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Increasing Performance of SQRD Decoding for LST Systems

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Abstract

A number of decoding schemes have been proposed for Layered Space-Time systems, such as the Ordered Successive Interference Cancellation and the Sorted QR Decomposition. We describe here a new addition to that group, increasing the performance of Layered Space-Time decoding by using the Sorted QR Decomposition technique to construct a list of constellations to be passed to a Maximum Likelihood decoder.

This paper shows that significant performance improvement can be obtained for symmetric systems, where there is an equal number of transmit and receive antennas. It shows that the proposed scheme, has a roughly linear increase in complexity compared to SQRD. To overcome this increase in computational complexity, an adaptive system is described that has similar performance with reduced complexity.

1. Introduction

In recent years, the demand for wireless communications has been increasing at a rapid pace, with more emphasis to provide higher rates, and improved quality in terms of reliability. It was shown, [1] [2], that employing multiple antennas both at the transmitter and receiver promises huge capacity increases in a multipath fading environment. Indeed, the capacity increases about linearly with the number of transmit and receive antennas.

The complexity of Maximum Likelihood (ML) decoding of such systems increases exponentially with transmit antenna numbers and constellation size. A number of sub-optimal decoding schemes with lower computational complexity have been proposed such as Zero Forcing (ZF), the Ordered Successive Interference Cancellation (OSIC)[3] [4] and the QR Decomposition [5].

These decoding systems perform best when the number of receive antennas is greater than the number of transmit antennas, while performance is less-optimal when antenna numbers are equal. This is due to the loss of diversity in the decoding process.

The paper is ordered as follows. Section 2 gives a brief system description of Layered Space-Time codes and Maximum Likelihood decoding. In Section 3 we review Layered Space Time system decoders such as ZF and OSIC, while Section 4 describes the Sorted QR Decomposition (SQRD). Section 5 introduces a method that uses the SQRD to produce a list of symbol combinations which is used by an ML decoder. Section 6 introduces an adaptive Reduced SQRD (RSQRD) and Section 7 compares the complexity of previous schemes to the fixed and adaptive RSQRD.

2. System Description

The Layered Space-Time Processing approach was first introduced by Lucent's Bell Labs, with their BLAST family of Space Time Code structures [6]. An uncoded Vertical Bell Laboratories Layered Space-Time (VBLAST) scheme, where the input bit stream is de-multiplexed into n_t substreams, is considered in this paper. Let n_t be the number of transmit and n_r be the number of receive antennas, where $n_r \geq n_t$, and $s = (s_1, s_2, ...s_{n_t})^T$ denote the vector of transmitted symbols in one symbol period. The received vector $Y = (Y_1, Y_2...Y_{n_t})^T$ is

$$Y = Hs + n \tag{1}$$

where $n = (n_1, n_2, ...n_{n_t})^T$ is the noise vector of additive white Gaussian noise of variance σ^2 equal to $\frac{1}{2}$ per dimension. The $n_r \times n_t$ channel matrix

$$H = \begin{pmatrix} h_{1,1} & . & . & h_{1,n_t} \\ . & . & & . \\ . & & . & . \\ h_{n_r,1} & . & . & h_{n_r,n_t} \end{pmatrix} \tag{2}$$

contains independent identical distribution (i.i.d.) complex fading gains $h_{i,j}$ from the j^{th} transmit antenna to the i^{th} receive antenna. We assume quasi-static flat fading where H is constant over L symbol periods.

Maximum Likelihood decoding is achieved by minimising

$$\| Hs - Y \|^2 \quad (3)$$

for all elements of s, which are symbols of constellation of size C. This would produce a search of length C^{m_t}, which for a system using 4 transmit antennas and 16 QAM gives 65536 possibilities, far beyond being practically decoded in real time. This leads to a search for methods of decoding with a reduced computational complexity.

3. V-BLAST Detector

The sub-optimal but less complex V-BLAST detector was proposed [3] [4] as a reduced complexity method to decode Layered Space-Time systems. A nulling (ZF) process was first introduced, which uses a pseudo inverse of H to produce estimates, $\widetilde{s}$, of the individual symbols, which are then passed to individual decoders. Conceptually, each transmitted symbol is considered in turn to be the desired symbol and the remaining symbols are treated as interferers.

$$\widetilde{s} = H^{\dagger} Y \quad (4)$$

where †is the Moore-Penrose pseudo inverse [3].

ZF nulling has the disadvantage that some of the diversity potential of the receiver antenna array is lost in the decoding process. To take advantage of the diversity potential, nonlinear techniques, such as Ordered Successive Interference Cancellation (OSIC) have been introduced [6] and shown to have superior performance.

The OSIC decoding algorithm uses the detected symbol $\widetilde{s_i}$, obtained by the zero forcing, to produce a modified received vector with $\widetilde{s_i}$ canceled out. This modified received vector has fewer interferers and better performance due to a higher level of diversity. This process is continued until all n_t symbols have been detected. Obviously an incorrect symbol selection in the early stages will create errors in the following stages. Therefore the order in which the components are detected becomes important to the overall system performance.

Figure 1 shows the Symbol Error Rate (SER) of Zero Forcing, OSIC, and Maximum Likelihood decoding for a system using 16-QAM with 4 transmit and 6 receive antennas. At a SER of 10^{-4} the difference between ZF and OSIC is approximately $5dB$, while the difference between OSIC and ML decoding is $2dB$. This demonstrates that there is only a small difference between OSIC and ML when the number of receive antennas is 50% more than the number of transmit antennas.

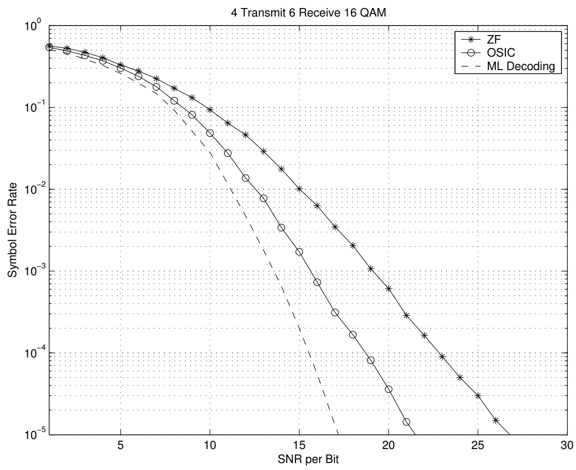

Fig. 1. Performance of ZF, OSIC and ML decoding with 4 Transmit, 6 Receive antennas and 16 QAM.

4. Sorted QR Decomposition

The QR decomposition of the channel matrix H was introduced in [5] as another method to decode Layered Space-Time systems. The $n_r \times n_t$ channel matrix H is factorised into the unitary $n_r \times n_t$ matrix Q and the upper triangular $n_t \times n_t$ matrix R.

$$H = Q.R \quad (5)$$

By denoting the column i of H by h_i and column i of Q by q_i, the decomposition in equation (6) is described columnwise by

$$(h_1 ... h_{n_t}) = (q_1 ... q_{n_t}) \begin{pmatrix} r_{1,1} & . & . & r_{1,n_t} \\ & & . & . \\ & & . & . \\ 0 & & & r_{n_t,n_t} \end{pmatrix} \quad (6)$$

By multiplying the received vector Y with the complex conjugate of matrix Q, an $n_t \times 1$ modified received vector

$$X = Q^H Y = Rs + \eta \quad (7)$$

is created from the $n_t \times 1$ received signal vector Y. The upper triangular matrix R has the lowest layer (transmit signal s_{n_t}) described by

$$x_{n_t} = r_{n_t,n_t} s_{n_t} + \eta_{n_t} \quad (8)$$

The decision statistic x_{n_t} is independent of the remaining transmit signals and can be used to estimate $\widetilde{s}_{n_t}$

$$\widetilde{s}_{n_t} = ML \left[\frac{x_{n_t}}{r_{n_t,n_t}} \right] \quad (9)$$

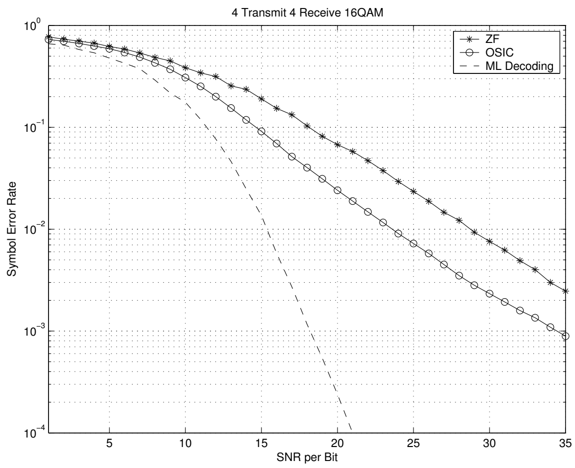

Fig. 2. Performance of ZF, OSIC and ML decoding with 4 Transmit, 4 Receive antennas and 16 QAM.

where ML is the Maximum Likelihood detector. This symbol is then used, by substitution, to detect $\widetilde{s}_{n_t-1}$ from the equation

$$\widetilde{s}_{n_t-1} = ML\left[\frac{x_{n_t-1} - r_{n_t-1,n_t}.\widetilde{s}_{n_t}}{r_{n_t-1,n_t-1}}\right] \quad (10)$$

This method of detection and substituting into upper layers is continued until all symbols are detected.

A number of techniques are based on using the QR decomposition [7] [8]. One such, the Sorted QR decomposition is proposed in [9]. SQRD is based on the modified Gram-Schmidt algorithm [10]. The columns of H, Q, and R are reordered in each orthogonalisation step to minimise the magnitude of the diagonal elements of R. This method ensures that symbols with larger channel co-efficients h_i are detected first while symbols with smaller h_i are detected later to reduce error propagation.

In each detection step $i = n_t \ldots 1$ a diversity of $G_{div} = n_r - i + 1$ is achieved. The SQRD algorithm has been shown in [9] to have similar performance to OSIC with lower computational complexity.

OSIC and SQRD have both been designed and shown to have performance similar that of ML decoding when the number of receive antennas is greater than than the number of transmit antennas. Figure 2 shows the performance of SQRD for a symmetric system where the number of transmit and receive antennas are equal to 4. At a SER of 10^{-3} the difference between SQRD and ML decoding is over $15dB$, compared to $2dB$ for the system of 4 transmit and 6 receive antennas shown in Figure 1. This loss in performance for symmetric systems is due to the first decoding stage having a $G_{div} = 1$ and hence a higher SER which creates error propagation in other layers [9].

5. Increasing performance of symmetric systems

Both the OSIC and SQRD decoding approaches have the disadvantage that some of the diversity potential of the receiver antenna array is lost in the decoding process, particularly when $n_t = n_r$ and in early detection stages of decoding. To overcome this, a scheme called Reduced search using Sorted QR Decomposition (RSQRD) is introduced. The proposed scheme uses the SQRD method to produce a reduced constellation list, which is then used by the ML detector to determine which combination of symbols is the most likely.

The assumption made for RSQRD is that if $\widetilde{s}_i$ (the symbol estimate independent of other interfering symbols) is chosen incorrectly by the ML detector, then the correct solution will be close to $\widetilde{s}_i$. Therefore, a local search around $\widetilde{s}_i$ is performed to find the order of closest symbols.

In the lowest layer of SQRD, where the symbol being detected is independent of all other symbols the closest k symbols to $\widetilde{s}_i$ are found. By choosing k symbols rather than just one symbol per layer, as with standard SQRD, the effect of wrong symbol selection producing error propagation is reduced. These k symbols are then in turn, substituted into the next highest layer to find the nearest symbol for each k symbol. Once the list is generated, the scheme performs the ML detection over all combinations in the list. In symmetric systems the majority of error propagation is caused by the first detected symbol because it has a $G_{div} = 1$. Therefore the largest gains obtained by RSQRD is when the constellation size is increased in the first detection stage.

6. Adaptive RSQRD

In Section 5 it was described that the greatest improvements of the RSQRD were made by finding the k most likely symbols in the first detected stage, where there was a reduced level of diversity, and then finding the combination for each value of k . The size of k was fixed to give a certain performance. This meant even when the correct combination of symbols was found the algorithm continued until the k^{th} time.

Instead of finding k combinations of symbols and then performing a Maximum Likelihood calculation, it would be far more efficient to perform the ML calculation for each combination of symbols after they have been detected and continue the search only if the ML solution is not found.

The Adaptive RSQRD algorithm works as follows: If the result of combination of symbols in (3) is less than

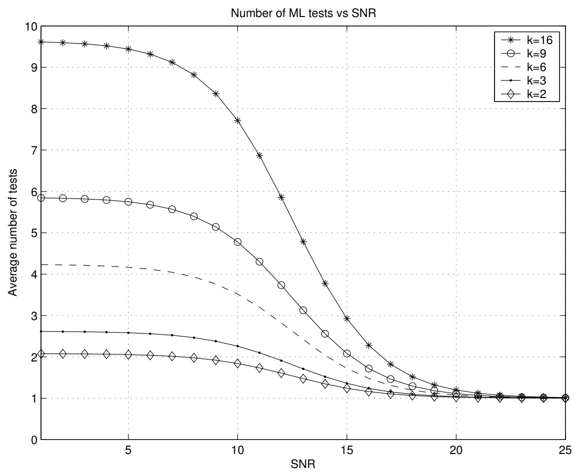

Fig. 3. Average number of tests for various values of maximum k vs SNR for 16-QAM.

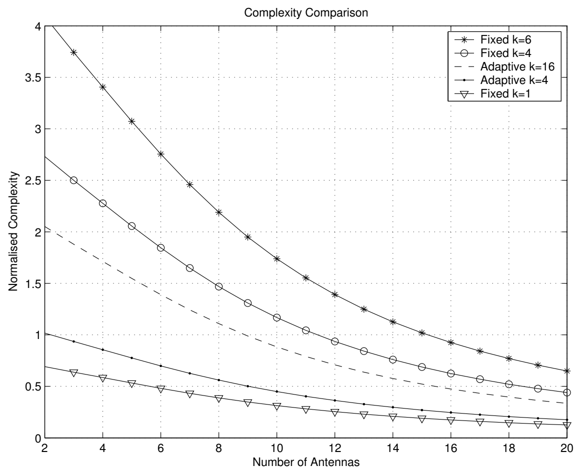

Fig. 4. Normalised Complexity comparison for fixed and Adaptive schemes, when $L = 100$ and $SNR = 15dB$.

a '$Threshold$' value the search is stopped, otherwise the search is continued to find the next combination of symbols. If after k times no combination is selected, the combination with the smallest result from (3) is chosen. The important question being what is the optimal value of the '$Threshold$' variable?

The noise variance (σ^2) and standard deviation (σ) were trialled as the '$Threshold$' value and were both found to reduce the number of ML tests. Using the noise variance substantially reduced the number of ML tests for $SNR \leq 10dB$, but increased the number of ML tests for higher $SNR's$. The standard deviation of noise was found to be optimal for $SNR > 10dB$. Since the greatest performance increase of RSQRD is when the $SNR > 15dB$, as shown in Figure 3, using the noise variance σ for the '$Threshold$' value is proposed for the adaptive scheme.

7. Comparison of the complexity of different schemes

Figure 3 shows average number of Maximum Likelihood tests versus SNR for k =16, 9, 6, 3 and 2. The system uses 4 Transmit 4 Receive antennas and 16-QAM. It can be seen from Figure 3 that for $SNR > 20dB$ the number of ML tests approaches one for all values of k. We used of Monte Carlo simulation technique to find the number of ML tests for 16-QAM for various values of k factor and SNR ranging from 0 to $25dB$. After applying the non-linear least mean squares curve fitting method[11], the formula approximating the number of ML tests for 16-QAM is:

$$N_{MLtests} = \frac{k \times 0.54}{1 + e^{(SNR-12.5)/2}} + 1 \qquad (11)$$

It was published in [12] that the decoding complexity of V-BLAST is approximately $\frac{27}{4}n_t^4$, while QR Decomposition based schemes have a decoding complexity of $\frac{29}{3}n_t^3$ [12]. These values are for systems with $n_t = n_r$ and do not take into account the assumption that the system has quasi-static flat fading, and H is constant over L symbol periods.

For this reason the computational complexity formulae of [7] were used as the basis for comparing RSQRD and V-BLAST. A single value was obtained for the computation complexity by counting real valued additions, multiplications and divisions as one floating point operation. The value of k is an integer for the fixed scheme and is equal to $N_{MLtests}$ from (11) for the Adaptive scheme. The ratio of complexity of RSQRD and V-BLAST is given by:

$$\frac{C_{RSQRD}}{C_{VBLAST}} = \frac{12n_t^3 + 18n_t^2 + Lk\left[(12n_t + 2)n_t\right]}{25n_t^4 + L\left[(18n_t)n_t\right]} \qquad (12)$$

Figure 4 shows the relationship between the RSQRD and V-BLAST schemes for a number of different values of k, using (12) at $SNR = 15dB$. It can be seen that the complexity of the Adaptive RSQRD with $k = 4$ has a complexity lower than one (i.e. less than V-BLAST) for all antenna numbers, while the complexity of the Adaptive RSQRD with $k = 16$ has a complexity greater than one for antenna number lower than 11.

8. Simulation Results

Monte Carlo simulations were used to compare the performance of the proposed scheme and the standard Layered Space-Time detection algorithms.

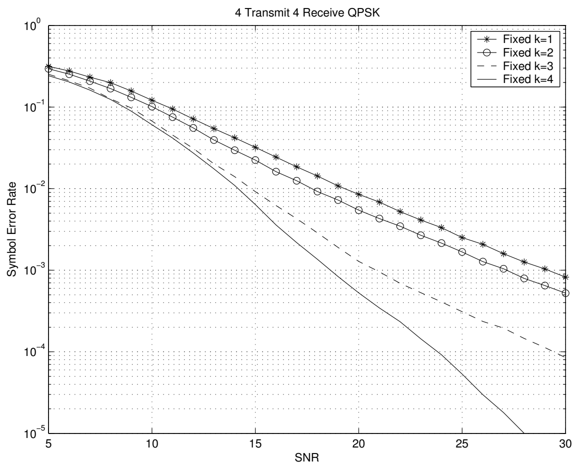

Fig. 5. 4 Transmit, 4 Receive QPSK RSQRD with constellation size k=1, 2, 3 and 4.

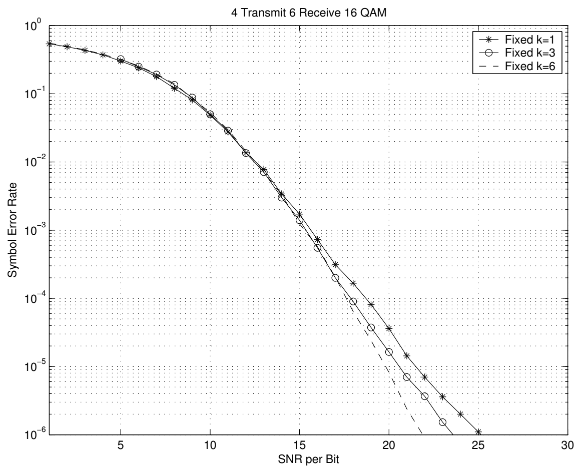

Fig. 6. 4 Transmit, 6 Receive 16-QAM SQRD with constellation size k=1, 3 and 6.

The simulation results presented in this paper are as follows: Figure 5 shows the comparison of the Reduced SQRD with different values of k for a system with $n_t = 4$ and $n_r = 4$ antennas using QPSK, while Figure 6 shows the comparison of the same decoders for a system using $n_t = 4$ and $n_r = 6$ antennas. Figure 7 illustrates the comparison of the reduced constellation SQRD for a system with $n_t = 8$ and $n_r = 8$ antennas using QPSK, with various values of k, while Figure 8 shows the comparison of the same decoders for a system using $n_t = 4$ and $n_r = 4$ antennas and 16-QAM.

It can be seen, from Figure 5, that there is a significant increase in performance between $k = 1$ (standard SQRD) and larger constellation size of 3 and 4 for symmetric systems. Approximately $6dB$ gain between SQRD and the proposed scheme using $k = 3$ and $11dB$ for $k = 4$ at SER of 10^{-3}.

Figure 6 shows that there is only a small increase in performance between $k = 1$ and a larger constellation size of 3 and 6 for an asymmetric system. Approximately $1dB$ gain between SQRD and proposed scheme using $k = 3$ and $2dB$ for $k = 6$ at a SER of 10^{-5}. Increasing the size of the lowest layer when $n_r > n_t$ brings only a small improvement because even the lowest layer, for the $n_t = 4$, $n_r = 6$ system, has a diversity level of 3.

From Figure 7, it can be seen that there is a significant increase in performance between $k = 1$ (standard SQRD) and larger constellation sizes of 3 and 4 for symmetric systems. Approximately $8dB$ gain between SQRD and the proposed scheme using $k = 3$ and $10dB$ for $k = 4$ at a SER of 10^{-3}, while there is only a small increase of $2dB$ for $k = 2$. Also of note is the result showing indistinguishable performance of the fixed and Adaptive systems with $k = 4$. This result was found to be the same for all k using 16-QAM.

Figure 8 shows the increase in performance between $k = 1$ and larger constellation size of 3 and 6 for a system using $n_t = 4$ and $n_r = 4$ antennas and 16-QAM. Approximately $5dB$ gain between SQRD and proposed scheme using $k = 3$ and $10dB$ for $k = 6$ at a SER of 10^{-3}. The Adaptive system with $k = 16$ has the same performance as the fixed $k = 16$ system, with an increase of $14dB$ over the original SQRD system at a Symbol Error Rate of 10^{-3}.

9. Conclusion

We have described a new improvement to increase the performance of Layered Space-Time systems, such as V-BLAST, by using the Sorted QR Decomposition technique to construct a list of constellations to be passed to a Maximum Likelihood decoder. It was shown that a significant performance increase can be obtained by increasing the constellation size for the lowest layer. In addition, it was shown that while at high SNR's there is improvement when $n_r > n_t$, greatest improvement in performance is for symmetric systems, i.e. when $n_r = n_t$. This due to a unity diversity level for the first detected symbol which is then used to detect other symbols.

To overcome the increase in computational complexity an adaptive system was shown to have similar performance with a reduced complexity. By testing the combination of symbols after each detection step and varying the size of k with the SNR, a computation complexity comparable to that of V-BLAST can be achieved with substantial performance increase.

The adaptive scheme is not dependent on the SQRD

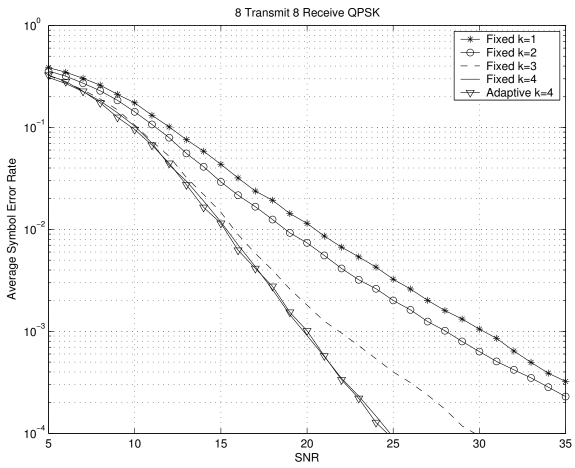

Fig. 7. 8 Transmit, 8 Receive QPSK, RSQRD with constellation size 1, 2, 3, 4 and Adaptive with k=4.

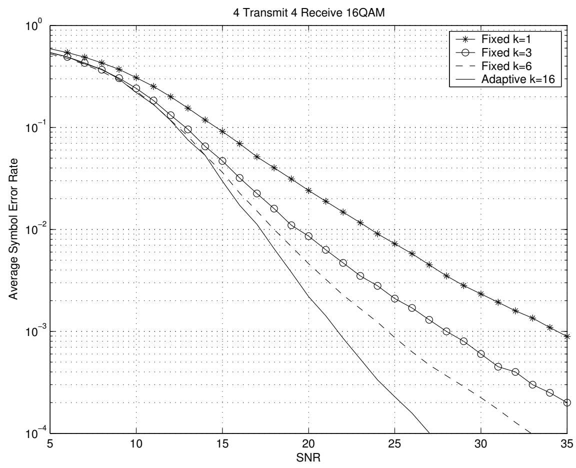

Fig. 8. 4 Transmit, 4 Receive 16 QAM, RSQRD with constellation size 1, 3, 6 and Adaptive with k=16.

scheme and could be implemented on a V-BLAST decoder described by [3], as well.

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