] B. L. Hughes, “Differential space-time modulation”, *IEEE Trans. Inform. Theory*, vol. 46, no. 7, pp. 2567-2578, Nov. 2000.

[3] B. M. Hochwald and W. Sweldens, “Differential unitary space-time modulation”, *IEEE Trans. Commun.*, vol. 48, no. 12, pp. 2041-2052, Dec. 2000.

[4] V. Tarokh and H. Jafarkhani “A differential detection scheme for transmit diversity”, *IEEE J. Select. Areas Commun.*, vol. 18, no. 7, pp. 1169-1174, July 2000.

[5] D. Gore and A. Paulraj, “Space-time block coding with optimal antenna selection”, *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol 4, pp. 2441-2444, 2001.

[6] M. Katz, E. Tiirola and J. Ylitalo, “Combining space-time block coding with diversity antenna selection for improved downlink performance”, *IEEE Veh. Technol. Conf. VTC’ 2001*, vol. 1, pp. 178-182, 2001.

[7] L. C. Tran, T. A. Wysocki and A. Mertins, “Improved antenna selection technique to enhance the downlink performance of mobile communications systems”, *Proc. 7th International Symposium on Signal Processing and Its Applications ISSPA’03*, Paris, France, 1-4, July 2003.

[8] A. V. Geramita and J. Seberry, *Orthogonal designs: quadratic forms and Hadamard matrices*, Lecture notes in pure and applied mathematics, vol. 43, Marcel Dekker, New York and Basel, 1979.

[9] G. Ganesan, *Designing space-time codes using orthogonal designs*, Doctoral dissertation, Uppsala University, Sweden, 2002.