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Charged-Particle Multiplicity $pp\bar{p}$ Collisions at $\sqrt{s} = 1.8$ TeV

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We report on a measurement of the mean charged-particle multiplicity of jets in dijet events with dijet masses in the range 80–630 GeV/c², produced at the Tevatron in p̅p collisions with √s = 1.8 TeV and recorded by the Collider Detector at Fermilab. The data are fit to perturbative-QCD calculations carried out in the framework of the modified leading log approximation and the hypothesis of local parton-hadron duality. The fit yields values for two parameters in that framework: the ratio of parton multiplicities in gluon and quark jets, \( r = N_{\text{partons}}^{\text{charged}} / N_{\text{partons}}^{g} \), and the ratio of the number of charged hadrons to the number of partons in a jet, \( K_{\text{LPHD}}^{\text{charged}} = N_{\text{hadrons}}^{\text{charged}} / N_{\text{partons}} \).

\[ N_{\text{hadrons}} = K_{\text{LPHD}} \times N_{\text{partons}}. \]  

A simple interpretation of Eq. (1) is that each parton produced during the perturbative QCD stage picks up a color partner from the vacuum sea at the end of parton branching and turns into a hadron, so that \( K_{\text{LPHD}} = N_{\text{hadrons}} / N_{\text{partons}} \approx 1 \). Then, for charged-particles only, one expects from simple isospin counting that the constant \( K_{\text{LPHD}}^{\text{charged}} = N_{\text{hadrons}}^{\text{charged}} / N_{\text{partons}} \) should be approximately 2/3 (e.g., \( \approx 0.60 \) as suggested by [3]). In MLLA, the multiplicity \( N_{\text{partons}}^{g} \) of partons in a gluon jet of energy \( E_{\text{jet}} \), and within an opening angle \( \theta_{e} \), is a function of \( E_{\text{jet}} \sin \theta_{e} / Q_{\text{eff}} \) [1]. The multiplicity of partons in a quark jet has exactly the same energy dependence and differs from a gluon jet only by the factor \( 1/r \), predicted to be the ratio of color charges \( 1/r = C_{F} / C_{A} = 4/9 \) [4].

Recent and more accurate solutions [5–7] of the same primary set of QCD evolution equations that forms the basis of the MLLA have resulted in corrections to both \( N_{\text{partons}} \) and \( r \). Reported results for the next-to-MLLA correction factor for \( N_{\text{partons}}^{g} \) are \( F_{n\text{MLLA}} = 1.13 \pm 0.02 \) [5], \( 1.50 \pm 0.08 \) [6], and \( 1.40 \pm 0.01 \) [7]. The parameter \( r \) takes the values \( 1.75 \pm 0.05, 1.60 \pm 0.05, \) and \( 1.79 \pm 0.07 \), respectively. For all three calculations, both \( F_{n\text{MLLA}} \) and \( r \) show little energy dependence and were treated as constants in this analysis. The uncertainties in the numbers quoted above correspond to the range of dijet masses in our sample.

Experimentally, early measurements of \( r \) were consistent with 1.0 [8]. More recent results from LEP and SLAC range from 1.0 to 1.5 with typically quite small errors [9]. The spread of the results motivates an independent measurement performed by different methods and in a different environment. Analyses of charged-particle momentum spectra at LEP yield \( K_{\text{LPHD}}^{\text{charged}} = 1.28 \pm 0.01 \) [10](about twice the expected value). These measurements are obtained assuming \( F_{n\text{MLLA}} = 1 \) and \( r = 9/4 \). If one uses \( F_{n\text{MLLA}} = 1.3 \pm 0.2 \) (the range suggested by [5–7]) and \( r = 1.5 \) (the most recent measurements from LEP [9]), one arrives at \( K_{\text{LPHD}} \approx 0.67 \).

At the Fermilab Tevatron, dijet events are a mixture of gluon and quark jets. Denoting the fractions of gluon and quark jets as \( \epsilon_{g} \) and \( \epsilon_{q} = (1 - \epsilon_{g}) \), one can derive an expression for the multiplicity of charged-light partons in the mixed jets:

\[ N_{\text{hadrons}}^{\text{charged}} = K_{\text{LPHD}}^{\text{charged}} \left( \epsilon_{g} + (1 - \epsilon_{g}) \frac{1}{r} \right) F_{n\text{MLLA}} N_{\text{partons}}^{g}. \]  

The current analysis is based on 95 pb⁻¹ of p̅p collisions at \( \sqrt{s} = 1.8 \) TeV recorded by the Collider Detector at Fermilab (CDF). The CDF detector is described in detail in [11], and references therein. Here, we will focus on those elements of the detector that are directly related to this analysis: the vertex detector (VTX), the central tracking chamber (CTC), and the full set of electromagnetic and hadronic calorimeters.
The VTX is a time-projection drift chamber and determines the z position of the primary vertex (or vertices in the case of multiple p̅p interactions in the same bunch crossing). The CTC is an open-cell drift chamber designed for measuring particle trajectories. Determination of a particle’s momentum is based on the curvature of its trajectory in the solenoidal magnetic field. In our analysis, we considered particles falling in restricted cones around the jet axis and determined the angular parameters of their trajectories with the CTC.

The jet energy and direction were measured in the central lead-scintillator electromagnetic (CEM) and iron-scintillator hadronic (CHA) calorimeters. The CEM and CHA both have 2π azimuthal coverage. In pseudorapidity |η| > 1.0. The segmentation of both detectors is 15° in φ and 0.1 unit in η.

CDF defines a cone using a cone algorithm, full details can be found in [13]. The algorithm searches for cones of radius $R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} = 0.7$ around the calorimeter seed towers (any tower with transverse energy $E_T > 1$ GeV) and adds towers with $E_T > 0.1$ GeV. If two or more adjacent seed towers are found within $R = 0.7$, they are merged. The coordinates of the jet axis are calculated as $E_T$-weighted sums of the $\phi_i$ and $\eta_i$ of towers assigned to the jet. Merging continues until a stable set of clusters is found. Corrections were applied to compensate for the nonlinearity and nonuniformity of the energy response of the calorimeter, the energy deposited inside the jet cone from sources other than the parent-parton, and the parent-parton energy that radiates out of the jet cone.

Approximately 100,000 dijet events were accumulated using single-jet triggers with transverse jet energy thresholds of 20, 50, 70, and 100 GeV, the first three triggers being prescaled by 1000, 40, and 8, respectively. To select dijet events, we required the presence of two high-$E_T$ jets, well balanced in the transverse direction: $|E_{T1} + E_{T2}|/(E_{T1} + E_{T2}) < 0.15$. To avoid biases, 3- and 4-jet events were allowed as well, if the nonleading extra jets were very soft, $(E_{T1} + E_{T2})/(E_{T1} + E_{T2}) < 0.05$. Only events with both leading jets in the central region of the detector were taken into account in the data analysis. These cones collected statistically the same backgrounds as the cones around jets. The absolute scale of this correction, for the largest opening angle $\theta_0 = 0.47$ around the jet axis, was almost independent of the jet energy and amounted to about 0.5–0.6 tracks per jet.

Finally, a small fraction of tracks coming from $\gamma$ conversions that were not removed by the vertex cuts was subtracted. The Herwig Monte Carlo event generator (version 5.6) [14] was used to evaluate the number of remaining $\gamma$-conversion tracks. The scale of this correction was 0.3 (0.8) tracks per jet for the lowest (highest) dijet mass data samples (for cone size $\theta_c = 0.47$).

The major sources of systematic uncertainties were as follows (for $\theta_c = 0.47$): (a) background track removal, 6%–7%, (b) uncertainties in CTC track reconstruction efficiency, 2%–6%, (c) jet energy measurement errors including both resolution and overall scale errors, 0.4%–3%, and (d) errors in the jet direction determination based on energy deposition in the calorimeter, 0.7%–1.2%. The uncertainties from a given source are strongly correlated between different dijet mass samples. These correlations were taken into account in the data analysis.

Table I summarizes the multiplicities for the 3-jet opening angles and all dijet mass data samples. Figure 1 shows the charged track multiplicity (per jet) in a cone $\theta_c = 0.47$ as a function of the dijet mass. To show the trends, we also plotted curves corresponding to the function (2)
with different values of the ratio $r$. Equation (2) implies knowledge of the relative fractions of quark and gluon jets in our dijet samples. These fractions were extracted from the Herwig Monte Carlo with CTEQ4M [15] parton distribution functions (PDFs), as well as with CTEQ4HJ [16]. The fraction of gluon jets was found to decrease from $\epsilon_q \sim 61\%$–$63\%$ of all jets at $M_{jj} = 80$ GeV/$c^2$ to $23\%$–$26\%$ at 630 GeV/$c^2$ (the variations result from using different PDFs).

Because of the correlations between $Q_{\text{eff}}$ and $K_{\text{LPHD}}^{\text{charged}}$, average multiplicity measurements alone do not allow the extraction of all three parameters $Q_{\text{eff}}$, $r$, and $K_{\text{LPHD}}^{\text{charged}}$. Therefore, we fixed $Q_{\text{eff}} = 240$ MeV, as obtained in our studies of charged-particle momentum distribution shapes [17], and fitted the data with the function (2) for two free parameters: $r$ and the combination $K_{\text{LPHD}}^{\text{charged}} F_{n\text{MLLA}}$. The fit yielded the following results: $r = 1.7 \pm 0.3 \pm 0.0 \pm 0.0$, for the ratio of multiplicities, and $K_{\text{LPHD}}^{\text{charged}} F_{n\text{MLLA}} = 0.74 \pm 0.04 \pm 0.06 \pm 0.04$.

The first uncertainty comes from statistical and systematic experimental errors (as discussed above and summarized in Table I), the second comes from variations of $Q_{\text{eff}}$ by $\pm 40$ MeV, and the third comes from using different PDFs. The choice of $Q_{\text{eff}}$ and PDFs had little effect on the measurement of $r$. This value agrees well with the three most recent theoretical predictions mentioned above.

Assuming $F_{n\text{MLLA}} = 1.30 \pm 0.20$, the data yielded $K_{\text{LPHD}}^{\text{charged}} = 0.57 \pm 0.06 \pm 0.09$. The first uncertainty includes all statistical and systematic uncertainties discussed above, while the second comes from the theoretical uncertainty in $F_{n\text{MLLA}}$. The result is consistent with the LPHD hypothesis of approximately one-to-one correspondence between final partons and observed hadrons.

Figure 2 shows how the average charged-particle multiplicity in three restricted cones changes with $M_{jj}$ and how it compares to the Herwig Monte Carlo that uses resummed perturbative calculations similar to MLLA for parton branching and a cluster model of hadronization.

### Table I. Measured values of inclusive charged-particle multiplicity per jet for tracks falling in restricted cones with opening angles $\theta_c = 0.17$, 0.28, and 0.47. The first error is statistical and the second is total systematic uncertainty. Systematic uncertainties are strongly correlated.

<table>
<thead>
<tr>
<th>Dijet mass (GeV/$c^2$)</th>
<th>$N_{\text{events}}$</th>
<th>Mean charged-particle multiplicity per jet (Cone $\theta_c = 0.17$)</th>
<th>Mean charged-particle multiplicity per jet (Cone $\theta_c = 0.28$)</th>
<th>Mean charged-particle multiplicity per jet (Cone $\theta_c = 0.47$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>82</td>
<td>4148</td>
<td>2.9 $\pm$ 0.03 $\pm$ 0.2</td>
<td>4.5 $\pm$ 0.04 $\pm$ 0.3</td>
<td>6.1 $\pm$ 0.05 $\pm$ 0.5</td>
</tr>
<tr>
<td>105</td>
<td>1968</td>
<td>3.4 $\pm$ 0.04 $\pm$ 0.3</td>
<td>5.1 $\pm$ 0.05 $\pm$ 0.4</td>
<td>6.9 $\pm$ 0.06 $\pm$ 0.5</td>
</tr>
<tr>
<td>140</td>
<td>3378</td>
<td>4.0 $\pm$ 0.04 $\pm$ 0.3</td>
<td>5.8 $\pm$ 0.05 $\pm$ 0.4</td>
<td>7.5 $\pm$ 0.05 $\pm$ 0.6</td>
</tr>
<tr>
<td>182</td>
<td>12058</td>
<td>4.9 $\pm$ 0.04 $\pm$ 0.3</td>
<td>6.8 $\pm$ 0.04 $\pm$ 0.4</td>
<td>8.7 $\pm$ 0.04 $\pm$ 0.6</td>
</tr>
<tr>
<td>229</td>
<td>31406</td>
<td>5.2 $\pm$ 0.04 $\pm$ 0.4</td>
<td>7.3 $\pm$ 0.04 $\pm$ 0.5</td>
<td>9.4 $\pm$ 0.05 $\pm$ 0.6</td>
</tr>
<tr>
<td>293</td>
<td>23206</td>
<td>6.0 $\pm$ 0.05 $\pm$ 0.4</td>
<td>8.2 $\pm$ 0.05 $\pm$ 0.5</td>
<td>10.3 $\pm$ 0.05 $\pm$ 0.7</td>
</tr>
<tr>
<td>378</td>
<td>7153</td>
<td>6.7 $\pm$ 0.06 $\pm$ 0.5</td>
<td>8.9 $\pm$ 0.06 $\pm$ 0.7</td>
<td>11.3 $\pm$ 0.09 $\pm$ 1.0</td>
</tr>
<tr>
<td>488</td>
<td>1943</td>
<td>7.4 $\pm$ 0.08 $\pm$ 0.6</td>
<td>9.7 $\pm$ 0.09 $\pm$ 0.8</td>
<td>12.2 $\pm$ 0.10 $\pm$ 1.0</td>
</tr>
<tr>
<td>629</td>
<td>416</td>
<td>7.5 $\pm$ 0.14 $\pm$ 0.7</td>
<td>9.6 $\pm$ 0.16 $\pm$ 0.9</td>
<td>12.5 $\pm$ 0.18 $\pm$ 1.3</td>
</tr>
</tbody>
</table>

![Figure 1](https://example.com/fig1.png)

**FIG. 1.** Average multiplicity of charged-particles per jet within a cone of size $\theta_c = 0.47$ in dijet events (points with error bars) vs dijet mass. A set of MLLA curves (normalized to the first data point) correspond to different values of $r$ (from top to bottom $r = 1, 1.2, 1.4, 1.6, 1.8, 2.0,$ and $2.25$). The two-parameter MLLA fit is represented by the solid line in the inset.

![Figure 2](https://example.com/fig2.png)

**FIG. 2.** Average multiplicity of charged-particles within cones $\theta_c = 0.17, 0.28$, and 0.47 in dijet events (symbols with error bars) compared to the Herwig predictions including detector simulation (lines), scaled by a factor of 0.89. Data errors are dominated by systematic uncertainties.
The error bars are statistical and the systematic uncertainties are added in quadrature. Herwig was found to be above the data by approximately 11%. A fit of the data to the Herwig predictions, where the overall Herwig normalization was treated as a free parameter $N$, and which took into account all systematic errors and their correlations, resulted in $N = 0.89 \pm 0.06$ (illustrated in Fig. 2).

In summary, we have measured the inclusive charged-particle multiplicity in dijet events for a wide range of dijet masses $80 - 630 \text{ GeV}/c^2$. The data were compared to calculations carried out in the framework of the modified leading log approximation complemented with the hypothesis of local parton-hadron duality. Assuming that multiplicity evolves with energy as prescribed by MLLA, we have fit two parameters of the model and found the ratio of parton multiplicities in gluon and quark jets $r = N_{\text{partons}}^{g-jet}/N_{\text{partons}}^{q-jet} = 1.7 \pm 0.3$ and the LPHD conversion constant $k_{\text{LPHD}} = 0.57 \pm 0.11$. The Herwig Monte Carlo was found to reflect the major trends observed in data, although an overall scaling of the Monte Carlo multiplicities by a factor of 0.89 is preferred.

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[1] Yu. Dokshitzer and S. Troyan, in Proceedings of the XIX Winter School of LNPI (LNPI, Leningrad, 1984), Vol. 1, p. 144; A.H. Mueller, Nucl. Phys. B213, 85 (1983); B241, 141(E) (1984); Yu. L. Dokshitzer, V.A. Khoze, and S.I. Troyan, Int. J. Mod. Phys. A 7, 1875 (1992); Z. Phys. C 55, 107 (1992). In MLLA, the parton multiplicity in a gluon jet is given by $N_{\text{partons}}^{g-jet} = \Gamma(B)(z/2)^{-B+1}I_{B+1}(z)$, with $z = \sqrt{16N_{e}b/(\ln(E_{jet}\sin\theta/Q_{eff})}$, and where $I_{B+1}(z)$ is the modified Bessel function of order $B + 1$. For the number of colors $N_{c} = 3$ and the number of flavors of light quarks $n_{f} = 3$ used in this analysis, $B = 101/81$ and $b = 9$.


[3] A straightforward estimate of the fraction of charged-particles with respect to all particles in jets can be obtained by measuring the energy fraction carried by charged-particles. It was reported to be around $0.61 \pm 0.02$; see, e.g., JADE Collaboration, W. Bartel et al., Z. Phys. C 9, 315 (1981); CELLO Collaboration, H.J. Behrend et al., Phys. Lett. B 113, 427 (1982); TASSO Collaboration, M. Althoff et al., Z. Phys. C 22, 307 (1984); HRS Collaboration, D. Bender et al., Phys. Rev. D 31, 1 (1985).


[12] The pseudorapidity $\eta$ is defined as $-\ln[\tan(\theta/2)]$, where $\theta$ is the polar angle measured relative to the outgoing proton beam. The transverse energy $E_T$ is defined as $E \sin \theta$.


