
Yijun Lu
University of Nebraska-Lincoln, yijlu@cse.unl.edu

Xueming Li
University of Nebraska-Lincoln, xli@cse.unl.edu

Hong Jiang
University of Nebraska-Lincoln, jiang@cse.unl.edu

Follow this and additional works at: http://digitalcommons.unl.edu/csetechreports

Part of the Computer Sciences Commons

This Article is brought to you for free and open access by the Computer Science and Engineering, Department of at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in CSE Technical reports by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.

Yijun Lu¹, Xueming Li²,¹, and Hong Jiang¹

¹Department of Computer Science and Engineering, University of Nebraska-Lincoln
{yijlu, jiang}@cse.unl.edu
²School of Computer Science and Engineering, Chongqing University, China
xli@cse.unl.edu

Abstract

With the increased popularity of replica-based services in distributed systems such as the Grid, consistency control among replicas becomes more and more important. To this end, IDF (Inconsistency Detection Framework), a two-layered overlay-based architecture, has been proposed as a new way to solve this problem—instead of enforcing a predefined protocol, IDF detects inconsistency in a timely manner when it occurs and resolves it based on applications’ semantics.

This paper presents a comprehensive analytical study of IDF to assess its performance and provide insight into its design. More specifically, it develops an analytical model to characterize IDF. Based on this model, we evaluate the successful rate of inconsistency detection within the top layer, which directly impacts the performance of IDF. In addition, this model helps derive a unified formula to characterize a wide range of applications, providing practitioners and protocol designers with quantitative insights into the IDF design that can be potentially optimized to specific applications.

1. Introduction

With the increased popularity of Internet-scale distributed systems, such as Grid and wide-area e-business applications, replica-based systems have gained momentum. With multiple replicas, the system can improve both the availability—a user can access a nearby copy—and scalability—there is no bottleneck and single point of failure [10]. However, with the presence of multiple replicas, consistency control becomes an important issue.

Conventionally, the consistency problem is solved as follows: before the system starts to run, the administrator deploys a pre-defined consistency protocol, such as an optimistic protocol [4, 9] or strong consistency protocol [1]. However, in an environment where multiple applications with different consistency requirements run concurrently, this simple approach is not versatile enough: sometimes a pre-defined protocol is insufficient, while at other times it can be overkill [2, 6, 7].

IDF (Inconsistency Detection Framework) provides an alternative [6, 7]. Instead of deploying a pre-defined consistency protocol, IDF lets the system run and detect any inconsistency of the system in a timely manner. When an inconsistency is detected, IDF resolves the inconsistency based on applications’ semantics: it neglects the inconsistency if the consistency level is still acceptable and resolves the inconsistency otherwise.

In IDF, timely inconsistency detection is the key because it ensures the system’s performance [7]. IDF achieves timely detection through a two-layer system infrastructure. The top layer includes all the hot writers and the bottom layer includes all the network nodes. In the best case, all the inconsistency would be detected by the top layer, which is significantly faster than the bottom layer.

While previous research has shown that IDF achieves the design goal with minimal bandwidth cost [7]—it can be supported by dial-up connections—and the semantic-based inconsistency resolution works with real applications [8], it is still not clear what the precise performance of IDF would be and how a practitioner can adjust or optimize the design for specific applications when adopting IDF. Addressing these issues will be the focus of this paper.

In particular, this paper tries to answer two questions. First, what is the successful rate at which an
inconsistency can be detected by the top layer, which in turn results in much faster detection? Second, how can practitioners relate the successful rate to a particular application, so that they can determine how to adopt or even optimize their IDF designs for specific applications?

We approach these questions by first developing an analytical model to characterize the main principles of IDF, and then deriving the successful rate of inconsistency detection by the top layer in all possible scenarios. Based on this information, a unified formula is then derived to relate these success rates to specific applications.

The rest of the paper is organized as follows. Section 2 presents an overview of IDF. Then Section 3 presents the analytical model and analyzes the success rate of the top-layer detection by considering all possible scenarios. Section 4 derives a unified formula to explore the implications of IDF design to particular applications and offers insight into possible ways to fine tune IDF to suit particular applications. Finally, Section 5 concludes the paper.

2. An Overview of IDF

2.1. The work flow

A logical diagram of IDF is shown in Figure 1. In this framework, multiple applications share data and services through the support of the Internet-scale middleware and inconsistencies among them are detected by the detector. Upon detection, the detector consults with the inconsistency-level monitor (step 1 and step 2) that reflects the consistency requirements of specific applications before any reaction is initiated. Based on applications’ semantics, if the inconsistency is tolerable, the detector does not react; otherwise, the detector informs the inconsistency resolution module to resolve this inconsistency (step 3).

The arrow from the middleware to the detector module signifies that the detector gets information from the middleware, and the arrow from the inconsistency resolution module to the middleware implies that the module can influence the middleware. The two arrows between the consistency-level module and the middleware indicate that the former can obtain the consistency levels for applications from the middleware and potentially help the applications adjust their consistency levels.

As we can see, the key to this framework is the timely inconsistency detection mechanism, which will be discussed next.

Figure 1. Architecture of the Inconsistency Detection Framework

2.2. Timely inconsistency detection

The basic idea of a timely detection is to build an overlay on top of the underlying network based on nodes’ updating history. Because the top layer is based on nodes’ updating history, or updating temperature, it is referred as “temperature overlay”. The bottom layer of gossip-based inconsistency detection [3] is used as a backup and only triggered when the top layer does not find any inconsistency.

In the temperature overlay, each node tracks its own updating history and exchanges this information with others through the RanSub [5] protocol periodically. When a node commits an update, this update is propagated in the temperature overlay in such a way that the nodes that update this file most frequently are visited first. The rationale behind this design is that a user usually works on a file for a certain period of time. For example, he/she may edit a report for 10 minutes, then debugs, thus updates, a C++ file for 20 minutes.

Protocol-wise, the top layer is formed by frequently running RanSub to distribute nodes’ updating history. Overtime, a top layer for each file will be formed and will keep evolving. When a node starts to modify a file for the first time, it will make its update visible to a random set of nodes, thus increasing the chance that other simultaneous writers will be aware of it. For more detailed description of the protocol, please refer to [6, 7].

2.3. How to analyze IDF?

A key measure of the performance of IDF is the success rate that an inconsistency can be detected by the top layer, thus avoiding triggering the bottom layer that is much slower. If the success rate is high, IDF can use top layer to detect most inconsistency in a timely manner, which translates into better performance—faster detection.

Another important performance issue is the impact of different applications. Since different applications
may exhibit difference updating patterns as well as user distributions, it is highly likely that, for different applications, the performance of IDF can be different. To give practitioners better understanding of how to best benefit their particular applications from adopting IDF, we need to put the performance of IDF in the context of a wide range of applications.

Thus, we will analyze IDF from two aspects: its success rate of the top-layer detecting an inconsistency and how to translate this success rate into positive performance impact on real applications.

3. The analytical model

3.1 Assumptions and definitions

The goal of this analysis is to derive the success rate of the top-layer inconsistency detection. In this analysis, we assume that there are \( n \) nodes in this system and, after a warm-up period, the top layer associated with a given file \( f \) has been formed. This can be visually represented by Figure 2.

For the purpose of analysis and simplicity, we assume that there are two concurrent writers \( A \) and \( B \) for file \( f \), and each writer can be from either top layer or bottom layer. While there can potentially be more than two concurrent writers, the analysis can be simplified to the case of two writers without the loss of generality because, as far as inconsistency detection is concerned, it is a matter of two writers—the writer that triggers the detection in the first place and whether its inconsistency (in the form conflicting updates) is detected when its update conflicts with that from the first concurrent writer, thus all other concurrent (but later) writers do not matter in this case.

Now let us first define some terms and parameters that we will use throughout the rest of the paper, starting with the following seven events.

1. \( E \): IDF fails to detect the inconsistency between \( A \) and \( B \) in the top layer.
2. \( e_i \): writers \( A \) and \( B \) are both from the top layer.
3. \( e_2 \): one writer comes from the top layer, and the other comes from the bottom layer.
4. \( e_3 \): writer is from the top layer, and the other comes from the bottom layer.
5. \( d_1 \): IDF fails to detect inconsistency between \( A \) and \( B \) in the top layer given event \( e_1 \).
6. \( d_2 \): IDF fails to detect inconsistency between \( A \) and \( B \) in the top layer given event \( e_2 \).
7. \( d_3 \): IDF fails to detect the inconsistency in the top layer given event \( e_3 \).

Figure 2. An abstract view of the two-layer system for a given file

It must be noted that, when we say that IDF fails to detect inconsistency in the top layer, we do not mean that IDF cannot detect the inconsistency. Actually, IDF can detect all the inconsistency eventually through the bottom layer, albeit it will be much slower than the detection in top layer. Thus, if IDF fails to detect an inconsistency in the top layer and the bottom layer has to be triggered, at the expense of performance, not the correctness of IDF.

As a measure of the performance of IDF, we denote the probabilities of several events as follows.

1. The overall success rate of the top layer detecting the inconsistency is denoted as \( P_{\text{succ}} \).
2. The probability of event \( E \) is denoted as \( P \).
3. The probability of event \( e_i \) \((i = 1, 2, 3)\) occurring is denoted by \( h_i \).
4. The probability of event \( d_i \) \((i = 1, 2, 3)\) occurring is denoted by \( p_i \).

Clearly, \( P_{\text{succ}} \) is the overall success rate of the top layer detecting an inconsistency. Intuitively, we have

\[
P_{\text{succ}} = 1 - P = 1 - \sum_{i=1}^{3} h_i p_i
\]  

(1)

3.2. The analysis

In this section, we analyze the performance of IDF by deriving the formulas for \( p_i \) \((i = 1, 2, 3)\)—the failure rates—and use these values as an indicator of the performance of IDF (success rate = 1 - failure rate). We need to note that the value of \( p_i \) is an abstract value. In other words, these values do not directly relate to particular applications. In order to translate \( p_i \) to the performances with regard to particular applications, we need to derive \( P_{\text{succ}} \) as indicated in formula (1) in which \( h_i \) \((i = 1, 2, 3)\) is determined by the applications’ characteristics. The derivation of \( P_{\text{succ}} \) will be presented in Section 4.

3.2.1. Derivation of \( p_i \)
Recall that \( p_1 \) is the probability of event \( d_1 \), which in turn implies that event \( e_1 \) happens. When event \( e_1 \) happens, concurrent writers \( A \) and \( B \) are both from top layer. According to the system design [7], the top layer is able to detect inconsistency whenever the two writers can find each other. In this case, they can find each other through the top layer, which is visible to both of them. Thus the probability that the system fails to detect the inconsistency is 0. So we have the:

\[
p_1 = p(d_1) = 0
\]

### 3.2.2. Derivation of \( p_2 \)

Note that \( p_2 \) is the probability of event \( d_2 \), which in turn implies event \( e_2 \). In this case, one of the two writers comes from the top layer and the other is from the bottom layer. Without loss of generality, let us suppose that \( B \) is from the top layer while \( A \) is from the bottom layer.

Let us assume that there is a set \( S_{\text{exist}} \) of \( n_1 \) nodes in the current top layer, then, from the assumption, we know that \( B \) belongs to the set \( S_{\text{exist}} \) and \( A \) is from the bottom layer. After \( A \) starts the update operation, its update will be distributed to certain nodes in the network overtime. Without loss of generality, we assume that there is a set \( S_{\text{new}} \) of \( n_2 \) nodes that have learned that \( A \) has become an active writer of the file \( f \) by the time both \( A \) and \( B \) become concurrent writers.

Because there is no particular requirement of these \( S_{\text{new}} \) nodes, it is possible that, probabilistically speaking, some nodes in \( S_{\text{new}} \) are from \( S_{\text{exist}} \). If sets \( S_{\text{exist}} \) and \( S_{\text{new}} \) do not intersect, the top layer will fail to detect this inconsistency because \( A \) and \( B \) cannot meet each other and the bottom layer has to be triggered (Figure 3(a)). On the other hand, if sets \( S_{\text{exist}} \) and \( S_{\text{new}} \) intersect, this inconsistency can be detected in the top layer because, according to the protocol, as long as \( A \) and \( B \) can meet each other, the inconsistency can be detected (Figure 3(b)).

Assuming that there are a total of \( n \) nodes in the system, we have

\[
p_2 = p(d_2) = \frac{C_n^m C_{n-m}^{n_2}}{C_n^{m_1} C_{n-m}^{n_2}}
\]

To calculate \( p_2 \), we use the Sterling formula, which states that

\[
n! \approx \sqrt{2\pi n} e^{-n} n^n
\]

Thus we have

\[
p_2 = p(d_2) \approx \frac{(n - n_1)(n - n_2)}{n(n - n_1 - n_2)} \frac{(n - n_1)^{n_1-n_2}}{n_2^{n_2-n_1}}
\]

### 3.2.3. Derivation of \( p_3 \)

The value of \( p_3 \) is the probability of event \( d_3 \), which in turn implies the occurrence of event \( e_{3w} \), meaning that both \( A \) and \( B \) are from the bottom layer. Following the same analysis in Section 3.2.2, we can assume the existence of three sets \( S_{\text{exist}}, S_{\text{new}} \) and \( S_{\text{new}} \), because both \( A \) and \( B \) are from the bottom layer.

Depending on whether the three sets intersect with each other, there are six cases that need to be considered. The six cases are illustrated in Figure 5 (from 4(a) to 4(f)).

For simplicity of analysis, we use \( n_1, n_2 \) and \( n_3 \) to denote the sizes of \( S_{\text{exist}}, S_{\text{new}} \), and \( S_{\text{new}} \), respectively. According to the IDF protocol, the top layer will fail to detect any inconsistency in cases (a), (b) and (c); and will be able to detect inconsistency in the remaining cases (d, e, and f) where \( A \) and \( B \) can meet each other eventually.

Let \( g_1, g_2 \) and \( g_3 \) denote the probabilities of cases (a), (b) and (c), respectively. Then we have

\[
g_1 = \frac{C_n^{m_1} C_{n-m_1}^{n_2} C_{n-m_2}^{n_3}}{C_n^{m_1} C_{n-m_1}^{n_2} C_{n-m_2}^{n_3}}
\]

\[
g_2 = \frac{C_n^{m_1} \left[ C_{n-m_1}^{n_2} C_{n-m_1}^{n_3} - C_{n-m_1}^{n_2} C_{n-m_1}^{n_3} \right]}{C_n^{m_1} C_{n-m_1}^{n_2} C_{n-m_1}^{n_3}}
\]

\[
g_3 = \frac{C_n^{m_1} C_{n-m_1}^{n_2} C_{n-m_1}^{n_3}}{C_n^{m_1} C_{n-m_1}^{n_2} C_{n-m_1}^{n_3}}
\]

\[1\] For the purpose of easy calculation, we can use log operation, such as

\[
\log p_2 = \frac{1}{2} \left( \log(n - n_1) + \log(n - n_2) - \log n - \log(n - n_1 - n_2) \right) + \left( (n - n_1) \log(n - n_1) + (n - n_2) \log(n - n_2) \right) - (n \log n + (n - n_1)(n - n_2) \log(n - n_1 - n_2))
\]

At this point, \( \log p_2 \) is very easy to calculate and \( p_2 \) can be calculated accordingly.
To simplify the formula, we can assume that $n_2$ is equal to $n_3$, which is reasonable for analysis purpose because $A$ and $B$ have equal status and behave similarly. Then we have

$$g_3 = \frac{C_n^{n_1} C_n^{n_2} C_n^{n_3} - C_n^{n_1} C_n^{n_2} C_n^{n_3}}{C_n^{n_1} C_n^{n_2} C_n^{n_3}}$$

Finally, we have

$$p_3 = p(d_3) = g_1 + g_2 + g_3$$

(4)

To calculate the final value of $p_3$, we can approximate the values of $g_1$, $g_2$, and $g_3$ by the Sterling formula and log computation as what we have done in Section 3.2.2.

**4. Application characteristics and design implications**

In this section, we explore the performance and design implications of IDF to specific applications. In particular, we relate the failure rate of the top-layer detection (the values of $p_1$, $p_2$, and $p_3$) to real applications by deriving a unified formula. We then show how to adjust two parameters in the unified formula to reflect a particular application’s user distribution, as well as usage pattern. In this way, a practitioner can adjust certain IDF protocol parameters to potentially optimize the IDF performance for his or her particular applications.

As discussed in Section 3, to derive a unified formula for IDF as a whole, $P_{\text{succ}}$, thus $P$, has to be derived (see formula (1) in Section 3). In turn, we have to derive $h_i$ ($i = 1, 2, 3$) first ($h_i$ is the probability that event $d_i$ occurs).

This analysis is conducted in three steps. In the first step, we assume that all nodes have the same probability of issuing the next update request for a particular file $f$, which implies that the updating operation is truly random and uniform. In the second step, we relax this assumption and consider different updating patterns. In the third and last step, we derive a unified formula to reflect the characteristics of particular applications.

Finally, we discuss the performance and design implications to specific applications that can help
practitioners choose the best IDF parameters to suit their particular needs.

4.1. Random and uniform updates

In step 1, we assume that all nodes have an equal probability of updating, thus the probability of a writer coming from the top layer is decided by the relative size of the top layer. For the same reason, the probability of a writer coming from the bottom layer is decided by the relative size of the bottom layer.

In practice, if all nodes in the top layer have more update requests than those in the bottom layer, updating requests are more likely from bottom layer. This is possible because, while the current top layer reflects the history, it is still possible that the future pattern is different from the history. For example, if there is a forum in which its user base keeps changing and number of requests from new members is larger than that from current members, $F_2$ would be larger than $F_1$.

After incorporating the frequencies of updating operations of the top layer and bottom layer nodes, the frequency of event $d_1$ can be expressed as $F_1^2 C_n$, the frequency of event $d_2$ as $F_1 F_2 C_n^1 C_{n-n_1}$, and the frequency of the event $d_3$ as $F_2^2 C_{n-n_1}^2$. Hence, we have

$$h_1 = \frac{F_1^2 C_n}{F_1^2 C_n^2 + F_1 F_2 C_n^1 C_{n-n_1} + F_2^2 C_{n-n_1}}$$

$$h_2 = \frac{F_1 F_2 C_n^1 C_{n-n_1}}{F_1^2 C_n^2 + F_1 F_2 C_n^1 C_{n-n_1} + F_2^2 C_{n-n_1}}$$

$$h_3 = \frac{F_2^2 C_{n-n_1}}{F_1^2 C_n^2 + F_1 F_2 C_n^1 C_{n-n_1} + F_2^2 C_{n-n_1}}$$

4.2. Non-uniform updates between two layers

In this step, we relax the random and uniform assumption made in Step 1 above by assuming that the updating frequency of the nodes in the top layer is $F_1$ and that of the nodes in the bottom layer is $F_2$. However, it is implied that within each layer, all nodes are equally likely to issue update requests for a given file.

In practice, if $F_1$ is larger than $F_2$, it means that nodes in the top layer have more update requests than the nodes in the bottom layer, which is the case for applications like online gaming in which active gamers keep playing the game for a long period of time (thus they will form a top layer and keep issuing updating requests).

If $F_2$ is larger than $F_1$ instead, it means that new updating requests are more likely from bottom layer. This is possible because, while the current top layer...
consideration. The two aspects, their meanings, and their relationships with the parameter of the formula for \( h_i \) (\( i = 1, 2, 3 \)) are summarized as follows.

- **User distribution:** This aspect characterizes the distribution of the active writers, i.e., how many users are active writers in the system. This aspect directly affects the size of top layer (the \( n_1 \) value).
- **Usage pattern:** This aspect characterizes where the new updates are most likely coming from. For example, if the updating operations are highly concentrated on a small group of dedicated users, then most new updates would come from the top layer; otherwise, more updates would come from the bottom layer. This aspect determines the value of \( f_1 \), as well as \( f_2 \) because \( f_2 = 1 - f_1 \).

To formally represent the two aspects in the formulas, we do the following substitutions. First, we use \( \alpha \) to substitute \( n/n \) and \( n_1 \) in the formulas can be then represented by \( n \alpha \). Second, we use \( \beta \) to substitute \( f_1 \) and \( (1 - \beta) \) to substitute \( f_2 \). Please note that now both \( \alpha \) and \( \beta \) are in the range of \((0, 1)\). Given the total number of nodes in the system—the value of \( n \)—the performance of IDF for different applications can be obtained by adjusting the \( \alpha \) and \( \beta \) values.

Now we can finally represent \( h_1 \), \( h_2 \), and \( h_3 \) as

\[
\begin{align*}
    h_1 &= \frac{\beta^2 C_{a}^{2} + \beta(1-\beta)C_{a}^{1}C_{n-\alpha}^{1} + (1-\beta)^2 C_{n-\alpha}^{2}}{\beta^2 C_{a}^{2} + \beta(1-\beta)C_{a}^{1}C_{n-\alpha}^{1} + (1-\beta)^2 C_{n-\alpha}^{2}} \\
    h_2 &= \frac{\beta(1-\beta)C_{a}^{1}C_{n-\alpha}^{1}}{\beta^2 C_{a}^{2} + \beta(1-\beta)C_{a}^{1}C_{n-\alpha}^{1} + (1-\beta)^2 C_{n-\alpha}^{2}} \\
    h_3 &= \frac{(1-\beta)^2 C_{n-\alpha}^{2}}{\beta^2 C_{a}^{2} + \beta(1-\beta)C_{a}^{1}C_{n-\alpha}^{1} + (1-\beta)^2 C_{n-\alpha}^{2}}
\end{align*}
\]

Now that we have \( h_1 \), \( h_2 \), and \( h_3 \), finally we get a unified formula to evaluate the performance of IDF based on formula (1) in Section 3.1. This unified formula can reflect the different characteristics of particular applications by adjusting the values of \( \alpha \) and \( \beta \), which are in the range of \((0, 1)\).

### 4.4. Results and design implications

In this section, we show results from the analytical model and offer insights about how much benefit particular applications can gain from adopting IDF and, more importantly, how a practitioner can tune the IDF’s parameters to tailor it for his or her particular needs.

First of all, a set of results based on the formulas we derived are summarized in Table 1 that lists several important variables including the detection success rate. From this table, we can see that the lowest success rate is 91.4% while in the vast majority cases the rate is more than 98%, which we believe is very encouraging because it states that the top layer is indeed able to detect most inconsistencies.

Now, we investigate how the parameters of IDF can affect the system performance for different applications. In the four parameters we discussed—\( n_1 \), \( n_2 \), \( \alpha \), and \( \beta \)—\( n_1 \) and \( \alpha \) correlate \((n_1 = n^* \alpha)\). Besides, \( \beta \) is entirely application-dependent. However, \( \alpha \) is adjustable because the protocol can lower the threshold of the required temperature for the top layer participants, which in turn can increase the size of the top layer; \( n_2 \) is also adjustable because it is a parameter in IDF.

Now, we investigate the impact of the two values, \( \alpha \) and \( n_2 \), as follows. We first fix \( \alpha \) and \( \beta \), then calculate \( Psucc \) based on different values of \( n_2 \). The result is shown in Figure 5(a). Two \((\alpha, \beta)\) pairs and four different \( n_2 \) values are used in this calculation.

Then we fix \( n_2 \) and \( \beta \) and calculate \( Psucc \) based on different values of \( n_1 \). The result is shown in Figure 5(b). Two \((n_2, \beta)\) pairs and four different \( \alpha \) values are used in this calculation.

From the two figures, we can see that the success rate keeps increasing with the size of \( n_1 \) and \( n_2 \) values. This makes sense because, with the increased top layer size \((n_1)\) and the limited broadcasting size of a new writer \((n_2)\), there are more chances for concurrent writers to meet. But this comes at a cost as well: slower detection delay due to the larger top layer size (in the case of increased size of \( \alpha \)) and increased network bandwidth overhead (in the case of increased size of \( n_2 \)).

Based on this information, the practitioners can fine tune the parameters as follows. If the system has enough bandwidth available, they can increase the value of \( n_2 \); if the response time of IDF is better than the required value, it is also possible that they can increase the value of \( \alpha \). While simple, we believe that this suggestion can at least help practitioners to tune IDF in the best way to achieve the best performance of their particular applications.

### 5. Conclusions

In this paper, we presented an analytical study of IDF, an inconsistency detection framework. More specifically, it developed an analytical model to characterize IDF and calculates the success rate of the top-layer detecting an inconsistency (reflected through the failure rate), which directly impacts the
Figure 5. Tuning IDF parameters with n = 1000

Table 1: Success rate (in percentage) when n is 1000 with 18 sets of parameter values

<table>
<thead>
<tr>
<th>( n_1 = n + a )</th>
<th>( n_2 )</th>
<th>( \beta )</th>
<th>( \beta_x )</th>
<th>( \beta_y )</th>
<th>( h_1 )</th>
<th>( h_2 )</th>
<th>( h_3 )</th>
<th>( P )</th>
<th>( P_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
<td>50</td>
<td>0.2</td>
<td>35.487</td>
<td>4.252</td>
<td>0.002</td>
<td>1.011</td>
<td>98.986</td>
<td>4.586</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>50</td>
<td>0.5</td>
<td>35.487</td>
<td>4.252</td>
<td>0.038</td>
<td>3.924</td>
<td>96.038</td>
<td>5.476</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>50</td>
<td>0.8</td>
<td>35.487</td>
<td>4.252</td>
<td>0.542</td>
<td>13.971</td>
<td>85.487</td>
<td>8.593</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>80</td>
<td>0.2</td>
<td>18.556</td>
<td>0.032</td>
<td>0.002</td>
<td>1.011</td>
<td>98.986</td>
<td>0.220</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>80</td>
<td>0.5</td>
<td>18.556</td>
<td>0.032</td>
<td>0.038</td>
<td>3.924</td>
<td>96.038</td>
<td>0.759</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>80</td>
<td>0.8</td>
<td>18.556</td>
<td>0.032</td>
<td>0.542</td>
<td>13.971</td>
<td>85.487</td>
<td>2.620</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>50</td>
<td>0.2</td>
<td>7.198</td>
<td>1.004</td>
<td>0.017</td>
<td>2.566</td>
<td>97.417</td>
<td>1.163</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>50</td>
<td>0.5</td>
<td>7.198</td>
<td>1.004</td>
<td>0.245</td>
<td>9.510</td>
<td>90.245</td>
<td>1.590</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>50</td>
<td>0.8</td>
<td>7.198</td>
<td>1.004</td>
<td>2.968</td>
<td>28.772</td>
<td>68.260</td>
<td>2.756</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>80</td>
<td>0.2</td>
<td>1.385</td>
<td>0.003</td>
<td>0.017</td>
<td>2.566</td>
<td>97.417</td>
<td>0.038</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>80</td>
<td>0.5</td>
<td>1.385</td>
<td>0.003</td>
<td>0.245</td>
<td>9.510</td>
<td>90.245</td>
<td>0.134</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>80</td>
<td>0.8</td>
<td>1.385</td>
<td>0.003</td>
<td>2.968</td>
<td>28.772</td>
<td>68.260</td>
<td>0.400</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>50</td>
<td>0.2</td>
<td>0.484</td>
<td>0.064</td>
<td>0.072</td>
<td>5.265</td>
<td>94.663</td>
<td>0.084</td>
</tr>
<tr>
<td>13</td>
<td>100</td>
<td>50</td>
<td>0.5</td>
<td>0.484</td>
<td>0.064</td>
<td>0.591</td>
<td>18.018</td>
<td>80.991</td>
<td>0.133</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>50</td>
<td>0.8</td>
<td>0.484</td>
<td>0.064</td>
<td>9.387</td>
<td>42.667</td>
<td>47.947</td>
<td>0.222</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>80</td>
<td>0.2</td>
<td>0.015</td>
<td>0.060</td>
<td>0.072</td>
<td>5.265</td>
<td>94.663</td>
<td>0.001</td>
</tr>
<tr>
<td>16</td>
<td>100</td>
<td>80</td>
<td>0.5</td>
<td>0.015</td>
<td>0.060</td>
<td>0.991</td>
<td>18.018</td>
<td>80.991</td>
<td>0.003</td>
</tr>
<tr>
<td>17</td>
<td>100</td>
<td>80</td>
<td>0.8</td>
<td>0.015</td>
<td>0.060</td>
<td>9.387</td>
<td>42.667</td>
<td>47.947</td>
<td>0.006</td>
</tr>
</tbody>
</table>

References


