Design Optimization of a Linear Generator With Dual Halbach Array for Human Motion Energy Harvesting

Wenjia Zhao

University of Nebraska-Lincoln, wenlovejia@yahoo.com

Follow this and additional works at: http://digitalcommons.unl.edu/elecengtheses

Part of the Electrical and Electronics Commons

http://digitalcommons.unl.edu/elecengtheses/65

This Article is brought to you for free and open access by the Electrical & Computer Engineering, Department of at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Theses, Dissertations, and Student Research from Electrical & Computer Engineering by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.
DESIGN OPTIMIZATION OF A LINEAR GENERATOR WITH
DUAL HALBACH ARRAY FOR HUMAN MOTION ENERGY HARVESTING

by

Wenjia Zhao

A THESIS

Presented to the Faculty of
The Graduate College at the University of Nebraska
In Partial Fulfillment of the Requirements
For the Degree of Master of Science

Major: Electrical Engineering

Under the Supervision of Professor Liyan Qu

Lincoln, Nebraska

July, 2015
A linear generator for human motion energy harvesting is designed and optimized in this study. The linear generator can be used to convert human motion energy to electrical energy to charge portable electronics, such as cellular phones, wireless internet devices, etc. The use of dual Halbach magnet arrays in the proposed generator provides a significant improvement in power density. Different dual Halbach arrays were studied, and a novel Halbach array with nonuniform magnet blocks is proposed to enhance the air-gap flux density in the linear generator for a given size and weight of permanent magnet material. Sensitivity analyses were performed to identify significant design variables for further optimization. Then, an optimization model was constructed with the objective of maximizing the output power of the linear generator. The proposed linear generator was compared with two other different linear generators for human walking motion energy harvesting. The effectiveness of the proposed methods was verified by two-dimensional finite element analysis simulation results.
Table of Contents

Abstract .......................................................................................................................... i

List of Tables ............................................................................................................... iv

List of Figures ............................................................................................................. v

Nomenclature ............................................................................................................. vi

Chapter 1: Introduction ........................................................................................... 1

1.1 Background and Motivation .................................................................................... 1

1.2 Objective .................................................................................................................. 2

Chapter 2: A Literature Review on Linear Generators for Human Energy Harvesting ................................................................................................................... 3

Chapter 3: Design Considerations of the Proposed Linear Generator .............. 10

3.1 The Dual Halbach Magnet Array Stator ................................................................. 11

3.1.1 Traditional 4-piece and 8-piece Halbach Arrays ............................................ 10

3.1.2. The proposed non-uniform Halbach array ................................................. 12

3.1.3. Sensitivity analysis ...................................................................................... 13

3.1.4. The initial design of non-uniform Halbach array ....................................... 16

3.2 The Moving Part .................................................................................................... 19

3.3 The Supporting Part ............................................................................................... 19

Chapter 4: Optimization of the Proposed Linear Generator ............................. 21
Chapter 5: Simulation Results ................................................................. 23

Chapter 6: Conclusions ........................................................................ 28

References ................................................................................................. 29
List of Tables

TABLE I. COMPARISON OF THE HUMAN ENERGY HARVESTING LINEAR GENERATOR PROTOTYPES FROM THE LITERATURE REVIEW ................................................. 8
TABLE II. DESIGN VARIABLES OF THE PROPOSED LINEAR GENERATOR .............. 14
TABLE III. VARIABLES AND CONSTANTS FOR THE OPTIMAL DESIGN.................... 22
TABLE IV. PARAMETERS OF THE OPTIMAL DESIGN WITH THE PROPOSED NON-UNIFORM ARRAYS ........................................................................................................... 23
TABLE V. PARAMETERS OF THE OPTIMAL DESIGN WITH THE UNIFORM ARRAYS ...... 25
TABLE VI. COMPARISONS BETWEEN THE PROPOSED LINEAR GENERATOR AND TWO DIFFERENT DESIGNS ............................................................................................... 26
List of Figures

Figure 1. Diagram of the proposed linear generator with dual Halbach array .......... 11

Figure 2. The flux distribution of (a) four- and (b) eight-piece Halbach array .......... 13

Figure 3. The comparison of the magnetic fields (x-axis component) on the enhanced side at different distances for one period ................................................................. 13

Figure 4. The proposed eight-piece, non-uniform Halbach array ......................... 14

Figure 5. Sensitivity index of the average absolute value of the magnetic flux density at 0.5g.............................................................................................................. 16

Figure 6. The variation of average absolute value of the flux density at the middle airgap due to the length-to-width ratio................................................................. 18

Figure 7. The variation of average absolute value of the flux density at the middle airgap due to the orthogonal magnet length ratio ............................................ 18

Figure 8. The flux distribution of the optimal linear generator .............................. 24

Figure 9. The no-load induced voltage of the optimal linear generator with the non-uniform Halbach array .................................................................................. 24

Figure 10. The no-load induced voltage of the optimal linear generator with the traditional uniform array ...................................................................................... 25
Nomenclature

A. Design Variables

\( N_m \) \hspace{1cm} \text{The total number of PM blocks}

\( W_m \) \hspace{1cm} \text{The magnet width}

\( L_{m1} \) \hspace{1cm} \text{The length of normally magnetized magnets}

\( L_{m2} \) \hspace{1cm} \text{The length of tilted magnetized magnets}

\( L_{m3} \) \hspace{1cm} \text{The length of horizontally magnetized magnets}

\( \theta_t \) \hspace{1cm} \text{The tilted angle for the magnet}

\( D \) \hspace{1cm} \text{The machine depth}

\( L_c \) \hspace{1cm} \text{The coil slot length}

\( W_c \) \hspace{1cm} \text{The coil slot width}

\( L_b \) \hspace{1cm} \text{The bobbin length}

\( L_{sp} \) \hspace{1cm} \text{The spacer length between the phases}

\( d_w \) \hspace{1cm} \text{The diameter of wire}

\( N_c \) \hspace{1cm} \text{The number of turns in each coil}

\( g \) \hspace{1cm} \text{The air gap length}

B. Constants and Parameters

\( J \) \hspace{1cm} \text{The current density}

\( J_{\text{max}} \) \hspace{1cm} \text{The maximum current density}

\( k_{\text{fill}} \) \hspace{1cm} \text{The fill factor}

\( \omega_{\text{mov}} \) \hspace{1cm} \text{The frequency of human body’s vertical movement during walking}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$</td>
<td>The natural frequency of the mass-spring system in the generator</td>
</tr>
<tr>
<td>$k$</td>
<td>The equivalent spring stiffness</td>
</tr>
<tr>
<td>$m$</td>
<td>The mass of moving part</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background and Motivation

Over the past few decades, portable electronics devices have been widely used in daily life. The conventional power sources for portable electronics devices, such as batteries, are reaching their limits. The increasing demand for solutions for recharging portable electronic devices outdoors has aroused renewed interest in human energy harvesting systems.

The basic transduction mechanisms that can be used for converting vibration to electricity in human energy harvesting systems include piezoelectric, electrostatic, and electromagnetic transductions [1]. Piezoelectric and electrostatic transductions are suitable for mounting on a human body since they are small and use lightweight materials. Piezoelectric generators are the simplest harvesting applications [2]. However, a microgenerator based on piezoelectric transductions was proposed in [3]; and its maximum output power was 15mW due to the high impedance of the material and the current flowing out during walking. In addition, piezoelectric materials are brittle [2].

Several electrostatic micropower generators were discussed in [4]-[6]. The electrostatics are easier to integrate without any smart materials [2],[4]. However, the output power of a microgenerator in [6] was 6.5 mW at 10 Hz; and the output power of other generators in [5] was 116 µW or less. In addition, the electrostatic generator
requires precharge voltage to operate [5].

The output power of these two types of generators does not go far enough for current portable device power levels that are typically up to 1 W [7]. In order to obtain enough power to charge portable electronic devices, electromagnetic transduction has been studied [8]. In particular, linear generators based on electromagnetic transduction are promising for human energy harvesting applications due to their simple structure, high output power, low cost, and stable output voltage.

1.2 Objective

In order to improve the power density of linear generators for human motion energy harvesting, a novel linear generator with dual Halbach magnet arrays is proposed to improve the flux density in the air-gap for a given size and weight of permanent magnet material. Different dual Halbach arrays were studied, and a novel Halbach array with nonuniform magnet blocks is proposed. Sensitivity analyses were performed to identify significant design variables for further optimization. Then, an optimization model was constructed with the objective of maximizing the output power of the linear generator. The proposed linear generator was compared with two other different linear generators for human walking motion energy harvesting. The effectiveness of the proposed methods was verified by two-dimensional finite element analysis simulation results.
Chapter 2
A Literature Review of Linear Generators for Human Energy Harvesting

In 2012, J.D. Power and Associates conducted a survey of batteries on 4G devices [8] which revealed that batteries still were not satisfying the utility of cellphones due to low battery life. The same issue occurs with other portable electronic devices. People have tried to find convenient and efficient ways to recharge portable electronic device batteries and are concerned with protecting the environment. Consumers are focused on Green energy, making the topic of a linear generator for human motion energy harvesting a hot topic.

The majority of research on linear generators for human energy harvesting is focused on using center-of-mass motion or foot horizontal motion as an energy source [10]-[22]. The oscillatory frequency of center-of-mass motion and foot horizontal motion is 2 Hz and 1.75 Hz, respectively. These body motions can drive a linear generator directly without any gear boxes. Therefore, human energy harvesting systems have simple structures and low costs.

Traditional linear generators for human energy harvesting are one degree of freedom (1-DOF) and generate energy only from a single axis. For 1-DOF harvesters, the orientation of the harvester on the body greatly impacts the output power. A two-degree-of-freedom (2-DOF) harvester consisting of two orthogonal 1-DOF harvesters has been studied in [11]. The results have shown that the 2-DOF harvester can maintain a more constant power output with rotation. Thus, 2-DOF harvesters are
more reliable than 1-DOF harvesters. Actually, three or multiple degree-of-freedom (3- or multi-DOF) energy harvesters can be constructed with 1-DOF harvesters to further reduce the effect of the harvester orientation on the output power. Since the harvesters on the body cannot be too heavy or too big to where they interfere with normal human motion, 3- or multi-DOF energy harvesters are not recommended for human energy harvesting systems.

The linear generators for human energy harvesting can also be categorized into two types: flat (planar) and tubular (cylindrical). The flat linear generator has simple structure and is cost effective. Compared to the flat linear generator, the tubular structure has a small amount of leakage flux, leading to high efficiency. The tubular linear generator is constructed with tubular permanent magnets either on the inner tubular housing or on the mover. However, the tubular linear generator has a higher manufacturing cost due to its geometrical complexity compared with a flat linear generator. A brief review of the development of linear generators for human energy harvesting is provided in this section.

In 2003, S. Turri et al. [14] proposed a tubular linear generator which was designed to hang at a person’s waistline. The permanent magnets (PMs) were assembled on the stator, and the windings were mounted on the moving part. In addition, the authors proposed that the natural motion of human walking could be modelled as a quasi-sinusoidal function. The maximum output power of the generator in [14] was 0.15 W, and its volume was 374 cm$^3$. After two years, M. Ruellan and S. Turri [16] assembled the linear generator in [14]. The average output power of the
generator prototype was about 0.014 W at 15 Hz because the spring with spring stiffness resonance at 2 Hz led to difficulties in manufacturing. The volume was 72 cm$^3$, and the weight was 0.9 kg.

In 2004, M. Duffy et al. [17] described a tubular linear generator in shoe soles, which included three separate coils wound axially on a stator tube one after another and two 15 mm disc magnets as moving parts inside the stator tube. Its volume was 3.5 cm$^3$. The measured output voltage was only 250 mV at 5 Hz.

In 2006, P. Niu et al. [10] proposed a flat linear generator for human energy harvesting to charge cell phones. The proposed topology had a PM stator with back iron, while the windings were mounted on the mover. The maximum output power achieved in the experiments was 3 W, and its volume was 96 cm$^3$. However, in order to reduce the weight of the PM material, light ceramic magnets were used in [11] which also reduced the magnetic field in the machine. The resulting power density of the machine was relatively low. The results proved that electromagnetic transduction was an effective method to convert human motion energy to electrical energy by using the proposed linear generator. The powerful center-of-mass motion was recommended as the energetic source of the linear generator.

Another tubular linear generator was presented by C.R. Saha [18] in 2008. This linear generator was placed in a rucksack and applied to charge sensors and actuators. The coils were wound on the stator tube; and the cylinder PM blocks, as the moving parts, were inside the stator tube. The maximum output power of 2.46 mW was measured, and its volume was 12.5 cm$^3$. The weight was 0.045 kg.
In 2010, a three-phase, flat linear scavenger was reported by I. Stamenkovic et al. [19]. Horizontal foot motion was used as the energy source for the proposed scavenger. The proposed topology had a stator with six PM blocks and back iron, while the windings were mounted on the mover. The output power obtained from a six-pack MOSFET rectifier was 70 mW. The volume of the scavenger was 10 cm$^3$.

One flat linear generator, which used horizontal foot motion as the energy source, was presented by P. Zeng et al. [20] in 2011. This linear generator had a moving-magnet mover with soft magnetic spacers and single-phase armature windings fixed on laminated silicon steel with slots stator. The linear generator reached 0.83 W, and the volume was 116 cm$^3$. The weight was 0.86 kg. In 2012, Z. Yang et al. [12] studied the same linear generator structure using center-of-mass human motion as the energy source and put the linear generator in a backpack. However, material for the stator changed to ferromagnetic material; and the mover spacers became iron. The obtained maximum output power was 7.2 W, the volume was 520 cm$^3$ and the weight was 3.84 kg. A significant disadvantage of the two generators described above was that the salient pole structure introduced a cogging force, which could degrade performance.

In 2012, E. Papatheou et al. [21] assembled a backpack energy harvester with hardware-in-the-loop simulation. This harvester was able to test different settings regarding the spring stiffness and damping coefficient of the backpack to optimize the system’s electrical energy output. The mover in the harvester was supported by a timing belt with two pulleys instead of springs and an armature-controlled DC motor.
was applied in the loop-controlled system to control the force on the mover. In this way, the total force on the mover could always remain equal to the ideal force. The mover could always be resonant with the human body to generate the maximum energy.

In 2013, a handheld tubular linear generator was reported by M.H. Mohammadi et al. [22]. The topology of this linear generator was similar to the one in [18] except that the moving part was changed to triple axial ring magnets. The three ring magnets were connected vertically. This type of magnet was larger and constructed the best flux linkage in the axial magnetization direction compared to other shaped magnets. The maximum output power measured at the shake frequency of 10 Hz was 1.6 W. The volume was about 25 cm$^3$, and the weight was 0.15 kg.

J. Shen [14] presented a tubular linear generator using the horizontal foot motion as the energy source in 2013, which had a moving-magnet mover and three-phase armature windings fixed on an air-core stator. Design considerations of the tubular linear generator were discussed, and the proposed generator was optimized to maximize the back electromotive force (EMF) by manually varying the design parameters in certain ranges. However, the design was not optimal. The objective of this linear generator was to charge AA batteries, and the output voltage required was higher than 1.2 V. Due to the high resistance of the windings, the output power was low at 0.23 W; and the volume was 30 cm$^3$.

In 2014, H. Huang et al. [11] proposed a 2-DOF tubular linear generator. This generator had two tubes: inner and outer. Two ring PMs were fixed at the top and
bottom of the two tubes. A cylinder PM was inside the inner tube as the mover. The coils were wound on the outer tube as the stator. This design incorporated two orthogonal 1-DOF inertial energy harvesters. The linear generator could convert the human energy associated with two-directional motions to electrical energy. The output power was 17.7 mW. The weight of the linear generator was 0.422 kg, and the volume was 336 cm$^3$.

In order to seek the relationship with size and weight of linear generator, there are two proportional parameters in table I. Weight is a major subject in the linear generator; therefore, power density is a linear relationship with weight. Table I lists some of the prototype details discussed in this paper.

**Table I. Comparison of the Human Energy Harvesting Linear Generator Prototypes from the Literature Review**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Volume (cm$^3$)</th>
<th>Mass (kg)</th>
<th>Power (W) (Shaking condition)</th>
<th>Power density (W/kg)</th>
<th>Structure</th>
<th>Energy source</th>
</tr>
</thead>
<tbody>
<tr>
<td>H. Huang et al. [8]</td>
<td>336</td>
<td>0.422</td>
<td>0.0177 (1-7Hz)</td>
<td>0.042</td>
<td>Tubular</td>
<td>Knees</td>
</tr>
<tr>
<td>C.R. Saha et al. [15]</td>
<td>14</td>
<td>0.045</td>
<td>0.00246 (2.75Hz)</td>
<td>0.0547</td>
<td>Tubular</td>
<td>Center-of-mass</td>
</tr>
<tr>
<td>M. Ruellan et al. [13]</td>
<td>36.6</td>
<td>0.9</td>
<td>0.2 (15Hz)</td>
<td>0.222</td>
<td>Tubular</td>
<td>Center-of-mass</td>
</tr>
<tr>
<td>P. Niu et al. [7]</td>
<td>903</td>
<td>2</td>
<td>0.784 (sin(12.55t) m/s)</td>
<td>0.392</td>
<td>Flat</td>
<td>Center-of-mass</td>
</tr>
<tr>
<td>I. Stamenkovic et al. [16]</td>
<td>10</td>
<td>0.096</td>
<td>0.059 (5m/s)</td>
<td>0.615</td>
<td>Flat</td>
<td>Foot Horizontal</td>
</tr>
<tr>
<td>Reference</td>
<td>Volume (cm³)</td>
<td>Mass (kg)</td>
<td>Power (W) (Shaking condition)</td>
<td>Power density (W/kg)</td>
<td>Structure</td>
<td>Energy source</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------</td>
<td>-----------</td>
<td>-------------------------------</td>
<td>----------------------</td>
<td>------------</td>
<td>----------------</td>
</tr>
<tr>
<td>P. Zeng et al.[17]</td>
<td>116</td>
<td>0.86</td>
<td>0.83 (1.75Hz)</td>
<td>0.965</td>
<td>Flat</td>
<td>Foot Horizontal</td>
</tr>
<tr>
<td>Z. Yang et al.[9]</td>
<td>520</td>
<td>3.25</td>
<td>7.2 (1.95Hz)</td>
<td>2.215</td>
<td>Flat</td>
<td>Center-of-mass</td>
</tr>
<tr>
<td>J. Shen et al. [12]</td>
<td>30</td>
<td>0.0701</td>
<td>0.23 (1m/s)</td>
<td>3.28</td>
<td>Tubular</td>
<td>Foot Horizontal</td>
</tr>
<tr>
<td>M.H. Mohammadi et al. [19]</td>
<td>41.6</td>
<td>0.15</td>
<td>1.6 (10Hz)</td>
<td>10.7</td>
<td>Tubular</td>
<td>Handheld</td>
</tr>
</tbody>
</table>
Chapter 3

Design Considerations of the Proposed Linear Generator

As shown in Fig. 1, the proposed linear generator consists of three parts: the stator, the moving part, and the supporting part. The two outside parts represent the dual Halbach magnet array stator, and the middle part represents the moving and the supporting parts. Due to the fact that the Halbach array is self-shielding, an air-core structure is used. The moving part is constructed with three-phase windings and plastic bobbins with copper coils winding around them. The moving part is suspended between two springs, which are mounted on the two ends of the case as the supporting part. The design considerations of the proposed linear generator are discussed in the following subsections.
3.1 The Dual Halbach Magnet Array Stator

The Halbach array is constructed by arranging the magnetization direction of the PM blocks to augment the magnetic field on one side of the PM array, while cancelling the field to approximately zero on the other side. The Halbach array has a much stronger field for a given size and weight of PM material than the magnet array with alternating polarity [23]. In addition, the magnetic field of the Halbach array can be concentrated without the use of a back iron; and the elimination of back iron on the stator reduces the total weight of the generator and eliminates the hysteresis and eddy current losses associated with the iron. The power density and the energy conversion efficiency can be improved.
3.1.1 Traditional Four-Piece and Eight-Piece Halbach Arrays

The configurations of the four- and eight-piece Halbach arrays [24] and their flux distributions are shown in Fig. 2. The magnetization direction is shown using red arrows in the figure. As shown in Fig. 2, the magnetization of the magnets in the four-piece array is oriented at 90° to the magnetization of the adjacent magnet; and the rotation angle is 45° in the eight-piece Halbach array.

Since the induced voltage of the linear generator depends on the normal component of the magnetic field, \( B_x \), only the normal component is studied for the four- and eight-piece Halbach arrays in this paper. To have a fair comparison between the two arrays, the air gap is set as the same value of \( g = 5 \text{mm} \); and the four- and eight-piece arrays are built with the same size square PM blocks. Figure 3 shows the magnet field flux densities \( (B_x) \) of the four- and eight-piece square block Halbach arrays at different distances (0.1, 0.25, 0.35, and 0.5\( g \)) from the magnet surface for one electrical period. It can be seen that the magnetic field flux density of the eight-piece dual Halbach array is greater than that of the four-piece array for all distances to the array surface; and the eight-piece array has better self-shielding performance. Therefore, the eight-piece dual Halbach array is employed in this paper.
3.1.2 The Proposed Nonuniform Halbach Array

Traditional Halbach arrays have uniform PM blocks, and each PM block is the same size. In this work, a novel eight-piece Halbach array with nonuniform magnet blocks is proposed. As shown in Fig. 4, $L_{m1}$, $L_{m2}$, and $L_{m3}$ represent the lengths of the
PM blocks of normally magnetized, tilted magnetized, and horizontally magnetized, respectively. The magnetization of the magnet blocks in the traditional eight-piece Halbach arrays is oriented at 45° to the magnetization direction of the adjacent magnet. In this work, the rotation angle ($\theta_t$) is also considered as one of the independent design variables of the proposed nonuniform Halbach array at the beginning of the design. Combined with the other design variables (except for $L_m$) shown in Fig. 1, the number of PM blocks ($N_m$), the machine depth ($D$) (the length in y-axis), the number of turns per phase ($N_t$), and the diameter of the copper wire ($d_w$), the total number of independent design variables is 14, as listed in Table I.

![Diagram of a linear generator](image.png)

Fig. 4. The proposed eight-piece, nonuniform Halbach array.

### TABLE II

**DESIGN VARIABLES OF THE PROPOSED LINEAR GENERATOR**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_m$</td>
<td>The total number of PM blocks</td>
</tr>
<tr>
<td>$W_m$</td>
<td>The magnet width</td>
</tr>
<tr>
<td>$L_{m1}$</td>
<td>The length of normally magnetized magnets</td>
</tr>
</tbody>
</table>
Parameter | Description
---|---
$L_{m2}$ | The length of tilted magnetized magnets
$L_{m3}$ | The length of horizontally magnetized magnets
$\theta_t$ | The tilted angle for the magnet
$D$ | The machine depth
$L_c$ | The coil slot length
$W_c$ | The coil slot width
$L_b$ | The bobbin length
$L_{sp}$ | The spacer length between the phases
$d_w$ | The diameter of wire
$N_c$ | The number of turns in each coil
$g$ | The air gap length

### 3.1.3 Sensitivity Analysis

To study how each design variable of the proposed nonuniform PM blocks affects the magnetic field generated in the air gap, sensitivity analyses were performed first. In addition, the computational cost required for further optimization can be reduced due to the design space reduction through the sensitivity analyses [26]. The sensitivity was measured by a sensitivity index that reflects the relative importance of the variable alone and is described as follows:

$$s_{si} = \frac{V_{x_i}(E_{x-i}(y|x_i))}{V(y)}$$  \hspace{1cm} (1)

where $E_{x-i}(y|x_i)$ is the average of $y$ taken over $x_{-i}$ (all factors except for $x_i$) when $x_i$ is fixed, $V_{x_i}(E_{x-i}(y|x_i))$ is the conditional variance of $E_{x-i}(y|x_i)$, and $V(y)$ is the variance of $y$. A higher sensitivity index indicates a larger influence to $y$ by $x_i$.

The variance-based sensitivity index of the magnetic field generated at a distance of 0.5g to the five design variables ($\theta_t$, $L_{m1}$, $L_{m2}$, $L_{m3}$, and $W_m$) was evaluated and the results are shown in Fig. 5. According to Fig. 5, $L_{m1}$, $L_{m3}$, and $W_m$ are significant
design variables for the linear generator optimization. The changes in $L_{m2}$ and $\theta_t$ have negligible effects on the magnetic field generated in the air gap. The values of $L_{m2}$ and $\theta_t$ can be set as constants based on other design considerations and were not selected as design variables in the subsequent optimization process. For example, $L_{m2}$ can be set to a relatively low level (e.g., 1 mm) to reduce the use of PM material; $\theta_t$ was set to 45°, the rotation angle of traditional eight-piece Halbach arrays. To compare with the linear generator presented in [13], the same type and amount (volume) of PM material was used in the proposed design. Then, the machine depth, $D$, depended on $L_{m1}$, $L_{m2}$, $L_{m3}$, and $W_m$.

Fig. 5. Sensitivity index of the average absolute value of the magnetic flux density at 0.5g.
3.1.4 The Initial Design of Nonuniform Halbach Array

After analyzing the variable parameters of the novel eight Halbach arrays, in a parametric analysis was conducted for the nonuniform Halbach arrays to determine their properties.

To fairly compare traditional Halbach arrays to nonuniform Halbach arrays, the air gaps in the latter were set to the same value of g = 5 mm. The nonuniform arrays were built to have the same volume of PM as that in the four- and eight-piece arrays. The $L_{m2}$ was set at its lowest bound to allow a larger variation of other design variables, and $\theta_t$ was set at 45° for simplicity. The three design variables, $L_{m1}$, $L_{m3}$, and $W_m$, discussed in the last subsection need to be determined.

To present the relationship of these three design variables, the parametric studies of the length-to-width ratio ($L_{m1}/W_m$) and the length ratio ($L_{m1}/L_{m3}$) of the orthogonal magnets were performed, and the results are shown in Fig. 6-7, respectively. Fig. 6 shows that the absolute average flux density increased with the increase in $W_m$, while the variation of $L_{m1}/W_m$ did not significantly change the absolute average flux density. The ratio of $L_{m1}$ and $W_m$ is 1 to maintain a relatively large flux density in the air gap. Fig. 7 shows an optimal $L_{m1}/L_{m3}$ for the nonuniform Halbach array, and the ratio was adjusted due to the volume limitations. According to the length of $L_{m3}$, the optimal ratio of $L_{m1}/L_{m3}$ changed. Therefore, the final optimal geometry structure of a nonuniform Halbach array is based on the optimization of all variables.
Fig. 6. The variation of average absolute value of the flux density at the middle airgap due to the length-to-width ratio, $L_{m1}/W_m$.

Fig. 7. The variation of average absolute value of the flux density at the middle airgap due to the orthogonal magnet length ratio, $L_{m1}/L_{m3}$. 
3.2 The Moving Part

Depending on the winding structure, the linear generator can be either single-phase or three-phase. Considering the size of capacitor required in the ac-to-dc converter, a three-phase winding structure was adopted in this design. The coil slot length, $L_c$, the bobbin length, $L_b$, and the spacer length between the phases, $L_{sp}$, are constrained by (2)-(4), according to the configuration of the Halbach array, to generate a three-phase voltage. The air gap is constrained by (5) while the distance between the coil and the array is simply set as 1 mm. The current density, $J$, is limited as (6) due to thermal constraints with limit operating temperature of the generator; and $J_{\text{max}} = 6 \, \text{A/mm}^2$ is the maximum allowable current density selected for the linear generator in this paper [26]. The coil slot space includes the net available space for the copper wires plus the slot lining or insulation. Usually, the fill factor $k_{\text{fill}}$ is constrained by (7).

$$L_c + L_b = L_{m1} + 2 \times L_{m2} + L_{m3}$$  \hspace{1cm} (2) \\
$$L_b > 1 \, \text{mm}$$  \hspace{1cm} (3) \\
$$2L_c + L_b + L_{sp} = \left(1 + \frac{1}{3}\right) \times \left(2 \times L_{m1} + 4 \times L_{m2} + 2 \times L_{m3}\right)$$  \hspace{1cm} (4) \\
$$g = W_c + 2 \, \text{mm}$$  \hspace{1cm} (5) \\
$$J \leq J_{\text{max}}$$  \hspace{1cm} (6) \\
$$30\% \leq k_{\text{fill}} \leq 60\%$$  \hspace{1cm} (7)
3.3 The Supporting Part

The human body’s vertical movement during walking can be modeled as a quasi-sinusoidal function with a frequency of about $\omega_{\text{mov}} = 2$ Hz. To yield the maximum recoverable mechanical power, the natural frequency of the mass-spring system in the generator, $\omega_0$, is designed to have a value of $\omega_{\text{mov}} = \omega_0 = \sqrt{k/m}$, where $m$ is the mass of the moving part and $k$ is the equivalent spring stiffness.
CHAPTER 4

Optimization of the Proposed Linear Generator

All independent design variables, constants, and their values or ranges for optimal design of the linear generator are listed in Table III. The optimization model can be formulated as:

\[
\max_x y(x) = \frac{U(x)^2}{R(x)}
\]  \hspace{1cm} (10)

subject to: (4)-(9)  \hspace{1cm} (11)

\[x_i \leq x_i \leq \bar{x}_i, \ i=1, 2, \ldots, 6\]  \hspace{1cm} (12)

where \(x = [L_{m1}, L_{m3}, W_m, L_c, W_c, d_w]\) is the vector of design variables, \(U(x)\) is the RMS value of induced voltage per phase, and \(R(x)\) is the winding resistance per phase. The constraints can be converted to the following form by using the Lagrangian relaxation technique [28],

\[
\max_x y(x) = \frac{U(x)^2}{R(x)} + p(x)
\]  \hspace{1cm} (13)

subject to: (12)

where \(p(x)\) is the penalty function for constraints (2)-(7). If any one of the constraints is not satisfied, a negative value of \(p(x)\) will be added to the objection function; otherwise, the value of \(p(x)\) is zero. The optimization model (12)-(13) will be used for the proposed linear generator optimization. The optimal design will then be obtained by solving the optimization problem (12)-(13) using the genetic algorithm provided by the ANSYS Maxwell [29].
TABLE III

VARIABLES AND CONSTANTS FOR THE OPTIMAL DESIGN

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Type</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{m1}$</td>
<td>Variable</td>
<td>mm</td>
<td>1-4</td>
</tr>
<tr>
<td>$L_{m3}$</td>
<td>Variable</td>
<td>mm</td>
<td>1-4</td>
</tr>
<tr>
<td>$W_{en}$</td>
<td>Variable</td>
<td>mm</td>
<td>2-4</td>
</tr>
<tr>
<td>$W_c$</td>
<td>Variable</td>
<td>mm</td>
<td>1-12</td>
</tr>
<tr>
<td>$d_w$</td>
<td>Variable</td>
<td>mm</td>
<td>0.1-1</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Variable</td>
<td>mm</td>
<td>2-10</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>Constant</td>
<td>deg</td>
<td>45</td>
</tr>
<tr>
<td>$L_{m2}$</td>
<td>Constant</td>
<td>mm</td>
<td>1</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Constant</td>
<td>N/A</td>
<td>82</td>
</tr>
</tbody>
</table>
CHAPTER 5
Simulation Results

The design optimization of the proposed linear generator based on the nonuniform Halbach array was performed using the design optimization framework. The dimensions of the optimized design of the proposed linear generator with nonuniform Halbach array are given in Table IV. As shown in Table IV, the optimal design of the proposed generator is much different from the initial design obtained from the parametric study. This is mainly because that the initial design for the nonuniform Halbach array was focused on the nonuniform Halbach array geometric construction study, and the objective is to achieve a high flux density in the air gap. In the optimal linear generator, the objective changes to: achieve a high output power by optimizing the nonuniform Halbach array combined with the windings. A finite element analysis (FEA) model of the optimized linear generator was created, and the flux distribution obtained from the FEA is shown in Fig. 8.

**TABLE IV**
PARAMETERS OF THE OPTIMAL DESIGN WITH THE PROPOSED NONUNIFORM ARRAYS

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Optimal Values</th>
<th>Design Variable</th>
<th>Optimal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_m$</td>
<td>82</td>
<td>$W_m$</td>
<td>2 mm</td>
</tr>
<tr>
<td>$L_{m1}$</td>
<td>2.75 mm</td>
<td>$L_{m2}$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$L_{m3}$</td>
<td>2.35 mm</td>
<td>$\theta_t$</td>
<td>45 deg</td>
</tr>
<tr>
<td>$D$</td>
<td>12.77 mm</td>
<td>$L_c$</td>
<td>4.96 mm</td>
</tr>
<tr>
<td>$W_c$</td>
<td>6.82 mm</td>
<td>$L_b$</td>
<td>4.53 mm</td>
</tr>
<tr>
<td>$L_{sp}$</td>
<td>10.82 mm</td>
<td>$d_w$</td>
<td>0.32 mm</td>
</tr>
<tr>
<td>$N_c$</td>
<td>315</td>
<td>$g$</td>
<td>8.821 mm</td>
</tr>
</tbody>
</table>
Fig. 8. The flux distribution of the optimal linear generator.

The no-load induced three-phase voltage waveforms of the optimal linear generator with the proposed nonuniform arrays are shown in Fig. 9.

Fig. 9. The no-load induced voltage of the optimal linear generator with the nonuniform Halbach array.
In order to show the improvement attained by applying the nonuniform eight-piece Halbach array, an optimal linear generator with the traditional eight-piece Halbach array was also created; and its dimensions are given in Table V. In both cases, the amount of PM material was the same; and the mover speed was set at 1 m/s.

TABLE V
PARAMETERS OF THE OPTIMAL DESIGN WITH THE UNIFORM ARRAYS

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Optimal Values</th>
<th>Design Variable</th>
<th>Optimal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_m$</td>
<td>82</td>
<td>$W_m$</td>
<td>2.00 mm</td>
</tr>
<tr>
<td>$L_m$</td>
<td>2.36 mm</td>
<td>$\theta_t$</td>
<td>45 deg</td>
</tr>
<tr>
<td>$D$</td>
<td>9.74 mm</td>
<td>$L_c$</td>
<td>4.91 mm</td>
</tr>
<tr>
<td>$W_c$</td>
<td>6.80 mm</td>
<td>$L_b$</td>
<td>4.53 mm</td>
</tr>
<tr>
<td>$L_{sp}$</td>
<td>10.82 mm</td>
<td>$d_w$</td>
<td>0.287 mm</td>
</tr>
<tr>
<td>$N_c$</td>
<td>391</td>
<td>$g$</td>
<td>8.8 mm</td>
</tr>
</tbody>
</table>

The no-load induced three-phase voltage waveforms of the optimized linear generator with the traditional, uniform eight-piece arrays are shown in Fig. 10.

![Fig. 10. The no-load induced voltage of the optimal linear generator with the traditional uniform array.](image-url)
It can be seen that the peak value of the induced voltage in the proposed nonuniform design is 17.5% lower than that in the uniform design. However, the winding resistances of nonuniform linear generator and uniform linear generator are 3.18 $\Omega$ and 5.481 $\Omega$, respectively, because the nonuniform design uses less total length of copper wires and larger diameter copper wires. The maximum output power of the linear generator can be estimated by

$$P_{out\_max} = \frac{3U^2}{R}$$

(12)

The calculated maximum output power of the proposed nonuniform design is 8.33% higher than that of the uniform design.

The comparison between the optimal proposed design and the linear generators presented in [10] and [13] under the same motion speed is shown in Table VI. In order to compare this design with the linear generator presented in [13], the same type and amount (volume) of PM material was used in this investigation. The optimal proposed linear generator is much smaller than the linear generator presented in [10], and the resulting power density is increased by a factor of about 25. Compared to the linear generator presented in [13], the total weight of the linear generator based on the proposed non-uniform Halbach array is decreased by 16.83% owing to the elimination of the back iron, although the weight of the copper used in the proposed linear generator is increased by 11.76%; the power produced by the proposed linear generator is increased by 522% while using the same volume of PM; and the power density is increased by 652%. 


<table>
<thead>
<tr>
<th>Design</th>
<th>Peak Induced Voltage (V)</th>
<th>Maximum Output Power (W)</th>
<th>Copper Weight (kg)</th>
<th>PM Weight (kg)</th>
<th>Steel Weight (kg)</th>
<th>Resistance per Phase (Ω)</th>
<th>Power Density (W/cm³)</th>
<th>Mover Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design in [4]</td>
<td>16.7</td>
<td>1.97</td>
<td>0.5850</td>
<td>0.9260</td>
<td>0.496</td>
<td>70.8</td>
<td>0.98</td>
<td>1.0</td>
</tr>
<tr>
<td>Design in [13]</td>
<td>1.80</td>
<td>0.23</td>
<td>0.0272</td>
<td>0.0279</td>
<td>0.015</td>
<td>19.8</td>
<td>3.28</td>
<td>1.0</td>
</tr>
<tr>
<td>Uniform Design</td>
<td>2.25</td>
<td>1.247</td>
<td>0.0358</td>
<td>0.0279</td>
<td>N/A</td>
<td>5.48</td>
<td>19.57</td>
<td>1.0</td>
</tr>
<tr>
<td>Nonuniform design</td>
<td>1.79</td>
<td>1.43</td>
<td>0.0301</td>
<td>0.0279</td>
<td>N/A</td>
<td>3.18</td>
<td>24.66</td>
<td>1.0</td>
</tr>
</tbody>
</table>
CHAPTER 6
Conclusions

A flat linear generator for human energy harvesting utilizing dual Halbach magnet arrays has been studied in this work. A novel nonuniform Halbach array has been proposed to enhance the air-gap flux density for a given amount of PM material. The optimization model has been constructed with the objective of maximizing the output power of the proposed linear generator. The optimal design has been obtained by solving the optimization problem using the genetic algorithm provided by the ANSYS Maxwell. The results have shown that the power density of the proposed linear generator has been significantly improved by using the proposed nonuniform dual Halbach array. The effectiveness of the proposed methods has been verified by 2D FEA simulation results.
References


[29] ANSYS Maxwell. [online]. Available:

http://www.ansys.com/Products/Simulation+Technology/Electronics/Electromechanical/ANSYS+Maxwell.