A Reformed College Algebra Course: Understanding Instructors' and Students' Beliefs About Teaching and Learning Mathematics

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A Reformed College Algebra Course: Understanding Instructors’ and Students’ Beliefs About Teaching and Learning Mathematics

by

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Reforms of undergraduate mathematics (e.g. Bressoud & Rasmussen, 2015; Laursen et. al, 2011) are changing the practice of teaching and learning within their courses. Prior research has established strong connections between practices and beliefs (Brickhouse, 1990; Raymond, 1997; Aguirre & Speer, 1999), therefore changing the practices within these courses may be affecting the beliefs of those tasked to enact the reformed practices. Thus, part of the work of the reforms in undergraduate mathematics is to learn how and why these beliefs may or may not be changing in this culture of reform.

In this qualitative case study, I analyzed the beliefs of two instructors, one who was the Graduate Teaching Assistant (GTA) and the other who was the Learning Assistant (LA), and two freshman level students. All four participants were part the same section of a semester-long College Algebra course at the University of Nebraska-Lincoln, which had been reformed to include more student-centered practices. The central question of this study is: What happens to instructors’ and students’ beliefs about teaching and learning mathematics when taking a reformed College Algebra course? Data analyzed included pre and post surveys, interviews using the pre and post surveys,
interviews using video clips of moments from their classroom, and observation notes of
the class.

Analysis suggests that beliefs held by these participants change in different ways.
One kind of change observed reflected a transformation from believing one set of beliefs
to a very different set of beliefs. For other participants, their beliefs changed in more
subtle ways. Learning from these kinds of change are important and necessary for
reforms to become sustainable and successful (Cohen & Hill, 2001). I also find that
making teaching decisions public was a significant catalyst for why beliefs were changed.
Acknowledgements

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and engage in school. There were some crucial moments where I desperately needed to escape school and my friends were always there to provide me some much-needed breaks. There were other moments where my friends challenged my thinking in ways that helped me engage more deeply in school. I needed my friends and they were there for me on a day-by-day basis.

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Chapter 1
Framing the Problem

Learning algebra has become a gatekeeper for many people on their road to success. A post secondary degree is rapidly replacing a high school diploma as the minimum level of education required for the majority of jobs (The White House, n.d.). People need college degrees in order to find and keep a job that keeps them out of poverty. In order to get a high school diploma and a college degree, people need to learn basic algebra. Hence, if students cannot master courses in algebra, their chances of getting a high school or college degree are seriously threatened. Reforms in K-12 mathematics (e.g., Spillane & Zueli, 1999) and in higher education mathematics classrooms (e.g., Seymour, 2002) have been and are continuing to help students learn algebra.

This vignette offers a look into the teaching and learning of the College Algebra classroom studied for this dissertation. Sally is a doctoral student in mathematics and hired as a Graduate Teaching Assistant (GTA). As the lead instructor, she sets up a task about inverse functions. By this point in the semester, her students are very accustomed to working together at tables. Each group ranges between three to six students.

A Vignette: Learning the Rules of Logarithmic Functions

March 3, 2015. At the front of the classroom, Sally has this written on the board:

- The inverse of an exponential function with base \( b \) is the _______ function with base \( b \).
- \( \log_b(1) = ___ \) because \( b^1 = 1 \)
- \( \log_b(b) = ___ \) because \( b^1 = b \)
- \( F(x) = \log_b(x) \) and \( G(x) = b^x \) are inverse functions.

---

\(^1\) Instructor and student names are pseudonyms.
Thus, \( \log_b(b^x) = \_ \) and \( b^{\log_b(x)} = \_ \).

Note that Sally has organized this task to carefully stagger students’ thinking about the relationship between exponentials and logarithmic functions. By writing the rules with blanks and the corresponding ‘because’ statements with blanks, she is encouraging students to reason that \( \log_b(1) = 0 \) because \( b^0 = 1 \) and \( \log_b(b) = 1 \) because \( b^1 = b \). And her hint about \( F(x) \) and \( G(x) \) being inverse functions provides a connection to students’ previous knowledge about inverse functions leading and recalling that the composition of inverse functions should give them back \( x \).

Just after Sally was done writing this on the board she tells the class to think with their tablemates about what the blanks should be. As she walks to the side of the room, groups begin reading the problems aloud and reasoning what the answers should be. At one table a student is heard saying, “Isn’t the second one 0? Because \( b^0 = 1 \). Or is it 1?” Later on at this table another student says, “It’s got to be 0. Because \( b^0 = 1 \) and \( b^1 = b \).” Students look back and forth through their notes and the questions on the board.

Sally says nothing and intently observes the groups as they think out loud. After about a minute and half, Sally says, “I heard lots of good things. So what do we think goes here {points to the first blank}.” Several students respond with the answer of ‘logarithmic.’

Sally: Ok. So \( \log_b(1) \) is…
Students: 0.
Sally: Because?

Note that even though Sally says she heard good things meaning she probably heard the correct answers, she is still using teaching moves to elicit answers from the students and push them to reason.

Students: \( b^0 \) is 1
Sally: How about \( \log_b(b) \)?
Students: 1

Sally: Good. Because when I raise \( b \) to the first power, I get \( b \). Ok. How about this last one? [She rereads the last statement on the board]
Sally: Then \( \log_b(b^x) = \_ \) gives me?
[short pause]
Student A: $G(x)$?
Sally: Ok. So notice I plugged my $G(x)$ into my $F(x)$, so this first part is $F(G(x))$.
   {small pause} Look at problem 6 from your worksheet.
   {Papers shuffling}
Student B: Oh. Is it just $x$?
Sally: Yeh. When I take the logarithm then the exponent, it’s the same as doing nothing at all. It just gives $x$ back. How about this guy then [points to the second blank in the statement.]
   {long pause with some students mumbling}
Sally: [Points to the blank they just talked about] So if I took an exponent then took a logarithm, that undid each other, right? How about if I do a logarithm then do the exponent? Should that undo each other as well?
Student C: Yes.
Sally: So then what should this [point to last blank] give me?
   {Short pause}
Student D: You just get $x$ again.
Sally: Uh huh. Yeh. They undo each other. Ok. So let’s talk about this one [points to this last blank] Does anyone have a way they can explain this one?
Sally: I have to think of a good way to say this. [She underlines the $\log_b(x)$ in the exponent.] This is going to tell me what power I need to raise $b$ to get $x$, right? [Pause. Then points to the base $b$.] So then when I take $b$ to that power, I just get $x$ back.

Even though Sally answers her own question, she at least prompts the students to think about why “they undo each other.” And the explanation she gives relies heavily on the procedural aspect of the problem; Sally is making an effort to give an explanation that moves beyond just the procedures.

*Sally wraps up by asking the class if this makes sense. She then gave them directions to move on and work on their worksheets.*

Properties about functions and inverse functions are usually aspects of mathematics people believe they just need to memorize, but how Sally created the task and the moves she made as a teacher moved the learning of this subject beyond memorization. Sally intentionally set up information about the inverse functions, exponentials and logarithms, in ways where she gave the students in her class time to think and actively engage with the content. First, she wrote the information with blanks for the students to fill in and gave them time to think about what should or should not go
in these blanks. The middle two statements have blanks that are structured to reinforce the rules of why one gets particular answers when one plugs 1 or b into the logarithmic function with a base of b. Because of this intentional way of setting up the task, students were given time to reason whether an answer was 0 or 1 with each other. They did this without an immediate intrusion from the teacher: she did not tell them how to do the problem.

When enacting this task with her students, Sally made different teaching moves that promoted engagement. She asked students multiple times what they thought should go into the blanks and, at times, asked them to explain their reasoning to the whole class. Her use of wait time and redirecting them to their worksheets were other ways she avoided just telling the students the answers. Even when she gave the students the explanation, she attempted to do it in a way that helped students reason about the problem. Sally could have written everything on the board with the blanks filled in and expected her students to memorize the information. She could have given the students the answers instead of asking for them one by one. Instead, she carefully organized and enacted this task of information in ways that allowed students to make sense of these properties.

In Sally’s class, this idea of working on mathematics to make sense of it is a strong, consistent notion she imparts to her students. Her students have learned to work on making sense of mathematics with one another regularly throughout the class and with Sally or the other instructor at sporadic times during the class. These intentional teaching decisions and ways students are engaging in the mathematics are reflections of the recent
reforms sweeping across undergraduate mathematics departments (Bressoud & Rasmussen, 2015; Laursen et al., 2011).

**Reforms in Teaching and Learning in College Mathematics**

The number of colleges enacting reforms in teaching and learning college level mathematics has significantly risen in the last ten years (Bressoud & Rasmussen, 2015; Seymour, 2002). One significant catalyst for these reforms is the public visibility (e.g., Small, 2002) of poor success rates of students in college mathematics courses (Cortes & Sandiford, 2008; Gupta, Harris, Carrier, & Caron, 2006; Stage & Kloosterman, 1995), helping faculty in mathematics departments come to the conclusion that the kind of teaching that is currently going on within their department is not evoking the kind of learning it should or could. The nature of the implementations of resultant reform varies widely and depends on the institution making the changes. For example, Reyes (2010) looked at the difference in success rates between an 8-week and a 16-week College Algebra course to explore whether the duration of the course influenced student success. Another study used pre- and post-tests for assessing whether students’ access to an online tutoring program had an effect on student achievement in College Algebra (Kersaint, Dogbey, Barber, & Kephart, 2011). These two examples reveal that one way in which departments were reforming their courses was by focusing on logistical factors, such as length of the course or access to online tutoring.

**Guidelines available.** To help guide departments in making reform decisions, efforts have been made to make guidelines public for lower-level courses and upper-level courses. Ferren and McCafferty (1992) conducted informal surveys to 13 institutions interested in reforming their courses in ways that addressed proper student placement,
course requirements, and perceptions of student attitudes. Even though the institutions were all addressing the same issues of undergraduate mathematics courses, they were making very different changes as to how they were addressing the common issues (Ferren & McCafferty, 1992). For example, most schools offered placement exams, but some chose to write their own exam, and still other schools used an exam distributed by the Mathematical Association of America. While the authors found other schools who shared the same concerns, the mismatched efforts for addressing these concerns meant there were not readily apparent solutions to help guide faculty in their decision-making process.

**Guidelines for a lower-level course.** In response to the ongoing lack of success in College Algebra, a lower-level course, and the faculty of universities recognizing the need for reform, in 2007, a group of faculty from large and small mathematics departments across the country attended a three-week workshop entitled *Curriculum Renewal Across the First Two Years* (CRAFTY). The focus of this specific workshop was on discussing the issues of linking content with understanding instead of memorization, unclear course goals, and changing the pedagogy behind the instruction of College Algebra (Ganter & Haver, 2011). The outcome of this discussion was a consensus regarding overall goals of the course, an agreed upon set of competencies and an emphasis on pedagogy (Fahringer & Yarrish, 2011). The guidelines for College Algebra that were endorsed by the Committee on the Undergraduate Program in Mathematics included the following goals:

1. Involve students in a meaningful and positive, intellectually engaging, mathematical experience;
2. provide students with opportunities to analyze, synthesize, and work collaboratively on explorations and reports;
3. develop students’ logical reasoning skills needed by informed and productive citizens;
4. strengthen students’ algebraic and quantitative abilities useful in the study of other disciplines;
5. develop students’ mastery of those algebraic techniques necessary for problem solving and mathematical modeling;
6. improve students’ ability to communicate mathematical ideas clearly in oral and written form;
7. develop students’ competence and confidence in their problem-solving ability;
8. develop students’ ability to use technology for understanding and doing mathematics; and
9. enable and encourage students to take additional coursework in the mathematical sciences. (Ganter & Haver, 2011, p. 45)

Goal 1 describes the need for a positive learning environment, while goals 2 and 3 give a picture of how students should be engaging with the mathematical content. Goals 4 and 5 give expectations of students’ mathematical knowledge. Goal 6 states a discourse expectation, whereas goals 7 and 8 describe how students should be linking their knowledge to the real world. Finally, goal 9 ends the list of goals with the message that students in College Algebra should be encouraged to learn more mathematics.

Participants of this workshop then chose a modeling-based College Algebra text, which embodied all of the recommendations of the CRAFTY project and developed a course based on this text. The essence behind a modeling-based approach involves immersing students in problems in which they use the scientific process, and which often involves students collecting data and coming up with their own models to approach solving a math problem. Researchers at one institution conducted a study comparing College Algebra taught using the modeling-based approach versus the traditional approach using lectures. Their results indicated there was no overall difference between the traditional instruction and the modeling-based approach in the passing rate, defined as achieving a C or better, but there were higher attendance rates and slightly lower
withdrawal rates in the modeling-based courses (Fahringer & Yarrish, 2011). These authors broadened the definition of success to include engagement, retention, and achievement.

Overall, there were nine institutions from the CRAFTY project that conducted studies examining the modeling-based approach. Edwards (2011) conducted an overall comparison of the results of these nine different studies. She noticed that six of the nine schools had at least one semester in which the modeling-based approach had a higher completion rate, but the variability from each school was so great that little could be generalized from that observation (Edwards, 2011). In Edwards’ (2011) overall look at the CRAFTY project, faculty involved concluded that this project allowed students to develop habits of mind necessary for life-long learning. This same group strongly suggested that courses with modeling-based approach that satisfy the guidelines were worthwhile and should be considered when revitalizing College Algebra (Edwards, Haver, & Small, 2011). Collectively, these studies not only show evidence of schools’ efforts at changing their College Algebra courses but, more importantly, they indicate the message that the teaching and learning of these courses needs to shift away from teaching by telling and learning by listening.

Guidelines for an upper-level course. In the Calculus Reform Movement (Steen, 1988), researchers have been working toward understanding how to teach Calculus courses in ways that would result in students understanding Calculus more deeply (Bressoud & Rasmussen, 2015). One study examined three Calculus courses which had the same mathematical content and were lecture based, but differed by how they delivered the assignments in the course. The authors concluded that students in the course
in which they could work with other students on the assignments scored higher on assignments than students who just received a paper assignment or a class in which students in the class used an online web program with unlimited attempts (Dedic, Rosenfield, & Ivanov, 2008). Kwon, Rasmussen, and Allen (2005) showed evidence of long-term retention for students in a Differential Equations courses using inquiry-oriented methods.

In an effort to describe what researchers were learning about how to teach Calculus, Bressoud and Rasmussen (2015) conducted case studies at five universities with successful Calculus courses. When describing the sites of the study the authors remarked,

In addition to productive disposition and improved retention rates, the five also had noticeably higher grades, cutting the D-F-W rates from 25 percent across all doctoral universities to only 15 percent at the case study sites. The difference was in B’s and C’s. The five case study universities actually gave out a slightly lower percentage of A’s than the overall average. (Bressoud & Rasmussen, 2015, pp. 144-145)

From their study of these five universities, Bressoud and Rasmussen (2015) identified seven characteristics of successful Calculus courses. These characteristics are:

1. Regular use of local data to guide curricular and structural modification.
2. Attention to the effectiveness of placement procedures.
3. Coordination of instruction, including the building of communities of practice.
4. Construction of challenging and engaging courses.
5. Use of student-centered pedagogies and active-learning strategies.
6. Effective training of graduate teaching assistants.
7. Proactive student support services, including the fostering of student academic and social integration. (Bressaud & Rasmussen, 2015, p. 145)
These characteristics represent what departments are doing that significantly aid in generating a successful Calculus course. In an additional note, Bressoud and Rasmussen (2015) acknowledge that these characteristics are also present, when applicable, in 12 other case study sites not included in this study. “What was common among all of the successful calculus programs was attention to the support of all students and a willingness to monitor and adjust the programs designed to help them” (p. 146). Thus, there was a general agreement among all of the sites studied to focus on students in ways that supported their learning of Calculus.

Despite having these seven common characteristics, Bressoud and Rasmussen (2015) point out that there were differences among the five universities. Not all of the universities steered away from using lecture as their primary method of instruction. All calculus courses at three universities and one section at a fourth university used less lecture “and much more use of students working together, holding discussions, and making presentations” (p. 146). In making this shift, these universities were emphasizing the idea that knowledge is constructed and thus they moved away from supporting the notion that knowledge is passively received. Other small differences included whether and how to use universal online homework and whether students were allowed to use a graphing calculator on exams. But the “most striking difference between these five universities and the overall survey was the number of instructors who ask students to explain their thinking” (p. 146). Thus, even though all five universities were considered successful in teaching Calculus, and their Calculus courses possessed the seven common characteristics, there were still some fundamental differences among them. Those universities in which students were asked to explain their reasoning were supporting
opportunities for deeper levels of engagement of the content and were portraying to students the belief that learning involves constructing knowledge through engagement.

It is unclear at this point whether these examples of overarching visions for both kinds of classes are known to or used by every college or university, but they offer a representation of teaching and learning in undergraduate mathematics courses. They suggest that teaching involves teachers setting up opportunities for learning and that this learning involves students actively thinking about mathematics. As in the vignette, Sally set up a carefully planned task and enacted the task in ways that pushed her students to think about the content. Her students were given time and space to question what the answers were and why the answers made sense. And while Sally had great intentions of giving her students more control of their learning, this was something not necessarily automatic for her. As was seen at the end of the vignette, Sally answered her own question of explaining why the composition “undid” one another. Sally’s answer relied heavily on the procedures of the composition. Given the demands placed on teachers and students, as indicated by the CRAFTY guidelines and insight from the vignette, Considering the details of what students should be doing as described by the CRAFTY guidelines, it is clear that this kind of teaching needs to be supported and developed. Two areas, active learning and supporting GTAs, are common themes of reform in higher education and will be discussed further.

**Good teaching and learning of college mathematics involves active learning.**

Many institutions trying to shift away from primarily using lecture as a format for classroom teaching are moving towards incorporating active learning, which is “generally defined as any instructional method that engages students in the learning process”
(Prince, 2004, p. 1) within their courses. Because of this general definition, many different instructional activities can be classified as part of active learning. Having students work in groups towards a common goal, referred to as collaborative learning, or working together in groups but still being assessed individually, referred to as cooperative learning, could both be considered examples of active learning (Prince, 2004). Another form of active learning is with problem-based learning where relevant problems are introduced and used to provide context and motivation for learning (Prince, 2004). Researchers at the University of Michigan (O’Neal & Pinder-Grover, 2005) describe some of the strategies for active learning to include:

- Think-Pair-Share (Johnson & Johnson, 1999)
- Informal Groups (e.g., Kagan, 1995)
- Group Evaluations (Wolsko, Park, Judd, & Bachelor, 2003)
- Inquiry Learning (Hmelo-Silver, Duncan, & Chinn, 2007)

Many of these strategies involve intentionally and sometimes dramatically changing the instructional approach. For example, using think-pair-share involves intentionally pausing and letting students think first by themselves then pair up and share what they were thinking with another student. For instructors used to providing a lecture the whole class period, think-pair-share pushes them to be intentional and depending on how rooted in lecturing they feel, think-pair-share may represent a dramatic change for them. One common approach involves using Inquiry Based Learning (IBL), which is an active learning strategy and often defined differently by those who are using it. Like Bressoud and Rasmussen (2015), Laursen and colleagues (2011) came up with a set of
commonalities among courses using IBL strategies in their mathematics courses. These commonalities, described by the author as seven characteristics of IBL classrooms.

1. **The main work of the course was problem-solving:** students solved challenging problems alone or in groups, in and out of class. In class they shared, evaluated and refined their own and each others’ solutions.

2. **The course was driven by a carefully built sequence of problems or proofs, rather than the textbook.** The pace of the course was set by students’ movement through this sequence, rather than pegged to a pre-set schedule.

3. **Course goals tended to emphasize development of skills** such as problem-solving, communication, and mathematics habits of mind, as much or more than covering specific content.

4. **To accomplish these goals, most of class time was spent on student-centered instructional activities.** Students or groups of students played a leading role in guiding these activities. Most activities were conducted for just a few minutes at a time: class work tended to change gears often and switched frequently between activities.

5. **Instructors’ main role was not to lecture.** They might give mini-lectures to shape or sum up the day’s work, set up a group activity, or provide context for a set of theorems. Instructors (as well as other students) might offer impromptu explanations to respond to a comment of question. They might ask questions to clarify student thinking, refine a presented solution, give feedback, or to elicit such comments from other students.

6. **Student voices were heard in the classroom:** presenting, explaining, arguing, asking questions. Their active participation meant that students themselves had much more control over how class time was spent and how fast the class moved through the material.

7. **This joint responsibility for the depth and progress of the course fostered a collegial atmosphere** that placed value on respectful listening and critique and invited every class member to contribute fruitfully to the mutual development of mathematical ideas. (Laursen et al., 2011, p. 43)

Active learning is a movement in higher education (Meyers & Jones, 1993; Prince, 2004). Colleges and universities are shifting towards expecting their students to be actively engaged in classrooms. And as Laursen et al. (2011) point out, active learning in undergraduate mathematics classes involve engagement that includes most of the class time is spent on solving problems for the purposes of developing mathematical habits of minds, and learning to communicate mathematical ideas with one another.
**Increased support for graduate teaching assistants.** At many colleges and universities, especially larger Research I Institutions, the primary resource for instructors of many undergraduate mathematics courses are graduate students within mathematics departments (Lutzer, Rodi, Kirkman, & Maxwell, 2007; Speer, Gutmann, & Murphy, 2005). GTAs simultaneously hold the role of being a learner in their own graduate level mathematics courses while also being the primary teacher in an undergraduate course. They possess a deep understanding of content knowledge and have encountered great success in learning mathematics. Traditionally, GTAs have had little or no formal training in teaching (DeChenne, Enochs, & Needham, 2012; Hardre & Burris, 2012; Marincovich, 1998; Tice, 1997). Thus, they began college teaching with only the images of teaching they have observed and experienced to draw on (Lortie, 1975) for how to teach. These images typically rely on lecture as a method of instruction which means they are mimicking the way they were taught math (Speer, Gutmann, & Murphy, 2005) and as successful math majors, this mode of instruction has been a fruitful way of learning for them.

Only in the last decade have colleges and universities begun thinking seriously about what kind of support or professional development they should be offering their GTAs (Speer, Gutmann, & Murphy, 2005) in order to support their teaching efforts in college math. There is growing recognition by colleges and universities of the problem of undergraduate mathematics courses being staffed with novice teachers unprepared in teaching techniques. One way GTAs have been supported is through the addition of Learning Assistants (LA). LAs are undergraduate students hired to help facilitate instruction within a course (Otero, Finklestein, McCray, & Pollock, 2006; Philipp, 2013).
These undergraduates are successful with the content in the course they are helping with and may even be pursuing a teaching certificate in mathematics (Philipp, 2013). While using undergraduate students to support courses in higher education is not necessarily a new entity, the Learning Assistant model from the University of Colorado was developed to address national challenges in higher education and using undergraduates to assist faculty “in making their courses student-centered, interactive, and collaborative” (Otero et al., 2006, p. 445) is new for the field of higher education.

Like GTAs, LAs possess high content knowledge and are put in a position of being an instructor in which they rely on their years of observing teachers when they themselves were learners. These observations become the images they use for deciding what method to use for teaching. Adding in the consideration that GTAs and LAs are often instructors of courses, like College Algebra, where students have not done well learning mathematics in the past, teaching and supporting teaching become even more problematic. The teachers, who have high content knowledge and have been successful in learning mathematics, are responsible for helping students who lack content knowledge and have not been successful learning mathematics. Moreover, these GTAs and LAs tend to rely on the methods they are most familiar with, which are methods that have specifically helped them learn, but are not necessarily methods that have helped the students in their classes learn. Thus by the very nature of who the instructors are and who the students are, there is a need to focus on more ways for supporting the GTAs and LAs teaching undergraduate mathematics courses.
What It Means to Learn and Teach: Past Knowledge and Research of Teaching and Learning Mathematics

Reforms in undergraduate mathematics education are growing (Ferren & McCafferty, 1992; Seymour, 2002), but reforms in K-12 mathematics education have been around for many decades (Ball & Cohen, 1996; Cohen & Hill, 2001; Lampert & Ball, 1999), and it is from these reforms that pivotal lessons about teaching and learning mathematics for deep understanding have emerged. While it remains unclear how reforms in undergraduate mathematics are using these past lessons from K-12 research, these lessons provide important information and insight into teaching and learning mathematics.

**What it means to teach mathematics.** In the last 20 years, our knowledge about teaching mathematics has been nurtured and enhanced by many significant insights by researchers. One notable idea is that mathematics teachers use different kinds of knowledge when teaching mathematics. Before this notion, many believed teaching mathematics involved only using one’s mathematical content knowledge and knowledge about teaching, in which these kinds of knowledge are considered to be separate and distinct. Now, it is widely accepted that teaching math also involves using another type of knowledge that comes from the integration of mathematical content knowledge and knowledge about teaching, referred to as pedagogical content knowledge (Shulman, 1986). Ball, Thames, and Phelps (2008) added to Shulman’s theory by proposing an elaborated model of the knowledge for teaching which included: Common Content Knowledge, Specialized Content Knowledge, Knowledge of Content and Students, Knowledge of Content and Teaching, Knowledge of Curriculum. The first two of these
types of knowledge encompass the subject matter knowledge a teacher uses whereas the last three categories describe a teacher’s pedagogical content knowledge.

Ball et al. (2008) also describe what a mathematics teacher could or should be doing in their “Mathematical Tasks of Teaching” (p. 10). These tasks, represented below, describe “something teachers routinely do” and “imply that teachers need to know a body of mathematics not typically taught to students” (p. 10).

- presenting mathematical ideas,
- responding to students’ “why” questions,
- finding an example to make a specific mathematical point,
- recognizing what is involved in using a particular representation,
- linking representations to underlying ideas and to other representations,
- connecting a topic being taught to topics from prior or future years,
- explaining mathematical goals and purposes to parents,
- appraising and adapting the mathematical content of textbooks,
- modifying tasks to be either easier or harder,
- evaluating the plausibility of students’ claims (often quickly),
- giving or evaluating mathematical explanations,
- choosing and developing useable definitions,
- using mathematical notation and language and critiquing its use,
- asking productive mathematical questions,
- selecting representations for particular purposes, and
- inspecting equivalencies. (Ball, Thames, & Phelps, 2008)

Taken with the different categories of knowledge needed for teaching mathematics, this gives a more clear idea of what teachers need to know and do when teaching mathematics. These ideas have helped form an idea of what good mathematics teaching entails when is it not centered on the teacher teaching by telling. Some of these tasks are tasks Sally used in the vignette. For example, she set up the task to connect with the students’ prior knowledge of inverse functions and gave a mathematical explanation for why the composition of functions “undid” one another.
Considered by themselves, the mathematical tasks of teaching and different
categories of knowledge needed for teaching help define what mathematics need to know
and do, but they do not provide a picture of what it looks like when teachers have and use
the knowledge and enact tasks within the classroom. Lampert (2001) and Heaton (2000)
wrote about their experiences teaching mathematics with the goal of teaching in ways
that shifted away from teaching by telling. In each of their accounts, readers are presented
with pictures of children as active mathematical thinkers and teachers who use the
mathematical tasks to cultivate students’ thinking. Lampert zooms in on individuals or
small groups of students to highlight detailed mathematical thinking and reasoning while
also zooming out to view the classroom culture nurtured by the whole group of students.
From these different view points, readers become acquainted with students making and
reasoning their assertions, like the student who reasoned through why he thought 22
groups of 12 equaled 10 groups of 6.

In Heaton’s experiences, readers are also presented with images of students
learning through active engagement with the mathematics and with each other. In
answering the question of what makes a rectangle a rectangle, one student John
responded with, “If two ends are the same size, like if two ends are little and two ends are
littler than the other two ends” (Heaton, 2000, pp. 72-73). His classmate Luke challenged
this statement which in turn caused John to become flustered enough to ask to go to the
board and draw a rectangle then use the drawing to help re-explain what he meant. This
caused Luke to respond with, “Oh. That’s not what you said,” and John answered back
with, “Well, I tried to say it.”
While both Heaton (2000) and Lampert (2001) offer rich descriptions of what it looks like when children are deeply engaged in learning mathematics, it is their use of the mathematical tasks of teaching that invokes and sustains this rich learning. Lampert posed the specific problem, “10 groups of 6=____ groups of 12,” as a way to understand how her students were reasoning about multiplication. This fruitful problem pushed students to fill in the blank and thus, make a mathematical claim. Like Lampert, Heaton (2000) started this conversation between the students by asking a productive mathematical question, “What makes something a rectangle?” By asking this specific question, Heaton acknowledged the difference in level of difficulty between identifying a rectangle and articulating the properties of a rectangle. She knew her students could identify a rectangle, but what she wanted to give her students were opportunities to make and reason about claims of a rectangle. Given their setting of elementary classrooms, the exact ways Heaton and Lampert used mathematical tasks of teaching may not be how instructors of undergraduate mathematics should use mathematical teaching, but there is certainly something to be learned by these instructors from these rich descriptions.

**What it means to learn mathematics.**

**What it means to learn.** To understand what it means to learn mathematics, it is necessary to examine what it means to learn in general. One model of learning is the Unified Learning Model, which capitalizes on recent research in neurobiology and synthesizes existing theories of learning (Shell et al., 2010). This model is founded on three principles: (a) learning is a product of working memory allocation, (b) working memory’s capacity for allocation is affected by prior knowledge, and (c) working memory allocation is directed by motivation. Learning is defined as the relatively
permanent change in a neuron. The basic idea is that every person has neurons that are arranged in ways that help them retrieve knowledge they have stored in their long-term memory and when someone learns something new the neuron or neurons change to reflect what was learned.

According to the Unified Learning Model, learning is a process. First, the information has to be in someone’s working memory, then it has to transfer to the person’s short-term memory. Depending on the connections and strength of connections for the information, the information can transfer to the person’s long-term memory and cause a change in the neuron or neurons. Of course whether learning occurs or not is complex and dependent on a number of factors, as indicated by the second two principles of the Unified Learning Model. Bransford, Brown, and Cocking, the authors of How People Learn, support the ideas of the Unified Learning Model and point out “it is clear that there are qualitative differences among kinds of learning opportunities” (p.127). In their words, “Different parts of the brain may be ready to learn at different times” (p. 115) and there are different reasons for this. For example, maybe the external environment is causing the person to become distracted and thus they are not able to hold the information very long in their working memory or short-term memory. Thus, learning in general is a complex process that depends on many factors.

**Self regulated learners.** While there are many different kinds of learners, theory unanimously agrees that the most effective learners are self-regulating (Butler & Winne, 1995). Butler and Winne (1995) explain

In academic contexts, self-regulation is a style of engaging with tasks in which students exercise a suite of powerful skills: setting goals for upgrading
knowledge; deliberating about strategies to select those that balance progress toward goals against unwanted costs; and, as steps are taken and the task evolves, monitoring the accumulating effects of their engagement. (p. 245)

In addition to these specific skills a self-regulating student possesses, Zimmerman (2002) points out that “self-regulation is not a mental ability or an academic skill; rather it is the self-directive process by which learners transform their mental abilities into academic skills” (p. 65). Together, these two descriptions of self-regulating learners imply that this kind of learner is someone who uses a process of skills to moderate what and how they are learning.

Not every student is a self-regulating learner, thus researchers have become interested in discovering ways to help learners become more self-regulating. Zimmerman (2002) provides insight into this interest by describing ways teachers fail to let students practice self-regulating skills. He argues that

Students are seldom given choices regarding academic tasks to pursue, methods for carrying out complex assignments, or study partners. Few teachers encourage students to establish specific goals for their academic work or teach explicit study strategies. Also, students are rarely asked to self-evaluate their work or estimate their competence on new tasks. Teachers seldom assess students’ beliefs about learning, such as self-efficacy perceptions or causal attributions, in order to identify cognitive or motivational difficulties before they become problematic. (p. 69)
Thus, teachers who give students choices in pursuing academic tasks, provide encouragement for goal setting, let students evaluate their own work, and assess students’ beliefs are providing ways for the students to develop their own self-regulating processes. It is these chances to practice that help self-regulating learners develop internal feedback, which “describes the nature of outcomes and the qualities of the cognitive processing that led to those states” (Butler & Winne, 1995, p. 246). To put it more simply, learners who get a chance to make their own choices and monitor what they are seeing and doing can develop or enhance their consciousness of what is helping them, which in turn lets them remember what helps or does not help them learn. Classes which include more active learning strategies are giving students more freedom to engage with whatever they are learning and with this additional sense of freedom, students need to become more cognizant of what and how they are engaging. Without this higher level of awareness, the learning is less likely to make a lasting impact on the students (Bell & Kozlowski, 2008).

From learning in general to learning mathematics. The phrase “learning mathematics” is often interchangeably used with “doing mathematics.” When students are asked what they are learning in mathematics, they will often describe what they are doing in their mathematics classroom. For example, a student in a College Algebra classroom may say he or she is solving equations when he or she is learning about quadratic functions. The emphasis in the student’s response is on what the student is doing as part of learning about quadratic equations. Therefore, to consider what it means to “learn mathematics” should include what it means to “do mathematics.”

One perspective to consider when thinking about what “doing mathematics” means is from a mathematician’s standpoint. “Doing mathematics” is a primary job
characteristic of a mathematician. In Singh’s *Fermat’s Enigma* (1997), pictures are painted of what it means to “do mathematics.” From a young age, Andrew Wiles became fascinated with solving the famous mathematical problem, Fermat’s Last Theorem, posed by the French mathematician, Pierre de Fermat. At the age of ten, Wiles began working on this problem that was over 300 years old by performing calculation after calculation. These calculations kept leading to a dead end, causing Wiles to search in school books for more information, ultimately leading him to change his perspective to start learning from the mistakes of others who have tried to solve this problem.

Wiles went on to graduate school in mathematics and put his work on Fermat’s Last Theorem on hold. When he returned to regularly working on the problem, Wiles said I used to come up to my study, and start trying to find patterns. I tried dong calculations which explain some little piece of mathematics. I tried to fit it in with some previous broad conceptual understanding of some part of mathematics that would clarify the particular problem I was thinking about. Sometimes that would involve going and looking it up in a book to see how it’s done there. Sometimes it was a question of modifying things a bit, doing a little extra calculation. And sometimes I realized that nothing that had ever been done before was any use at all. Then I just had to find something completely new – it’s a mystery where that comes from. (pp. 207-208)

From Wiles’ story, a picture of “doing mathematics” involves trying calculations, looking for patterns, changing perspectives when needed, studying what is not working as well as what does work, and thinking about problems piece by piece. Students in undergraduate mathematics classes need these opportunities. Like in the opening
vignette, they needed a space and time to consider on their own what made sense and what did not make sense.

In 2010, the Common Core initiative released the Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These eight practices, written by mathematicians, “describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (Common Core State Standards Initiative, 2016, para. 1). Furthermore, they “describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years” (Common Core State Standards Initiative, 2016, para. 9). The practices are represented below.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

From the past lessons of teaching and learning in K-12 classrooms (e.g., Ball et al., 2008; Shulman, 1986) and the current reforms in undergraduate education (e.g., Bressoud & Rasmussen, 2015; Laursen et al., 2011), there emerges a direction that teaching and learning is progressing towards. “Good” mathematics teaching involves the teacher being a facilitator of learning and involves interacting with students in ways that stimulate their thinking. “Good” mathematics teachers utilize various kinds of knowledge in planning and enacting mathematical tasks. “Good” learning of mathematics involves
students interacting with mathematics in ways that resemble what mathematicians do by executing the mathematical practices. “Good” learners of mathematics are conscious of what methods are and are not working and this level of consciousness helps them change strategies when needed and remember what and how the mathematics connects. All of these conceptions of “good” represent a larger shift of believing knowledge is constructed.

If these ideas of “good” mathematics teaching and “good” mathematics learning are what reforms are shifting classrooms towards, then this means they are shifting away from other conceptions and practices of mathematics teaching and learning. But what if teachers and students have different beliefs about what constitute “good” mathematics teaching and learning? Do beliefs matter when practices are changing? It is not clear yet how these ideas of “good” mathematics teaching from K-12 research translate to undergraduate mathematics, but they are certainly worthy of considering as resources for reforms in higher education. The next part of this chapter is dedicated to examining beliefs people hold about teaching and learning mathematics and the degree to which teachers’ and students’ beliefs matter for changing the practices of teaching and learning mathematics.

Beliefs

Within every person lies a set of beliefs pertaining to all facets of his or her life that he or she believes, in the midst of constant retesting, to be true (Philipp, 2007). “In our most mundane encounters with new information and in our most sophisticated pursuits of knowledge, we are influenced by the beliefs we hold about knowledge and knowing” (Hofer, 2004, p. 3). Whether it is a mundane task like talking about the weather
with a stranger on the street, or an active pursuit like an in-depth studying of Biblical passages, people are constantly presented with new information that must be processed and made sense of. It is a persons’ beliefs and their ramifications that help shape who he or she is and how he or she engages in the world where the experiences encountered fall between trivial and significant. Philipp (2007) said, “the way one makes sense of his or her world not only defines that person for the world but also defines the world for that person” (p. 257).

Beliefs differ from one another in terms of importance and the strength to which that person holds onto a belief. Thompson (1992), a notable scholar in research on beliefs in mathematics education, gives a description of how beliefs are differentiated. Primary beliefs are those someone holds of high importance and derivative beliefs are those that would not be considered less important. For the second dimension, central beliefs are beliefs someone holds onto strongly, and peripheral beliefs are the beliefs that are more weakly held. Thus, a belief someone holds can be classified as either primary or derivative, and central or peripheral. For example, the belief that all students can learn mathematics could be a primary, central belief for one person but a derivative, peripheral belief for another. Beliefs are never held in isolation and these dimensions can be used to consider the relationship among them (Philipp, 2007).

Because a person holds multiple beliefs about similar and different topics, it is understood that these beliefs do not act in insolation of one another and are thought of as behaving in a system (Leathem, 2006). Many beliefs a person holds are about some kind of result or process, such as how to make the best kind of chocolate chip cookies or how the world was created. At the same time, that person holds particular beliefs about their
own ability to produce a particular result or engage in a particular process. Those beliefs, referred to as a person’s self-efficacy, can and should be thought of as a subset of beliefs in his or her system of beliefs. A person’s self perceptions of their capabilities play a crucial role in his or her motivation and behavior (Bandura, 1982).

One core experience that unites most people is that of learning. Most people have experiences with some sort of formal schooling, such as public education or home schooling, which means people have had years of developing beliefs about school through their own observations (Lortie, 1975); this leads them to ideas of what “good” teaching and learning looks like. Every person has had years of making sense of his or her school, which not only defines his or her role in school but also defines school for him or her. Thus, every person has beliefs pertaining to how he or she connects to school and also has beliefs about school. Since most conceptions of school involve some kind of teaching and learning, every person has beliefs pertaining to teaching and learning. Furthermore, every person has beliefs about their own ability to learn mathematics and possibly beliefs indicating how they could teach mathematics. It is these beliefs about teaching and learning, particularly conceptions of “good” teaching and learning, that shape how teachers and students act and interact within mathematics classrooms, namely their practices.

A long history of believing in teaching by telling and learning by listening. Strongly rooted in our nation’s history is the belief that teaching mathematics means “teachers’ central task is to provide clear, step-by-step demonstrations of each procedure, restate steps in response to student questions, provide adequate opportunities for students to practice the procedures, and offer corrective support when necessary” (Smith, 1996,
Smith (1996) goes on to argue that when students do not master a skill, the teacher should repeat his or her demonstration while also providing opportunities for students to refresh their previously mastered skills. Therefore, the belief that teaching is telling poses the teacher as almighty holder of power and knowledge whose main job is to reinforce the procedural nature of mathematics.

Teaching mathematics by telling means believing students learn mathematics by listening and watching teachers demonstrate the procedures, then practicing these procedures until mastered, which means that solving problems involves only recalling and applying the appropriate steps (Smith, 1996). Since there is such emphasis on recall, memory is required as well as student effort and innate ability to use this memory (Smith, 1996). This emphasis reinforces the picture that learning is repetition to the point of memorization of rules and procedures. Hence, students are passive learners whose main responsibility is to regurgitate and replicate what the teacher is telling them; if they cannot do this, then they have not had enough practice, or their memory, effort, and ability is at fault. Because there is a clear link between what the teacher is telling and what the students are doing, teachers who teach by telling “can feel efficacious when their students accomplish the reasonable tasks of remembering facts and computing with the standard procedures” (Smith, 1996, p. 393).

These beliefs that teaching mathematics mostly involves telling and students learn by listening and practicing revolve around the belief that mathematics is a “fixed set of facts and practices” (Smith, 1996, p. 390). In fact, several studies suggest that beliefs about teaching and learning mathematics revolve around a particular conception of mathematical content (Beswick, 2012; Smith, 1996). In Schoenfeld’s (2004), “The Math
Wars,” he recounts the nation’s battle with perspectives of mathematical teaching and learning. Schoenfeld points out that only a small percentage of Americans were being challenged with rigorous mathematics curriculum before the American loss to the Soviets in the race to space. After the loss, there was a great push for more Americans to learn mathematics and science, which meant curricula now included more rigorous topics, such as set theory and symbolic logic. Schoenfeld quickly points out that this movement did not last long and quickly turned into a “back to basics” shift where, as it sounds, the curriculum was “watered-down” and did not emphasize mathematics beyond procedures (Schoenfeld, 2004). Considering this shift back to “back to basics,” it makes sense that the dominant belief of teaching mathematics involves telling and learning involves performing procedures.

Schoenfeld (2004) makes it clear that two important events happened in the 1980’s which resulted in a shift in our nation’s thinking about teaching and learning mathematics. The first of these events was the National Council of Teachers of Mathematics (NCTM) publishing a report, *An Agenda for Action* (1980), which spelled out eight recommendations for school mathematics in K-12 that shifted away from only emphasizing basics to encompassing a larger idea of mathematics involving problem solving. As Schoenfeld (2004) puts it, “an exclusive focus on basics was wrongheaded, and that a primary goal of mathematics curricula should be to have students develop problem-solving skills” (p. 258). The second event was the *A Nation at Risk* (Gardner, 1983) report by the National Commission on Excellent in Education, which terrified the public into thinking that “the educational foundations of mediocrity” (p. 6) of our population “threatens our very future as a Nation and a people” (p. 6) unless there was a
serious change in how people were educated. These two events combined caused the public to pay attention and to see that teaching and learning math needed to be changed, but it was still unclear how it needed to change.

This is analogous to mathematics departments agreeing that teaching and learning need to change in their undergraduate classes, but there are no clear steps for what exact changes to make and how to make them. Faculty will have to decide what can be changed, what can not be changed, and what order to make these changes.

**Changing beliefs and beliefs changed.** With the goal of shifting teaching and learning of mathematics towards a problem solving approach, Ernest (1989) points out that this shift depended on institutional changes, mainly curricular changes, and teachers changing their approach to teaching mathematics. He goes on to argue that this shift relies on deeper changes that “depend fundamentally on the teachers’ system of beliefs, and in particular, the teacher’s conception of the nature of mathematics and mental models of teaching and learning mathematics” (Ernest, 1989, p. 99). Instrumentalists view mathematics as a set of facts, rules, and skills. Platonists view mathematics as a unified body of knowledge that is discovered whereas a third view sees mathematics as a “dynamic, continually expanding field of human creation” (Ernest, 1989, p. 99). Ernest goes on to explain how a person’s view of the nature of mathematics effects their beliefs (see Figure 1).

In Ernest’s (1989) eyes, a person’s view of mathematics not only directly impacts their views of teaching and learning mathematics, but it impacts how they teach and learn mathematics. A Platonist teacher is an explainer whereas a teacher who views mathematics as a dynamic and expanding field needing to be created is a facilitator. For
teaching and learning math to change towards a problem solving approach, our view of mathematics must shift away from only viewing it as a set of facts and skills to be memorized. This indicates a relationship between beliefs and practices. At this point, the question becomes how do we change our view of the nature of mathematics?

Existing definitions of beliefs are ill-defined; in his efforts to clean up the construct of beliefs, Pajares (1992) describes beliefs as part of a “filter through which new phenomena are interpreted” (p. 325). This notion that beliefs affect what one sees suggests that the beliefs themselves need to be changed first, in order for someone to have any kind of different interpretation of the world, thus changing how he or she behaves in this new world. It would be like going on a road trip and using a map as a guide --not just any kind of map, but one of those “trip ticket” maps AAA used to give

Figure 1. Ernest’s (1989) model of the relationship between beliefs and practices (p. 100).
people where an exact route was given and the only aspects of the road the travelers can see on the map are the exits right off the road. This kind of map acts like a person’s beliefs in that it informs him or her of the terrain he or she will encounter, and the course he or she will travel will be dictated by the map. Any chance of experiencing the terrain differently from this map is unlikely as the map is your only way of viewing the terrain. Thus, this point of view suggests practices are predetermined and beliefs are what are needed to change them. Grant, Hiebert, and Wearne (1998) supported this idea that beliefs act as filters with their study that involved nine teachers asked to observe specially designed instruction over a period of time. By comparing the teachers’ beliefs of teaching and learning mathematics with the teachers’ evaluations of the observations, these researchers were able to see that beliefs the teachers held affected whether and how they could interpret the lessons. For example, two of the nine teachers were able to recognize and interpret the features, such as goals, of the instruction the researchers thought were important.

Another approach for how teacher beliefs change was proposed by Guskey in his 1986 paper on the effects of staff development. He argues that when teachers are involved in quality professional development with carefully thought out and targeted activities, then the teachers change their practices within the classroom causing different student outcomes and thus changing the teachers’ beliefs and attitudes. Guskey admits that this model for teacher change “does not explain or account for all the variables” (p. 7), but that it is a model that he thinks will help researchers better understand teacher change. Unlike the first approach, Guskey argues that instead of changing beliefs first, changing teachers’ practices will change their beliefs.
Considering these two different approaches, researchers and teachers have asked the question, “Do we change beliefs in order to change practice or do we change practices in order to change beliefs?” Two math education researchers, Bay-Williams and Karp (2010), have referred to this as the “chicken-or-egg question” and said that the answer to this question is “yes.” Both cases happen and sometimes changes in beliefs and changes in practices happen simultaneously. In a study involving teachers participating in a professional development that analyzed videos of teaching on a regular basis, researchers found that the teachers were changing their beliefs and instruction at the same time (van Es & Sherin, 2010). Regardless of whether researchers and educators are seeing teachers’ practices changing their beliefs or their beliefs changing their practice or their practices and beliefs changing at the same time, there is an undeniable connection between beliefs and practices (Aguirre & Speer, 1999; Brickhouse, 1990; Raymond, 1997).

**Inconsistencies among beliefs and practices.** In Ernest’s (1989) diagram depicting how he views the relationship between a teacher’s beliefs and practices, it is easy to believe that there is direct influence between the two, meaning what a teacher believes does inform and perhaps even controls what the teacher does in the classroom. For example, a teacher who believes students learn best by practicing would give his or her students worksheets filled with practice problems for the students. In the pursuit of understanding the relationship between a teacher’s beliefs and practices, researchers have discovered there can be inconsistencies between what a teacher believes and how the teacher behaves in the classroom (e.g., Basturkmen, Loewen, & Ellis, 2004).

Raymond (1997) observed and measured the beliefs of Joanna, a new teacher, from March of Joanna’s first full year teaching until the December of her second year of
teaching. In Raymond’s analysis of Joanna’s classroom practices and beliefs, she noticed that there were some significant inconsistencies. Joanna’s view of mathematics was strongly aligned with the traditional view, meaning she believed the nature of mathematics was fixed, predictable, absolute, certain, and applicable. But her views about teaching and learning mathematics were less traditional, meaning she believed in ideas such as “good mathematics teaching involves both the teacher’s and student’s input” and “learning math involves a balance of independent and group work.” In her observations of Joanna’s classroom practice as a teacher, Raymond discovered many examples in which Joanna’s practices were inconsistent with how she believed mathematics should be learned and taught, thus yielding multiple inconsistencies among beliefs and between beliefs and practice. For example, even though Joanna indicated she believed that learning mathematics involves a balance of independent and group work, she rarely had her students engage with mathematics in groups. Joanna’s practices as a teacher aligned more closely with her beliefs about the nature of mathematics rather than her beliefs about how mathematics should be taught and learned (Raymond, 1997).

In efforts to explain inconsistencies between beliefs and practices, researchers have offered many explanations. In a case study of a teacher chosen from a large survey study on teacher beliefs and practices, the teacher suggested that the cause or factors of these inconsistencies came from classroom situations, prior experiences, and social norms (Barkatsas & Malone, 2005). In the case of Joanna, Raymond (1997) argued that Joanna’s traditional view of mathematics was a central belief whereas her less traditional beliefs about how to teach and learn mathematics were more surface level and so the central belief more heavily influenced Joanna’s practices. Aguirre and Speer (1999)
reasoned that the inconsistencies they were seeing were because the teacher’s goals for her students were shifting in response to how her students were reacting during the lesson, thereby bypassing particular beliefs that teacher held. While researchers do not agree on one single explanation for the inconsistency between practices and beliefs, they do agree that the relationship between a teacher’s set of beliefs and his or her practices is very complex.

The problem. Mathematics departments are choosing to participate in reforms regarding how their mathematics courses are taught. They are recognizing that the kind of teaching and learning that is dominating their mathematics courses is not generating success, typically measured in whether students pass or not, thereby not producing the kind of learning the faculty in the department want. Thus, faculty in mathematics departments conduct some research about what kind of changes they could make and how they might make them. For the departments who agree with the trends of active learning and providing more support for their instructors, they will make changes reflecting the progressive conceptions of “good” teaching and learning reflecting the past lessons learned about mathematics education in K-12 classrooms and from recent departments who are already in the process of reforming their courses. And departments who are already in the process of reforming their courses will continue to make changes in efforts to continue shifting their teaching and learning toward these conceptions of “good.”

All of the changes currently being made by departments in the midst of reform, or those who will soon be undergoing reform, will change the practices of teaching and learning. Research confirms the presence of a strong connection between practices of
teaching and learning and beliefs about teaching and learning. This means that the
students and teachers who are enacting these practices, which reflect specific conceptions
of teaching and learning, hold their own beliefs about teaching and learning mathematics.
These beliefs may be consistent with the practices or they may be inconsistent.

Considering these teachers and students are products of mathematics classes from the last
twenty years, it is likely they have been greatly influenced by our nation’s rooted belief
that teaching involves telling and learning involves listening. This might influence
whether there is an inconsistency between the practices of the reform and the beliefs of
the teachers and students. Besides the possibility of inconsistencies, researchers indicate
that changing practices sometimes causes beliefs to change and vice versa. Thus, when
the practices are changed, does this mean the beliefs of the participants automatically
change or can they stay the same?

Thinking back to the vignette at the beginning of this chapter, it has already been
established that the teaching moves Sally made were intentional and engaging. If this was
the only glance into Sally’s teaching, would it be clear whether Sally made these choices
because they matched her beliefs about teaching and learning or because she felt this is
how the Department of Mathematics expected her to teach? Even if her beliefs matched
her practices, did her practices or beliefs have to change prior to this teaching episode? If
she felt as though she had to proceed this way because of Department expectations, then
what were her beliefs and how were they different than those beliefs embedded in the
Department’s expectations?

A similar line of questions can be asked about the students in the vignette. It was
clear the students were engaging in the problem posed, but was it obvious they were
engaging in the task because they believed it was helping them learn or because they thought that was what expected of them? If it were the former, how were they seeing this kind of engagement helpful to their learning? If it were the latter, what part of this kind of engagement were they seeing as unhelpful or even indifferent to their learning? Posing ‘or’ questions for Sally and the students makes it understood that these participants in this reformed classroom could be embracing or resisting reforms, but in reality there are more ways they could be responding to the reform ideas.

In their analysis of successful reforms, Cohen and Hill (2001) argue that reforms require learning. It is not enough to mandate reform policy, change teaching practices to match the policy, and then expect learning to occur. There must to be opportunities created and taken advantage of for teachers and students to learn how to enact the reforms and for policy makers of the reforms to learn from these teacher and student learning opportunities. For reforms in undergraduate mathematics departments, this means that it is just as necessary for the faculty behind the reforms in the department to learn how participants of the reform are responding as it is necessary for those participants to learn how to respond to the reforms. While this need for learning encompasses many facets of a reform, one facet often focused on are the practices and one facet often overlooked are the beliefs behind the practices.

It is clear that there exists a connection between the teachers’ and students’ beliefs with the practices they are enacting in reformed mathematics courses. It is unclear how this connection exists and affects the teachers, students, and reform practices in the classroom. Ignoring this connection could negatively affect the reforms of the departments or the people within them. Addressing this question gives mathematics
departments insight into how teachers and students are both embracing and resisting features of the reform. The purpose of this study is to investigate what happens to instructors’ and students’ beliefs when they are involved in a reformed mathematics course. The research questions that guide this study are the following:

What happens to instructors’ and students’ beliefs about teaching and learning mathematics during a reformed College Algebra course?

(1) How do instructors and students define “good” mathematics teaching?

(2) How do instructors’ and students’ beliefs about “good” mathematics teaching interact with one another during a semester of College Algebra?
Chapter 2

Statement of the Problem

Aligned with the research questions posed at the end of Chapter 1, this is a qualitative study of an entry-level undergraduate mathematics course.

Findings from this study will add to and influence existing research in the following areas: (a) a needed focus on College Algebra, (b) help in departments’ reform of undergraduate mathematics courses, (c) focus greater attention on beliefs in the context of reform, and (d) addressing limitations of research on teaching and learning in undergraduate mathematics.

A Needed Focus on College Algebra

Among courses within a mathematics department, there tends to be a divide. How the divide is talked about differs from department to department. For this dissertation, the divide can be thought of and referred to as upper-level and lower-level courses. Upper-level mathematics courses usually include the Calculus sequence, Differential Equations, Linear Algebra, and other courses someone with a STEM major needs for his or her degree. These courses tend to be taught by faculty. Lower-level mathematics courses include courses like College Algebra and Intermediate Algebra. Lower-level courses, often taught by GTAs, include the kinds of mathematics courses non-STEM majors need to take in order to satisfy a mathematical component of their degree. If STEM majors are taking lower-level mathematics classes, then students are taking them to help prepare for taking upper-level courses for the future. Thus lower-level mathematics courses either serve as a minimum requirement for a student’s degree or are a gateway to upper-level mathematics courses.
Considering this divide between kinds of courses offered in mathematics departments is important in that it provides a way to think about how attention in research has been paid to research in undergraduate mathematics. Furthermore, it leads to the claim that more attention, in both reforming and researching the reforms, has been paid to upper-level mathematics courses compared to lower-level courses.

As mentioned in Chapter 1, the Calculus Reform movement has existed for a few decades (Steen, 1988), and is thought to be one of the most notable reforms of undergraduate mathematics education (Hurley, Koehn, & Ganter, 1999). At the 1992 International Congress of Mathematics Education, Tall recorded and published notes from the meetings of the working group describing the difficulties in Calculus. These difficulties described what students found challenging from a topical standpoint. For example, there is an entire section within the published notes devoted to what students find difficult about limits and infinite process (Tall, 1993). Partly in response to these documented difficulties, Tall (1996) argued for widening the examining of functions within Calculus from using just a symbolic approach to incorporating different representations and provides a spectrum of representations for functions, derivatives, and integrals. These were just a few examples of the part of the Calculus reform movement that include a deep look into student thinking about individual topics within Calculus and possible ways to expand these topics.

By the fall of 2010, Bressoud, Carlson, Mesa, and Rasmussen (2013) conducted a large-scale study of undergraduate Calculus using a national survey sent to a random sample of two-year and four-year colleges and universities. From this survey study came
several insights about calculus students and the impact of instructors’ characteristics on student learning:

behaviors such as connecting with the students by listening carefully to their questions and probing their understanding, appropriately pacing the course, presenting multiple methods for solving problems, encouraging students to come to office hours and keeping the expectations of the course challenging but achievable. (pp. 696-697)

Along with this list of desirable characteristic behaviors of instructors, Bressoud, Carlson, Mesa, and Rasmussen acknowledged that students who take Calculus are, “highly motivated and consider themselves well prepared” (p. 697), but that “Calculus is the primary gateway for most students heading into the technical and scientific fields” (p. 697). In an effort to explain why students were switching out of Calculus, Rasmussen and Ellis (2013) argued that, “instructional variables such as actively engaging students, having students explain their reasoning, etc. may make a difference in retaining STEM majors” (p. 79). What these studies reflect is a major effort to understand what is happening with the teaching and learning of Calculus at the post-secondary level, whether they are reforming their courses or not, and an intention to better understand the students and their experiences taking Calculus.

One final point to observe about the Calculus Reform movement is that it has evolved enough to have a synthesized view of common characteristics of successful Calculus courses. As mentioned in Chapter 1, Bressoud and Rasmussen (2015) conducted case studies of successful Calculus courses at five different universities. In their analysis of these case studies, seven characteristics were identified as commonalities of the
successful Calculus courses studied. This study is significant because not only does it formally acknowledge there are successfully reformed Calculus courses, but it also identifies and publicizes what is making these reformed courses successful, thereby giving support to others who aim to reform their Calculus courses. Thus, research on the Calculus Reform movement has encompassed investigations into difficulties related to students’ learning of the content within it, describing the population and experience of its students, and synthesizing what is making the reforms more successful. While it could be assumed that the findings from the study of these improvements in the teaching of Calculus would be transferable to the teaching of non-Calculus courses, the study of the improvement of non-calculus courses would be a fertile ground to explore for future research.

Research and reforms of other upper-level mathematics courses have not been as extensive as the Calculus Reform, but they still offer significant opportunities for examining the teaching and learning within them. Linear Algebra is one such course. Researchers have described points of difficulties learning the content (e.g., Carlson, 1993), the emotional experiences of students taking the course (e.g., Martinez-Sierra & Garcia-Gonzalez, 2016), and multiple ways Linear Algebra is being reformed for increased success in teaching and learning (e.g., Wawro, Rasmussen, Zandieh, & Larson, 2013). Similar points can be made about the research of Introductory to Proofs courses (Mariotti, 2006; Weber, 2001, 2004) and Differential Equations courses (Rasmussen & King, 2000; Rasmussen, Kwon, Marrongelle, & Burtch, 2006). Researchers of these upper-level classes are making careful, in-depth investigations, but it is not clear from the available literature that this same type of qualitative investigations have occurred with
lower-level courses like College Algebra. While these researchers would agree that there is much more to learn from studying upper-level courses, the fact that more attention has been paid to them is at least part of the reason why. Since nowhere near the same effort of study has been given to lower-level courses, there may be an implicit message that studying lower-level courses is very different than studying upper-level courses.

To understand the differences and similarities between the study of teaching in the two levels of courses, upper and lower, it is necessary to consider what research says about College Algebra. One of the first notable recurring themes across the research on College Algebra is the poor success rates of the students. Passing rates have long been resting around 50% (Cortes-Suarez & Sandiford, 2008; Hembree, 1990; Weinstein, 2004). Considering that the content of College Algebra is included in the content of Middle School and High School mathematics classes, College Algebra students are at least on their second attempt of learning these ideas of basic Algebra. Reyes (2010) found that up to 20% of the population of College Algebra students ends up repeating College Algebra three times. So it is clear from the research that students are struggling to learn the content of College Algebra, but it is unclear why. Perhaps this is because it is being taught in ways that support the notion that knowledge is transmitted, not constructed. Or perhaps teachers are using teaching strategies that do not align with how students learn, or teachers are using strategies incorrectly. Or perhaps it is because students in the course lack learning skills like self-regulation, or they are not given opportunities to reflect on their learning (Bell & Kozlowski, 2008). Or perhaps it is because the reason students need to learn this specific content has not been made clear to them (Herriot & Dunbar,
2009). It is most likely the case that there is more than one reason why students have been struggling.

Along with poor success rates, research on College Algebra shows different universities are trying different methods of reforming this course. Several of these methods relied on changing the method of instruction. For example, some universities have changed to an Emporium model (Twigg, 2011), where students use an interactive textbook and notebook to learn primarily on their own while instructors are available to answer questions (Hodges & Kim, 2013; Williams, 2005). Other universities have tried some version of using computer-based instruction (Sandruck, 2003; Snyder, 2006). One graduate student’s dissertation work studied the difference and impact between College Algebra being taught traditionally and being taught as a flipped classroom (Overmyer, 2014), which is when the instruction is done outside of the classroom and the practice is enacted during class. Focusing on changing the method of instruction does not necessarily mean quality learning will occur. While there are many examples of universities and colleges trying different methods of reforming their courses, specifically focusing on changing the method of instruction of College Algebra, many of these reforms do not reflect the quality of the changes made in upper-level courses. Few researchers have described departments creating College Algebra courses so that students are being actively engaged by being asked to explain their thinking, or in which instructors are using behaviors mentioned in the report on Calculus reforms (Bressoud & Rasmussen, 2015).

There appear to be fewer research studies about College Algebra courses being reformed in similar quality ways as upper-level courses have been reformed. College
Algebra courses need to be reformed in order to study how the teaching and learning can be different. One possible reason is that College Algebra is not being widely reformed in these ways. Another possible reason is that College Algebra is a different enough kind of course from upper-level courses so that reforming these courses in the same ways is challenging, therefore slowing down the number of research studies associated with it. In the lower-level courses like College Algebra, students who are taking these courses are usually not pursuing a degree in a science-related field (Herriot & Dunbar, 2009). This makes it harder for departments to understand how College Algebra will be helpful for individual students in their future, and has some people asking why College Algebra needs to be required at all (Hacker, 2012).

Many of the reforms of the upper-level courses use Inquiry Based Learning (IBL) or techniques of IBL, but those who support and run IBL’s main webpage are not actively trying to reform College Algebra using IBL. Under the section of “Target IBL Courses” (Academy of Inquiry Based Learning, n.d.) the authors claim, “While any course is suitable for IBL teaching methods, attendees are encouraged to select an upper-level math course (if possible), because it is easier to implement IBL at this level.” Thus, at least some of those people mostly intimately acquainted with IBL are implying that these techniques are harder to implement in a College Algebra course. Recall Laursen’ and colleagues’ (2011) list of characteristics of an IBL classroom (see pp. 12-13) from Chapter 1. Thinking of how College Algebra can be a course that uses IBL means departments must consider how these characteristics, like students solving challenging problems and making students voices heard, translate to a College Algebra course. For example, consider the first characteristic describing the main work of the course was
spent on problem-solving. What does a ‘challenging problem’ look like in College Algebra? How do departments help set up and enact an expectation of College Algebra students solving these problems both in and out of class? What does it mean for College Algebra students to share, evaluate, and refine their own thinking as well as other students’ thinking? The lack of answers to these kinds of questions in research and the lack of reformers asking these questions suggests both that there is a need for the answers and that they are difficult to attain.

The faculty in the department in which this College Algebra course has been reformed intentionally started with reforming the lower-level courses. Not only did this department begin with these courses, it purposefully decided to focus on changing the courses in ways that incorporated active learning strategies (Prince, 2004; O’Neal & Pinder-Grover, 2005). The upper-level courses had not yet received the same kind of attention that had been paid to reforming the lower-level courses. Because of this intentional focus on the teaching and learning of College Algebra and the notion that research has not paid the same kind of attention to College Algebra as it has with upper-level courses, this study decided to capitalize on such a fruitful ground for reforming lower-level courses. As established above, there is a need to focus on what is happening with reforms of lower-level courses and this study responds to this need. While the need to focus on lower-level courses encompasses many facets, this study will use the study of teachers’ and students’ beliefs as a way to provide insight into the student population of College Algebra and how these reforms are being experienced. More details about findings will help understand reforms is discussed in the next section.
Help Departments Reform Undergraduate Mathematics Courses

As mentioned above, reforms of both upper-level courses and lower-level courses are occurring. This means that departments are making intentional choices in how to change their courses, and then the teachers and students are enacting these choices. While there are varied guidelines depicting standards or characteristics of courses (Bressoud & Rasmussen, 2015; Ganter & Haver, 2011; Laursen et al., 2014), these resources do not offer insight into how to achieve what they are depicting. Thus, many departments and colleges turn towards their own sources of knowledge for what and how to change rather guide their decisions with research (e.g., Johnson, 2004) about how to proceed.

Despite active learning becoming a popular way of reforming courses (Prince, 2004), it remains unclear how departments and universities are deciding what to change and how to assess the impact they have on teaching and learning. Figure 2 was created by researchers at the University of Michigan (O’Neal & Pinder-Grover, 2005), which illustrates examples of active learning arranged along a spectrum based on complexity and classroom time commitment.

While this figure shows many different strategies instructors can use to stimulate active learning, it also indicates that these strategies are unequal in the complexity of their design and the ease with which they can be implemented. For example, an instructor could bring the class to a bottling factory for the purpose of letting them investigate which variables and limitations they would need to consider if they were to construct simple models of the cost to bottle a product. Another example could be an instructor deciding to make finishing worksheets a game, thus whoever finished first gets bonus points on his or her worksheet. While the first example represents a rich way of using the
active learning strategy, site visits, the second example lacks this same richness because it only involves a shallow feature of completing the

![Active Learning Strategies](image)

*Figure 2.* O’Neal and Pinder-Grover’s (2005) arrangement of different learning strategies according to complexity and classroom time commitment.

worksheet first to win the game. The first example engages and enriches the material for the students whereas the second example encourages a hurried look at the content reinforcing the negative mindset that quicker is better when learning mathematics. This means that institutions cannot randomly choose particular strategies and assume it will take the same amount of time and effort to implement. Active learning is a complex way of teaching and learning and there are not clear directions for how to achieve this kind of complex teaching and learning in mathematics classrooms.
Furthermore, it is the teachers and students who are enacting and engaging in these active learning strategies. With most reform choices, especially reforms embodying active learning, GTAs who are considered novice teachers (Hardre & Burris, 2012; Speer, Gutmann, & Murphy, 2005) are being expected to plan for and carry out complex teaching strategies. Students are expected to naturally engage in these strategies, which are likely new to them in the context of mathematics, in order to engage in deeper kinds of learning. As math education research in K-12 classrooms has learned, this kind of teaching (Chapin & O’Connor, 2009; Smith & Stein, 2011) and learning (Clements & Sarama, 2004; Stein, Smith, Henningsen, & Siver, 2009) is challenging for teachers and learners. Therefore, there is great potential that the kinds of reforms colleges and universities intend to implement require a high level of cognitive demand from the teachers and students involved. This idea suggests that department chairs and faculty at higher education institutions not only need help in making good choices for their respective institution, but also there is work involved after these decisions are made in figuring out how to support the teachers and students in these reformed classrooms.

This study will help department chairs and faculty with these decisions by focusing on how instructors and students in one section are experiencing and thinking about the reformed classroom. How instructors and students are experiencing and thinking about reforms will let departments know how instructors and students are embracing and resisting particular parts of the reform. Which tasks of teaching will instructors find easier to enact? Which tasks will instructors struggle with enacting? Which tasks of teaching do instructors think help students learn? Which tasks of teaching do instructors think students struggle with? Which tasks are students finding easy or
difficult? Answers to these questions will not only give the department insight into the embracing or resisting of the reforms, but it will also push instructors and students to be more cognizant of their teaching and learning, helping them to identify strengths and limitations in their practices as teachers and learners.

Findings from this study will generate an example of the kind of insight into what is needed to create and sustain successful change within mathematics courses. This study intentionally positions instructors and students in ways that position them to reflect on their beliefs and at times their practices that otherwise they might not notice. Making this often unnoticed and invisible work of the reforms visible and at the forefront of department chairs, faculty, instructors, and students minds gives opportunities to learn in and from reform. This kind of learning is necessary for successful implementation of reforms (Cohen & Hill, 2001). There are many complex components, like choosing and implementing active learning strategies, involved in reforming an undergraduate course. There need to be different ways of examining how these complex components are support (or fail to support) student learning.

**Focusing on Beliefs**

Research on beliefs has long been substantiated as a rigorous field of study in education (Thompson, 1992), and much is known about beliefs. Looking at beliefs in the context of reform, particularly in higher education, will add new knowledge to the already robust field of study of beliefs. Specifically, this study will address gaps in the methodology of studies on beliefs and how beliefs play a role in reforms of higher education.
Researchers studying beliefs of teachers and students in math education typically use case study methodology or some kind of assessment instrument (Philipp, 2007). Case study designs allow researchers to generate rich data sets: “these rich data sets are important for theory building, inasmuch as they enable researchers to consider interrelationships in the complex world of teaching” (Philipp, 2007, p. 268). The richness allows researchers to make deeper and hopefully more accurate inferences about what the participant believes, therefore leading to more accurate pictures of what each participant believes. Researchers using assessment instruments rely on closed data, typically in a form of a Likert scale (Philipp, 2007), for gathering information on teachers’ beliefs. While this approach faces the typical challenges of quantitative studies, such as getting a large and appropriate sample size, the findings from these kinds of studies also rely on researchers making generalizable inferences from the data. Because results from both kinds of approaches typically used in studying beliefs rely too closely on making inferences, researchers have begun to more seriously question the intricacies of these methodologies. In his paper on better defining the research on beliefs, Pajares (1992) states:

It is also clear that, if reasonable inferences about beliefs require assessments of what individuals say, intend, and do, then teachers’ verbal expressions, predispositions to action, and teaching behaviors must all be included in assessments of beliefs. Not to do so calls into question the validity of the findings and the value of the study. Traditional belief inventories provide limited information with which to make inferences, and it is at this step in the
Hofer and Pintrich (1997) have noted this concern of inference, and called for researchers to gather more naturalistic evidence with more ecological validity, such as using videotapes or audiotapes for recall purposes. In their opinion, enriching the data in this way will give insight into evidence for how beliefs connect to student learning. Other researchers call for using both quantitative and qualitative techniques for gathering data on beliefs (e.g., Palak & Walls, 2009). According to Philipp (2007), “studying teachers’ perspectives involves the investigation of the relationships among teachers’ beliefs, knowledge, values, intuitions, feelings, and practices” (p. 310).

This dissertation will intentionally use survey data as the basis for some interviews of participants in a case study methodology in ways that compliment each other in order to make accurate inferences about beliefs from the data. Survey data pushes the participants to make closed kinds of judgments then interviews with the same participants allow them to expand these judgments to include the richness of reasons why they believe what they do. Likewise, video records of the classroom, which include practices of both the teachers and students, are used in two interviews. Another two interviews with participants are centered on the participant’s answers on a quantitative measure in order to gather more open-ended data. In this assembling of different sorts of evidence, the flaws of inferential judgments are minimized, though not excluded, and a more accurate connection between beliefs and the classroom experience is better characterized.
Because reforms have existed longer in K-12 classrooms than in higher education, much has been learned about how teachers’ beliefs connect with reform efforts. One significant area of this research has been focused on teachers who hold different beliefs than those of the reform efforts (Cuban, 1993; Drake & Sherin, 2006; Szatijn, 2003). Battista (1994) is one such researcher who warned the public of this incompatibility between teachers’ beliefs and the beliefs of the underlying reform efforts. One example he uses are vignettes of teachers who “possessed a view of mathematics that is totally incongruous with that of the current reform movement” (p. 467). What is clear from this line of research is that teachers’ beliefs may not necessarily match those of the reforms or the practices embedded within them.

Considering this disparity between the beliefs of the reform and those who are enacting them, researchers have turned the scope to understanding how beliefs change in response to reforms. One possible response, which is favorable to reformers, is that teachers’ beliefs drastically change to match those of the reforms, thus helping teachers become more effective using the teaching strategies of the reform (e.g., Remillard, 1999). A way of describing this change is a ‘transformation of consciousness’, in which a person experiences a change in consciousness (Ackerman-Anderson & Anderson, 2001). Some metaphors that have been used to describe this kind of change are: from darkness to light, from caterpillar to butterfly, and dying and being reborn (Metzler, 1986). In the context of teaching, this type of change might be a teacher who transitions from directly showing students procedures for solving mathematical problems to letting students solve problems on their own. Within this transition, the teacher dramatically changes her practice to purposefully let students reason about the mathematical content first as opposed to her
former practice where she immediately limits their thinking by showing them steps to solve problems.

Another possible response, which is less favorable to reformers, is that teachers’ beliefs do not change, increasing the probability the teachers are not enacting the strategies in ways aligned with the reforms (Battista, 1994). Sengen (1998) gave examples of observing three kinds of ways teachers were not enacting reform strategies effectively: teachers verbalized ideas of the change but did not incorporate them into their practice, new practices were tried out without much deliberation, and sometimes the reform practices were completely ignored. Thus, teachers had more than one way they resisted reform, whether these resistance efforts were intentional or not. A teacher might comprehend and accept the elements of reformed methods of instruction while at the same time find themselves unable to implement reform minded pedagogy due to the obstacles created by their past learning experiences.

Perhaps one of the most poignant lessons learned in the K-12 literature on beliefs related to reforms is the warning that if teachers’ beliefs do not support reform efforts then the reform efforts will fail. In his account of what happened with the release of NCTM’s (1989) Curriculum and Evaluation Standards for School Mathematics, Battista (1994) discusses ways ideas from this national document fail when teachers’ beliefs are not aligned with those of the new standards. He proposes ways in which schools, districts, and teacher education programs can better support teachers in the midst of reforms, then ends his argument with the warning, “All of our efforts to make the mathematics curriculum consistent with the NCTM Standards will fail if teachers’ beliefs about mathematics do not become aligned with those of the reform movement” (1989,
Beliefs in the context of reform need to be recognized and actively thought about, not ignored because, if ignored, beliefs can be the reason why reforms will not succeed.

When departments make choices as to how to reform their undergraduate mathematics courses, they are usually focused on the practices of teaching and learning within the courses. They take into account the practices that currently exist, the practices they wish to change, the practices they think they cannot or should not change, and the practices they wish to implement. But as lessons from K-12 research about beliefs and reforms indicate, departments should not only focus on the practice side of reforms. There is a need to also consider the beliefs of those enacting the reforms. Some studies are beginning to consider the students and instructors involved in the reforms of undergraduate mathematics (e.g., Treisman, 1992), but there is still a need to concentrate even more deeply on participants’ beliefs and whether or not these beliefs align with reforms.

In summary, beliefs in the context of reform need attention and, in general, beliefs need to be studied in ways that can capture their complexity. Lessons from K-12 research suggest that ignoring beliefs in the context of reform seriously jeopardizes the success of the reform. This study responds to these needs by focusing on beliefs in the context of reform. The overarching research question guiding this study is phrased in such a way that makes it open for the possibility that participants’ beliefs may or may not change. The sole focus on beliefs of the instructors and students and what happens to these beliefs will help mathematics departments become aware of how reforms initially align with participants’ beliefs and whether participants’ beliefs change in ways that align. At the
present time there are few studies of active learning being implemented in College Algebra, which means there are few opportunities to learn what happens to instructors’ and students’ beliefs as they are involved in their efforts to teach and learn College Algebra in meaningful ways.

**Addressing Limitations of Research on Teaching and Learning in Undergraduate Mathematics**

Existing research in teaching and learning mathematics in higher education varies widely in terms of quality and scope (Speer, Smith, & Horvath, 2010). In their review of research on improving teaching in college, Levinson-Rose and Menges (1981) conclude that the 71 studies they analyzed is a larger amount of research than they expected, but that the studies themselves were of “lower quality than we hoped” (p. 417). The following is an excerpt from their review detailing how they approached their analysis:

Changed teacher behavior may be assessed using data of several types from students, professors, or observers. We have identified five data categories: teacher attitude from self-report, teacher knowledge from tests or observer, teacher skill from observer, student attitude from self-report, and student learning from tests or observer reports. The strongest evidence for most interventions is impact on students (the last two categories), and the weakest is self-reported opinion of participants (the first category). Much of this research fails to go beyond data collected on the spot from participants (first and second categories), and those studies have been excluded from the review. (p. 403)

Considering their five categories, it is no wonder that Levinson-Rose and Menges (1981) rate few studies as highly valuable. Two of the categories are based on self-reports
and two other categories are controlled by observers with no use of systematic or rigorous means such as validated observation tools. Given that this body of existing research is rooted in deficient sources of data, there are considerable limitations in research within this field of study. These deficiencies make it difficult to trust the accuracy and usability of research from these studies. Thus, from Levinson-Rose and Menges’ standpoint, there is a need to make more rigorous and system inquiries because without those kinds of studies researchers in this field face great difficulty making any kind of trustworthy progress.

A more recent example from the field illustrates how narrow the scope can be for researchers. Kay and Kletskin (2012) evaluated the use of problem-based video podcasts designed to improve Calculus readiness for first year university students in mathematics classes. While their analysis emerged from a sample of 288 students enrolled in an undergraduate Calculus course, the type of data collected relied on student self-report. Thus, not only were Kay and Kletskin relying on self-reports, the only variable they were considering for evaluating was whether students were more prepared for Calculus with the podcasts. In their summary, they asserted the large claim that podcasts were readily accepted by first year university students and using the podcasts did help improve understanding of precalculus concepts (Kay & Kletskin, 2012). In reality, their data does not support the latter part of this claim since their data did not include any evidence pertaining to learning. To truly investigate a student’s understanding of a mathematical content topic is a complex and multifaceted task requiring more analysis and different kinds of observational data than this analysis of video podcasts has undertaken. This
dissertation study is choosing to focus on one facet of understanding reform implementation, namely beliefs.

This field of study has been and continues to be dominated by quantitative studies (Bressoud, 2011; Levinson-Rose & Menges, 1981). Whether it involves looking at student test scores (e.g. Freeman et al., 2014) or survey data (e.g. Laursen et al., 2011), researchers in this field have relied on quantitative measures and methodology to generate understanding about the teaching and learning of mathematics within higher education. While quantitative studies of good quality provide generalizable findings, they often lack detail and fall short of being useful (Creswell, 2013; Merriam, 2009). For example, Li, Uvah, Amin, and Okafor (2010) conducted a study assessing whether the changes made in a College Algebra course helped students become more successful in college. This study used measures from grade point averages, both high school and college, and test scores from College Algebra and standardized tests such as the ACT and or the SAT. With these measures, the authors developed correlations among these variables that indicate there are strong relationships between the changes faculty made, such as adding in web based homework, in College Algebra and students becoming more successful in college (Li, Uvah, Amin, & Okafor, 2010). At first glance, this seems promising to consumers, but there are some obvious questions that arise. How and why did the changes to College Algebra happen? Can these changes be made to other College Algebra courses at other institutions? If yes, should they be the exact same changes? The answer to these questions cannot come from close-ended measures. These are the typical questions faculty in charge of reforms should be asking when reading literature on reforms for College Algebra. This study will offer a close-up picture of what has been
happening with one College Algebra reform effort that reflects many of the qualities of upper-level changes, which will help faculty think about the answers to these questions and provoke them to ask even more detail oriented questions.

One facet of undergraduate mathematics education is the teaching practices of these mathematics courses. Speer, Smith, and Horvath (2010) conducted a search of articles and books about undergraduate mathematics education and reviewed them for the purpose of determining what these written accounts reflect about the teaching practices of undergraduate mathematics. They were looking for record of “what teachers do and think daily, in class and out” (p. 99). In their thorough review of these written accounts, they found that “research on collegiate teachers’ actual classroom teaching practice is virtually non-existent” (p. 99). The following table from their article summarizes what they found.

The written accounts (see Table 1) Speer, Smith, and Horvath analyzed varied between being considered empirical research or anecdotal memoirs or reflections, but very rarely described collegiate teaching practice. Anecdotal reflections provide some evidence of what college mathematics teachers are doing on a daily basis, but researchers trying to investigate or use research about teaching practice can not rely on anecdotal data indefinitely. Even the more research-oriented accounts are of limited help to fellow researchers since they are not actually describing teaching practices. The authors admit that their work characterizes “prior research and scholarship about collegiate teaching” (Speer, Smith, & Horvath, 2010, p. 111), but that there is a strong “need for more empirical research that examines and describes the work of teaching collegiate mathematics in detail” (p. 111).
Table 1

*Categories of Research about Collegiate Mathematics Teaching and Their Characteristics*

<table>
<thead>
<tr>
<th>Scholarship Category</th>
<th>Research?</th>
<th>Empirical?</th>
<th>Descriptive of practice?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memoirs of famous mathematicians</td>
<td>No</td>
<td>No</td>
<td>Sometimes</td>
</tr>
<tr>
<td>Analytic reflections on particular courses</td>
<td>No</td>
<td>No</td>
<td>Sometimes</td>
</tr>
<tr>
<td>Research on impact of instructional activities</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Research on impact of calculus reform</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Effects of reshaping classroom norms</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Prescriptive analysis of instruction</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Source: Speer, Smith, & Horvath (2010, p. 103).

Mathematics teachers are doing on a daily basis, but researchers trying to investigate or use research about teaching practice cannot rely on anecdotal data indefinitely. Even the more research-oriented accounts are of limited help to fellow researchers since they are not actually describing teaching practices. The authors admit that their work characterizes “prior research and scholarship *about* collegiate teaching” (Speer, Smith, & Horvath, 2010, p. 111), but that there is a strong “need for more empirical research that examines and describes the work of teaching collegiate mathematics in detail” (p. 111).

It is clear that the field of research in undergraduate mathematics needs more quality research, specifically research using qualitative methods to investigate facets of undergraduate research that have not been carefully examined. There needs to be more systematic, qualitative inquiries. The work of
this dissertation uses a systematic, qualitative inquiry designed to look into the work of teaching and learning through the context of participants’ beliefs. Because beliefs and practices are strongly related, this study will help researchers gain insight into the practices as well as beliefs of teachers in undergraduate mathematics courses. Although this study cannot address all of the needs of this field of study, it answers the basic need for rigorous qualitative research.
Chapter 3

Methodology

The central aim of this study is to understand what happens to instructors’ and students’ beliefs about teaching and learning mathematics as they participated in one semester of a reformed College Algebra course. The organization of this chapter is separated into two major parts: the background of the study and an explanation of the methods of data analysis. In the background of the study, details about the study, such as what kind of data were collected, are discussed in depth. Then, the three phases of the data analysis, including methods of analysis and the reasons guiding each phase, are detailed in the second part of the chapter.

Background of the Study

Researcher positioning. My interest in the topic of this study grew out of my own trajectory of teaching mathematics and involvement with committees focused on reforming college level mathematics courses. After obtaining an undergraduate degree in mathematics, I began working on my masters in mathematics at the University of Nebraska-Lincoln (UNL). I was hired as a GTA and my job was to teach mostly lower-level courses. This began my formal teaching of mathematics in college. With little formal training and limited ongoing support throughout the semester, I began as a teacher who tried to ‘perfect her telling’ (Smith III, 1996) with the goal of helping students gain procedural knowledge. Over time, I began to realize my teaching style of lecturing was not successful, because my students needed more than just procedures and ways to help them learn, other than me telling them directly. These new ideas about different kinds of knowledge helped me begin changing my practice from doing less telling and more
listening, and responding in ways that kept the focus on what my students were trying to understand.

Because I became fascinated with learning how to change my own practice, and my students were responding favorably to me as their instructor, I was invited to participate as a master’s student in committees in the Department of Mathematics at UNL intending to fundamentally change the pedagogical approach of teaching in the mathematics courses offered. In addition to helping these committees make decisions about how to best change these courses, I was hired over the summer of 2012 to write common lesson plans for Intermediate Algebra for GTAs to use that upcoming fall. Another GTA was hired to write lesson plans for College Algebra. My goal in writing these lesson plans was to help change the pedagogical approach by providing fellow GTAs with mathematically richer problems to use with their students, and to make more visible to students the conceptual ideas rooted within the traditional mathematical procedures, in hopes that the GTAs would be better prepared to facilitate understanding rooted both in procedural and conceptual knowledge (NCTM, 2014; Star, 2005).

**The Department of Mathematics’ college algebra.** The site of this study is one College Algebra class situated within the context of the Department of Mathematics at UNL. To better understand the richness of the semester-long College Algebra course, it is necessary to describe the reform-minded changes which were made prior to the semester of data collection.

**Summary of the changes.** In the summer of 2012, the Department of Mathematics decided to begin the transformation of their mathematics courses by focusing on and changing the first two lower-level courses, Intermediate Algebra and
College Algebra. Many decisions were made by the faculty about what should be taught in these courses and how they should be taught. The Department was using the University of Michigan’s ‘Michigan Math In Action’ as a source for ideas about how to change the practices within these courses. For example, one of the changes the Department made was to encourage groups of students to discuss problems with each other, much like what was seen in the vignette at the beginning of Chapter 1. With this change, the Department of Mathematics was trying to shift what should be taught to include students’ reasoning about answers. These changes were put into effect fall 2012 and were continuously reexamined and appropriately modified every academic year by a committee of faculty members within the department. This means the lessons plans that were written by me and the other GTA in the summer of 2012 had been revised multiple times by the time of this study.

Because College Algebra and Intermediate Algebra include similar content and students often move from one directly to the other, the decisions made for both courses are very similar, and thus I will choose to focus on only describing decisions made for College Algebra, which is the course examined in this study. To help provide a more complete picture of the department and better understand these changes for College Algebra, in Table 2, I have used the current syllabus (Wakefield, 2014) and the welcome letter from the chair of the mathematics department (Walker, 2014) to identify and organize these changes in a way which represents the shifts (NCTM, 2000) that were happening.
<table>
<thead>
<tr>
<th>Type of Change</th>
<th>Shifting away from</th>
<th>Shifting towards</th>
</tr>
</thead>
</table>
| Changes in Classroom Experience   | Students sitting in individual desks lined up in rows, listening to and watching the instructor work through mathematics problems | * Group work on a regular basis during class  
* Group homework assignments that involve writing more in depth solutions and explanations  
* Engaging in whole classroom discussions  
* Working on math problems that emphasize math in every day life and applications for physical and social science  
* Completing homework problems through an online program, WebWork |
| Changes in Course Policy          | Instructors creating their own regular exams and enacting their own policies about how students should behave in class | * Common grading across all sections on all exams  
* An active participation grade which represents a student’s behavior during class  
* Students engage in a Course Readiness Activity to review prerequisite material on Day 1 of the course (and have the first two weeks to master the material if they do not succeed on the first day)  
* A strong expectation of the students to read the textbook before class |
| Changes in Support for the Instructors | Instructors choosing on their own which problems to work and what to say from the textbook | * Before the academic year, instructors attend a multi-day training sponsored by the Mathematics Department  
* Instructors are given pre-written lesson plans helping them decide what problems to pick and how/when to do group work  
* New instructors attend a weekly pedagogy class devoted to developing their pedagogical knowledge of teaching  
* Instructors attend a weekly meeting focused on logistics of the course  
* There is an undergraduate Learning Assistant in each section intended to assist the main instructor  
* Teaching mentors assigned to instructors regularly observe instruction and offer supportive feedback |
A few of these changes, such as moving to a community grading system, were put into place before 2012, but most of them were decided and enacted only in the last several years. It should also be noted that before these changes, the normal routine for teaching College Algebra in this department can be described as graduate students being given a College Algebra textbook told which chapters they needed to cover, and given a few policies to which they needed to adhere. The graduate students were given complete discretion as to how they wanted to teach, which meant that many resorted to lecturing. This meant students’ experience in learning College Algebra in this department was as passive observers (Smith, 1996). Students were perhaps given the opportunity to repeat solving a similar problem the GTA just modeled on the board.

Considering this picture represented the normal experience of teaching and learning College Algebra before 2012, the changes described in Table 2 depict a definite shift intended to encourage both the students and instructors to actively engage in the teaching and learning of College Algebra. In a welcome letter to UNL students taking a math course in the fall of 2014, the chair of the mathematics department informed the students they would experience a course which had been quoted as being “leaner, livelier, and more relevant to real-life problems” while also being held to different expectations than they might have had in high school (Walker, 2014). This statement from the chair was intended to help students realize they would be holding different responsibilities as students than they might have been accustomed to previously. And though GTAs no longer had complete control of how or what they would teach, they remained involved and had access to ongoing resources intended to support how and what they were required to teach. One of the resources for GTAs was the weekly
pedagogy class, in which GTAs learned about different learning theories. I regularly observed this pedagogy class, and while it gave me a richer sense of the support provided to the GTAs, it was not used in the analysis. Another resource for GTAs was another weekly meeting organized by the convener and attended by the instructors in which the logistics of the course they were teaching were discussed.

**Connecting the changes to the literature.** One common theme across these changes reflected the Mathematics Department’s desire to incorporate more active learning in their courses. Recall that active learning is defined as any instructional activity which engages students in the learning process (Prince, 2004). Recall Figure 2 from Chapter 2 (p. 47) which describes different strategies for active learning.

The Department of Mathematics in this study incorporated many of these strategies in both the structure of the course and how they encouraged the GTAs to carry out the teaching of the course. Students in the course are seated in groups every class period and were required to complete quizzes in these groups regularly throughout the semester (Wakefield, 2014). Thus, the mathematics department was imposing the active learning strategies, group evaluations and informal groups. GTAs were told not to lecture the entire duration of the class period, but mini-lectures were to be at most ten minutes. These mini-lectures and other activities chosen by the GTA were encouraged to be interactive for the students. Some GTAs might choose to have large group discussions or use think-pair-share as a strategy (O’Neal & Pinder-Grover, 2005) for helping the students to be more actively engaged in the class. Of course, how much and what the GTAs chose to do to make their lectures more interactive differs by every GTA. Supports like the weekly meetings were put into place to help the GTAs navigate these decisions.
Overall, this means that there was flexibility in how the GTAs could choose to spend some of the class time, but there were still expectations of how it should not be spent.

The changes to the mathematics courses as outlined in Table 2 suggest to the instructors and students the Department of Mathematics’ images of what “good” mathematics teaching and “good” mathematics learning is: it does not involve a teacher who comes in and lectures, and students who listen quietly. Rather, the changes indicate the idea that the department views “good” teaching and “good” learning of mathematics as instructors helping small groups of students work through and think about problems that reflect everyday life.

These conceptions of “good” teaching and learning resemble many ideas from the characteristics of successful Calculus and IBL courses. One example is the idea that the main work of the course was spent on students solving problems either alone or in groups, and that the instructor’s main role was not to lecture (Laursen et al., 2011). Considering that the department added an LA to every section and was providing more support outside of class to the instructors meant that the Department of Mathematics acknowledges that this idea of “good” teaching and “good” learning is complex (Lampert, 2001; Cochran-Smith, 2003), and that a GTAs’ mathematical knowledge alone (Ball, Thames, & Phillips, 2008) may not have been enough to successfully support students in this kind of learning environment (Bressoud & Rasmussen, 2015). While all the details the Department of Mathematics sees as being necessary for such “good” teaching and learning may not be clear, it is clear that there is a shift toward practices which support the idea that knowledge is constructed and practices within courses should help students construct their own knowledge.
Data collected. To understand the significance of the data collected, this section will first describe what methodology was chosen and why it is appropriate. Then in-depth looks at the different kinds of data will be discussed.

Choosing case study as a methodology. This study has a primarily qualitative design. As Merriam (2009) argues, quantitative studies typically intend to determine a cause or predict similar events while qualitative researchers “are interested in understanding how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences” (p. 5). Because the main purpose of this study is to understand what happens to instructors’ and students’ beliefs at the beginning and end of the semester, it is primarily a qualitative study. There is no intention to predict what will happen with other instructors and students in reformed undergraduate mathematics courses. Rather, this study hopes to gather an in-depth understanding of what kinds of beliefs instructors and students hold when they are part of a reformed mathematics course over a semester. Given that these participants’ beliefs might be different than those ideas of the reform, or that the participants might be accustomed to different practices of teaching and learning, it is this study’s focus to investigate how these participants are attributing meaning to their experiences from the mathematics classroom. By focusing on understanding what happens with instructors’ and students’ beliefs, mathematics departments can use this information to learn about how such reforms are being embraced and resisted. While this study did collect quantitative data, they were collected for qualitative purposes.

Considering the main objective of this study was to generate an in-depth understanding (Creswell, 2013; Merriam, 2009; Stake, 1995) of individuals’ beliefs over
time, a case study methodology was chosen. A case can be something concrete, such as a small group of people, or less concrete like a relationship (Creswell, 2013). In this study, the case was chosen to be about instructors’ and students’ beliefs and it is bounded by only considering their beliefs about teaching and learning mathematics during a single semester. Even though the participants in this study are sharing a common experience much like a phenomenology, the intent of the study is only looking at one aspect of this shared experience and thus is not trying to describe the entire essence of the shared experience. In order to generate an in-depth understanding, this case study collected a variety of different kinds of data (Stake, 1995; Yin, 2009), which will all be explained further in sections below.

Tables 3, 4, and 5 provide a brief summary of the data that were collected for this study. This study was approved by UNL’s Institutional Review Board. Each instructor and student provided informed consent by signing a consent form to participate in the research.

Table 3

Pre- and Post-survey Data Collected

<table>
<thead>
<tr>
<th>Pre- and post-survey Information (n = 18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 students enrolled in the course.</td>
</tr>
<tr>
<td>25 student responses on pre-survey (20 of them signed consent forms)</td>
</tr>
<tr>
<td>26 students responses on post-survey (21 of them signed consent forms)</td>
</tr>
<tr>
<td>18 students with signed consent forms and matching pre- and post-surveys</td>
</tr>
</tbody>
</table>
Descriptive statistics were calculated for comparing the 18 pre- and post-surveys. These calculations were not used in the analysis and are located in Appendix A.

Table 4

*Observations of the College Algebra Class*

<table>
<thead>
<tr>
<th>Month</th>
<th>Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>20, 28, 29</td>
</tr>
<tr>
<td>February</td>
<td>3, 12, 19, 26</td>
</tr>
<tr>
<td>March</td>
<td>3, 5, 10, 11, 12, 17, 19,</td>
</tr>
<tr>
<td>April</td>
<td>2, 7, 16, 23</td>
</tr>
</tbody>
</table>

Total Number of Observations: 18

Note: Bold dates reflect observation notes and video were collected.

My goal was to be in the classroom at least once a week. I also tried to be in the classroom a few times for consecutive class days.

Table 5

*Interviews with the Four Main Participants*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Dates of Interviews</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sally</td>
<td>Jan 20, <strong>Feb 23, Apr 3, May 7</strong></td>
</tr>
<tr>
<td>Cara</td>
<td>Jan 27, <strong>Feb 20, Apr 9/Apr 16, May 7</strong></td>
</tr>
<tr>
<td>Brad</td>
<td>Jan 28, <strong>Feb 24, Apr 3, May 7</strong></td>
</tr>
<tr>
<td>Michael</td>
<td>Jan 29, <strong>Feb 27, Apr 7, May 7</strong></td>
</tr>
</tbody>
</table>

Total Number of Interviews: 16

Note: Bold dates reflect interviews with video clips.
These interviews were spaced out intentionally to span the semester. To be able to conduct the interviews in the February and April, I needed to make sure I had been in the classroom enough to gather sufficient video footage for choosing clips (video clips described below). I also wanted to make sure the interviews with the four main participants were completed such that they were interviewed at approximately the same times during the semester. Each main participant had four interviews, except for Cara. During Cara’s third interview on April 9th, the audio recorder malfunctioned; Cara agreed to repeat the interview on April 16th.

**Pre- and post-surveys.** All students and instructors in one section of College Algebra were invited to complete a pre-survey and a post-survey which had two dimensions, (a) self-efficacy, and (b) beliefs about teaching and learning mathematics. The items on the pre-survey and post-survey were intentionally kept the same so as to better make a comparison of changes (Goedhart & Hoogstraten, 1992). The purposes of using quantitative survey measures were to gather a basic description of the class, use the initial survey measures to pick student participants, and use the survey measures as means for gathering richer interview data with participants. Using quantitative measures in these ways has enhanced and enriched the additional qualitative data collected (Belli, Shay, & Stafford, 2001; Caracelli & Greene, 1993; Greene, Caracelli, & Graham, 1989).

**Dimension 1: Self-efficacy.** According to Bandura (1982), a person’s self-perceptions of their capabilities play a crucial role in his or her motivation and behavior. Therefore, instructors’ confidence in their capabilities should and does influence their actions as teachers. Likewise, students’ confidence in their capabilities to learn influences...
their actions as students. Because such a strong connection or relationship exists between a person’s beliefs in his or her own capabilities of teaching mathematics (Smith, 1996), learning mathematics (Pajares & Miller, 1994), and his or her behaviors as a teacher or a student have been established, it is necessary to include questions for participants about self-efficacy.

To assess the instructors’ self-efficacy, I used measures from Midgley, Feldlaufer, and Eccles’ work on change in teacher efficacy and student beliefs during the transition to junior high school. By using the five items they used, I was able to focus on the instructors’ personal teaching efficacy, rather than a general teaching efficacy (Midgley, Feldlaufer, & Eccles, 1989). Because the other dimension on the pre- and post-survey intended to elicit general beliefs about teaching mathematics, it was important to include questions that shifted the focus to a more personal standpoint for each instructor.

Measuring student self-efficacy involved using measures from May’s (2009) dissertation work, in which she explored the relationship between student self-efficacy and anxiety within mathematics. Her work involved using and modifying the widely used Mathematics Self-Efficacy Scale (Betz & Hackett, 1983); she found seven of those 14 questions, when used in isolation, were “reliable, relatively valid, and efficient” (May, 2009, p. 58). These seven questions were used to measure students’ self-efficacy on both the pre-survey and post-survey. Similarly to the instructors’ survey with personal teaching efficacy, I intended to capture students’ self-perceptions; using these questions with “I” in them allowed me to elicit specific information about how students felt about themselves rather than their more general thoughts about teaching and learning mathematics.
Dimension 2: Beliefs about teaching and learning. The surveys for both the instructors and students included the same questions assessing general beliefs about teaching and learning mathematics. To capitalize on prior research concerning beliefs about teaching and learning mathematics, I took the framework from *Principles to Actions* (NCTM, 2014) to describe productive and unproductive beliefs about teaching and learning mathematics. The ideas within *Principles to Actions* (see Table 6) use the evolution of standards for mathematics coupled with research studies using data from K-12 classrooms to articulate a “unified vision of what is needed to realize the potential of educating all students” (NCTM, 2014, p. vii). Even though NCTM focuses on research and evidence of K-12 classrooms, the content of College Algebra is repeated content from middle and high schools. Also, a large percentage of students taking College Algebra are freshmen, and thus only recently were in high school. Therefore, the beliefs outlined in *Principles to Actions* have at least reasonable if not strong overlap with beliefs about teaching and learning mathematics held by teachers and learners in College Algebra.

To use these beliefs on the pre- and post-survey, I constructed six separate questions with each productive and unproductive belief placed on opposite endpoints of a line representing a spectrum. The instructions participants read were, “Below are a list of different beliefs about teaching and learning mathematics. For each question, mark an ‘x’ anywhere on the line indicating where your belief lies in relation to the two beliefs.” See an example of one of the questions in Figure 3.
Table 6

**Beliefs about Teaching and Learning Mathematics**

<table>
<thead>
<tr>
<th>Unproductive Beliefs</th>
<th>Productive Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics learning should focus on practicing procedures and memorizing basic</td>
<td>Mathematics learning should focus on developing understanding of concepts and</td>
</tr>
<tr>
<td>number combinations.</td>
<td>procedures through problem solving, reasoning, and discourse.</td>
</tr>
<tr>
<td>Students need only to learn and use the same standard computational algorithms and</td>
<td>All students need to have a range of strategies and approaches from which to</td>
</tr>
<tr>
<td>the same prescribed methods to solve algebraic problems.</td>
<td>choose in solving problems, including, but not limited to, general methods,</td>
</tr>
<tr>
<td>Students can learn to apply mathematics only after they have mastered the basic</td>
<td>standard algorithms, and procedures.</td>
</tr>
<tr>
<td>skills.</td>
<td>Students can learn mathematics through exploring and solving contextual and</td>
</tr>
<tr>
<td>The role of the teacher is to tell students exactly what definitions, formulas, and</td>
<td>mathematical problems.</td>
</tr>
<tr>
<td>rules they should know and demonstrate how to use this information to solve</td>
<td>The role of the teacher is to engage students in tasks that promote reasoning and</td>
</tr>
<tr>
<td>mathematics problems.</td>
<td>problem solving and facilitate discourse that moves students toward shared</td>
</tr>
<tr>
<td>The role of the student is to memorize information that is presented and then use</td>
<td>understanding of mathematics.</td>
</tr>
<tr>
<td>it to solve routine problems on homework, quizzes, and tests.</td>
<td>The role of the student is to be actively involved in making sense of mathematics</td>
</tr>
<tr>
<td>An effective teacher makes the mathematics easy for students by guiding them step</td>
<td>tasks by using varied strategies and representations, justifying solutions,</td>
</tr>
<tr>
<td>by step through problem solving to ensure that they are not frustrated or confused.</td>
<td>making connections to prior knowledge or familiar contexts and experiences, and</td>
</tr>
<tr>
<td></td>
<td>considering the reasoning of others.</td>
</tr>
<tr>
<td>An effective teacher provides students with appropriate challenge, encourages</td>
<td>Students can learn mathematics through exploring and solving contextual and</td>
</tr>
<tr>
<td>perseverance in solving problems, and supports productive struggle in learning</td>
<td>mathematical problems.</td>
</tr>
<tr>
<td>mathematics.</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Taken from *Principles to Actions*, p. 11, by National Council of Teachers of Mathematics, 2014, Virginia: The National Council of Teachers of Mathematics Inc.

*Figure 3.* An example of one of the questions on the pre- and post-survey using NCTM’s productive and unproductive beliefs.
Tick marks were placed on the line in order to better quantify where participants indicated their beliefs fell. Using tick marks instead of a typical Likert scale better represented a spectrum because it steered participants away from choosing a number. The idea behind this choice was this modification allowed participants to pick a place on the line that best reflected their belief in relation to the spectrum. When participants were making their choice for each question, they were not confronted with discrete numbers, instead they had this line with tick marks, which better represented a continuous pathway between the productive and unproductive belief.

**Interviews.** The instructors and eight students were selected to participate beyond completing the pre- and post-survey. While eight students were selected, only two students agreed to participate beyond the pre- and post-surveys. More about that part of the data selection will be discussed in the analysis section below. These four participants, whom I refer to as the main participants, were interviewed four times over the course of the semester. Table 7 gives basic information of the four main participants. The first and the last interviews were focused on elaborating the person’s thinking when he or she answered the pre- and post-survey. During both semi-structured interviews, the participant and I always had the participant’s answers to the survey in front of us. Questions for the pre-survey interview mainly came from the pre-survey itself, with additional probing questions asked. For the post-survey interview, I did general comparisons of the participant’s pre-survey answers with the post-survey answers. One of these comparisons included noting for each question how the answer changed or did not change. I also reexamined and coded the transcript of each participant’s pre-survey interview in order to organize their beliefs into several larger categories for generating
interview questions and prepare myself to ask different probing questions. This process ended with notes for each participant concerning all or most of the following areas: views about mathematics, how they viewed the role of the instructor, how they again viewed the role of students or opinions of students, views of themselves in their role in a math class, and beliefs they admitted to being unsure of at that point of the semester. I created specific questions from this document for each participant for the post interview.

Table 7

*Basic Information for Four Main Participants*

<table>
<thead>
<tr>
<th>Name</th>
<th>Level of School</th>
<th>Role in College Algebra</th>
<th>Prior Teaching/Learning Experiences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sally</td>
<td>2nd year PhD student</td>
<td>Main Instructor (GTA)</td>
<td>* Instructor of College Algebra prior semester * Calculus Recitations * Tutoring in undergraduate school</td>
</tr>
<tr>
<td>Cara</td>
<td>Sophomore undergraduate</td>
<td>Support Instructor (LA)</td>
<td>* LA of College Algebra prior semester * tutoring in high school</td>
</tr>
<tr>
<td>Brad</td>
<td>Freshman other undergraduate</td>
<td>Student</td>
<td>N/A</td>
</tr>
<tr>
<td>Michael</td>
<td>Freshman other undergraduate</td>
<td>Student</td>
<td>* Took Intermediate Algebra at UNL previous semester</td>
</tr>
</tbody>
</table>

For the two interviews in the middle of the semester, I chose to focus the interview on specific instances of teaching and learning that occurred in their College Algebra course. To do this, I videotaped class sessions, then edited those recordings to short two- to three-minute video clips. The intention of using video clips as the center of these interviews was to focus the conversations on common interactions within the classroom (Speer, 2008), rather than letting participants openly reflect. Using video clips
for teacher development is a common way to help teachers reflect more deeply on their own practices (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Sherin, 2002; Sherin & van Es, 2009). Hartman’s (2010) dissertation study involved teachers using videos of themselves when they were students in a mathematics course to support reflection on themselves as learners. Thus, by using video clips for these interviews, I sought to help both the instructors and students reflect more deeply on their practices as an instructor or as a student.

For each interview in the middle of the semester, each participant and I watched three video clips and then using a semi-structure interview technique (Creswell, 2013). I gathered more in-depth data from the participant about their beliefs. Watching each clip several times, I then picked certain points during the video, created video clips, to direct the conversation towards and created questions for these semi-structured interviews using these video clips. Once a point in the video was chosen, I would create correlating questions for each participant. For example, in one video clip, a point was identified in which a student presented a solution and the GTA asked the student to further explain her solution. For this point in the video, I asked the GTA why she chose to ask the student to explain further, then I also asked the LA and two students why they thought the GTA chose to push the student for more of an explanation. At the end of this process I had a list of questions for the GTA, the LA, and one list for the students (see Appendix B). Figure 4 denotes the general timeline in which these four interviews occurred for each main participant.
Table 8 gives a brief summary of each video clip used and why it was picked.

There were a variety of clips used in the two interviews. Two of the six video clips focus on Sally talking directly about beliefs and reasons behind teaching decisions.

**Participant selection.**

**Choosing one section of college algebra.** The only criteria for picking this section of College Algebra to study was having a main instructor who was a GTA with less than one year in teaching experience. I met with the Director of First Year Mathematics Programs to ask for his suggestions of which section to contact first. He gave me two initial sections to contact because he believed both of the GTAs would be willing to be open and thoughtful about their ideas. The first GTA I contacted indicated she would be out of town for about three weeks during the semester, but was willing to let me study her section. I decided to choose the second section whose GTA was also willing to participate and did not intend to be gone for a substantial period of time. Her section met three consecutive days of the week, each for a period of 1 hour and 15 minutes, giving a total contact time of 3 hours and 45 minutes per week. The classroom
## Summary of Video Clips Used in the Middle Interviews

<table>
<thead>
<tr>
<th>Interview #2: Focus on 3 Video Clips</th>
<th>Mathematical Focus</th>
<th>Brief Summary of Clip</th>
<th>Why I chose this Clip</th>
</tr>
</thead>
</table>
| **The Bridge Problem**              | Problem solving using the Seven Bridges of Königsberg problem | Sally presented a solution to The Bridge Problem, a problem students had been working on the previous class session. During her presentation, Sally used a specific representation of The Bridge Problem and asked students questions. Afterwards, Sally got on a “soap box” where she explained her reasoning behind giving this type of problem. | * It was a challenging problem with no solution  
* I was not sure whether the students had come up with the representation Sally used  
* Sally explained to the whole class her reasons behind this teaching decision |
| **Cara Helping A Group of Students**| Graphing and interpreting asymptotes | Cara initially helped two students with a calculator issue then helped a different group of students working on a question about asymptotes. | * Cara directly intervened when helping the students with the calculator  
* Cara was obviously trying to lead the 2nd group of students by making leading statements |
| **Composition of Functions with Pizza** | Composition of Functions | After giving a brief overall picture of what composition of functions mean, Sally worked through a specific example of composition of functions using the price a pizza parlor charges and that cost that is required to make a pizza. While Sally worked through this example, the students seemed lost and there was a moment where Sally tried to clear up why people might be lost. | * Sally gave a conceptual picture of the content first  
* I really wanted to see what each participant saw as reasons for confusion  
* This was a moment where Sally, Cara, and many students were engaged in one conversation |

Table 8 continues
<table>
<thead>
<tr>
<th>Interview #3: Focus on 3 Video Clips</th>
<th>Mathematical Focus</th>
<th>Brief Summary of Clip</th>
<th>Why I chose this Clip</th>
</tr>
</thead>
</table>
| **The Magnitude Discussion** | The meaning of exponents | A student presented her solution to the whole class. Sally asked her to explain more details about the solution, but the student got hung up. Other students and Cara tried to help explain more details of the solution. | * A student presenting was something relatively new in this class  
* The student’s struggle is a typical kind of error seen in College Algebra  
* The problem and Sally were pushing students to think beyond the procedures  
* Cara interjected in the group conversation |
| **Exploring Even and Odd Functions** | Examining properties of even and odd functions | In a whole group discussion, Sally started leading the class through the example of what happens at x=0 when you reflect an even function across the y axis. The class came to the consensus that x could be anything when you reflect an even function across the y-axis. Then the class examined what happens when an odd function is reflected across the y-axis. Unlike the even function, the class sees that x had to be 0 when the odd function is reflected. | * The students appeared to be really engaged in this discussion  
* Sally set up the discussion so that the students really had to think about what happened at x=0 for both problems |
| **Sharing Mid-Semester Feedback** | Not Applicable | Sally shared the feedback she received on mid-semester evaluations she previously gave. The feedback was positive but also contradictory at points where students described what was helping them learn and other things they wanted to help them learn. | * Another instance where Sally is being public about her teaching decisions  
* Insight into student feedback |
in which they met was a recently redone room with a white board covering most of the walls, which were a light grey color all around. The room was filled with seven brand new tables; each table had between four to seven new rolling chairs around them. It was a recently renovated classroom, unlike older classrooms with chalkboards and rows of individual desks.

**Initial information of the main participants.** The main participants in this study included two instructors and two students. The main instructor, Sally, was a second year doctoral student in the mathematics department. Before this Spring 2015 semester, she had only been the main instructor of College Algebra in the previous semester; the rest of her teaching experiences included being a recitation instructor for Calculus and being engaged in tutoring during her undergraduate years. Cara, the Learning Assistant (LA), was a sophomore undergraduate student majoring in mathematics who was the LA with the main instructor in the previous semester. The LA had also been a tutor previously, during her later years in high school. Both instructors were young females who were outwardly enthusiastic about mathematics and approachable. While both instructors had great intentions of being helpful to others, they differed slightly on how they enacted these intentions. If a person were to ask Sally for help with a mathematics problem, she would kneel down next to that person and first ask him or her what he or she has already tried. Then she would wait for the person to respond and follow up with more questions like the first one she asked. If a person were to ask Cara for help with the same mathematics question, she would ask the person what he or she has tried but then almost immediately ask him or her more questions or make more statements that hint more strongly at the answer. I quickly found out from both participants after choosing this
section for this study that Cara and Sally taught together in College Algebra the prior semester, Fall 2014. Throughout the semester, both of them often referred to this semester together and believed that it had significant impact on this semester of Spring 2015.

Using the two dimensions from the pre-survey, the following four categories were created as a way for picking different types of students: low self-efficacy and traditional beliefs, high self-efficacy and traditional beliefs, low self-efficacy and progressive beliefs, and high self-efficacy and progressive beliefs. Eight students, two from each of the four categories, were initially contacted to be part of the study, but only two agreed to further participate beyond the pre-survey. The two students scored high on the self-efficacy part of the pre-test, but differed in terms of how progressive their views were on the beliefs portion of the pre-test. Table 9 summarizes these four categories and how the students were selected. Also, both students self-identified as the “freshman other” level of undergraduate education meaning they were not within one year of being out of high school and only possessed enough credit hours to be considered a freshman.

Table 9

<table>
<thead>
<tr>
<th>High self-efficacy, traditional beliefs</th>
<th>High self-efficacy, progressive beliefs</th>
<th>Low self-efficacy, traditional beliefs</th>
<th>Low self-efficacy, progressive beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student A</td>
<td>Student B</td>
<td>Student C</td>
<td>Student E</td>
</tr>
<tr>
<td>Brad</td>
<td>Michael</td>
<td>Student D</td>
<td>Student F</td>
</tr>
</tbody>
</table>
From his pre-survey responses, Michael indicated high confidence in his mathematical abilities and identified more progressive beliefs about teaching and learning mathematics. Brad also identified high confidence in his mathematics abilities, but differed from Michael: Brad held more traditional beliefs about teaching and learning mathematics. Both students were fairly shy upon first meeting but indicated that they wanted to help me out by participating. Because of this willingness, it became more important to me to spend more effort keeping them involved in the study than to try to fulfill the original plan of getting several students with different levels of self-efficacy and beliefs about teaching and learning mathematics. Not having students from the other categories in Table 9 is a limitation for this study.

**Data Analysis**

Analysis of the data occurred in three separate phases. Phase 1 involved analyzing the pre- and post-surveys primarily for the purpose of collecting richer qualitative data, which will be explained in further detail below. Phase 2 is where the interviews, observation notes, and video clips were analyzed for understanding what happened to the instructors’ and students’ beliefs about teaching and learning mathematics. Interviews from the beginning and end of the semester, as well results from the first phase were used to gather a sense of what happened to the participants’ beliefs. Then, data from the middle of the semester is used to support what was declared to have happened with each participants’ beliefs. After Phase 2, I realized the analysis had only been considering participants’ beliefs individually, which meant all four participants’ beliefs had not been considered together. From this realization emerged Phase 3, which involved zooming in on three video clips, and the participants’ responses to specific questions within these
video clips to understand the second research question about how participants’ beliefs might be interacting during the semester.

**Phase 1: Focus on survey data.** The primary purpose of analyzing the survey data were to generate richer interviews with the primary participants (Belli et al., 2001; Caracelli & Greene, 1993; Greene et al., 1989). An initial analysis of the pre-survey responses allowed me to identify ranges for the following four categories of students: high self-efficacy and progressive beliefs, low self-efficacy and progressive beliefs, high self-efficacy and traditional beliefs, and low self-efficacy and traditional beliefs. These categories were used for identifying and contacting eight students who represented a wide range of students to participate in the interviews.

**Giving students labels.** For identifying ranges relating to the self-efficacy dimension, I calculated a median score for each question and an overall median score of the cumulative scores for each student who gave consent. I used the overall median score to make a cut-off point for a high self-efficacy range and a low self-efficacy range. With a cut off point in place, I labeled each student as having either high self-efficacy or low self-efficacy. For identifying ranges relating to the progressive or traditional beliefs, I enacted the same process, except I had to place numeric integer values, 1 through 5, on the tick marks to generate scores for calculating the median. When I assigned values for participants’ marks, I used this rule of thumb: if they only placed marks on tick marks, then I only used integer values 1 through 5; if they placed at least one mark in between tick marks and one mark on a tick mark, then I acknowledged their recognition of those being different marks and counted their marks in between as halves, such 2.5; if they only placed marks inbetween tick marks, I only gave them 1.5, 2.5, 3.5, or 4.5. Once a median
score was established, I gave each person another label of progressive beliefs if his or her score was greater than the median, and traditional beliefs if his or her score was less than the median. Individual scores for questions were used as tiebreakers if the individual’s cumulative median score matched the cut off median score.

Comparing pre- with post-surveys. To generate a richer final interview with each participant, I did a comparison of scores for the whole class then also for each of the four main participants. The comparison of the whole class involved 18 total students after accounting for informed consent and matching pre-surveys with post-surveys. Because of this small sample size, I only used basic descriptive statistics when making comparisons. First, I repeated the same calculations described above for the post-survey. Once I had those scores and labels of whether individual students had high or low self-efficacy and progressive or traditional beliefs, I created a table with the median, mean, and standard error for both the pre-survey and post-survey. I also created a table describing changes or no changes in student’s labels. For example, did a student who had an initial label of low self-efficacy and traditional beliefs end the semester with the same label or did he or she get a new label like low self-efficacy and progressive beliefs? These tables are located in Appendix A.

After completing this overall look at all of the students’ scores from the pre- and post-survey, I concentrated on preparing the pre- and post-surveys of the four main participants for our final interview. For each main participant, I took his or her post-survey and, with a different color, marked his or her response from the pre-survey onto his or her post-survey. I used this to help generate final interview questions. I also
brought this marked up survey with me to the final interview so that the participant and I could physically look at the two sets of scores together.

**Phase 2: Focusing on the beliefs of the four main participants separately.**

The primary purpose of this part of the data analysis was to understand what happened to each main participant’s beliefs. Most of the analysis occurred after all of the data were collected, but some of it occurred during the semester.

*Analysis during the semester.* Using pre-survey responses and the transcript from the first interview, the first step of this analysis involved constructing a summary sheet for each main participant reflecting his or her beliefs at the beginning of the semester. To construct this summary sheet, I did an initial round of open coding then used the six categories from the beliefs dimension (NCTM, 2014) as a frame for organizing these codes into categories. Because of the way the participants responded to some questions, these categories were at times modified or combined to better represent what the participant said. In the end, each main participant had the following four categories: Role of the Instructor, Role of the Student, How He or She Views Math, and How He or She Views Himself or Herself. I also asked each participant whether he or she could think of beliefs he or she might be wavering about or whether there were struggles he or she could already identify at this early point in the semester. If the participant gave me an answer to either of these questions, I added a note to his or her summary sheet. For example, Sally told me she was already worried about how quiet this particular class was this early in the semester. Along with the analysis done with the pre-surveys in the first phase, I used
these summary sheets as a source of data for constructing final interview questions for each participant.

**Analysis after the semester.** In this part of the analysis, I used all interviews, transcripts of interviews, observation notes, analysis from Phase 1, and video clips to develop a more complete picture of each main participant that answered the ‘what happened’ question. I first constructed a timeline for each participant using the four interviews as time points across the semester. In this timeline, I identified specific common themes each participant talked about somewhat consistently across the semester then used these themes as a way to describe what the participant said about that theme at each point across the semester. Figure 5 shows an outline of a timeline that represents what the outcome of this analysis looked like.

<table>
<thead>
<tr>
<th>Emergent themes</th>
<th>Interview #1 (January)</th>
<th>Interview #2 (February)</th>
<th>Interview #3 (April)</th>
<th>Interview #4 (May)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theme 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theme 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theme 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Figure 5. Analysis tool to create overview of themes in a participant’s timeline.*

The themes that were used for each participant are shown in Table 10.

It should be noted that there was not evidence for every theme at every time point in participants’ timelines, but for most themes there was evidence across at least three of the time points. The information gained from the first step gave me insight into individual
characteristics of each main participant. For example, I noticed Brad was a student who made frequent statements questioning whether or not he understood particular solutions to math problems, whereas Michael was a student who commented about times he lost focus when working with peers by talking about weekend plans or joking around.
Table 10

Each Main Participant’s Emergent Themes

<table>
<thead>
<tr>
<th>Participant</th>
<th>Emergent Themes</th>
</tr>
</thead>
</table>
| Sally       | * How She Views Herself as an Instructor  
              * Relationship With Cara  
              * How She Views Students  
              * How She Defines Limitations of Teaching and Learning  
              * Past Experiences of Learning She Brings Up  
              * Struggles She Admits She Is Having |
| Cara        | * How She Views Herself as an Instructor  
              * Relationship With Sally  
              * How She Views Students  
              * How She Defines Limitations of Teaching and Learning  
              * Past Experiences of Learning She Brings Up |
| Brad        | * What an Effective Teacher Looks Like  
              * How College Algebra Effects Students  
              * How He Likes To Learn  
              * What Is He Worrying About? |
| Michael     | * What an Effective Teacher Looks Like  
              * How College Algebra Effects Students  
              * How He Likes To Learn  
              * Will He Use Math In His Future?  
              * Exhibiting Carefree Quality |

The next step involved focusing on participants’ beliefs at the beginning and end of the semester. While the step before in this Phase 2 gave me insight into the individuals, I still needed to consider another way of looking at what happens in order to more deeply immerse myself in their individual stories. To help change my lens of looking at the participants’ beliefs from the beginning of the semester to the end, I used Lampert’s (2001) adaptation of Hawkins’ *I, Thou, and It* as a way to focus on core components of
teaching and learning which in turn helped analyze the beliefs differently. Hawkins (1967/2002) initially developed and wrote about the relationship between a teacher (“I”), student (“Thou”), and content (“It”) using a triangle with the teacher, student, and content at separate vertices of the triangle and two way arrows. This representation symbolizes the importance of and the interaction of the three relationships. Teaching is not only about the relationship between the teacher and the student, rather there needs to be an “It” that is of interest to both the teacher and the student (Hawkins, 1967/2002).

Lampert (2001) expanded upon Hawkins’ theory by considering the work of teaching and the work of learning. The arrows between the teacher and student, student and content, and content and teacher represent the kind of practice that occurs within teaching and learning. Students practice includes studying the content. The practice between, “teachers and students have different purposes for their collaboration, but they must work together to accomplish them” (p. 31). The practice between teachers and content, “requires working in relationships with the content of the school curriculum” (p. 31). Then there is a fourth arrow representing the practice of the teacher that occurs by examining and responding to how students are interacting with the content. By identifying the kinds of practice that exist between the teacher, student, and content, Lampert is developing the complexity of what “teachers can do to address the fundamental problems in teaching” (p. 31) (see Figure 6).

In building on this complexity Lampert (2001) notes that these “practice arrows are not separate and distinct sites for a teacher’s work” (p. 33). Because of this simultaneous work of the teacher, Lampert adds a banner across the triad representing the
Figure 6. Lampert’s triad relationship.

practice of the teacher, indicating spanning the individual practices among teacher, student, and content. Putting a banner across these arrows of practice, Lampert emphasizes that the work of a teacher involves attending to all of these different practices at once (see Figure 7).

Figure 7. Lampert’s triadic relationship with the practice of teacher.

Lampert’s model of practice enriched my analysis by providing a different lens for looking at teaching and learning. Because my data did not capture the individual
practices as closely as Lampert describes in her model, I adapted the framework to only include the practice of the teacher, and evidence I had for individual practices were incorporated in the boxes for teacher, student(s), and content. Therefore, the framework I used to look at participants’ beliefs is reflected in Figure 8: describing how the participants were viewing the teacher, students, the content, and the practice of the teacher.

Figure 8. The adapted framework I used from Lampert and Hawkins.

Separate frameworks were created for each participant at both the beginning and end of the semester using only the surveys and first and last interviews. I placed notes about what a participant believed or described as the role of the teacher, student(s), and content in each corresponding box. Then I copied and pasted a quotations from the corresponding transcript into the banner labeled practice, which I believed embodied that participant’s notion of the teacher’s practice. For the instructors, these quotations were sometimes how they were describing their own work as a teacher or what they believed
the instructor should be doing. For the students, these quotes varied between what they believed the instructor should be doing and how they described the work of the teacher within this College Algebra course.

Once I had a representation for a participant’s beliefs at the beginning of the semester and the end of the semester, I put them side-by-side to make comparisons. This side-by-side comparison (Creswell & Clark, 2007; Starks & Trinidad, 2007) helped me gather an initial answer to what happened with this participant’s beliefs. Once I made an initial hypothesis about what happened, I looked back at the participant’s timeline and other data to refine my hypothesis. To revise my hypothesis, I made several iterations of looking at the two beginning and end of the semester representations, the participant’s timeline, and other data. When I had a solid working hypothesis, I then concentrated on filling in an understanding of why the hypothesis made sense for this participant. Yin (2009) refers to this analytic technique as “explanation building” used as a means for explaining, “‘how’ or ‘why’ something happened” (p. 141). This shift in concentration led to developing what happened during the semester in order to support the claim of what happened at the beginning of the semester compared to the end of the semester.

**Phase 3: Focusing on the beliefs of the four main participants together.** For this phase of the data analysis, I shifted my concentration towards answering the research question about how the participants’ beliefs were interacting with one another. It is very uncommon for people’s beliefs to directly interact with one another unless they are explicitly talking about them with other people, but it is common for people to reflect on happenings and it is these reflections which are the focus of this chapter because reflections are often driven by beliefs. Moreover, the idea of beliefs interacting was being
operationalized by considering how much participants’ reflections connected with one another. For example, I separately asked participants why they thought the student in the video clip, The Magnitude Discussion, struggled with explaining her solution. One participant said he thought this student struggled because she was very nervous, whereas another participant said he thought the student did not understand the solution and therefore could not explain it. These two responses from the participant about why the student struggled represent very different views of why the student struggled, thus the students had no connection in their responses.

**Identifying instances.** I chose three of the six video clips I used in the two middle interviews. Although each video clip had instances where all four main participants were talking about the same questions, I picked the three video clips where participants were talking about their beliefs or reflections in rich ways related to the question asked. The three video clips and the questions, which I call instances, are in Table 11.

Each video clip had between three to four instances of interaction, which I defined as places where all four main participants reflected on the same idea. These questions were carefully and purposefully constructed before interviews with participants in effort to capture when and how participants’ beliefs might be interacting. After selecting a clip to use in the interview, I identified separate instances I wanted to draw participants’ attention to then I constructed parallel questions for each participant about these specific instances. For example, if the instance was about why Sally picked a problem to discuss as a whole class, I asked Sally why she picked this problem to discuss as a whole class and I asked Cara, Brad, and Michael why they thought Sally picked the problem to discuss as a whole class.
Table 11

*Video Clips and Instances Used in Phase 3*

<table>
<thead>
<tr>
<th>Name of Video Clip</th>
<th>Brief Description of Video Clip</th>
<th>Interview Questions Asked about Video Clips of Interaction</th>
</tr>
</thead>
</table>
| The Magnitude Discussion      | A student presented her solution to the whole class. Sally asked her to explain more details about the solution, but the student got hung up. Other students and Cara tried to help explain more details of the solution. | * Why did Sally ask the student to say more?  
* How do you think the student was feeling?  
* Why do you think Cara decided to interject?  
* Why do you think the student got so hung up on solving for $W_o$?                                                                                                                   |
| Sharing Mid-Semester Feedback | Sally shared the feedback she received on mid-semester evaluations she previously gave. The feedback was positive but also contradictory at points where students described what was helping them learn and other things they wanted to help them learn. | * What was Sally thinking when she was sharing this feedback?  
* Did any of this feedback surprise you?  
* Why do you think there was that contradictory feedback?                                                                                                                      |
| The Bridge Problem            | Sally presented a solution to The Bridge Problem, a problem students had been working on the previous class session. During her presentation, Sally used a specific representation of The Bridge Problem and asked students questions. Afterwards, Sally got on a “soap box” where she explained her reasoning behind giving this type of problem. | * What did you think about the dot representation Sally used when explaining the solution?  
* Why do you think Sally gave this problem?  
* Why do you think Sally went on her “soap box”?  
* What were you thinking while Sally was on her “soap box”?                                                                                                                      |

*Analyzing instances.* For each instance, I created a document with the question prompting the instance at the top then I copied and pasted the specific parts of participant’s transcripts where they were answering this question. Every instance looked like this:
After a document was created for an instance, I did an initial round of coding where I summarized each participant’s reflection about a particular question. Once I had a summary of each person’s reflection, I looked across the four participants’ summaries and decided how much their reflections connected defined as how much they aligned or did not align with one another, then placed a code at the top of each paper to denote this look across the participants. After I had completed this process for all 11 instances, I looked across the analysis for commonalities and found five different categories of connections:

- No Connection
- Students Connected, Instructors Connected
- Instructors Connected, Students Different
- Instructors and 1 Student connected, 1 Student Different
- Connection Across Students and Instructors

A category of ‘No Connection’ means that all four participant answers are fundamentally different. In the example described above where two participants responded to the question of why the student struggled to explain her solution, they responses were fundamentally different. One participant said he thought the student struggled because she was nervous, whereas the other participant said he thought the student struggled because she did not understand the solution. These reasons for why the student struggled
are very different and do not share any similar reasoning for why the student struggled, hence they are fundamentally different. This example is an example of responses within the ‘No Connection’ category. A category of ‘Connection Across Students and Instructors’ indicates that some connections could be made between all four participant answers. An example of this kind of connection can be seen when I asked the participants why they thought Sally gave The Bridge Problem to the students. One participant said the problem was given to, “make us think a little bit more,” and another participant said, “to critically think and try to like push ourselves.” Both of these participants believed Sally gave this particular problem to students in order to get them to think more, meaning their responses overlap and thus they share a connection.

**Organization of the Next Three Chapters**

Phase 2 of the analysis focused on individual participants’ beliefs and from this phase it became clear to me that the students’ beliefs and the instructors’ beliefs were easier to think about together. For example, the students focused a lot on describing the kind of instructor they want or thought was the most effective, whereas the instructors focused on this topic as well, but added many more details or concerns in their descriptions. Also, the instructors taught together the prior semester, thus making their beliefs more intertwined with one another than the students. Because of this decision, Chapter 4 describes a case of the two students’ beliefs and Chapter 5 illustrates a case of the two instructors’ beliefs. Then Chapter 6 uses Phase 3 of the analysis to contextualize the main participants’ beliefs in practice in order to gain insight into how their beliefs might be interacting.
Chapter 4

Transformations and Subtleties: A Case of Two Students’ Beliefs

Upon first glance, Brad and Michael might have seemed like the same kind of College Algebra student. They were both young males who seemed very polite and approachable. After my first interview with them, I asked both of them why they emailed me back and agreed to be a participant, to which both responded that they thought it was a good thing to help other people out, and that they would hope someone would help them out if they needed it. It was tempting to expect that two young males with similar dispositions in the same math course would want, think, and do similar things as learners in this course, but of course initial impressions rarely tell a complete and accurate picture.

After analyzing the data from Brad and Michael, it became evident that Brad and Michael were indeed different from one another in terms of what they believed about teaching and learning mathematics, particularly at the beginning of the semester. To describe what happened to each of their beliefs, I have organized this chapter as follows: a first glance at what happened to Brad’s and Michael’s beliefs, a concentrated look at Brad’s beliefs, a concentrated look at Michael’s beliefs, and a final comparison and summary of Brad’s and Michael’s beliefs.

A First Glance at What Happened

After the pre-survey and accompanying first interview with each student, it was clear that Brad and Michael were indeed not the same exact student. Analysis of the data across the entire semester revealed Brad and Michael held different beliefs about teaching and learning mathematics at the beginning of the semester, and their beliefs were still different at the end of the semester. Many of their differing beliefs were about minor
aspects of teaching and learning mathematics, but what really made Brad and Michael different was their vision of what an effective math teacher looked like. At the beginning of the semester, they had very different views of what an effective mathematics teacher looked like. By the end of the semester, their views seemed more similar, but were still different. Figure 9 summarizes how Brad’s and Michael’s views of an effective mathematics’ teacher and how they changed after being part of the reformed College Algebra class.

As Figure 9 shows, there are changes within the individual student’s conceptions across the semester and, when compared to each other, the students’ beliefs were different in the beginning and the end of the semester. Brad and Michael started the semester with two different conceptions of an effective mathematics teacher. In Brad’s views, he saw an effective teacher as someone who directly told him what do, as indicated by a one-way direction arrow, in a step-by-step fashion. Michael’s vision of an effective mathematics teacher incorporates “working together,” as indicated by the two-way direction arrow, with Michael and with other students. In Michael’s eyes, effective mathematics teaching includes not just him and the teacher, but also other students in the classroom, which is something Brad did not see at this point in the semester. Michael also started the semester with a belief that he should work not only with the teacher and other students, but that he should be able to work by himself. One commonality between Michael and Brad’s initial vision of effective teaching was the belief that the teacher
Brad’s Views of an Effective Mathematics’ Teacher

Michael’s Views of an Effective Mathematics’ Teacher

Figure 9. A side-by-side comparison of Brad’s beliefs with Michael’s beliefs.
should use direct instruction. Brad thought direct instruction was needed all the time, whereas Michael thought direct instruction should be used with students who were struggling.

By the end of the semester, Brad and Michael’s visions were much more alike. They both saw the mathematics teacher as using less direct instruction and holding a facilitator role, as indicated by the bracket in Figure 9. By the end of the semester, Brad included working with other students in the classroom in his vision, and Michael placed even more emphasis on this relationship of working with the other students in the classroom. Also, how Brad and Michael characterized what is involved in learning mathematics became more aligned in that they both view doing problems as a central practice for learning, and subsequently requiring little or no emphasis on memorization or mastery of basic facts. While Brad’s idea of what learning mathematics entails changed the most, Michael’s idea of learning mathematics did not change at all even though his views on the roles of the major participants in the class did change. Despite being more aligned, it is clear from Figure 9 that Brad’s and Michael’s visions of effective mathematics teaching at the end of the semester were not exactly the same.

Figure 9 gives a snapshot of what happened to Brad’s and Michael’s beliefs about teaching and learning after being involved in their reformed College Algebra class, but there are still many questions left to be answered. Why did their beliefs change or not change the way they did? Were they noticing and responding to the same moments in the College Algebra class? While it appears they ended up with more similar visions at the end of the semester, were their visions really similar or driven by the same ideas? Further
individual analysis of Brad and Michael’s experiences reveals insights into these questions, and complete a more detailed picture beyond the initial glance from Figure 9.

**Understanding what happened to Brad’s beliefs**

**Brad’s beliefs at the beginning of the semester.** At the beginning of the semester, Brad was very clear about what he thought about teaching and learning mathematics. For every response on his pre-survey that I questioned him about, he was quick to answer and never wavered on what he said. Figure 10 uses his pre-survey answers and first interview data to summarize Brad’s views about teaching and learning mathematics at the beginning of the semester.

*Figure 10. Brad’s beliefs at the beginning of the semester.*
At the beginning of the semester, Brad saw the teacher as someone who helps him learn mathematics by showing him how to do problems in a step-by-step fashion. He explained that this belief stems from this being the only kind of math teacher he had ever had. He said, “I’m more used to being taught like the way to do it and then getting something to do” (Interview #1, 1/28/15). This comfort of familiarity in a direct teacher is mentioned several times throughout the initial interview.

When asked about the role of the students, Brad said:

I put that [his mark] in the middle because I feel like when she [the teacher] asks questions or when she [the teacher] does a problem in class, like if you get engaged and help her try to figure it out, it helps you walk through it better so you can further understand, or better understand how to do it. But then I also was kind of in the middle because you have to memorize certain things for tests, quizzes, homework and stuff like that as well. Which is like those formulas and stuff like that. That’s why I was right in the middle. I guess you have to memorize it but if you help in class and stuff like that, and being interactive and engaged, it helps you memorize it. (Interview #1, 1/23/15)

Brad believed students have a responsibility to actively engage in the course, especially when the teacher is up at the board, but he also acknowledged another role of the student is to memorize. In the quotation above, Brad indicates at first that he thinks memorizing can be a gatekeeper by saying, “you have to memorize certain things for tests, quizzes, homework and stuff ...” (Interview #1, 1/28/15) but then adds on that students who are putting effort into engaging in the class should find memorizing easier. At this point in the semester, Brad believes students need to know basic facts, especially for tests and
quizzes, and if students do not know these facts then it is their responsibility to learn
them. His conception of learning is that it is transmitted to him either by actively paying
attention to the teacher or by memorization.

At the end of the first interview, I asked Brad whether he had an inclination about
whether any of his beliefs might change after taking this College Algebra course. His
response was this:

They might. I mean, I guess we’ll see after what this class does. But it’s hard to
change something after you’ve been doing it for so long. So right now I’d
probably say no. It’s probably not going to change, but who knows. Maybe it will.
Never say never, right? (Interview #1, 1/28/15)

While Brad did not waiver on how he replied to this question, what he said indicated he
felt certain in what he believes about teaching and learning mathematics but did not want
to commit to saying his beliefs will not change. But from Figure 10, it is clear that Brad’s
beliefs did change.

Across all the interviews, Brad was often articulate about aspects of the class that
were either very helpful or unhelpful in helping him learn mathematics, because he
always held the goal of not wanting to be the student in the group or the classroom who
“didn’t get it” (Interview #1, 1/28/15). This goal helped shape Brad’s conception of
effective mathematics teaching. Having a mathematics teacher directly showing him how
to do a problem first was helpful in his eyes. Being shown multiple ways to do a problem
was not helpful and made him “just block it out” (Interview #1, 1/28/15). Having a
mathematics teacher walking around, being available to answer his questions was helpful
because he could get his question answered, but also unhelpful because he felt more timid
in asking questions in front of his peers. Analysis of observation notes, video clips, and
transcript data reveal that this worry of his was a theme behind Brad’s change in beliefs.
Brad’s worry helped drive him to do what he believed was best for his own learning of
mathematics, subsequently causing him to pay closer attention to his surroundings.

Brad’s beliefs a month and half into the semester. In the second interview,
Brad admitted to liking The Bridge Problem and putting forth great effort in solving it
during class, but “the second day we just kind of wanted to know the answer to it...”
(Interview #2, 2/24/15) For this same video clip, the teacher got on what she referred to
as a “soap box” and explained to the class that she gave The Bridge Problem because she
wanted to stress to the students that the thinking behind the problem was more important
than just finding the answer. In response to this explanation from the teacher, Brad said:

I guess kind of the same thing that I am now, like I’m open to her idea with the
worksheets and stuff like that but I’m still kind of the way I was when I wanted to
know like the formulas, know how to do the stuff and then do the worksheet or
something like that. But, I mean, I get her point more now that she wants us to try
and work through it ourselves so we have to figure it out. (Interview #2, 2/24/15)

Brad acknowledged that he was trying to be open to the idea of learning in a different
way, yet he was still adamant about wanting to learn in the ways that he was used to. He
admitted that he tried working on The Bridge Problem without direct intervention from
the teacher, but he also admitted that by the second day he reached the point of wanting a
direct answer.

Also in the second interview, Brad noted a confusing point in one of the video
clips which would have ‘messed’ him up more. He referred to the moment in the
Composition of Functions with Pizza video where Cara, the Learning Assistant, interjected during a group discussion about a formula for the cost of making a pizza. Cara responded to a particular student’s question with a statement intended to redirect students to think about how pizzas are sold as slices. Brad said “that would have just messed me up more so I probably would have just not listened to what she was saying” (Interview #2, 2/24/15) because “if I listen to them it might make me change what I’m thinking...” (Interview #2, 2/24/15). Brad was confident in his own understanding of the problem, and really believed that his understanding might shatter if he listened to different ways of thinking.

At this point of time in the semester, Brad was still grasping onto these beliefs of wanting direct help and only one way of doing problems. His reluctance to listen to other ways came from the fact that if he listened, it might change what he was thinking, meaning he might no longer “get it” (Interview #1, 1/28/15). Yet, he also continued to show signs that he was paying attention to the differences in this kind of classroom, and was not completely shutting down the possibility of those differences helping him. Acknowledging these differences and even rejecting them as things that might help them imply Brad has some level of cognition about how he was learning.

**Brad’s beliefs two and a half months into the semester.** During the first part of the third interview, Brad continued to show signs that he was paying attention to details of the teaching and learning while also trying to convince himself to be open to differences. When watching the clip, The Magnitude Discussion, in which one of his fellow students struggled to present and answer questions about her solution in front of the whole class, Brad shared compassion for her and said
If she [the teacher] would have just came up and helped right at the beginning . . . if I was her [the student], that’s what I would have wanted her [the teacher] to do instead of just sitting there looking like an idiot, but... (Interview #3, 4/3/15).

He was again reiterating his familiar feeling of wanting direct help and not wanting to be the one who does not “get it” (Interview #1, 1/28/15).

Minutes later in the conversation, Brad reiterated his desire to know whether he was doing a problem right because he was worried about getting that problem correct on a test. He said, “I always just, if I don’t understand a problem then I ask about it because that could be the problem that’s on the test and then I don’t know because I didn’t ask, so...” (Interview #3, 4/3/15). As part of his preparation for the exam, Brad admitted he put together a table with the necessary information about translations of functions then asked the instructor if the table was correct. While Brad was again hinting he wanted direct help, in this instance he did the work himself of putting together information he thought he should know rather than asking the teacher directly what he should know for the test. It is unclear whether Brad realized that he was starting to change his practice as a student by constructing and organizing first instead of waiting for direct instructions from the teacher.

Perhaps the most notable reflection in this third interview came when Brad was watching the final video clip, The Bridge Problem. In this final video clip, the teacher is relaying to the whole class the feedback she received from them on the mid-semester evaluations and connecting the feedback to reasons why this class is being taught in this different manner. Brad responded to questions about the instructor’s public reasoning:
Brad: But, and she was explaining how the class was taught which now listening to her and looking at it in retrospect it’s kind of like yeah that does work a lot better and it has worked pretty good for me. Um, and that we do the questions that she gives us and of course they’re a little bit harder questions ‘cause she said she wants us to work on the harder questions so we can solve problems. Um, and then if we don’t get it or something like that then we do an example or a review of that to more understand it. And then like she was talking about the understanding of brains thinking like computers or whatever, that I guess you kind of make sense when you think about it too because if you just see an equation or something like that which that’s the beginning of the semester I said that that’s how I wanted it was just an equation and then I’ll do it, but if you do an equation and you know why this equation has to work for this or something like that, it’s easier to remember when you get to a test or something like that, so…. I mean, in retrospect I think what I said earlier this semester was not right, now. And I’ve changed my mind on how the class is taught, so...

Interviewer: Huh.

Brad: But, yeah.

Interviewer: Does that surprise you?

Brad: It does. I thought I was going to stick to my guns, but she changed my mind. But if she wouldn’t have said anything, I probably would have not even realized that I changed my mind but, she said something and then she kind of put it in layman’s terms I guess and made I understandable for all
of us to understand how the class is set up and how she teaches and then it made sense and kind of changed my mind. (Interview #3, 4/3/15)

Right before this part of the interview, Brad acknowledged how nice it was to see the teacher being so sincere, honest, and open with the class. Now, listening to what the teacher was saying about how people learn during the video clip helped Brad understand how this kind of teaching helped him learn more. For example, when he pointed out that understanding why an equation works made it easier for him to remember. Brad also resonated with the teacher’s connection of the cognitive process with learning. Although he admitted he was surprised, Brad made a major shift in recognizing what he believes had changed. What he had said previously in other interviews about what helping him most to learn is no longer true.

**Brad’s beliefs at the end of the semester.** Brad appeared to make a major shift in his thinking about three-quarters of the way into the semester, but whether or not this shift could be permanent was not confirmed until the final interview. In this final discussion with Brad, it was clear that changing his mind was not a fleeting, momentary decision. His initial impressions of the teaching and learning in College Algebra included, “I think the way they taught was good, it was different for me but… I adapted to it and then it was, it worked out,” and “it opened up to me saying like 101 is not such an easy class. It still has stuff that I didn’t know” (Interview #4, 5/7/15). Using his post-survey answers and interview transcripts, Figure 11 reflects Brad’s views about teaching and learning mathematics at the end of the semester.
**Figure 11.** Brad’s beliefs at the end of the semester.

The kind of mathematics teacher Brad describes at the end draws on what he was noticing throughout the semester. His appreciation of Sally’s commitment to being transparent and giving reasons behind her actions shows as he describes an effective teacher and includes these qualities in this description. Also part of his description of an effective teacher includes the teacher’s actions as a facilitator, rather than the direct instructor he wanted at the beginning of the semester. The quotation in the center of figure 11 reflects Brad’s central view of what he thinks the teacher should be doing with the students when they are learning mathematics. This quotation, which is aligned with observation and survey data, provides evidence that Brad now sees the teacher stepping back and letting students try the problems first on their own.

Brad still thinks students need to engage in class, but he is able to articulate more specific behaviors he thinks students should be doing in pursuit of learning. One
significant change in Brad’s views about students is that he now believes working with other students is a significant part of learning in a math class. In the last interview he said:

At the beginning I just kind of wanted to do my own worksheets, you know, and then asked Sally if I was right or something like that but later on I was doing them and then I would ask the other two what they got on the questions to see if they got ‘em, ‘cause then I started working with them a little bit more, but at the beginning, I definitely didn’t and I feel like you should because working with the people at your table is a lot faster first of all, since you know, they’re right there. You don’t have to wait for a teacher, and then second of all they [other students] can show you like how they got their answer and then you can show them how you got yours and figure out cause then we’d have some discrepancies and then I would have to ask Sally. (Interview #4, 5/7/15)

From this quotation, Brad still shows signs of wanting to be right, yet now he understands there are other ways he can convince himself that he is right or that he “gets it” (Interview #1, 1/28/15). Moreover, learning for Brad involves becoming less stressed about memorizing and knowing basic facts, because if he “work(s) through them” then that helps more than doing a bunch of “easy questions” (Interview #4, 5/7/15). In sum, Brad believes “I don’t have to memorize stuff anymore, I just need to do problem solving and figure it out” (Interview #4, 5/7/15).

**Summarizing Brad’s changes in beliefs.** To help visualize and summarize what happened to Brad’s beliefs, part of Figure 12 will be reexamined.
Brad went from wanting a teacher who directly showed him how to do problems first to believing that a teacher who lets him work with others and try problems first was better for his own learning. He thought he needed to focus less on memorizing facts or basic skills and focus more on engaging with problems. These changes reflect an overall shift in Brad’s views of the nature of learning mathematics. Brad is starting to believe that learning mathematics means he must construct his own knowledge of mathematics. This is a big shift away from his previous belief that learning mathematics happens when it is transmitted directly to him.

**Understanding what happened to Michael’s beliefs**

**Michael’s beliefs at the beginning of the semester.** Just like Brad, Michael was pretty clear about his ideas of what effective mathematics teaching and learning looked like...
Like at the beginning of the semester. While he was not always as quick as Brad to answer my questions, he always answered in ways that emitted confidence. Figure 13 uses his pre-survey answers and first interview data to summarize Michael’s views about teaching and learning mathematics at the beginning of the semester.

Figure 13. Michael’s beliefs at the beginning of the semester.

Michael viewed an effective mathematics teacher as someone who let students engage with problems first, but would step in and directly help them if they were struggling. He admitted, “I do like the step by step helping but I don’t like when they always like [give] step by step help” (Interview #1, 1/29/15). Unlike Brad, Michael started the semester with the view that students should work with the teacher, other students, and work by themselves. He said:

Like if someone shows me like right there like how to do it, I’ll be fine. Or just kind of like figuring it out on my own or like with a group. Contemplating like
how to do this. But even if like none of us actually know how to do the whole problem. Each one of us might know how to do a little piece. So we can kind of put it together and figure it out. That’s how we do it with my group. (Interview #1, 1/29/15)

His open attitude about with whom students should work continued when he described multiple ways to do problems, and that students should try different strategies because trying “the same thing over and over again if it’s not working” (Interview #1, 1/29/15) did not help lead to a solution. To Michael, learning mathematics involved no memorization because that meant students only “push stuff into your(‘their) mind,” but rather students should just “let it just kind of happen” (Interview #1, 1/29/15).

At the beginning of the semester, Michael appeared to be very open and even welcoming to learning mathematics in this new reformed College Algebra class. His beliefs as stated were that teaching mathematics involves letting students work on problems first and only directly intervening if there is a struggle. As a consequence to this view of teaching, Michael viewed learning mathematics as constructing knowledge. These ideas of teaching and learning strongly align with the Department of Mathematics’ conception of “good” teaching and learning. Because of this alignment, one might not expect his picture of effective mathematics teaching and learning to change by the end of the semester, but figure 9 reveals that Michael’s beliefs did in fact change by the end of the semester.

Like Brad’s common theme of worrying about not “getting it” (Interview #1, 1/29/15), a common theme behind Michael’s beliefs emerged in the subsequent interviews. In every interview with Michael, he made statements that in isolation would
make someone think he was very committed to being a student in College Algebra, yet he also made statements that suggested he, himself, was not as committed. I do not mean he was apathetic or did not care, but rather I mean he was usually easy-going about learning in College Algebra. It was this committed yet uncommitted attitude that shaped Michael’s conception of effective mathematics teaching. He had beliefs about what should be happening with the teaching and learning, but also made statements revealing that he might not be matching his practices as a learner with these beliefs. For example, in the first interview Michael suggested that students should be working with the teacher, with other students, and by themselves but then later admitted, “And we got like 2.5 pages of the 3 pages done. So we did pretty well. We were goofing around a little bit but that’s normal” (Interview #1, 1/29/15). This example shows that Michael said he believed students should be working, but then also indicated it was okay not to work sometimes.

**Michael’s beliefs a month and half into the semester.** When Michael and I watched the clip, The Bridge Problem, Michael recalled liking The Bridge Problem and spending a lot of time thinking about it inside and outside of class. In this particular clip, the teacher went on a “soap box” and explained her reasons behind the teaching decisions she was making. When I asked Michael why he thought the teacher gave the class that particular problem he said, “Probably to critically think and try to like push ourselves like, ‘cause usually when you do math you don’t go into the problem thinking it’s not gonna work” (Interview #2, 2/27/15). He admitted that this problem made him think, but when I asked about the teacher’s solution, he tried to explain it to me but then ended up stating, “it’s… I don’t really know how to explain it exactly” (Interview #2, 2/27/15).
And when I asked him what he thought about the teacher’s soapbox, he said did not really remember that part of the video. This also meant he did not remember this part of the class when it actually happened.

For the third clip, Composition of Functions with Pizza, Michael offered a reason for why students were not participating in the group discussion. He thought everyone was “bummed out that particular day” (Interview #2, 2/27/15) which is why they did not seem to engage more in solving the example about composition of functions. Later in the conversation, he said, “The laziest students. That’s what we are,” (Interview #2, 2/27/15) and that the students were silent because they were thinking about anything other than math in that moment. Here is an excerpt from the transcript where I asked Michael more directly about the mathematics in this particular clip:

Michael: I don’t think like anybody’s really been like my whole table wasn’t really confused with this, they just don’t want to do it. I don’t know why, like you just take one equation and put it in with the other. It’s like not hard at all, but I just don’t know why people see the f of x to the g or whatever. And then they just freak out, I guess. Uhh,

Interviewer: So do you think that’s what’s confusing about composition of functions? Maybe how they write it or?

Michael: I think they write it just fine I just think people see that it’s like, when you have to like combine these two equations to get one answer in the end, I think it just makes people like start to have like, they’ll automatically start over thinking and then they’ll think too much about the problem and then they’ll just give up on it pretty quickly and then just not even do it
cause I don’t know. I don’t know why, it just doesn’t. I could do this, like cause that’s the most, every day in real life that’s one of the types of math you would do a lot. Because when you’re trying to just like, if you were trying to figure out um, I don’t know let’s see. How many, uh, I don’t know like a good example, but it just like, a lot of problems in the real world are two different equations that come together to make one.

Interview #2, 2/27/15

In this excerpt, Michael claimed that the composition of functions is not difficult to learn but could not seem to explain many specific details about this topic. Then the rest of what Michael focused on were thoughts about the students and their behaviors, such as saying they ‘freak out’ or give up quickly. In this second interview, he never identified himself as a student who freaks out or who gives up quickly. In fact, he positioned himself as a student who did not give up quickly by describing how he worked on The Bridge Problem in and outside of class. These ideas contradict earlier statements when he described the other students, and even himself, as the laziest students. By saying the mathematics is not difficult to learn. and framing students as those who give up easily, Michael projects an idea that learning mathematics should be difficult, at least difficult enough to keep students engaged and working. There were times when he placed himself as a student who was engaged and working, and at other times suggesting he was not, either directly or by trivializing aspects of the classroom. For whatever reasons, Michael wanted to claim both the status of a committed student and also an uncommitted student.

Michael's beliefs two and a half months into the semester. Michael continued to trivialize aspects of the classroom that were brought up in our interviews. He said
multiple times that the other problems were not too hard and the ones in the video clips were the harder ones for that day. When discussing the scene in The Magnitude Discussion, where the student was having serious troubles explaining and working through her solution in front of the class, Michael posed the idea that maybe this particular student was not ready for college math and tested well on the day of the math placement exam. In comparing this statement with how Michael voluntarily explained how he was placed into this class as a result of having a bad day, Michael continues to avoid identifying himself as a particular type of student.

When reflecting on Sharing Mid Semester Feedback, where the lead instructor was recounting the mid-semester feedback she received from the class, Michael immediately started to make comparisons to his negative experiences from the class before. He said

I think it’s cool she actually gave feedback to what we said. She let us know like that she’s actually, the last class the guy didn’t really care what we said, he was just gonna keep doing what he did. (Interview #3, 4/7/15).

Michael stated further that even though his last teacher also collected feedback, he was pretty sure the teacher just threw the feedback in the trash. He even went on to talk about another teacher, his calculus teacher in high school and described him as the “laziest guy I ever knew” (Interview #3, 4/7/15). For Michael, describing what he thought was good teaching often involved describing past experiences of teaching and learning, which often were examples of what he described as bad teaching.

When I later asked him if he felt like he could ask the teacher in this College Algebra course anything, Michael replied that he did and also that
Like, I mean I’m not gonna be here tomorrow so I just like asked her if I can come in some day and she said yeah. Like no problem, she’ll get me caught up and stuff so… it’s like I’m not even really missing class too bad (Interview #3, 4/7/15).

He went on to say how much he liked this way of learning and that it was like the flipped history class he took in the past. He said,

And I learned more in that class than I have in any other class I’ve taken in my life, ‘cause you get a little prepared, …and then you ask them like why did this, like what happened with this. It was so interesting. It was way cooler than most history classes where it’s like on 1822 something something and this happened...

(Interview #3, 4/7/15)

Michael is committed to this way of learning, but also admitted he was missing class, something that might be a pattern since he added he was not missing class “too bad” (Interview #3, 4/7/15). Also, Michael describes liking this way of learning so much, but can not seem to remember specific details about what it is he liked. Someone who can describe specific details about a type of teaching they like is much more convincing than say someone who says he or she likes this kind of teaching but can not really articulate why he or she likes this kind of teaching.

**Michael’s beliefs at the end of the semester.** Up to the end of the semester, Michael’s beliefs did not appear to change at all. He continued to stress that he liked and learned a lot in this style of teaching. As mentioned at previous points along the semester, Michael appeared to be committed to particular ideas about teaching and learning, but then less committed when he talked about himself as a learner. It was a big surprise to see
Michael’s post-survey answers, because almost every single one of his answers reflected a negative shift in self-efficacy or more traditional beliefs about teaching and learning mathematics. I did not expect this at all. There was no indication in my observation notes about a difference in his actions between the third interview and when he took the post-survey. Because of these answers, I half-worried that he would not show up to the final interview because something bad had happened. In hindsight, I realize it was those moments when he seemed less committed which had been clues indicating Michael might not feel as solid in his beliefs and actions as a student.

Michael did show up to the final interview and was completely normal during the first part. He did not mention anything that suggested he was upset or was thinking very differently about teaching or learning mathematics. His general reflections on the course included that he thought all of the students were more confident in math and that he “didn’t see anyone freaking about the test “(Interview #4, 5/7/15). Without prompting, he relayed to me that all he needed to get on the final was 15 points out of 150 in order to get a 75% in the course. While he admitted this was not necessarily a “great way to do it” (Interview #4, 5/7/15), he “was confident enough to know I [he] wasn’t gonna fail a class” (Interview #4, 5/7/15). Only briefly did he say that the last few chapters in the course were difficult. When I asked him whether he thought the class changed the way he viewed mathematics, he said not really but maybe that it was easier to “do” mathematics in teams, in other words work with the other students at his table. This idea of working with the other students came up several times in this final interview.

Well over halfway through the interview, I finally asked him directly about his post-survey answers:
Interviewer: Alright, now I want to kind of transition to looking at your pre and your post tests, I highlighted some of the questions I want to ask you about, but how I’ve been doing this is we kinda put them right next to each other. Scoot over so we can look, so this one’s your pre, this one’s your post.

Michael: Yeah this is right before the final so I was a little bit freaking out and I was nervous.

Interviewer: So you went down on like almost all of them.

Michael: I went there ‘cause I was nervous, but . . . she said oh Chapter 11’s gonna be in like 20 some percent of your final and...

Interviewer: No but really she did say that right before so...

Michael: So that kind of freaked me out.

Interviewer: Cause at that point you didn’t really think you were doing very well on the last chapter?

Michael: I knew I was doing terrible on that. Yeah, so I got a little bit nervous.

Interviewer: Yeah, I mean, you literally I think on every question went down.

Michael: Yeah, well one reason like with this one is I actually changed majors.

(Interview #4, 5/7/15)

In this exchange, Michael explicitly admits he did ‘freak out’ about the final exam because he found out Chapter 11 represented a large portion of the final exam. It was still unclear how he viewed himself in relation to the other students in the course. Previously in the conversation he said he had not seen anyone ‘freaking out’ about the class, yet minutes later in the conversation he admitted he himself ‘freaked out.’
Just after the above exchange, Michael told me he changed majors from meteorology to a double major in journalism and German, and because of this switch, he was already behind in graduation but he no longer thought he would be required to take any more math classes. Because this fourth interview took place after the final exam, he felt confident in his final exam score and felt no longer anxious. His decrease in anxiety might also stem from knowing he would not need to take any more mathematics classes. We continued to talk about his answers on the post-survey. When explaining his shifts on the questions for self-efficacy, Michael attributed some of those changes to now thinking he would no longer need as much math in his future. For the shift concerned with thinking less like a mathematician, Michael believed he does math differently because he did not always have steps to get to answers and he feels “like a lot of mathematicians memorize a lot of things (Interview #1, 1/29/15).” These answers seem to contradict the beginning of this fourth interview where he said he was still “pretty decent at math.” These contradictions suggest that instead of believing Michael has a lower self-efficacy or a higher self-efficacy, he is in the process of reevaluating his self-efficacy; his confidence in learning mathematics seemed uneven.

When Michael and I started talking about his changes in answers for the questions about beliefs, Michael verbally edited his answers indicating this his beliefs did more align where he indicated they were at the beginning of the semester. For example:

Interviewer: Okay. And then this makes sense, what you’re saying ‘cause I made marks as to like where you put your mark in relationship to the post-test on the pre. So like, your pre-test you were all the way to the right and then you scooted a little bit on this first question. Was that intentional?
Michael: Um, maybe not. I think I meant like, I think I looked at boxes and then.

Interviewer: Okay, not on the line. So like this really in your mind would have been on the same mark?

Michael: Oh, anywhere on that line, yeah I probably would have just put it all the way to the right and left.

Interviewer: Okay, so you were the same then, on this first one...

Michael: Yeah, it should be the same on that one. (Interview #4, 5/7/15)

Because Michael indicated in the interview that his marks on the post-survey were not actually meant to be different, I put more emphasis on using the interview data instead of the survey data for creating Figure 14 summarizing his beliefs about teaching and learning mathematics.

The kind of mathematics teacher Michael describes at the end of the semester is not drastically different than the one he described at the beginning. An overall observation to note is that Michael is able to elaborate more about the qualities he believes teachers and students should have or display. For example, Michael articulates that he thinks the teacher needs to have good knowledge of the math, which is something
Figure 14. Michael’s beliefs at the end of the semester.

he did not articulate at the beginning of the semester. Also, Michael is able to provide a list of actions he thinks students should not be doing, such as waiting until the last minute to do homework, something Michael admitted to doing too often.

**Summarizing Michael’s changes in beliefs.** Michael’s beliefs did not change in drastic ways. He still believed students learn from doing problems themselves first, and that the teacher should be helping students by being available to answer questions. These core ideas Michael still held onto represent his continued alignment with the reform ideals of the Department of Mathematics. Instead of changing in drastic ways, Michael’s beliefs underwent more subtle changes. For example, at the beginning of the semester,
Michael indicated that he thought students should work by themselves, with the teacher, and with other students. Now, Michael emphasizes working with other students much more, meaning his view of the role of the teacher is less at the forefront. Describing how he thinks the teacher should interact with students, Michael says, “Um, pretty much the way they did where they just walked around and waited for people to ask questions ‘cause that was definitely the most helpful thing” (Interview #4, 5/7/15).

Figure 15 depicts Michael’s view of effective mathematics teaching.

![Figure 15. Michael’s view of effective mathematics teaching.](image)

The stronger emphasis on working with other students is reflected in the larger two-way arrow, and the teacher stepping back being ready to answer more questions is noted by the bracket.

Another subtle change of Michael’s beliefs is his ability to articulate more characteristics of the instructor’s role. As shown in Figure 15, Michael now believes the
instructor should be knowledgeable of math and be welcoming towards students. These additional characteristics Michael describes enhance his original vision of an instructor.

**Comparing Brad and Michael**

As stated at the beginning of this chapter, Brad and Michael appear to be very similar kinds of students at first. They are both confident, willing to talk to me, and seem to have an overall happy disposition towards learning mathematics. But as this chapter has revealed, Brad and Michael are not at all the same kind of student. They started off the semester with very different views of effective teaching and learning in mathematics. By the end of the semester, both participants’ views had changed in different ways and for different reasons. Brad’s beliefs went through a transformation, much like the ‘transformation of consciousness’ described in Chapter 2 (Ackerman-Anderson & Anderson, 2001), in that he went from believing in a more traditional view of teaching and learning to having a more progressive view that matched the Department’s ideas of “good” teaching and learning. Especially when compared to Brad’s belief, it is clear Michael’s beliefs did not undergo the type of transformation from one idea to very different ideas. Rather, Michael’s beliefs changed in subtle ways, such as being able to articulate more qualities desired in a mathematics teacher, or putting more emphasis on the value of working with other students.

One meaningful similarity between Brad’s and Michael’s beliefs is that by the end of the semester, they both describe seeing the teacher in mathematics holding a facilitator role (Laursen et al., 2011) rather than directly helping students. Because Brad came to understand that having the teacher step back and let him try problems first, Brad’s belief about the role of the teacher changed to support this way of learning he came to
appreciate. Michael was less certain than Brad about defending why he viewed the teacher this way. Michael wavered back and forth between different reasons that directly and indirectly positioned the teacher as a facilitator in his view of effective mathematics teaching. For example, when Michael repeatedly talked about working and goofing off with other students, he was subtly pushing the teacher to the back of the picture, making it less certain that he truly believes the teacher should hold a facilitator role for the same reason Brad does. Michael was never direct or convincing in describing why his belief of the role of the teacher became less concerned with direct involvement between him and the teacher.

The views of effective mathematics teaching Brad and Michael ended the semester with seem much more alike than they were at the beginning of the semester. One reason for this alignment was each student’s understanding of what learning mathematics means. For Brad, his definition of what learning meant shifted away from seeing learning as something transmitted directly to him, and shifted towards seeing that he needs chances to construct his own knowledge of mathematics (Laursen et al., 2011). Through these shifts, Brad redefined what learning mathematics meant and how this new definition changed the roles of students, teacher, and mathematics within the classroom. Michael did not redefine what learning mathematics meant, and it was less clear how his views on students’ learning should apply directly to him. Michael’s mismatching statements about learning, such as saying students should not procrastinate yet admitted he himself procrastinates, make it hard to be certain that Michael really believes in learning in the ways he claims.
When making these comparisons between Brad and Michael, it was easier to identify reasons why there were changes with Brad. One possible reason for this is the difference between how cognizant each student was of himself as a learner. Brad is a student who knows himself, knows what he understands, knows what he does not understand, and knows how to learn what he does not understand. Michael is a student who thinks he knows himself, but he does not know what he understands and what he does not understand. This makes it hard for Michael to see what he needs to learn and what he has already learned. This difference in level of cognizance suggests Brad has a more developed feedback loop that allows him to evaluate his own work and assess his own beliefs (Butler & Winne, 1995). With Michael it was harder to determine if he had a strong feedback loop because there were not any times he verbally reevaluated his own work or beliefs.

In thinking further about Michael and the inconsistencies between his espoused beliefs and descriptions of his own practice, there are more possibilities to consider. Maybe he was telling me what I thought I wanted to hear? Philipp (2007) argues that any inconsistencies seen between beliefs and practices are because the researcher does not know enough. Michael and I only had four interviews that were 30 minutes each. There are several possibilities that could accurately describe Michael and at this point it is the most accurate to be open to any of these possibilities.

**Final lessons learned from Brad and Michael.** Brad and Michael offer distinct, yet sometimes similar, views from the student perspective in College Algebra. From the individual analysis of each student along with the comparison of the two students, I end this chapter with these final lessons.
Determining how, how much, and why changes in students’ beliefs vary from student to student. As seen with Brad and Michael, one student’s beliefs changed much more than the other student’s beliefs. In the case of Brad, his beliefs went from a much more traditional view of teaching and learning mathematics to a more progressive view in which the teacher is not telling and the students are trying problems on their own. Brad was very explicit in explaining these different views. In a convincing manner, he explained these shifts happened because he felt like they were helping him learn more mathematics in an improved way. Michael’s beliefs, which were progressive and aligned with the reform’s beliefs, changed in more subtle ways compared to Brad’s, and it was harder to understand why his beliefs changed in the ways they did. It was only through my in-depth analysis that I detected the subtle changes. Michael was less articulate in explaining how his views of teaching and learning mathematics were the same or different. Because of this less explicit nature and his tendency to make mismatching statements between what he believed and how he acted as a student, it was even less clear why there were subtle changes in Michael’s beliefs.

Students’ beliefs may not be predictive of their behavior as students. If a College Algebra instructor were given Michael and Brad’s views of teaching and learning mathematics, then asked which student they would prefer to have in their reformed class, the instructor would likely pick Michael. Michael’s views at the beginning align more directly with the beliefs of the reformed College Algebra course. Michael positioned himself as someone who believed in working with groups of students and did not think the teacher should be telling him the answers, which are some of the core characteristics in this reformed College Algebra course. But as it turned out, Michael had a big
mismatch (Schoenfeld, 1989) between his beliefs of effective mathematics teaching and learning and how he described himself as a student. The data do not describe Michael as a bad student, but they does describe Michael admitting to being a different kind of student than he described in his beliefs. For example, Michael said that students should actively be trying to find different ways to solve problems, and if they are stuck then they should be trying different methods, not the same one over and over. But Michael could never go into much mathematical detail about specific problems when asked, admitted to giving up sometimes, and described on a few occasions goofing around with his classmates, even missing class. These inconsistencies suggest Michael might have to reconcile what he believes students should be doing with how he behaves as a student in a mathematics course.

When teachers make public their reasons behind making particular decisions, students may be prompted to reevaluate their beliefs in light of the public reasoning.

The main instructor was regularly vocal in giving reason behind certain teaching decisions she was making. This act of public decision making did not seem to ignite much thought with Michael, but it did seem to stick out to Brad on numerous occasions. At the beginning of the semester, Brad responded to one of the instructor’s public explanations with, “I’m open to her idea with the worksheets and stuff like that but I’m still kind of the way I was when I wanted to know like the formulas...” (Interview #2, 2/24/15). In the third interview, the instructor gave an explanation for how peoples’ brains work in the act of learning and linked this idea to why she was asking the students to learn in the ways they were. When watching this part of the clip, Brad said:
And then like she was talking about the understanding of brains thinking like computers or whatever, that I guess you kind of make sense when you think about it too because if you just see an equation or something like that which that’s the beginning of the semester I said that that’s how I wanted it was just an equation and then I’ll do it, but if you do an equation and you know why this equation has to work for this or something like that, it’s easier to remember when you get to a test or something like that, so... I mean, in retrospect I think what I said earlier this semester was not right, now. And I’ve changed my mind on how the class is taught. (Interview #3, 4/3/15)

Brad is a student who is very in touch with what he knows and does not know, so the instructor’s comment may not have been the only thing but it was a factor that helped Brad reconsider his beliefs. These interactions between the instructor’s public reasoning and Brad’s thinking suggest that College Algebra teachers should be more open to their students about why they are teaching in the ways they are teaching.

The study discloses an intimate look at a pair of students who appeared less different in the beginning only because less was known of them. The data accumulated showed one student in a mode of transformation of his beliefs about mathematics teaching, and another student whose changes likely appear to be subtle perhaps because he remained uncommitted to either his beliefs or his practices. Regardless of the nature or outcome of change observed, each kind of change is important to take note of and continue to learn more about.
Chapter 5

Growth with Independence and Within Mentoring:

A Case of Two Instructors’ Beliefs

This chapter reveals the analysis of the two instructors’ beliefs. It begins with explaining a significant shift in my own perspective for studying these instructors and how this impacted the study, then focuses on Sally’s and Cara’s beliefs at the beginning of the semester with insight of the impact from their relationship. Then follows Sally’s and Cara’s beliefs at the end of the semester, and final lessons learned from the analysis of their beliefs.

Realizing the Importance of Sally and Cara’s Prior Relationship

Just before the semester started, I approached Sally to ask her to be part of this study. She had been recommended to me as someone who fit my requirement of being a novice teacher, defined as having less than one formal year of being the lead instructor of a course. I had expected that picking a novice teacher would give me more of a chance to see change if change were to happen. She was also described as being open and expressive, qualities which would be helpful in the study. By this time the Department had already made the assignment of her learning assistant, Cara, to Sally’s section of College Algebra, but I had not met Cara yet. Sally agreed to be part of the study and we conducted our first interview, in which we focused on her answers to the pre-survey that first week of the semester. I intentionally stayed out of the classroom the first week of the semester for two reasons: to let Sally establish routines and relationships with her students without my presence, and because the Department of Mathematics has a policy by which students can move in and out of sections within the first week so being there the
second week would mean I would have a more reliable population of students staying in this particular section.

It was in my first visit to Sally’s classroom that I realized there was already an established relationship between Sally and Cara. At the beginning of the class, I made introductory statements about the study to the students and subsequently to Cara, who was sitting in the back. Then I proceeded to pass out the informed consent forms and pre-surveys to the students and to Cara. After collecting the forms and pre-surveys, I glanced at Cara’s responses noticing that Cara had views of teaching and learning mathematics that were more progressive than I anticipated for a novice teacher. Since I had already interviewed Sally, I noticed that Cara’s views seemed to align with Sally’s views. As the rest of this class session unfolded, I observed Cara exhibiting an air of confidence about what she should be doing and her interactions with Sally were already established. By the end of the class, I thought Cara could not be a new LA this semester, and most likely was not new to working with Sally since they seemed to have such natural interactions so soon in the semester.

My suspicions were confirmed in the following few days when I had my first interview with Cara about her responses to the pre-survey. Sally and Cara had worked together as main instructor and LA in the previous semester for a College Algebra section. While it might have been beneficial to have started with a ‘blank slate’ of a relationship between instructors and even a blank slate of formal teaching experiences, this was not an option this semester. Most of the other main instructors of College Algebra had already taught College Algebra the previous semester, and most of them had an LA who had been an LA before. Thus, there would be some kind of attention needed
to what these instructors learned in the previous attention regardless of who I chose for
this study. Although unexpected, I decided that this relationship between Sally and Cara
would not change procedures for data collection and most of my data analysis, but that it
would change how I framed and discussed my analysis.

To answer the central research question of what happened to the instructors’
beliefs about teaching and learning mathematics, I now had to consider what transpired
between Sally and Cara in the previous semester. Without this new consideration, I
would be leaving out what both participants described as a semester of significant
change. Answering the research question of what happened to Sally’s and Cara’s beliefs
must then include a lens in which I determined what they said happened in the previous
semester and determined how those parts of their past played a role in their relationship
this semester. Therefore, this analysis is written in a way that uses insight about their
relationship to explain why Sally and Cara started the semester with similar beliefs and
why their beliefs changed in the ways they did.

Sally’s & Cara’s Beliefs at the Beginning of the Semester

Before being instructors in College Algebra, Sally and Cara both had informal
experiences in teaching mathematics. Cara was a “lab assistant” for Algebra 1 during
high school, while Sally tutored her undergraduate peers and was a recitation instructor in
her first year in graduate school. These experiences revealed each instructor had an
interest in not just being the learner in a mathematics classroom, but were interested in
being a teacher as well, well before this semester of College Algebra. Although these
experiences allowed Sally and Cara to catch glimpses of teaching mathematics, they were
still not formal experiences in teaching where each person had autonomy in making moment-to-moment teaching decisions for a whole class of students.

**Sally’s beliefs at the beginning of the semester.** Analysis of Sally’s pre-survey answers and first interview revealed that Sally had a central set of beliefs about teaching and learning mathematics which included the teacher as someone who gave students opportunities to think about the content first and then continuing to focus on their thinking when helping them directly. This vision of teaching and learning mathematics reinforces the idea that Sally believed knowledge is constructed, therefore teaching and learning need opportunities for this construction. Figure 16 depicts Sally’s views of teaching and learning mathematics at the beginning of the semester.

![Figure 16. Sally’s beliefs at the beginning of the semester.](image)
Sally strongly believed that the teacher should be facilitating learning between students by positioning them where stronger students could be helping the weaker students, getting students to try problems on their own first before helping them, and then when helping a student directly, the teacher should make every effort to start from that student’s thinking. She recognized those students in her class who had “mental baggage” (Interview #1, 1/20/15) from their prior experiences, but then reasoned that because of these prior experiences these students had developed the habit of feeling comforted with doing mathematics a standard way. This was important to Sally, because while she acknowledged and accepted that there were reasons behind why students behaved in this repetitive, standard way, it was the teacher’s job, which was in this case her job, to pull students away from behaving in this standard comfortable way and think beyond the surface level of mathematics.

Understanding a bigger picture had been important to Sally in her own learning of mathematics, so it came as no surprise that she viewed seeing a bigger picture then moving down to the details as an important way to help students learn mathematics. She used this as a tool when helping students. She commented, “So my first approach is to guide them through some big picture “(Interview #1, 1/20/15). Using this big picture approach as a tool connected to Sally’s belief that concepts are more important “because if you can remember a simple rule, a simple idea and then recreate that as you do, that’s a lot more powerful than being able to recite the quadratic formula and it’s also more applicable to other areas” (Interview #1, 1/20/15). Memorization played little importance in Sally’s set of beliefs, both in the necessity for it and not wanting students to rely on it. To Sally, learning mathematics was an emergent process that began as problem-solving
by big concepts then discovering that the details fit with the larger picture, thereby negating any need for memorizing.

Sally responded to my predetermined questions without hesitation, implying an air of confidence, something I really did not expect to hear or feel from a new instructor. It made me begin considering whether she really did feel as confident and assured as she sounded. As the interview was drawing to an end, I asked Sally whether there were any beliefs she was waver ing about. Her initial response was:

That’s a good question. I guess, so one of the things that comes up frequently with other grad students is how much effort to put into students who are not making efforts themselves. About 2 years ago or so I very very strongly believed that I needed to do everything I could to bring them back. More recently I feel like I have been trying to cut my losses at a certain point. Partially so I stay sane. Partially because I care about teaching but that’s not my primary goal in life and also because there’s a lot of energy that you can pour into a student for very little reward. And while I feel it’s important that that happens for students eventually that somebody or something happens for them that they see that education [has a] value or like understand why they’re in college or something like that. I believe less strongly now that it’s my job to make sure that that happens for them. So yeah. I guess that’s one of things that I’m still trying to figure out how I fall on that. (Interview #1, 1/20/15)

Following this question of how much effort she should have put into teaching, Sally described struggling with “trying to make them[students] care and to actually work on the worksheet or to work with other table mates ...” (Interview #1, 1/20/15) because she
wanted to be approachable while also setting firm lines. While Sally said she believed not every student could be reached, these doubts suggested that she was still mulling this belief over. She believed there was a point for each student at which she, the teacher, could not induce change, but knowing where that point was for each student and whether that point really could or could not be moved were questions she still considered. It is as if she said she believed that not every student could be reached, but then at the same time wanted to believe something different.

**Cara’s beliefs at the beginning of the semester.** Analysis of Cara’s initial interview and pre-survey answers revealed that her set of beliefs about teaching and learning mathematics resembled Sally’s beliefs. Figure 17 displays Cara’s beliefs about teaching and learning mathematics at the beginning of the semester.
To Cara, teaching mathematics meant not telling the answer, but doing some explaining for aspects of learning mathematics such as for learning rules and definitions. This idea of not telling an answer is something Cara was trying to understand and apply even at the beginning of the semester. In the quotation that represents Cara idea of a teacher’s practice, she said that teachers should not directly point out steps but that they could be “adding a little more flavor” (Interview #1, 1/27/15) by doing a step and asking “Why do you think I did this?” (Interview #1, 1/27/15) or “What do you think would be the next step?” (Interview #1, 1/27/15) This idea of not giving students answers directly was strong for Cara and “adding a little more flavor “(Interview #1, 1/27/15) or asking the questions just mentioned are her ways of trying to teach in ways that do not give students direct answers.

Cara had ideas that learning mathematics involved little memorization and placed more emphasis on understanding concepts rather than procedures. In this initial interview, she did not go further into detail about why she believed these particular ideas, but she was convinced that they were worth believing. She said at one point during the interview, “We[the instructors] can’t just tell them to memorize things and they can’t just memorize things and expect to get away with it (Interview #1, 1/27/15).” Yet she also indicated that learning mathematics involves knowing basic mastery skills, like addition, subtraction, and division to understand how those essentially work... to be able to do more things. It’s hard to build on
something if you just are doing... then you can’t build all the way up. (Interview #1, 1/27/15)

It is not clear whether in this statement Cara believed the student must memorize these “basic mastery skills,” (Interview #1, 1/27/15) but she indicated that they are important in learning mathematics.

When Cara shared her views about students, she shared them in a way that revealed she was trying to understand them. For example, Cara believed students learn in different ways, but they decide early in math whether they like math or not, meaning they can not always be motivated. Cara’s belief that some students’ outlook cannot be changed indicated a more fixed mindset about students at this point in the semester. Yet, she mentioned noticing students who were willing to try different things and that some of them connected with her during office hours last semester

I mean with students that end up coming once or twice to my office hours and then they ended up seeing something beneficial out of it, they came back again. I could tell they were starting to enjoy the class more because they understood what was happening. (Interview #1, 1/27/15)

In this example, Cara talked about students in ways that move away from them having a fixed mindset.

**Comparing Sally’s and Cara’s beliefs at the beginning of the semester.** Sally and Cara shared many of the same beliefs: teachers should not directly tell answers, there is only so much that teachers can do to reach students, there is little need for memorizing in learning mathematics, and understanding concepts is more important than knowing procedures. However, just because they shared many beliefs does not mean they shared
those beliefs in the same ways. For Sally, many of these beliefs were central and strong. She reiterated these beliefs in different ways. For example, she spoke about teachers not telling answers and gave many ways that she tried to avoid doing this, such as using analogies, metaphors, or drawing pictures. In comparison, Cara declared these same beliefs, but then made statements indicating she might think about a belief in a different way than she previously stated. Much like her statement about students needing to know basic mastery skills, it is not clear whether Cara really believed students should not memorize. For Cara, these common beliefs were issues about teaching and learning she wanted to believe in, but she was still learning how to believe in them. Sally believed strongly in most of these issues, but might herself still be learning how to fully believe the idea that there is only so much teachers can do to reach students. Sally and Cara were similar in respect to what they said they believed, but they were different in how and how much they believed in their beliefs.

Ways their prior relationship impacted their beliefs. One significant reason why Sally and Cara had similar beliefs at this point in the semester was because of their previous relationship teaching College Algebra together the semester before. It was not until after seeing Cara in the classroom for my first visit that I realized this relationship, which means I did not explicitly ask Sally about it in our first interview. Except for the first interview with Sally, both Sally and Cara talked about their relationship throughout the study. When they mentioned their prior relationship, it was either about what had happened in the past or what was currently happening. It is from these excerpts that one can see important facets of this relationship and how it impacted their beliefs at the
beginning of the semester. Two outcomes of this relationship and the significance they hold on the relationship between Sally and Car are discussed below in further detail.

_Sally and Cara began meeting outside of class to discuss teaching and learning._

Cara first mentioned that she started meeting with Sally for about an hour once a week every week, which at first focused on ways to teach and support learning. But at some point, these meetings for this semester shifted towards discussing logistics of the class. According to Sally, these meetings were initiated because she wanted to help Cara move away from giving answers and “we had several quite long conversations about what it means to teach and what’s the most effective way to teach and what I want to happen in the classroom” (Interview #2, 23/15). Cara referred to this and said:

This is something especially Sally taught me which was really cool. She was saying that you don’t just necessarily launch in to the step-by-step so instead we’ll question through it like, ‘What did you do?’ or ‘Where do you go wrong?’ instead of me saying ‘Stop, this is how you do it.’ (Interview # 1, 1/27/15)

Sally admitted that now Cara was much better with guiding students, but that this was a big stressor for her last semester, at times more stressful than helping the students. By initiating these meetings with Cara and comparing Cara’s path of teaching to her own, Sally showed that she believed Cara could grow as a teacher and was someone worth investing her time in helping. Even though these meetings might have been stressful for both instructors at first, both admitted they were significant times for getting on the same page, which in essence meant Cara started adapting more of Sally’s beliefs about teaching and learning.
Sally and Cara observe each other during teaching and learning. Because of their relationships and regular meetings, Sally and Cara continued to be aware of one another during class sessions. For Cara, this meant that she was always keeping a keen eye on observing and thinking about how and why Sally was teaching. For example, she noted the differences in how Sally discussed The Bridge Problem this semester compared with how Sally discussed it last semester. In her comparison, Cara noted:

"So I don’t know, if she[Sally], at first just got ahead of herself to make them understand what was happening. But, or she felt that maybe they didn’t, or they weren’t thinking that hard so she had to give them a little bit more of a nudge." (Interview #2, 2/20/15)

In another observation of Sally during the Composition of Functions with Pizza discussion, Cara said, “And so I think that middle part is the part that really started chipping them [students] because then she [Sally] referenced, she was trying to be like ‘Hey, you want to go from area to diameter from there’” (Interview #2, 2/20/15). This observation indicates that Cara was not a passive observer of Sally as a teacher, but often tried to actively make sense of moves or shifts Sally was making.

With less intensity and a different lens than Cara, Sally admitted to observing Cara during class. Noticing Cara meant Sally was thinking about how Cara was responding to students, including what Cara might be thinking about while she was responding, and whether or how Sally might help Cara analyze instances of teaching and learning. When watching the clip Cara Helping a Group of Students, Sally noted that Cara did a good job making connections to previous problems from that day’s class session, but pointed out that Cara was mainly talking to only one of the students in the
group. This is a place where Sally wanted to help Cara think about responding to the whole group of students in hopes of getting them to interact with one another, rather than only looking to the instructor for answers. One of Sally’s overall observations was that Cara tried to make students think about problems, but “she focuses on how students are responding to it[problems] and whether that moment was effective or not” (Interview #3, 4/3/15), often leading to a pessimistic perspective.

In summation, the evidence accumulated shows that the prior relationship between Sally and Cara impacted each other in how they interacted with one another throughout the semester. This continued interaction is one reason why they had shared beliefs about teaching and learning at the beginning of the semester.

**Sally’s and Cara’s beliefs at the end of the semester**

With the student, Brad, there was significant and radical change in his beliefs about teaching and learning mathematics. His beliefs were altered from one end of the spectrum to the other. Initially Brad wanted a teacher directly telling him how to do a problem and then repeatedly practicing these directions, but by the end of the semester he wanted to try the problems first on his own or with other students and use the teacher as a resource for guiding him when he was stuck. This type of conspicuous and dramatic change did not occur with Sally’s and Cara’s beliefs this semester. Considering that both Sally and Cara talked about how different Cara was in the previous semester, there may have been this kind of change with Cara’s beliefs, but that is beyond what this dissertation could capture. But Sally and Cara did not end the semester having drastically different beliefs than they had at the beginning. Instead, their beliefs were changed in subtle ways that enhanced their original views of “good” mathematics teaching and
learning. The rest of this section is dedicated to examining the ways and reasons Sally’s and Cara’s beliefs subtly changed, respectively.

**Sally’s beliefs at the end of the semester.** Sally was a thoughtful, caring, and knowledgeable instructor at the beginning as well as at the end of the semester. Her core beliefs of seeing the teacher as a guide who focuses on students’ mathematical thinking, needing little memorization in learning mathematics, and characterizing College Algebra students as having “mental baggage” (Interview #1, 1/20/15) did not change. Remembering that Sally described herself struggling with determining how much to care about teaching and how to help students care about learning mathematics, I realized that while Sally’s core beliefs in isolation might not have changed, she spent the whole semester trying to figure out how these beliefs fit together. By trying to understand how Sally was trying to mesh these beliefs together, there were the ways that Sally started thinking a little bit differently about students and differently about the role of the teacher.

**Sally was tired.** One of the first things Sally mentioned to me in the final interview was how “fatigued” (Interview #4, 5/7/15) she felt. In response to my first question concerning her overall impressions of the semester, this conversation occurred:

Sally: I think it was overall pretty good. I wasn’t quite as happy with this semester as I was with my previous semester and I think that’s partially because I had a larger class size. So I had a harder time feeling like I was personally getting through to each student. But I still knew each student really well by the end of the semester. And I think we were all tired. Including me and the students. Didn’t really want to be there towards the end. But I’m happy with what they learned, I think that they did well as far as learning
the course material. It does feel like things started slipping a little bit in the sense of I didn’t really fight my students quite as much as I ideally would have.

Interviewer: You didn’t fight them?

Sally: When they tried to do things rhythmically and just started trying to memorize things. I didn’t fight that as much as I would have if I was more... like... rested I guess? I don’t know, I feel like I literally feel like I’m going into this battle with them with all of their preconceived notions of math. And like in the fall semester I feel like I did a better job keeping that up rather than this semester I would occasionally just let the students be more comfortable and just do things the way they wanted to do.

(Interview #4, 5/7/15)

In Sally’s eyes, the main repercussion of her being so tired was that she did not fight her students as she knew she could and did in the past, meaning she did not challenge them. She still had a positive outlook on what the students learned, but it was obvious this bothered her. Alongside the fight Sally had with the students, she was also fighting the question: How do I guide students with “mental baggage” (Interview #1, 1/20/15) who keep doing repetitive actions they have memorized from their previous experiences? It was her core beliefs that were in conflict with one another and it was this conflict that kept Sally struggling, making it a tiring experience to teach this semester, a tiredness that had likely been building since last semester.
**Sally started to see things a little bit differently.** The tension between Sally’s beliefs could be considered as a source for possible change. Figure 18 reveals a representation of her beliefs at the end of the semester.

![Diagram](image)

**Figure 18.** Sally’s beliefs at the end of the semester.

In comparison to her beliefs at the beginning of the semester, this representation shows Sally is beginning to see learning mathematics differently. Now she feels convinced that it is beneficial to practice problems and sometimes work from the details up to a bigger picture. This belief is somewhat different than her stance at the beginning of the semester, in which she was more convinced, partly because it was her personal preference, that starting with a bigger picture was most helpful. Also, her idea of working...
through problems at the beginning of the semester did not necessarily include practicing as part of working. While these ideas were different, they did not change Sally’s belief that learning mathematics is not a mechanized routine students perform through memorization. Instead, these elusive changes suggest Sally is opening her eyes to different ways students can learn that do not rely solely on memorization.

The quotation which represents Sally’s view of the teacher’s practice shows that she still strongly believed in the teacher being a guide, or a “master sensei” (Interview #4, 5/7/15) as she puts it now, and doing everything possible to support or create a “softer landing” (Interview #4, 5/7/15) for students when someone challenges them. What is different about this is that while her idea about the practice is not changing, Sally has started to view students and the role of the teacher in slightly different ways. Part of the “mental baggage” (Interview #1, 1/20/15) Sally saw now is the students’ inability to do basic skills, and their unrealistic expectations of the work associated in a math class. Thus, Sally began to understand and see more about what students’ “mental baggage” (Interview #1, 1/20/15) includes. In response to seeing more of their “mental baggage” (Interview #1, 1/20/15), Sally admitted to emphasizing steps more when working with students who were really struggling. While she felt disappointed to admit this change in practice, she rationalized that it was something that was helpful for some students. Even if it was not the way she initially believed she should help students, it was how Sally was adapting to students and their thinking. In her continued attempts to rationalize this change in practice, Sally said she did not do this all the time and always tried to connect the steps back to a larger picture to ensure that just emphasizing the steps was not the only way she helped students.
It is not clear whether Sally was aware of these subtle differences in her thinking about how students learn and the teacher’s practice, but she did have an epiphany in our last interview. In the beginning of the semester, Sally mentioned that this class was unusually quiet compared to last semester and this level of quietness added to her struggle of making students care and work with other students. When I asked her about this in the final interview she said this worry was much better this semester than she thought it would be and admitted to being surprised by this. Pushing further on this idea of her initial worry that this semester the students were very quiet, the following conversation transpired:

Sally: And, yeah, students were just much more willing to actually engage and work with each other. Or like...

Interviewer: Mmhm.

Sally: Think of it as teams rather than an individual effort.

Interviewer: Mmhm.

Sally: Which and again this is reminding me of this time social constructivism (laughs) that it really felt more this semester like they bonded with each other and thought of me as the outsider. Whereas in the fall semester, many more of them had more of a personal relationship with me like relationships ran through me. Instead of like just working with each other.

Interviewer: That’s an incredibly interesting and profound way to put, to think about this, right? ‘Cause earlier you were saying that you felt like you couldn’t personally interact with students.

Sally: Mmhm
Interviewer: Right? But now that you’re thinking about this and that it

Sally: Maybe it wasn’t a good thing (laughs)

Interviewer: Maybe it could be, cause when you’re talking about it before, it did

   seem like you thought that was a bad thing, right? That you couldn’t
   personally reach every student in the class. Cause it was so much bigger.
   But now you’ve got to a point where you’re acknowledging that maybe
   you being the outsider was helpful for them to engage with their table
   mates and care about working with other people. Other students.

Sally: So maybe... I do believe that, actually.

Interviewer: Maybe it’s not a this or that, like it has to be one or the other, it’s not,

   yeah... I don’t know.

Sally: I think it may have been better for them to work with each other. But it also

   makes sense that I was uncomfortable with it because I mean, I was
   basically setting myself up as an outsider and that’s just not as nice.

   (laughs)

Interviewer: No one wants to be the outsider, right?

Sally: But I am willing to fill that role if that’s what’s going to be different for

   them. (Interview #4, 5/7/15)

In this epiphany, Sally realized that her belief of personally reaching every student did
not mean she had to interact with them all of the time. Rather, Sally might be an
“outsider” (Interview #4, 5/7/15), someone who is carefully watching over students
working with other students, and who could enact this belief of reaching students. She
already believed students should be working with each other, but now she was seeing
other ways she could support this interaction.

Sally was the same devoted and caring instructor at the end of the semester as she
had been at the beginning. She was energetic and resourceful and in her openness, she
experienced a new dimension of her classroom teaching, one in which her role might not
always be central to the learning experience but would at the same time be no less
integral to the process.

**Cara’s beliefs at the end of the semester.** Like Sally, Cara started and ended the
semester with her beliefs intact. She was a bubbly, caring instructor who believed in
teaching students in ways that involved guiding them to an answer as opposed to telling
them directly. Her beliefs at the end were not drastically different; she was still struggling
to figure out how to enact these beliefs in her practice. Further examination of these two
aspects of Cara’s beliefs and practices reveal the subtle changes that took place.

**Cara learning why she believes in what she believes.** As mentioned earlier,
Cara’s beliefs at the beginning of the semester matched several of Sally’s progressive
ideas, but Cara did not offer reasons for why she believed them. For example, in the first
interview Cara said memorization was not important, but did not explain why she thought
this. She even made a statement about students needing to know basic skills that could be
interpreted as needing to be memorized. By the end of the semester though, Cara said
much more about her beliefs and gave detailed reasons for why she believed certain
things. Figure 19 summarizes Cara’s beliefs at the end of the semester.
Figure 19. Cara’s beliefs at the end of the semester.

For the example just given about memorization, Cara was able to explain that when students memorized, they were often not thinking about the problem deeper and thus gained little understanding. “Memorization I feel like to me implies that you don’t understand the meaning of it. You just do it because you were told to do it that way” (Interview #4, 5/7/15). In Cara’s view at the end of the semester, she saw that understanding is something more permanent for students and when students just memorize they are more likely to “brain dump” (Interview #4, 5/7/15), which meant they just got rid of that information from their brains. These ideas were much more formed than at the beginning of the semester when Cara only said memorization was not
important. This more-formed idea helped Cara see how and why students should learn mathematics more deeply, which was another belief she shared at the beginning of the semester.

Cara described students at the beginning of the semester as being not always able to be motivated, and that students’ outlook either could or could not be changed. This meant Cara believed there were certain limits in working with students. By the end of the semester, Cara provided more insight into these limits. She believed that students have the ability to learn math, but she was learning that students are constantly second-guessing themselves and would often “break math” (Interview #4, 5/7/15) meaning they would do something that was not mathematically sound. Even though Cara’s thinking might not be at the point that she could explain why students “break math” (Interview #4, 5/7/15) or how they were able to learn, these new observations helped her reason why students “break math” (Interview #4, 5/7/15). These new observations helped her map out a deeper understanding of student behavior thus helping her better define the limits in working them. It was in these more descriptive observations of students that Cara started to reason why she might not able to change a student’s outlook or why that student might lack motivation.

Not telling students answers was a very big idea to Cara at the beginning of the semester, but she did not go beyond this idea to describe how or why this was helpful in teaching mathematics. In this final interview, Cara described watching students use different ways to solve problems than she had previously learned, “Okay and it’s just, I don’t know, I kind of get the idea that it kind of opened me more up to think about the
fact that people teach math in very different ways...” (Interview #4, 5/7/15). Just after she made this statement, she went on to say:

Cara: But it was just in a different way. It was just very interesting and it becomes very apparent when you are on the other side and you are watching somebody do it a certain way. It is very interesting. It is almost like I have to hold my tongue when I am watching them[students] do it because like if you get the right answer great. I just I wouldn’t of done it that way but you can. It’s just more recognizing other people’s ways of getting to things. Especially.

Interviewer: Okay.

Cara: And how they get to the right answer. I don’t know if it is really, I just I might have seen math in a different way. It makes me realize what the difference is between how people are taught how to do a certain simple subject. (Interview #4, 5/7/15)

Even though Cara was fixated on students ending up with a correct answer, she began to see why she needs to “hold her tongue” (Interview #4, 5/7/15). Cara was starting to see how students learn differently by seeing them “do it a certain way” (Interview #4, 5/7/15), a way that was different than she had been taught. Not only was Cara starting to see and appreciate that there are different ways to solve problems, but this helped her see why teachers should not directly show students how to solve problems.

Cara is still trying to match her practices with her beliefs. Cara ended the semester still caring deeply about teaching, but analysis of interviews across the semester and observations of her practice as a teacher revealed that Cara was still learning about
her beliefs and how to teach in ways that reflect these beliefs. Although Cara understood more about why she believed teachers should not give answers directly, she struggled to do this in her own practice as a teacher. Near the beginning of the semester, I observed Cara interacting with students in ways where she was directly giving them an answer or pointing them to the answer. For example, one day I observed this interaction:

Student: Would this be right? Or is it completely wrong? [points to work on worksheet]

Cara: It’s not completely wrong. You are missing a few things.

[pause]

Cara: You are missing intervals. (Observation, 2/3/15)

In another interaction with a different student, Cara said, “We are here [points to step] and we want to go here [points to next step]” (Observation, 2/12/15). Both interactions revealed an inconsistency with what Cara said she believed teachers should be doing and what she actually was doing as a teacher. In the second example, Cara was not giving a direct answer but her remark is so pointed that it might as well be giving the student an answer. In the first example, Cara just blatantly told the student what was missing in her answer.

These kinds of interactions still occurred closer to the end of the semester, but there were also moments when Cara did less telling when helping students. For example, during a lesson on transformations of functions Cara had this interaction with a student:

Cara: What’s happening on the inside?

Student answers.

Cara: Ok. What does that look like?
Even though Cara answered her question by pointing to the board, she was using open-ended questions to guide the students thinking. Clearly she was trying to do less telling when she asked more of these open-ended questions. Her questioning demonstrates there is room for her to grow in learning how to guide students without telling answers directly, but Cara’s interactions with students show that she is learning more about the inherent difficulty of supporting students.

In addition to her teaching practice, Cara continued to sort out details related to her espoused beliefs. For her belief that it is okay for teachers to tell students definitions and rules, Cara now viewed these ideas as tools students need in their toolbox in order to be able to “build on top of that [basic skills]” (Interview #4, 5/7/15). She rationalized that it was okay for teachers to tell these things directly to students because without them, students would “fall down” (Interview #4, 5/7/15) and not “understand what happened” (Interview #4, 5/7/15). This is the point in which Cara began to sort out her beliefs about students needing basic skills, teachers telling definitions and rules, and what was involved in learning mathematics. Now Cara understood the relationship between these beliefs as conditional. If students had basic skills and they were given the tools, then they could learn mathematics. And, if this conditional statement held true, then Cara believed this would help students avoid the need to memorize. If either of the first two pieces in the conditional statement were not evident, then the determination of learning would become less certain. When students did not have basic skills or they did not use tools, it became harder for Cara to support students’ learning.
Cara remained an enthusiastic instructor committed to learning more about how to teach. Part of this process of learning involved Cara trying to match her practices as a teacher with her beliefs. Another part of Cara’s learning included building a more thorough and durable understanding of her own beliefs, particularly about why she believed in particular ideas.

**Lessons learned from comparing Sally’s and Cara’s beliefs**

To outsiders, Sally and Cara were the same kind of instructors at the end of the semester as they were at the beginning, but by closer examination they were different and changed. Both perspectives offer different comparisons and consequential lessons learned about what happens to instructors’ beliefs when teaching a reformed College Algebra course.

**Changes in instructors’ beliefs may mean there are subtle differences in how or what someone believes, but their core beliefs stay the same.** Both Sally and Cara ended the semester with a more descriptive picture of what and why they held certain beliefs about teaching and learning mathematics. Their picture or the outline of their beliefs did not change dramatically, rather it became more detailed and influenced by experience for each instructor. Rather than just stating that she thought teachers should avoid giving students direct answers, Cara could now articulate why teachers should incorporate this idea into her practice wholeheartedly. Cara learned that students were learning differently and doing mathematics in ways that she had not thought of because she held her tongue. Sally ended the semester with a clearer understanding about her role as a teacher in helping students work with each other. She realized that often she might not need to be the personal agent of students’ learning, but that her role could include
stepping back and letting students learn from each other. Both instructors held core beliefs that resemble the characteristics of IBL classrooms (Laursen et al., 2014) and these subtle changes of their beliefs reflect their learning about how to enact these characteristics.

**Placing support instructors who are intended to help students learn in a classroom may create an additional task for lead instructors. Lead instructors may adopt an additional belief of needing to mentor support instructors.** Support instructors, like LAs, are intended to help instructors especially with large classes (Otero et al., 2006). As the LA, Cara was intended to be a support for Sally both in helping students learn and helping Sally teach. Both Cara and Sally admitted that in the semester previous to this study there were significant conversations between them outside of class about teaching and learning. And according to both of them, these conversations helped Cara start to change the way she was interacting with students. These conversations were driven by Sally’s deeply held beliefs of a teacher being a guide to students actively engaging in problems. It appeared too that Sally had been quite perturbed by Cara’s initial interactions with students, and this was part of those conversations. To Sally, these interactions were not supporting her beliefs about teaching and learning mathematics so Sally felt like she needed share these beliefs with Cara. Thus, a significant and additional kind of relationship was already formed between Sally and Cara, which carried into this semester. From this insight, it is clear that putting a support instructor in a classroom may cause an additional task for the main instructor. Just by the very nature of the support instructor being in the room, the main instructor will have to keep track of and monitor what the support instructor is doing in the classroom. But as was seen with Sally and
Cara, the main instructor may take it upon himself or herself to be a mentor for the support instructor, especially if it is believed this mentorship will help the students learn.

**Novice instructors can hold strong, progressive beliefs about teaching and learning mathematics. Inconsistencies between novice instructors’ beliefs and practices can be viewed as their learning how to teach mathematics.** As was mentioned earlier in this chapter, it was not anticipated for Sally to have such strong, progressive beliefs about teaching and learning mathematics. I had anticipated a novice instructor who was struggling with thinking about what to believe and Cara more resembled this kind of instructor. Yet because of the previous semester and her relationship with Sally, Cara seemed somewhat clear and confident in what she believed in. Further analysis of Cara’s contradictory statements and inconsistencies between her espoused beliefs and practice as an instructor revealed Cara was not as initially clear and confident as she believed at the time. Cara wanted to believe in these ideas about teaching and learning mathematics and her struggles indicate that she was in the midst of learning why and how to believe in them. Research suggests novice mathematics teachers’ beliefs and practices may be inconsistent (Leatham, 2006) and their beliefs may align with more traditional views (Ball & Wilson, 1990). As this study shows, novice instructors can hold progressive beliefs and have a practice that often matches with their beliefs. And novice instructors can have major inconsistencies between their beliefs and practices and part of their learning to teach involves them trying to match these inconsistencies (Leatham, 2006).
Chapter 6
Contextualizing Beliefs in Practice

The previous two chapters gave individual descriptions of the instructors’ and the students’ beliefs. While these descriptions give insight to these participants as individuals, they do not describe what it means for Sally, Cara, Brad, and Michael to be interacting together in one class. Every mathematics class is filled with moments between teachers and students whose actions are often driven by their beliefs. These moments might be actual moments when a teacher is directly speaking with students or they could be moments when students are watching the teacher write a problem on the board. Whether teachers and students are directly or indirectly involved in these moments, they are often developing and reexamining their beliefs as a result of these moments. Consider the vignette below describing one class session of the College Algebra course studied:

A Typical Day in College Algebra (February 3rd, 2015)

It is 2:02 and Sally is standing at the front of the classroom going through her usual routine for starting class: taking attendance and asking an ‘off the wall’ question to start conversation. Today’s question was, ‘waffles or pancakes?’ Side conversations started off quiet and infrequent, but by the end of Sally taking attendance there were several students chiming in with strong opinions about which one they preferred or thought was superior. Pancakes soaked up more butter, but waffles had the natural ability to pool syrup in little pockets. After a vote was taken, Sally declared waffles the winner then smoothly transitioned everyone’s attention to today’s topic, composition and inverses of functions.
Before talking about a specific example, Sally went through a two-minute “lecture” in which she described a big picture view of what composition of functions meant and looked like.

After this big picture view, Sally launched into an example about a pizza parlor needing to decide how much to charge customers with the consideration of how much it cost them to make the pizza. She used questioning to engage the students, but as her questions progressed she was met with longer and longer periods of silence from the students. Sally was trying to guide them through the idea of starting with the cost in terms of area it takes to make a pizza, then use the area of a pizza to elaborate. There was an equation on the board that represented the cost of making a pizza: \( C = 0.5A \) when \( A \) is the area of a pizza and \( C \) is the cost. She reiterated that this equation meant it cost 50 cents per square inch of pizza, so if it was a 10 square inch pizza it would cost 5 dollars to make. Sally then asked the class what was the area of the pizza and no one responded. After a long period of silence, Sally asked, “Ok. Are people confused? What’s going on here? Where is it confusing?” One student responded to her question with a question...
about what was trying to be calculated in the example. Cara jumped in saying that pizzas are round and that we think of slices of pizza. The student nodded in response to Cara’s comment. Sally continued on with the example and when it was finished, she directed groups to start working on the accompanied worksheet.

The rest of the class session involved students working on the worksheets in their small groups. Some of the groups worked together on the same question in some methodical fashion. Some of the groups worked together by checking answers and then talking if there was a discrepancy in answers. In one group of four, they worked together in pairs first then collaborated when checking answers. No one worked solely by himself or herself. Sally and Cara circled around responding to students or groups who had their hands up. If no hands were up, Sally and Cara sauntered around the room eavesdropping on students’ conversation and would sometimes chime in with a question or a comment.

This vignette reflects a typical day in Sally’s College Algebra course. Most of the time was devoted to students working in groups on problems from the worksheet. About 15 minutes of the time was spent on Sally leading the whole class through a problem. There were moments of confusion, both by students and the instructor. There were moments for being “off topic,” led both by students and the instructor. All in all, it appears to be typical, simple day of classroom interactions between students and instructors. But any typical, simple day becomes less simple and more complicated when one thinks about what is driving or influencing the interactions that occur. For example, why did Cara decide to interject in the whole group conversation when she did? How did she decide what to say? What was Sally’s perspective of why Cara interjected? What
were the other students’ perspectives of why Cara interjected? Cara made the decision to interject and make a statement, and what makes this complicated is that she did not do this in a vacuum. She did this at a specific point in the conversation in front of students and Sally, who then make explicit or implicit inferences about this interaction.

The purpose of this chapter is to revisit these moments within the classroom and analyze how the four main participants were reflecting on them. Combining how these four participants were reflecting about the same moments with what was learned about their beliefs individually in Chapters 4 and 5 sheds light on what might be learned about these moments. This combination also allows the individuals beliefs of the participants to be contextualized in practice.

**Five categories of connections**

**Recalling Phase 3 of the data analysis.** Recall that Phase 3 of the data analysis focused on answering the research question posed about how the participants’ beliefs interact. Interaction of beliefs was then defined as how the participants’ beliefs relate to one another by looking at how their reflections relate to one another. Thus, Phase 3 focused on defining how participants’ reflections about common moments, that occurred in their College Algebra course, aligned or did not align with one another. I first identified these common moments using three of the six video clips. Table 11 from Chapter 3 (p. 93) displays the video clips used and the common moments of interaction identified within them.

Once these instances were identified, I constructed a document for every instance where each participant’s answer was copied and pasted, so that I could analyze the answers side-by-side. After summarizing each participant’s individual answer, then
making an overall observations how the participants’ answer related one another on the question in focus, I found 11 total observations of how the responses connected for each question. I then made observations among these 11 total observations to identify common themes about how these responses aligned or did not align, which led me to five different categories of connections:

- No Connection
- Students Connected, Instructors Connected
- Instructors Connected, Students Different
- Instructors and One Student Connected, One Student Different
- Connection Across Students and Instructors

**Looking at the five categories more closely.** These five categories suggest there are a variety of ways the instructors and students were thinking alike and different. Taking a deeper look at each category is necessary to understand what it looked like when instructors and students were thinking similarly and differently.

**No connection.** When an instance was labeled as ‘No Connection,’ it meant that the answers given by each participant were fundamentally different. In the video clip, The Magnitude Discussion, the student presenting had a very difficult time enacting simple procedures to solve for the ‘\(W_0\)’ when asked by Sally. When I asked the participants why they thought the student got so hung up solving for this variable, they all responded with different reasons. Sally said she thought, “it stems from sloppy notation, that students know that they have to do the same thing to both side of the equation, but then they actually don’t. They write pieces of the step, but not the whole step...” Cara thought the student did not know what she was looking for, then became stressed out.
Brad thought the student was “just getting frustrated and didn’t even want to finish it or do it anymore because I don’t know, she’s maybe getting a little embarrassed in front of her classmate and stuff like that.” Michael thought that maybe it was because she was supposed to be solving for the variable with the naught and that it looks different than what they usually solve. Additionally, Michael thought maybe this was a “too advanced for her” and “she’s not ready for it.”

All four of these answers are fundamentally different in that they each provide a certain reason for why the student struggled, and these reasons do not overlap. Sally’s and Michael’s reason involved notation, but Sally’s reason was about the student’s lack of using good notation in her answer, while Michael’s was about the notation of the problem. Cara’s reason involved the student getting stressed out because she did not know what she was looking for. While Brad’s reason involved the student’s emotions, it was about the student being frustrated enough to not want to finish presenting the solution. Cara thought the student did not know what she was looking for; Brad thought the student did know but was too frustrated to finish. Thinking about how participants’ reasons were different forces one to establish a core reason for why the student struggled for each participant:

- Sally: Student using poor, sloppy notation
- Cara: Student did not know what she was looking for
- Brad: Student was frustrated enough to not want to finish the problem
- Michael: It was a weird variable to solve for and student might not be advanced enough for this class.

When the core reason each participant provides is established, it becomes much clearer that their responses have no fundamental connection. The way they were describing why they thought the student struggled all relied on different explanations. Brad thought the
student had the ability to solve the problem whereas Michael doubted the student’s ability. Sally’s reason was rooted in the importance of being precise when writing your thinking about a mathematical problem. And Cara’s reason highlighted the need to have a more conceptual understanding of what a problem is asking for.

In the video clip, Sharing Mid-Semester Feedback, Sally shared several pieces of feedback she received from the students in the week before she gave them a mid-semester evaluation. When I asked each participant if he or she was surprised by any of the feedback, each participant responded differently. Sally said she was surprised to hear that most of the students said the pace of the class was “just about right” (Interview #2, 2/23/15). She said, “it was nice to know that at the very least, the majority of my class doesn’t think that we’re moving too slow” (Interview #2, 2/23/15). Cara responded to the question with “I mean, I wouldn’t say it was like ‘Oh my gosh, that’s so surprising,’ but it was kind of like, ‘Oh, that’s kind of fun.’” (Interview #2, 2/23/15). Brad was surprised “that most people were learning from doing group work and the worksheets and Web Work, which I was like ‘What the heck? I learned more from you doing an explanation on the board’” (Interview #2, 2/24/15). Michael was surprised that students wanted more examples explained on the board, which surprised him because “it takes away from, I feel like learning time, ‘cause I learn better just like doing it” (Interview #2, 2/27/15).

Just like the example before, these are four fundamentally different answers. Sally was surprised to hear students believed the pace of the course was “just about right” (Interview #1, 1/20/15) while Cara was not surprised by anything. Brad was surprised to hear people say they are learning the most doing problems because he wanted more examples. Just opposite of Brad, Michael was surprised people wanted to hear more
examples because he said he learns the most by doing the problems. While Brad and Michael’s answers were motivated by how they both thought they were learning, they were surprised by different messages from the feedback.

**Students connected, instructors connected.** For this category the students’ responses had a connection, whereas the instructors’ responses connected to each other, but were different from the students. This type of connection occurred only once in the Sharing Mid-Semester Feedback reflections. When I asked each participant what Sally was thinking when she was giving the mid-semester feedback, Michael and Brad both said they thought Sally was happy because the feedback was positive. Michael added to this idea by saying he thought Sally was giving the feedback in order to “make the relationship between instructor and students closer. So people feel comfortable talking to her” (Interview #3, 4/7/15). Even though Michael added an extension to his reasoning, both Brad and Michael’s responses connected because they both thought Sally was happy.

Cara thought Sally gave the feedback to help students realize why they are being taught differently because “they [students] still don’t realize the difference” (Interview #3, 4/16/15). Sally said she felt nervous standing in front of the class talking for that long but that she thought this was a way to help the students “keep thinking about it [how they learn]” (Interview #3, 4/3/15). Sally admitted that she spent a lot of time “thinking about math, and math education, and how students think, and how I can explain things to students, like I spent a lot of my time thinking about that. I know that they don’t” (Interview #3, 4/3/15). So to Sally, this was a way to help the students think more about teaching and learning mathematics. Both Cara’s and Sally’s responses connect in that
they share the idea of Sally wanting to get students thinking more about the operation or method of teaching. Even though Cara’s response indicated she thought Sally wanted students to see a specific difference and Sally’s response was more open, the main idea behind both responses is wanting students to think more about teaching itself.

**Instructors connected, students different.** If an instance was categorized as ‘Instructors Connected, Students Different’ then it meant that the instructors’ responses had overlapping parts, while the student’s responses were different from the instructors and each other.

In the video clip, The Magnitude Discussion, I asked the four participants why they thought Sally asked the student who was presenting to say more about her solution. Sally responded,

> So, partially because I really detest the idea that the only thing that’s important is the steps that you used to manipulate equations. What I wanted to emphasize before then is that you need to explain the whole picture, you need to explain why this is what you thought you should do and what the answer you got means.

(Interview #3, 4/3/15)

Like Sally’s answer, Cara said the prompt for more explanation was “Because it’s you [the student] actually explaining what your answer means. It’s not just getting an answer. But it’s actually interpreting what you just found” (Interview #3, 4/16/15). Cara’s response indicated that she thought Sally was trying to get the student to move beyond just replicating procedures, which is what Sally said she was trying to do. Both answers reflect an idea of helping the student think beyond the steps of a problem.
Brad thought Sally asked the student to say more about her answer “probably ‘cause it wasn’t the final answer yet. But she [Sally] wanted her to keep going, I don’t think she [the student] know she need to keep going though, which was why that question was probably picked...” (Interview #3, 4/3/15). Michael thought Sally prompted the student “to see if she actually understood it at all. Because that first explanation was not good at all” (Interview #3, 4/7/15). Brad’s and Michael’s answers are very different from one another. Brad thought Sally asked the student to say more because the student had not given a final answer whereas Michael thought the reason was because the student did not understand the problem, and the initial explanation was bad. These two different answers also do not connect to Sally’s and Cara’s shared idea that asking the student say more was because the student needed to think more about the problem and examine her own thought process.

Instructors and One student connected, One student different. When an instance was placed in this category, it meant that the two instructors and one of the students shared a connection in their reflections while the remaining student’s response was different. An example of this type of connection occurred was when the four participants responded to why they thought Sally went on her “soap box” in the clip, The Bridge Problem. Sally said she

wanted to say something about why I gave them this problem to think about. But in particular, why I said I wanted to get on my soap box was because of how outraged they were at me that there wasn’t a solution. Um, so I wanted to emphasize immediately that that was not the point of the problem. (Interview #2, 2/23/15)
Cara thought Sally wanted students to look back and think about the problem in a different way. She said, “So it’s kind of like getting them a way to feel like let’s do a little kick start, math isn’t just one direction. There could be ten thousand directions that lead you in all the different ways to the same answer” (Interview #2, 2/24/15). Cara also said,

Because the actual answer is unsatisfying and that’s what the students realize is what they are like, ‘This is stupid’ but she’s[Sally] like, ‘No, think about it.’ Think about what the answers means though, the answer, the process of how to get there that that’s the important part. (Interview #2, 2/20/15)

Brad believed Sally went on her soap box because she knows “none of us like doing these worksheets all the time or something like that so she [Sally] wanted to reinforce why she does the worksheets with us. That’s why I think she said it” (Interview #2, 2/24/15).

Thus, Cara and Brad both thought Sally went on her soap box to help students think more about why they are learning math the way they are in this class. Sally confirmed this idea in her response. These commonalities in all three participant responses reveal their reflections were connected.

When I asked Michael why he thought Sally went on her soap box, he said he did not even remember that part of the clip so he was not sure why Sally did this. Michael did not remember this part of the clip nor did he remember any of it from the actual class period. This answer and lack of remembering indicated Michael’s reflection had no connection with Sally, Cara, and Brad’s responses.
**Connection across students and instructors.** As the title of this category suggests, instances within this category mean that all four participant reflections share a connection. When all four participants were asked why they thought Sally gave The Bridge Problem to the students, they all responded with likeness. Sally said she gave the problem to help students learn to think more about problems rather than just solving them. Cara also shared this response when she said, “It’s more of having them think about the ins and outs of the bridges, because she[Sally] discusses it in a way to that promotes thinking about it” (Interview #2, 2/20/15). Brad said Sally gave the problem to “make us think a little bit more” (Interview #2, 2/20/15). Michael replied, “Um, probably to critically think and try to like push ourselves like, cause usually when you do math you don’t go into the problem thinking it’s not gonna work” (Interview #2, 2/27/15). Thus, all four participants’ reflection relied on the idea that the problem was given in order to push students’ thinking. This means that Cara and Brad and Michael were in tune with Sally’s intentions behind giving this problem.

**Looking at connections by the video clip.** Even though 11 instances is not a large enough sample size to conclude anything statistically significant, a look at the 11 instances across the three video clips does provide some insight in the kinds of connections happened among the four participants. Table 12 reflects instances, what category they were assigned, and which video clip they stemmed from.

One initial observation that can be made from Table 12 is that the most common connection was ‘No Connection.’ This type of connection occurred five times, almost half the number of instances found in these video clips. Along with being the category with the most occurrences, it is also the only category that occurred in all three video
clips. These two attributes of ‘No Connection’ point towards the notion that a lack of connection occurs often and at different times within a classroom, meaning that the instructors and students all have different perspectives about what is going on. While it is beyond the scope of this dissertation to understand what the lack of connection leads to in each instance, it is worth acknowledging that these kinds of moments exist in the classroom.

Another observation that can be made is that ‘Connection Across Students and Instructors’ and ‘Instructors and One Student Connected, One Student Different’ tied for the second most frequent category of connection. Each of these categories had two instances and each category’s instances occurred in separate video clips. Considering these two categories reflect a stronger sense of connection among participants, and that together they occurred a total of four times, indicates that there were moments when all or almost all participants were thinking in similar ways with one another. Further, comparing this with
Table 12

*Connections within the Three Video Clips*

<table>
<thead>
<tr>
<th>Name of Video Clip</th>
<th>Instance of Interaction</th>
<th>Verbal Description of Connection</th>
<th>Category of Connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Magnitude Discussion</td>
<td>* Why did Sally ask the student to say more?</td>
<td>Sally &amp; Cara on same page, Brad &amp; Michael different also different than Sally &amp; Cara.</td>
<td>Instructors Connected, Students Different</td>
</tr>
<tr>
<td></td>
<td>* How do you think the student was feeling?</td>
<td>Sally, Cara, Brad on similar page, Michael different</td>
<td>Instructors and 1 Student Connected, 1 Student Different</td>
</tr>
<tr>
<td></td>
<td>* Why do you think Cara decided to interject?</td>
<td>All different</td>
<td>No Connection</td>
</tr>
<tr>
<td></td>
<td>* Why do you think the student got so hung up on solving for ( W_o )?</td>
<td>All different</td>
<td>No Connection</td>
</tr>
<tr>
<td>Sharing Mid-Semester Feedback</td>
<td>* What was Sally thinking when she was sharing this feedback?</td>
<td>Sally &amp; Cara on one page, Brad &amp; Michael similar to each other but not with Sally &amp; Cara</td>
<td>Students Connected, Instructors Connected</td>
</tr>
<tr>
<td></td>
<td>* Did any of this feedback surprise you?</td>
<td>All different</td>
<td>No Connection</td>
</tr>
<tr>
<td></td>
<td>* Why do you think there was that contradictory feedback?</td>
<td>Everyone on a similar page</td>
<td>Connection across Students and Instructors</td>
</tr>
<tr>
<td>The Bridge Problem</td>
<td>* What did you think about the dot representation Sally used when explaining the solution?</td>
<td>All different</td>
<td>No Connection</td>
</tr>
<tr>
<td></td>
<td>* Why do you think Sally gave this problem?</td>
<td>Everyone on a similar page</td>
<td>Connection across Students and Instructors</td>
</tr>
<tr>
<td></td>
<td>* Why do you think Sally went on her “soap box”?</td>
<td>Sally predicted Brad’s response, Cara and Sally on a similar page, Michael was different than everyone else</td>
<td>Instructors and 1 Student Connected, 1 Student Different</td>
</tr>
<tr>
<td></td>
<td>* What were you thinking while Sally was on her “soap box”?</td>
<td>All different</td>
<td>No Connection</td>
</tr>
</tbody>
</table>
‘No Connection’ occurring five times suggests that there are almost as many moments when participants are connecting than moments in which participants were not connecting. Again, even though it is beyond the scope of this study to track and understand what these connections arise from and lead to, it is comforting to know that there were almost as many moments when the instructors and students were thinking about aspects of the classroom in a similar manner compared with moments in which they all thought differently.

There was only one instance where the instructors thought about a question in a similar manner and the students had different interpretations from each other and from the instructors. Also, there was only one instance where the students had a connection in their reflections and the instructors connected, but there was not a connection between the students and instructors. What must be gleaned from this is that there are times where connections happen based on what role a participant plays in the classroom. Compared to other kinds of connections that can happen, the idea that connections stem from the role of the participant (i.e., an instructor’s reflection matches only with another instructor’s reflection) does not appear to hold for these participants. Sally, an instructor, did not seem to always have the same kind of reflections as Cara simply because Cara was another instructor. And Brad, a student, did not seem to always have the same kind of reflections as Michael simply because he was another student. Rather, the variety of connections made between instructors and students indicate that there was an openness and synergy between Sally, Cara, Brad, and Michael.
Using the Responses to Contextualize Participants’ Beliefs in Practice

The previous two chapters provided a deeper look into the beliefs held by Brad, Michael, Sally, and Cara. Brad went through a transformation from wanting a teacher to tell him how to do a problem to realizing it was better for him to try the problems first. Michael had progressive beliefs about teaching and learning mathematics, yet his own self-described actions as a student did not always match these progressive ideas. Sally remained a confident, caring instructor whose core beliefs about teaching and learning mathematics did not change, but her understanding of how to enact some of those beliefs did change. Cara struggled to match her progressive beliefs, which largely came from Sally, with her practices as an instructor. In this struggle to match her practices with her beliefs, Cara learned more about why she should believe particular beliefs. Given these insights into each individual participant, a natural question arises: How does what was learned about each individual participant’s beliefs relate to what was learned about the connections made among them?

Understanding more about the individual participant’s beliefs gives insight into a participant’s response: knowing what a participant believed helps understand why they replied to questions in the ways they did. With this additional information about participants’ beliefs, more inferences can be made about how participants’ beliefs might interact beyond what has been said above, which primarily relied on participants’ responses. By combining the ideas from the previous two chapters with the ideas in this chapter, aspects of how individuals responded in this chapter can be gleaned and then used to make these deeper inferences about how their beliefs might be interacting.
**Considering participant responses and beliefs individually.** From Chapter 5, Sally believed deeply in challenging students in ways which made them think, and also to understand that learning mathematics meant much more than doing procedures. So it makes sense that Sally responded to her question about why she gave the students The Bridge Problem by saying she wanted to push the students to think more. And it also makes sense that she said she asked the student in The Magnitude Discussion to say more about her solution, because Sally believes learning mathematics is much more than doing simple procedures. Sally also said she hated when people speaking with her assumed she automatically knew what they were talking about. Consequently, she asserted the belief that people should try to be on the “same page (Interview #4, 5/7/15)” when talking with one another. It is this belief that seemed to push Sally into making public her decisions about teaching, like when she got on her soap box, and her explanations of “wanting to help make connections (Interview #4, 5/7/15)” about why the students are learning the way they are in this class.

Also from Chapter 5, Cara held a lot of Sally’s core beliefs about teaching and learning mathematics, but that she was learning more about how and why to believe in these particular beliefs. Sally’s relationship with Cara this semester and the previous semester was a significant source of learning for Cara. Cara admitted she closely observed Sally. These observations seem to help Cara be able to decipher why Sally made a particular teaching move, as evidenced by Cara’s and Sally’s responses often connecting in these particular instances. For example, Cara was able to describe why Sally went on the “soap box” in much the same way Sally described why she went on a “soap box”- to help students understand why they are learning the way they are learning.
Even though Cara still had much to learn about how to match her practices as a teacher with her beliefs, this ability to connect with Sally and, at times, the other two students, suggests that she was accurately observing and interpreting Sally’s teaching which in turn helped Cara learn more about her own beliefs.

Brad had a significant transformation in his beliefs. He evolved from wanting to be a student in a traditional style mathematics classroom to appreciating a classroom in which he tries problems first and the teacher is a nearby facilitator if needed. The three video clips analyzed in this chapter occurred at various points throughout the semester, making Brad’s transformation more plain, and displaying how his responses were rooted in his beliefs at these different times. When Brad said he was surprised by the feedback of students learning most by trying problems on their own, Brad admitted that he was surprised by this because he still wanted the teacher to explain problems at the board. At this point of the semester, Brad was still clinging to his beliefs that teaching and learning mathematics were most effective when teachers explained problems then students practiced those problems. Brad was able to reason why Sally went on her “soap box” was because she wanted to explain why the class was always working on the worksheets given. He admitted that he heard what was Sally was saying but that at that point he still preferred “the old way.” What is important here is that Brad was able to connect with Sally’s ideas while still considering his own beliefs which were different than Sally’s ideas. This acknowledgement of other ideas while simultaneously restating his own beliefs is what made Brad able to reconsider his beliefs. Without being able to connect to Sally’s reflections, Brad would have had a hard time voicing his changed thoughts about teaching and learning mathematics.
Michael was a committed yet uncommitted type of student. He was committed to progressive ideas about teaching and learning, but he also mentioned aspects of himself as a student that indicated he was not the kind of student he talked about in his progressive beliefs. Michael’s responses reflect his progressive beliefs and sometimes offer insight into the type of student he is based on how he portrayed himself. When he responded to the question about what surprised him about the mid-semester feedback, Michael admitted he was surprised that students wanted examples explained at the board. He further explained this by saying that this desire from other students took away from learning. This aligns with Michael’s belief that learning math involves students doing problems on their own first. In another instance, Michael admitted he did not pay attention to Sally when she went on her “soap box,” either in the classroom originally or in review of the video clip, which does not represent his idea of things he thinks students should be doing. This aloof attitude of Michael as a student shows up in the two instances that were categorized as ‘Instructors and One Student Connected, One Student Different.’ Michael was the student who was different both times, which corresponds with him being this aloof student who did not always fit in.

Considering participant responses and beliefs all together. Understanding how each participants’ beliefs related to his or her responses in the instances reveal a significant underlying idea: the participants were responding in ways that reflected or were motivated by their beliefs. As mentioned above, Cara and Brad gave evidence that they were both listening to Sally closely. These two ideas taken together suggest that Sally, Cara, and Brad not only had responses that were connecting, but that underneath those responses their beliefs were interacting. For a while, Brad’s beliefs were different
than Sally’s and Cara’s, but his close observations of them and what was going on around him helped him reconsider those beliefs in ways that aligned them more with Sally’s and Cara’s ideas. For these three participants, their responses were driven by their beliefs, or how they understood what was going on around them. Because Michael was somewhat uncommitted, he made fewer connections with these participants, suggesting that his beliefs interacted less with theirs.

If the participants’ responses were driven by beliefs, and their responses overlapping indicates that their beliefs were interacting, then what can be said if their responses had no connection? Does this imply their beliefs are not interacting? One answer is yes, their beliefs are not fully interacting. It is also correct to say that such a sweeping judgment is impossible given the instance being considered.

For many of the instances labeled ‘No Connection,’ both of these answers could be true. Consider the instance in which all four participants said different things surprised them about the mid-semester feedback. Sally said she was surprised that the students said the pace of the class was just right, but then Cara said nothing really surprised her. While Sally’s answer is motivated by her belief that she needs to understand her students, Cara’s answer does not seem to be motivated by a strong belief. This means there does not appear to be interacting beliefs between these two participants from this question.

Brad said he was surprised by students saying they learned the most doing problems first, and Michael said he was surprised by students saying the opposite and wanting more examples done at the board. As was said before, these two responses have no connection because they are both describing different aspects of the feedback. But these two responses were driven by each participant’s desire to learn at that point in the
semester. Michael was surprised because doing examples at the board is not how he liked to learn. Brad was surprised because he had not yet been convinced of the helpfulness of doing problems first. So while Michael and Brad were not saying the same things in their responses, their beliefs about how they liked to learn were still interacting at this point.

Lessons learned from contextualizing beliefs with practice

Thinking back to the vignette at the beginning of this chapter is a reminder that a classroom is filled with many moments of interaction. Sometimes these interactions occur publicly in front of the whole class. Sometimes these interactions occur within a small group. But, every interaction has someone in the classroom watching, which creates a dynamic in which students and instructors are constantly generating impressions about what is going on and, very importantly, these impressions are sometimes driven by their beliefs. Analyses from this chapter shed light on the impressions participants make about instances that occur within their class and how those reflections connect with each other.

The first take-away lesson is how and when instructors’ and students’ responses connect vary. Sometimes the instructors and students are all on the same page when reflecting on an instance. Sometimes the instructors and students are all on different pages when reflecting on an instance. For other instances there are varied combinations of instructors and students that share connections.

Another lesson is that it is difficult to tell when participants’ beliefs are interacting. For this analysis, the idea of interacting was operationalized into thinking about how participants’ responses to questions about clips in video clips connected. After analyzing the instances using this framing of connections, and comparing this analysis with what was learned about participant beliefs, there was evidence that participants were
responding at times with motivation from their beliefs. Because of this strong connection between the participant’s responses and his or her beliefs, there is evidence to say that clearly sometimes the participants’ beliefs were interacting, especially when their responses indicated a congruence. But there were times when participants were not responding in the same way, and it was hard to say beliefs were not interacting or they were interacting.
**Chapter 7**

**Conclusion and Implications**

In Chapters 4 and 5, I investigated individual students’ and instructors’ beliefs about teaching and learning. Using the combination of NCTM’s productive and unproductive beliefs (NCTM, 2014), survey questions about each participant’s self-efficacy, and conducting intentional interviews with video footage of the College Algebra classroom, allowed me to gather their individual beliefs about the role of the teacher and students, how to learn mathematics, and what constitutes a teacher’s practice.

For each individual student and instructor, I offer an explanation or theory describing what happened to their beliefs after being involved in a reformed College Algebra class (Chapters 4 and 5). In Chapter 6, I contextualized each individual’s beliefs in practice in the interviews using video clips by reexamining common places of reflection on specific moments from their class. While Chapters 4 and 5 mainly consider the four main participants as individuals, Chapter 6 considers what it means when all four main participants were instructors and students in the same class. The findings from each chapter are summarized in Table 13.

When reflecting on these findings, particularly from Chapters 4 and 5, change of participants’ beliefs was observed in different ways. Before the study, the initial conception of change included a stance of either it happened or it did not happen, and if it happened then there was a lot of change. Clearly, however, this is not in fact what happened with all of the participants. Thus, one overarching conclusion to this study is the need to view change differently from how it was conceptualized when the study was designed.
Table 13

*Summary of Findings from the Previous Analysis Chapters*

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Finding</th>
</tr>
</thead>
</table>
| Chapter 4 | Determining *how, how much*, and *why* changes in students’ beliefs vary from student to student.  
Students’ beliefs may not be predictive of their behavior as students.  
When teachers make public their reasons behind making particular decisions, students may be prompted to reevaluate their beliefs in light of the teacher’s public reasoning. |
| Chapter 5 | Changes in instructors’ beliefs may mean there are subtle differences in how or what someone believes, but their core beliefs stay the same.  
Placing support instructors who are intended to help students learn in a classroom may create an additional task for lead instructors. Lead instructors may adopt an additional belief of needing to mentor support instructors.  
Novice instructors can hold strong, progressive beliefs about teaching and learning mathematics.  
Inconsistencies between a novice instructor’s beliefs and practices can be viewed as the instructor learning to teach. |
| Chapter 6 | How and when instructors’ and students’ responses reflect connections vary.  
It is difficult to tell when participants’ beliefs are interacting. |

Another observation of the findings, specifically when comparing the insights gained about individuals with the insights when they were considered together, reveals that individual students and instructors can all be thinking about things differently one moment, and other moments they could be thinking almost exactly alike. Without looking at these instructors and students together, this observation would have been missed, implying that another overarching conclusion for this study is the need to consider the beliefs and practices of students and instructors together when studying undergraduate mathematics.
Considering Change Differently

In Chapter 2, the conception of change that was described was an ‘all or nothing’ kind of process. Either change happened or it did not happen. And if it happened, then it must resemble a great transformation (Ackerman-Anderson & Anderson, 2001) in which metaphors such as, ‘from darkness to light’ (Metzler, 1986), give an accurate description of the change. Before the data collection phase of this study, this conception of change seemed reasonable, yet now proves naïve. This was a study intended to focus on novice teachers and students in a context of teaching and learning most likely different than what their years of apprenticeship of observation (Lortie, 1975) had led them to expect. Accepting the context of this study in this way made it seem plausible some great kind of transformation might occur. For Brad just such a transformation did take place. His beliefs changed from believing that an instructor should show him how to do a problem before he practiced it himself, to wanting to try problems on his own or with other students first, then asking the instructor for help as needed. This kind of change from Brad is quite dramatic. He went from having beliefs about teaching and learning that did not align with those of the reform to beliefs that did align with those of the reform. This profound type of change, however, did not occur with Michael, Sally, or Cara.

The Merriam-Webster dictionary defines ‘change’ in the verb usage as “to become different; to make (someone or something) different; to become something else.” As a noun it means “the act, process, or result of changing” (www.merriam-webster.com) where ‘changing’ is defined as alteration, transformation, and substitution. When considering the answer to the central research question of ‘what happens’ to each individual’s beliefs about teaching and learning mathematics, then according to Merriam-
Webster’s definition of change, these individuals changed. Their beliefs became something different. The participants were different instructors and students than they had been when the course began. At the same time, it would be fair to say they were not overwhelmingly transformed. All of the main participants could elaborate more on what they believe. For example, Michael in the end could describe more in detail the kind of teacher he thought represented a “good” teacher, and Cara could articulate why she thought students should not rely on memorization. Both of these changes are not drastic, rather these changes were subtle in nature: the instructors and Michael held the same beliefs from when they started the course but by the end they could explain why they believed what they did.

Marshak (2002) calls into question the language of change, and claims it is ambiguous and imprecise. He says, “the generic term ‘change’ does not differentiate among sources, types or magnitudes of change” (p. 280). “Language is not a neutral medium. Language both enables and limits what and how we think and therefore what we do” (p. 285). Thus, the word ‘change’ is problematic, and there is a need for further ways of describing the types and magnitudes of change that naturally occur. For Michael, Cara, and Sally, thinking more deeply about the magnitude and type of their change will better describe what was so subtle.

Even though this study is interested in describing what happens to individuals’ beliefs when they are members of a reform culture, it is also accurate to say it is interested in shedding light on the more general question of what happens when these individuals are members of a culture of reform. Anthropologists have posed similar questions of describing what happens when people are acting within a culture.
D’Andrade (1992) describes this phenomenon with these questions: “Do people always do what their culture tells them to? If they do, why do they? If they don’t, why don’t they? And how does culture make them do it?” (p. 23). To help answer these questions, D’Andrade poses a theoretical argument in which he conceptualizes motivation in order to relate motivation to culture. His focus on motivation comes from the idea that, “motivation is necessary for the performance of cultural roles” (p. 23) meaning motivation is often an essential impetus for why people do what they do in a culture.

D’Andrade (1992) makes a connection between, “motivational force of cultural models and Spiro’s concept of internalization” (p. 36) in order to describe how “all parts of a culture are not held by people in the same way” (p. 36). Spiro described four levels of internalization. The first level is one in which a person is just a person just acquainted with some part of a cultural system, but is indifferent to or rejects the beliefs of the culture. The second level is one in which a person makes claims that align with the culture, but the person has not really acquired the beliefs behind the claims made. The third level is one in which a person has internalized the beliefs of the culture system, but these beliefs are not embodied. The fourth and final level is one in which a person has internalized the beliefs of the culture so deeply it has become salient, meaning the person holds these beliefs with conviction.

While D’Andrade (1992) discusses several other aspects that affect how people behave in a culture, these four levels of internalization adapted from Spiro seem very applicable to describing the four main participants’ behavior in this study. These four levels are situated in the idea that people behave in a culture, which for this study would be the reformed College Algebra classroom, and it is the type and magnitude of their
beliefs that describe how connected a person is in this culture. I propose that with replacing “culture system” with “reform culture,” Spiro’s four levels can be used to describe change observed in the instructors of this study. Figure 14 portrays the levels with this change.

Table 14

*Adaptation of Spiro’s Levels Centered on Reform Culture*

<table>
<thead>
<tr>
<th>Level</th>
<th>Reform Culture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1 (General Awareness)</td>
<td>A person is acquainted with at least some part of the reform culture but is indifferent or rejects the beliefs of the reform culture</td>
</tr>
<tr>
<td>Level 2 (Beliefs are external, but not internal)</td>
<td>A person honors the reform culture by making statements that reflect the beliefs of the reform culture, but they do not act in ways that support these beliefs.</td>
</tr>
<tr>
<td>Level 3 (Beliefs are external and internal)</td>
<td>A person honors and actively reflects the beliefs of the reform culture. Their beliefs in the reform culture influence their actions.</td>
</tr>
<tr>
<td>Level 4 (Beliefs with conviction)</td>
<td>A person honors and embodies the beliefs of the reform culture. Their beliefs in the reform culture are strong enough to influence their actions and emotions.</td>
</tr>
</tbody>
</table>

This framework is a way to contextualize their beliefs with their practice in the reform. When someone moves up the levels, their beliefs are deepening or strengthening, practices are matching beliefs, and explanations for beliefs and practice are expressed with confidence and articulation.

**Using the reform culture levels with Sally and Cara**

**Describing Sally with the reform culture levels.** Sally started the semester stating beliefs and using teaching practices that matched the reform. For example, she said at the
beginning of the semester that she believed a teacher’s job included finding appropriate challenges for the students and that a teacher should start with students’ thinking first. Then in the video clip, The Bridge Problem, Sally started the whole group discussion by soliciting students’ thinking first then as she admitted, “doing more telling” than she probably should have. Following the group discussion about the answer on the problem, Sally got on a “soap box” where she told them she gave them this problem to challenge them and “make them think.” Though Sally had the strong beliefs that matched the reform and acted in ways that matched the reform, she still had moments when her teaching practice did not always match her beliefs and the beliefs of the reform. An example of this was when she did more ‘telling’ in The Bridge Problem clip. Thus, Sally was mostly at a Level 3, but teetered sometimes in Level 2. Her beliefs and practices mostly matched, but there were times they mismatched.

By the end of the semester, some of Sally’s beliefs became more internalized. She had an epiphany about her role as a teacher, in which she realized it was okay for her to be an outsider when her students were working with each other. In the beginning of the semester, Sally admitted to being worried that her students were so quiet, it was hard for her to know whether to intervene or not. But with this epiphany, she became more comfortable with her students being quiet or working with each other instead of her intervening. She also became more open to helping students by using a bigger picture approach and starting with the details, instead of her preference with just using a bigger picture. By the end of the semester Sally more deeply honored the beliefs she already had and perhaps had begun to believe some of them with conviction. While her teaching practices may not always have reflected her beliefs, this growth in honoring beliefs made
Sally more squarely in a level 3, where her beliefs were more consistently matching her practice, than she had been at the beginning of the semester.

*Describing Cara with the reform culture levels.* Cara started the semester squarely at the level 2. She espoused beliefs that matched the reforms, but her practices as a teacher did not match these beliefs. For example, she made many statements that indicated she believed a teacher should not directly give students answers, and that he or she should make students think. Yet when observed interacting with students, Cara often made leading statements or questions, and sometimes directly told students where they went wrong. She did not always directly tell students answers, but making leading statements or telling where students went wrong in their solution are teaching moves that stifle students’ thinking.

Like Sally, by the end of the semester, Cara’s beliefs had become more internalized. For Cara, this meant that she began to articulate why she should believe in certain beliefs, like why students should not rely on memorization. At the beginning of the semester, Cara could just state this as something she believed in, but by the end of the semester she was able to reason that students should not rely on memorization because it removed all of the thinking within the problems. There was also evidence that Cara’s teaching practices started to better match her beliefs, as in the example in which she asked the student the following questions: ‘What’s happening on the inside?’ ‘What does that look like?’ ‘What else can we do?’ In building her more durable and thorough understanding of her own beliefs, which matched those of the reform, along with making conscious efforts to match her practice with her beliefs, Cara began the transition away
from level 2 towards level 3, a significant and important maturation in her teaching mindset.

**Using the reform culture levels with the students.** The levels require knowledge of a person’s practice within the reform culture and this study is limited in that there is little data reflecting the students’ practice. For example, how students solve and talk about problems on the worksheets and how they prepare for exams are not part of the data. Because of this, I can only hypothesize what levels Brad and Michael were, using what they said about their own practice and other comments they made that seem to relate to their own practice. Brad started the semester at a level 1 in that he was aware of the beliefs of the reform, but he admittedly rejected some of the beliefs and seemed indifferent to others. For example, Brad acknowledged the idea that the instructor was supposed to let students work on problems on their own, but he really wanted the instructor to show him how to do the problem first before he tried it. But by the end of the semester, Brad’s beliefs began to match those of the reform culture and he admitted that the characteristics of the reform were helping him learn. He then agreed that it was better for him to try the problems first before getting help from the instructor. Considering these changed beliefs and Brad expecting to get an A in the course, it appears that Brad ended the semester closer to a level 3, where his beliefs matched the reform and his practices as a student matched his and the reforms’ beliefs.

Much different than Brad, Michael started the semester with espoused beliefs that matched the reform culture. For example, he described the role of a teacher as being someone who does not give students the answer and should focus on engaging students’ thinking. And at the end of the semester, his beliefs largely remained the same but
became more detailed. His conception of a teacher engaging students’ thinking evolved into more concrete characteristics like the teacher needing good knowledge of mathematics and needing to have a personality that is not “straight laced.” Michael also made comments that suggested his own behavior as a student did not match his beliefs. This was best exemplified when he said, “The laziest students. That’s what we are,” and that he was not missing class “too bad.” At the end of the semester, he specifically remarked that he thought students should be putting a lot of effort into the course and not procrastinating, yet he admitted that he did these things. Taking these comments into account, with his admission that all he needed to get was 15 points on the final just to pass the class, suggests Michael’s practices did not match his beliefs. It is somewhat unclear exactly where in level 2 Michael started and ended the semester, but it appeared as though Michael fluctuated within level 2. He moved back and forth between saying he believed in the reforms and learned how to match his practices as a student with his beliefs.

**Two different kinds of change.** Using the levels to describe where the participants started and ended the semester reveals that two different kinds of change occurred: across levels and within a level. Brad’s change happened across levels. He made a huge transformation in which he acknowledged the reforms but was indifferent towards them, but then he bought into the reforms and saw them helping him learn so his beliefs changed to match those of the reforms. This kind of transformation took Brad from being at a level 1 to being in a completely different higher level. This kind of change did not happen with the instructors. Both Sally and Cara started in a level and ended in this level but with some movement toward the next higher level. Michael, too,
seemed to be moving back and forth within a level. These two kinds of subtle change suggest that sometimes growth can be linear, like Sally and Cara continuing to grow in ways that move them up the levels, and sometimes linear can be nonlinear, like with Michael who appears to be moving back and forth within a level. Using these levels to describe change allows outside observers to readily see what a person is doing in relation to the reform culture he or she are in and it provides a sense of direction of where he or she could change and grow into and where that person might regress.

**Considering Instructors and Students Together**

Chapters 4 and 5 have built individual descriptions of Brad, Michael, Sally, and Cara. These descriptions include what individuals believe and sometimes why they believe in what they believe. Brad is a student who went from wanting a teacher to show him how to do a problem to wanting to try problems on his own first because he came to realize trying problems without the teacher’s help first is what was helping him learn. Michael is a student who says he wants a teacher to push his thinking and not give him answers. Sally is a teacher who believes in continually challenging her students by focusing on their thinking. And Cara believes a teacher should not give students answers, yet she is learning how to do this in her own practice as a teacher. But these four individuals are in the same classroom, so what can be said about what it means for these four individuals to be in the same class? Chapter 6 offers insight into this question. When examining common moments of teaching and learning from their class, it became clear that there are times when all four of the participants were reflecting about a particular moment in different ways and then there were times when they were all reflecting about a particular moment in very similar ways. Without looking at their reflections of these
common moments together, this idea that instructors and students can think very differently or similarly would have been missed. This calls for a need to consider instructors and students together, not separately. I discuss two examples, one where when instructors and students were thinking alike and one where they were all thinking differently, then explain why missing these examples would have undervalued the classroom experience as a whole.

Recall from Chapter 6, in which I discussed the four participants’ responses to the question, “Why do you think Sally went on her “soap box”? Sally said she went on her soap box because she wanted to emphasize that the point of working some problems is to think about them, not just to find an answer. Cara thought Sally went on the soap box because she wanted the students to think about the answer and this process of thinking about the answer was important. Brad said he thought Sally got on the soap box because she wanted to reinforce why she does the worksheets with students so as to help them think. Michael did not remember Sally getting on the soap box from the video clip or from the actual moment in the class, so he could not give a reason. What is learned about comparing the reflections about this common moment is that Sally, Cara, and Brad all had similar reflections. In this case it means that Brad and Cara were successfully interpreting why Sally chose to get on her soap box. In thinking about what was learned about each individual in the previous analysis chapters, this alignment is not surprising. As the learning assistant, Cara admitted she was always observing Sally, trying to learn more about teaching. Brad made several statements indicating he was listening to Sally, even when he might not always have agreed with what she was saying. For Sally, this teaching decision was not out of character. She admitted that she hated talking to people
when they were not on the same page, so getting on a soap box trying to get on the same page with her students was very much in her character.

What would have been missed if Sally, Cara, Brad, and Michael had not been considered together in this particular moment? Perhaps the most significant observation that would have been missed is how what was learned about these individuals helped them think alike about this particular moment from their class. It was learned from the previous analysis that Cara and Brad were listening to Sally carefully, and from this analysis, it becomes clear that this is what helped them correctly decipher why Sally got on her soap box. This is significant because it reveals a sense of communal understanding about particular teaching decisions and provides evidence for how it exists.

Recall when participants were asked why the student presenting the solution in The Magnitude Discussion got so hung up solving for $W_o$. All four of the main participants’ responses indicated they were thinking very different reasons why this student struggled. Sally thought the student was challenged because she was using sloppy notation. Cara thought the student did not understand what she was looking for. Brad thought the student was frustrated and did not want to finish the problem. Michael thought that it was a weird variable to solve for and maybe this kind of mathematics was too advanced for this particular student. Connecting these responses back to what was learned about these individuals, their responses make sense. Brad believed students have the ability to learn mathematics whereas Michael made statements that indicated he questions students’ abilities. Sally believed being precise was very important in explaining mathematical thinking and Cara emphasized knowing a conceptual understanding behind a problem.
What would have been missed if Sally, Cara, Brad, and Michael had not been considered together in this particular moment? Like the moment before, the most significant observation that would have been missed is the linkage of what was learned about the individuals and how that knowledge connected to their responses about why this student was struggling. From considering their reflections together about this particular moment, it becomes evident that none of them were thinking alike but that they all had reasons for thinking the way they did. It makes sense that Brad did not want to think the student could not do the problem and would rather interpret her struggle as her just getting frustrated. And it makes sense that Michael thought maybe this level of mathematics was too difficult for the student. This observation is significant because in contrast to the prior episode, it reveals a lack of communal understanding and provides reasons for how it exists.

Reexamining these two examples not only reveals that there are times when instructors and students think alike and times when they think differently, but there are good reasons for these individuals thinking the way they do. Thus, a moment in which they are all thinking differently is not coincidental, and a moment in which they are all thinking similarly is not happenstance. These moments imply that there are, at times, a sense of communal understanding about what is happening within a common classroom, and at times there can be lack of communal understanding. Because teaching and learning are complex practices filled with numerous moments, a common sense of what is happening with both teaching and learning can help propel the momentum and direction within a class. Likewise, a lack of common understanding can impede the momentum and direction within a class. Thus, considering instructors and students together made
seeing these moments of communal understanding or lack of communal understanding possible. Without considering the instructors and students together, the full accurate history of that classroom experience could never be understood with the depth that analysis and advance in teaching requires.

Implications

Both of the overarching conclusions discussed above have implications for future research, teaching, and professional development in higher education.

Implications for future research. Viewing the instructors’ and students’ changes using the levels proposed opens up different ways of defining and looking at learning for both the instructors learning to teach and students learning mathematics. For at least the two instructors, it was clear they ended the semester at a place further up the levels than where they started, indicating a growth in internalizing the reforms. This indicates that for these instructors, this classroom experience was a place for their own learning and growth. And for one of the students, there appeared to be a major leap across levels suggesting a significant change in the student’s acceptance and internalizing of the reforms. These patterns suggest that there might be some kind of learning trajectory each of these individuals are following as they are learning to teach or learning mathematics in the context of a reformed classroom. Learning trajectories have been used in the contexts of describing young children’s mathematical learning and have been defined as descriptions of children’s thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks designed to engender those mental process or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of
supporting children’s of specific goals in that mathematical domain. (Clements & Sarama, 2004, p. 83)

Thus, learning trajectories have been used to better describe what it looks like for students to engage in a process of learning, which is being cultivated by instructional tasks, in hopes of understanding how to help the student achieve a specific goal. While it is certainly not clear how this directly translates to the learning witnessed in this dissertation study, there are some overlapping characteristics. Undergraduate reforms are using purposeful instructional tasks intended to cultivate learning from the students, and these instructional tasks are often embedded with particular beliefs of what “good” teaching and learning of mathematics looks like. Instructors are tasked with learning how to effectively use these instructional tasks, and possibly improvise to create other instructional tasks that coincide with the reforms idea of “good” teaching. This study captured instructors and students moving their practices towards goals of “good” teaching and learning aligned with the reform. Thus, there could be value in considering and further investigating whether or not there are learning trajectories that describe instructors’ and students’ learning processes. While this is a larger idea worth thinking about, I propose two specific studies needed to better understand students and two specific studies needed to better understand instructors in the context of reformed lower level courses. These specific future studies would help with further consideration of the learning trajectories.

One of the limitations of this study is the lack of data describing the students’ practice as learners. This was especially noticeable when it was not clear exactly what was going on with Michael’s beliefs. Because of this limitation, it could only be
hypothesized where Brad and Michael fall among the four levels. A future study could more intentionally collect data describing the practice of students within a lower level undergraduate course, as well as data reflecting their beliefs. This kind of study would allow for a more accurate understanding of where students fall among the levels, because it would keep a focus on students’ beliefs, but have evidence as to whether the students’ practice matched their beliefs. Another kind of data that could be added into a study like this dissertation study are data describing the students’ performance such as test scores. This dissertation intentionally did not collect any data reflecting students’ performance in the course, but adding this data to the kinds of data already collected in this study would give mathematics departments’ faculty insight into how a student’s set of beliefs might connect with or not connect with his or her performance in the class. This kind of knowledge would help future mathematics departments better distinguish what it means when students are embracing or resisting the reforms.

Both of the suggested studies would contribute to learning more about students like Michael, a student who said he believed in the reform ideals but simultaneously suggested he was not engaging in them the way he described in his beliefs. Was Michael a student who lacked motivation to learn? Was he just telling me what he thought I wanted to hear? Was Michael struggling with knowing how to learn in the ways he described? It would be easy for mathematics departments to say Michael lacked motivation, and then claim motivation is not something they can always control, and toss Michael aside. There is some truth in the idea motivation held by students is not always influenced by outside factors, like the mathematics department, but there is more truth in saying that there are likely multiple factors that explain why Michael held progressive
beliefs aligned with the reform, but yet he ended the class with a “just passing” grade. Understanding more about his practice as a student, meaning aspects like his mathematical thinking about problems and how cognizant of his thinking he is able to be, and his performance would help better explain what is happening with his beliefs and experiences in a reformed College Algebra course. The case of Michael calls for a need to more seriously consider what kind of data needs to be collected that would better explain what is going on with his beliefs.

Given their limited formal teaching experiences, both instructors were novices, yet they started the semester at different places in what they believed and how they reflected these beliefs in their practices of teaching. The main GTA, Sally, started the semester with strongly held beliefs that aligned with the reforms, and already had a rhythm of consistency between her beliefs and her practice, although this consistency was something Sally was continuing to learn about in her growth as a teacher. A needed future study is to intentionally focus on a novice teacher who does not have beliefs aligned with those of the reform they are teaching. This kind of study would help mathematics departments more clearly see the learning a GTA experiences from the immediate start of their involvement in a reformed class. Research supports the idea that novice teachers may hold traditional beliefs (Ball & Wilson, 1990), and thus a future study intentionally focused on finding a novice instructor with less-defined beliefs would help better see what it means for a novice teacher to learn to teach in a reformed undergraduate course. This is most likely a more common experience, considering GTAs within a mathematics department often have little or no formal teaching experiences (Hardre & Burris, 2012; Marincovich, 1998; Tice, 1997).
The GTA and LA described the relationship they had during the semester prior to this study as something that was significant and intentional to the learning from the LA. They decided on their own to meet regularly outside of class to talk about teaching decisions and theories. Both instructors said this was a period when the LA learned a lot from the GTA about how and why to teach in ways that attended to students’ thinking. And in the semester they both participated in this dissertation study, this kind of relationship continued. The LA repeatedly admitted to observing the GTA closely, and would make statements referring to what she learned last semester or how what she was learning now about teaching related to the previous semester. While this study gained some insight into what and how the LA was learning about teaching and learning from the GTA, a future study is needed to further focus on and explore what it means and for a GTA and LA to learn from one another.

**Implications for teaching and learning.** Teaching has been described as a practice often done in isolation where a teacher is making moment to moment (Lampert, 2001) decisions in his or her head all occurring behind four walls. The GTA in this study had a practice of teaching in which she at times intentionally made public reasons behind teaching decisions. Considering this study followed and analyzed the instructors and two students together, it became clear that this move of making her teaching public generated positive learning experiences for the LA, one of the students, and the GTA herself. These three different kinds of learning experiences, made possible by the public nature of the GTA’s teaching, suggest a larger implication about teaching at the undergraduate level: departments need to include ways for GTAs to make public their decisions about teaching.
Making visible the invisible for the LA. As has already been mentioned, the GTA and LA met outside of class regularly to discuss teaching. While I did not capture these meetings in my data, both the GTA and LA alluded to what happened in these meetings. The meetings included the GTA describing why she said or did certain moves in previous class periods, and also discussed what her goals were for teaching and for the students in upcoming class periods. Holding these meetings and talking about these parts of her teaching practice were ways the GTA was making visible the part of teaching that is often invisible to others who are watching. Making this invisible part visible helps people such as the LA see how teaching is systematic and intentional. Without this public kind of sharing, the LA would only have what she observed about teaching during class and the inferences she made about those observations. What this public sharing does is help the LA make more accurate inferences about the teaching she is so intently observing as well as aiding.

Helping students buy in. One of the students was very observant of the GTA. He was a student who started the semester being openly opinionated that his beliefs were very different than the reforms’ beliefs, but he acknowledged in several statements that he was listening to the GTA. It was through his listening of the GTA being public about her teaching that helped this student start to understand and accept learning mathematics differently. For example, when watching The Bridge Problem clip in which the GTA told the students she gave this problem because she wanted them to learn that thinking about problems matters just as much as finding a solution, this student made a statement acknowledging he heard her reasoning, but he was not convinced yet as he still just wanted a direct answer. When watching the Mid Semester Feedback clip, the GTA
described briefly the role of information processing in how students learn and when reflecting on what the GTA said about learning, the student admitted that he had changed his mind. Hearing this reasoning about how students learned helped him realize that trying problems first on his own or with others was what was helping him learn. Thus, the GTAs’ public reasoning of her teaching helped this student become more cognizant of what was helping him learn and what was not helping him learn, which in turn helped him buy into the reformed practices of the course.

**GTA is forced to reflect.** When the GTA made public the reasons behind her teaching decisions, either in a meeting with the LA or in front of the class, it forced her to confront her own thinking about her teaching. For example, when she met with the LA to talk about teaching, the GTA had to think through why she made certain teaching moves or said something in particular in order to share this reason with the LA. And when she stood in front of the class explaining why she gave them a particular problem, she had to think about the reason ahead of time. Most of the time, the GTA was intentional about making her teaching public, which means she had to think about what wanted to say before actually saying it. This intention forces her to consider and reflect on her own practice in deeper ways.

Thus, teaching in undergraduate mathematics needs to be made public. Making this teaching public helps other instructors learn about teaching in ways that would otherwise be unattainable. This public nature of teaching also helps students become more aware of the reasons why they are learning in particular ways and this greater awareness may help students buy into these new differences. Finally, an instructor who
makes public his or her reasons behind teaching decisions must reflect on his or her own teaching before deciding what to say publicly.

**Implications for mathematics departments.** Mathematics departments are tasked with making many decisions in how to reform their courses. If they are already well into a reform, part of their work revolves around assessing what parts of the reform are successful and which parts need attention. If mathematics departments are early in the reform process, part of their work involves deciding what needs to change and how it should or could be initially changed to fit the needs of their reform goals. Regardless of where they are in the process of reforming their undergraduate courses, mathematics departments hold a great deal of responsibility for the teaching and learning that happens within their courses. Findings from this study will help mathematics departments in two ways: (a) assess their own reform efforts, and (b) help them think about better supporting individual instructors and students within their reforms.

Cohen and Hill (2001) argue that learning is needed both in and of reforms. Therefore, there is need for reforms to learn from their own efforts. One way departments are trying to learn from their reforms is by gathering and analyzing student success scores (e.g., Marbach-Ad, Ziemer, Orgler, & Thomspen, 2014). While this type of analysis offers one perspective of how students are responding to reforms, it is limited in describing how students are experiencing these multifaceted reforms. Another possible way departments might be learning from their reforms is from analyzing student evaluations. Like the student success scores, this method will provide some insight, but overall lacks the depth needed for real sustainable change. Findings from this dissertation point to the need for teaching in undergraduate mathematics courses to be made public,
and suggest viewing the change among instructors and students using the levels proposed above. Both of these suggestions will help departments assess their own reforms. With more instructors making their decisions about teaching public, mathematics departments will have more evidence that reflects how these teachers are interpreting the reforms. Hearing these interpretations will help mathematics departments better understand how instructors are embracing or resisting reforms. Using the levels proposed above to situate where instructors and students are in a reform process would allow mathematics departments to find different ways of supporting different kinds of instructors and students.

**How to help instructors like Sally.** While Sally represents an instructor who held strong beliefs and practices that matched her beliefs, there is still more for Sally to learn about teaching. To a mathematics department, knowing that an instructor like Sally is at level 3, where her beliefs are starting to become more internalized, gives a sense of their practice in the context of the reform, and provides a direction or a reasonable goal in where the instructor could grow. Thus, the mathematics department could think about what opportunities they need to provide which would help this kind of instructor continue to match his or her practices with his or her beliefs and ways the instructor could start to believe in his or her own beliefs with more assuredness and passion. For example, one way a mathematics department could support this kind of instructor is to present him or her with a list of the mathematical tasks of teaching (Ball, Thames, & Phelps, 2008) and have the instructor reflect on which tasks he or she believes are doing well within his or her practice and which tasks the instructor might decide to work on more diligently. With this kind of support, the mathematics department is helping the teacher continue to take
ownership of his or her teaching practice and with further feeling of ownership the teacher might start believing in their beliefs with more conviction.

**How to help instructors like Cara.** Cara represents an instructor who is just starting to understand what she believes, and how to teach in ways that match her beliefs. To mathematics departments, Cara is like an instructor at a level 2 whose beliefs have not yet been internalized. A reasonable way to help an instructor at this level is to help the instructor better match their beliefs with their practices, a level 3, therefore pushing the instructor to see the benefits of what he or she believes. For example, the mathematics department could help this kind of instructor better evaluate this mismatch by having the instructor watch videos of his or her teaching. This type of noticing of a teacher’s own practices has been shown to be a good way for a teacher to seriously reconsider some of the practices they are currently using and think about how to incorporate new practices (Sherin & van Es, 2005).

**How to help students like Michael or Brad.** It has been hypothesized that Brad and Michael represent different kinds of students falling at different levels. With more evidence of the practices of these students, mathematics departments could be more sure of which level a student is, and where they might be pushed next. For example, if a student was at a level 2, in which his or her beliefs match those of the reform, but his or her practices as a student did not match the beliefs, then this might be a place where more the student needs more opportunities to develop self-regulation skills (Butler & Winne, 1995). This might mean the students have more times where they set up individual goals for learning or are asked questions that force them to explain how they learned a topic instead of just what they learned about the topic. For example, a list of questions like
‘Why was the answer wrong?’ and ‘What did I have to do or think differently to find a successful strategy?’ could be publicized in the classroom so that students have a regular reminder to ask these questions, in hopes of giving more attention to their monitoring skills.

Currently there are a few guidelines for mathematics departments to consider when making their choices about how to reform courses. The guidelines given in Chapter 1 for College Algebra describe intentions for teaching and learning whereas Bressoud’s and Rasmussen’s (2015) characteristics of Calculus courses as well as Laursen and colleagues (2011) characteristics of IBL classes are based off of actual observations of upper level classes. This study reveals that College Algebra can be taught in ways that reflect the many of the characteristics of the upper level classes, which means there needs to be more studies of reforms of College Algebra that more accurately captures how the intentions of College Algebra turn into characteristics of successfully reformed College Algebra courses.

Reforms in higher education are aiming to make learning algebra less of a gatekeeper for students pursuing college degrees. But for reforms to work, Cohen and Hill (2001) argue there needs to be both learning in and of reforms. Considering there are several major stakeholders’ perspectives (i.e., instructors, students, faculty), the learning both in and of reforms must be happening on different levels. This study gives insight into what it means for instructors to learn to teach and students to learn in the reform. Focusing on beliefs helped discover what instructors and students were learning of reform. More studies are needed to continue understanding how instructors and students can learn in and of the reforms they are teaching and learning in. Additionally, more
opportunities are needed for other stakeholders, like faculty within mathematics departments, to learn in and of the reforms within their department and other departments in the midst of reform. Making the learning of algebra accessible to all students is a challenging task that must be addressed in multiple ways and by all of the major stakeholders.
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APPENDIX A

Pre Survey Statistics

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<tr>
<td>Low</td>
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*SE means self-efficacy and B means general beliefs (NCTM, 2014)

Post Survey Statistics

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Comparing Pre Survey to Post Survey Change in Students’ Categories

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<td>--------------</td>
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<td>LL to H?</td>
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<td>LL to L?</td>
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<td>LH to LH</td>
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<td>LH to LL</td>
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*The first letter denotes self-efficacy, the second letter denotes beliefs. For example HL means high self efficacy and traditional beliefs and HH means high self efficacy and progressive beliefs.

**Summary of change:**

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APPENDIX B

QUESTIONS FOR INTERVIEW #4

*Before each participant’s list of questions are notes I made about their first interview. I used this to refer to if I wanted to push more specifically beyond the probing questions listed below.

Questions for Sally’s Final Interview

1) What are your overall impressions of what happened in the 101 class this semester?
2) Has teaching 101 this semester changed how you see mathematics as a subject?
   Probe: Do you still think the big picture is more important than the details? Do you think your students got that message this message? How do you think Claire sees the big picture?
3) Has teaching 101 this semester changed how you view students and what you think they should be doing?
   Probe: Do you think your students see math as important to their lives? What parts of the course do you think your 101 students bought into the course? What parts of the course do you think your 101 students didn’t buy into? What things were you seeing students do in the course? What things do you wish students were doing more in the course?
4) Has teaching 101 this semester changed how you view the role of the instructor?
   Probe: What sort of things do you think 101 instructors should be doing in this course? What sort of things do you think 101 instructors should not be doing in this course?
5) Has teaching 101 this semester changed how you see yourself?
   Probe: What kind of values were you trying to instill into the students? Do you feel like you did instill some of those values? You mentioned in the beginning interview that you struggle to make students care and work with their tablemates, how do you feel about that now? You also mentioned in the beginning interview that you struggle with balancing being approachable yet holding a firm line with some things, how do you feel about that now?
6) Did notice any changes in the students over the course of the semester? If yes, then what were they? Did you notice any changes in Cara of the course of the semester? If yes, then what were they? Why do you think they changed?
7) Look at pre and post survey.

Questions for Cara’s Final Interview

1) What are your overall impressions of what happened in the 101 class this semester?
2) Has teaching 101 this semester changed how you see mathematics as a subject?
   Probe: How do you see memorization relating to mathematics? How do you see building concepts relating to mathematics?

3) Has teaching 101 this semester changed how you view students and what you think they should be doing?
   Probe: Do you think your students see math as important to their lives? How do you think 101 impacted the impressions of math the students came with to 101? What do you think students should be doing in 101? What do you think the students should not be doing in 101?

4) Has teaching 101 this semester changed how you view the role of the instructor?
   Probe: How do you think instructors in 101 should work with students? How has this semester impacted what you think 101 instructors should be doing?

5) Has teaching 101 this semester changed how you see yourself?
   Probe: How do you compare high school students and college students learning mathematics? When and how do you think you connect the most with students?

6) Did notice any changes in the students over the course of the semester? If yes, then what were they? Did you notice any changes in Sally over the course of the semester? If yes, then what were they? Why do you think they changed?

7) Look at pre and post survey.

**Questions for Brad’s and Michael’s Final Interview**

1) What are your overall impressions of what happened in the 101 class this semester?

2) Has teaching 101 this semester changed how you see mathematics as a subject?

3) Has teaching 101 this semester changed how you view students and what you think they should be doing?
   Probe: How do you think this style of teaching helped students learn? How do you think this style of teaching prevented students from learning? What do you think students should be doing in 101? What do you think the students should not be doing in 101?

4) Has teaching 101 this semester changed how you view the role of the instructor?
   Probe: How do you think instructors in 101 should work with students? How has this semester impacted what you think 101 instructors should be doing?

5) Has teaching 101 this semester changed how you see yourself?
Probe: Do you still think you are pretty good at doing math? How would you like the instructors to help you out if you had a question? If there is more than one way to do a problem, would you want to know all of the ways to solve it?

6) Did notice any changes in the other students over the course of the semester? If yes, then what were they? Did you notice any changes in Sally or Cara over the course of the semester? If yes, then what were they? Why do you think they changed?

7) Look at pre and post survey.