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Honors Calculus: An Historical Approach

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When the honors program at the University of Arkansas-Fort Smith (then Westark Community College) began in 1990, it was decided that the honors mathematics course would be College Algebra, a decision based on the premise that our student population required an opportunity to earn honors credit at an introductory level, at least in mathematics. The course developed by the mathematics department met with some success by stimulating interest in algebra topics using an environmental modeling approach. The textbook, *Earth Algebra* (by Schuafele, Zumoff, Sims, and Sims), provided a wonderful opportunity for our students to learn the techniques of College Algebra through applications to "real-world" problems.

After offering Honors College Algebra for several years, it gradually became apparent that offering honors credit for College Algebra was excluding many students who tested into higher courses, particularly calculus. Not surprisingly, these higher placement scores often equated to honors students. Since Westark (a two-year college) became the University of Arkansas-Fort Smith (offering baccalaureate degrees) in 2002 and because mathematics became one of the first baccalaureate degrees offered by our new university, the mathematics department decided to develop an honors calculus course to reflect the changing demographics of our students. After discussing several ideas, the mathematics faculty decided to develop a first-semester calculus course centered on a study of the historical development of calculus. With a university enrollment of about 6500 students, and a small honors program (40–60 students), a single section of 15–20 students was planned. My interests and experiences in using history in the mathematics classroom led to my being "volunteered" to create the course, an assignment I enthusiastically accepted.

The premise for the honors calculus course I designed was that the students would learn the traditional topics in a Calculus I course—an important criterion for several reasons, not the least of which was that most of these students would continue on to Calculus II—while participating in an ongoing investigation of the historical development of calculus. The use of historical concepts to motivate learning in mathematics has many adherents. F. J. Swetz wrote, "History is commonly taught in schools to initiate the young into a community—to give them an awareness of tradition, a feeling of belonging, and a sense of participation in an ongoing process or institution. Similar goals can be advocated for the teaching of the history of mathematics." Swetz and many other authors (some of whom may be found below in the bibliographical essay) have found great success in motivating student learning and inspiring disinterested students by "humanizing" mathematics through its history. In addition to the strong belief from many in the mathematical community that history enhances the learning experience in mathematics, another motivation for this format comes from the goals of our honors program. Each honors course is designed to improve communication skills, both verbal and written, and honors faculty are encouraged to develop courses that address learning across the curriculum.

The catalog description of the new course encapsulates these ideas:

> This course will develop the standard topics in Calculus I from the perspective of the historical development of calculus and its reciprocating influence on society. Readings from original sources and extensive writing are required.

In addition to the typical requirements of a calculus course, the honors students are asked to read, discuss, debate, and write about issues and episodes in the history of calculus. Because the academic requirements (GPA and standardized test scores) are significantly higher for our honors students than the general student population, this rather accelerated first-semester calculus has never presented serious problems for most of my students. I have been able to weave historical content and assignments into the standard syllabus without causing noticeable distress in the students. By inserting reading assignments and discussions at appropriate times—for instance, students read and discuss the article "Fermat on maxima and minima" [see bibliographical essay below] while we are covering the traditional calculus subject of function extrema—I have attempted to create a course in which the historical perspectives fit seamlessly into the calculus curriculum. The study of the history of calculus culminates in a debate over who should be considered the "father" of calculus, Isaac Newton or Gottfried Leibniz. In essence, the addition of historical content entails that the honors calculus class requires more time and more work than the non-honors sections although the mathematical difficulty and expectations remain the same.

Has the course been successful? So far, there are not enough data to make any conclusions. Although the success rate (as measured by the number of students passing with a C or better) is significantly higher than in my non-honors calculus classes—around 85 percent for the honors course versus 60 percent for non-honors—this could very easily be attributed to the higher entrance requirements for honors students. As much as 20 percent of the final grade may come from the historical modules; however, this grade is not given easily and does not normally have a significantly positive effect on the final grade. Anecdotally speaking, I do have a strong impression that my honors students are more engaged than my other calculus students. Again, this could be the product of a higher quality student population, yet the group projects and the class interactions do seem to provide a more stimulating environment for learning. With a few notable exceptions, the vast majority of the students seem to enjoy the class format. Only a few complain that all the history research and discussion take away from their study of calculus. This essay details some of the activities I have found successful (as well as a few that were disasters!) over the past several years of teaching the course.

The honors component is centered on a series of projects that all relate to the history of mathematics in general and the history of calculus in particular. One interesting class project I also often use in my non-honors calculus classes comes from a volume of applications of calculus published by the Mathematical Association of America (MAA Notes Number 29, Volume 3). This project combines "real-world" application with an historical twist. The project involves using elementary calculus to find a solution to an arbitration dispute between management and labor in a fictitious company. The project is rather long and involved; although the mathematics is not terribly difficult for a first-year calculus student, it does consume the better part of a week's class time. Without going into detail, the solution of the problem involves an application of the Nash equilibrium theory, published by John Nash in the 1950s. Nash later won a Nobel Prize in Economics for his work. What is so interesting about John Nash? Some say he had *A Beautiful Mind*. Interestingly, the farther removed from the movie's premier we become, the fewer students I have who have heard of John Nash or even of the movie.

The majority of the readings and discussions come from secondary sources detailing the development of calculus and from primary sources by a few of the mathematicians who provided the foundations of calculus in the seventeenth and eighteenth centuries. (The only exception to this seventeenth- and eighteenth-century emphasis is the incredible accomplishments of Archimedes.) From the work of Johannes Kepler in approximating the volumes of wine barrels to the important contributions of Pierre Fermat to both the differential and integral calculus, the students read about, write about, and discuss in class the evolution of calculus before the arrival of Newton and Leibniz on the scene.

At first, I believed that honors students should be able to work together—without detailed direction from me—to understand the mathematics they were reading in the original sources. Of course, in retrospect, I should have known better. Reading mathematics in original sources is difficult not only because the mathematics itself might be on the outer edge of the students' abilities but also because the language and notation of the seventeenth century is appreciably different from that of the twenty-first century. Therefore, as I taught this course in subsequent semesters, I developed questions and guidelines to help students grasp the material they were reading (see Appendix for a few examples). With these guidelines, along with direction provided directly by me during class time, I found that students in small groups of three or four could successfully work through this difficult material and gain insight into the evolution of some central ideas in calculus.

THE GREAT DEBATE

Because the unifying theme of the course is the historical development of calculus, the major project that the students tackle is a debate titled "Who should be called the father of calculus, Newton or Leibniz?" For those not familiar with the story, I will provide a brief summary. The great English mathematician and physicist Isaac Newton developed calculus (although he called it the method of fluxions) in the late 1660s. Although some of the essential ideas of calculus were developed by earlier mathematicians such as Fermat, Cavalieri, Barrow, and others, Newton was able to pull some of these disparate methods together, add several crucial ideas of his own, and "invent" a completely new branch of mathematics. Although Newton wrote and circulated several tracts on his new discovery, none were published until a much later date. In the meantime, Gottfried Leibniz, a German philosopher who developed an interest in mathematics later in life, discovered his own version of calculus. Although he did not develop his ideas on the new mathematics until the mid-1670s (nearly ten years after Newton), Leibniz—unlike Newton—immediately published his results. A bitter priority dispute immediately arose between Newton and his supporters (primarily British) and Leibniz and his supporters (most of continental Europe). With national pride at stake, Newtonians accused Leibniz of plagiarism, claiming that he had seen the unpublished manuscripts containing Newton's calculus. Meanwhile, Leibnizians also accused Newton of plagiarism, maintaining that Newton did not complete his ideas on calculus until after Leibniz's publications appeared. Although the dispute was long and bitter, lasting even after the deaths of the two protagonists, today historians agree that both developed calculus independently and should both be considered the co-inventors of calculus.

At least six weeks before the debate is scheduled, students are divided into two groups, the Newtonians and the Leibnizians. With an average enrollment of somewhere between 12 and 16 students, this means that the groups themselves are rather large. A chief Newtonian and a chief Leibnizian are each instructed to distribute the workload among their groups. I provide a set of readings for both groups. The first set of readings consists of articles, or portions of books or articles, that provide an overview of the contributions of both men to the development of calculus. After students have read this assignment, we take class time to discuss the essential contributions of Newton and Leibniz. Then the meat of their research begins.

Each group is given several readings pertaining specifically to their man. The Leibnizians read both secondary sources and some portions of Leibniz's original works. At the same time, of course, the Newtonians are reading similar sources by and about Newton. I provide plenty of class time for the teams to talk with each other—and with me—about what they are reading and work through some of the problems they encounter. Some of these readings, especially the primary source materials, are rather difficult for first-year calculus students to follow. Although the calculus concepts are those they are studying in the course, the notation and the language sometimes make interpretation a challenge. It is not unusual for a group to discuss a short section of a reading for most of a class period until someone in the group finally says something like, "Wait, wait . . . that's just basically the product rule, isn't it?" Sometimes these epiphanies require a little prompting from me.

One fact that neither group knows is that I have planted a mole inside each group. Sometime near the beginning of the process, I pull aside a member of each group—someone I see is particularly engaged in the process—and hand each of them a copy of Bishop Berkeley's *The Analyst*. I explain that Berkeley attacked the foundations of calculus and those who professed to accept its tenets (the diatribe is sub-titled *A discourse addressed to an infidel mathematician*). The assignment for these two hand-picked students is two-fold. First, I want them to continue honest participation in their groups as they prepare for the debate. Secondly, I ask them to read Berkeley's essay and prepare their own refutation of the methods of *both* Newton and Leibniz. As the debate winds down, these two students will take the floor and accuse both sides of unsound thinking. Many times the debate that follows is more heated than the original priority debate as the Newtonians and Leibnizians bond together against the common enemy.

Of course, as with all college students, how the project is graded is of great concern. The majority of the grade for the project comes simply from participating. If both sides actively participate in the research, the preparation, and the debate itself—and if they do an acceptable job in each phase—the grading is not an issue; it is, after all, an honors course and for the most part the students are highly motivated and engaged in the project. If I feel that a particular student is not pulling his or her weight, I talk with the student and indicate that he/she may not receive the same grade as the rest of the team without improved participation. As an added incentive, the students are informed in the beginning that the team deemed the winner of the debate by the judge—that would be me—receives bonus points for the project. This alone stokes the fire of their competitive spirits even if the idea of a debate has not already done the trick.

I have now been through this assignment four times with an honors calculus class and twice more (in a slightly revised format) with a senior-level history of mathematics course. The day of the debate invariably proceeds in about the same way. Before the debate even begins, the classroom resembles a scene from *Westside Story*, with catcalls and challenges flying across the room. Under the rules of the debate, the first presentation from each side must address specifically the methods and procedures that their man used in developing and refining his version of the calculus. No mention of priority is allowed. This section is a test of the group's understanding of the calculus as written by Leibniz and by Newton. Each group is also expected to answer questions posed by me about the mathematics of their man.

After the technical portion of the debate ends, the fun begins. Each side is allowed a set time to present an overview of their claim for the priority of their mathematician in the calculus dispute. The basic argument by the Newtonians is always, "Newton did it first, and we're still not so sure Leibniz didn't steal some of his ideas." On the other hand, the Leibnizians counter with something along the lines of "Even if Newton was first with a complete calculus (a point that may not be completely true to begin with), he did not publish, teach, or disseminate his knowledge. Leibniz nurtured calculus and made it what we know it as today!" Dates fly back and forth, letters written by both men, travels and contacts of Leibniz, etc. I should note that I do not require the students to follow any customary rules of debating—I make up the rules and may even change them as the debate progresses. If one team makes a particularly interesting or contentious point, I might interrupt and ask the other group to respond. If the group requests a few minutes to talk about their response, the time is usually granted. My goal, of course, is not to teach proper debating etiquette but rather for the students to learn as much about the early development of calculus as they can. I think flexible debating rules help me in this endeavor.

In closing, each class brings its own surprises and new outlooks on a subject I have studied for many years. One particular episode stands out in my mind. Near the end of the debate, I told each group they had one minute to summarize their most important points. After the Newtonians reiterated that Newton "discovered" calculus before Leibniz did and therefore Newton should be called the father of calculus, the Leibnizians responded with an ingenious tactic. Reading from a dictionary, one student emphasized various definitions of "dad" that used terms like "nurturing parent" or "one who teaches and instructs offspring." They made the point that under this definition of "dad," Leibniz would obviously be the most qualified. Newton, they concluded, might lay claim to father, but never to "dad." At best, Newton could be called "the sperm donor" of calculus. The Leibnizians won the debate that year.

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BIBLIOGRAPHICAL ESSAY

There are countless readings addressing the history of calculus. These readings take the form of complete books, journal articles, edited collections of essays, and Internet articles. Each class is not necessarily assigned all of the readings discussed below. Following is a complete list of readings I have used at various times for the honors calculus course.

Of the numerous books that attempt to cover the entire history of the development of calculus, the three I have found most useful are Carl Boyer's *The History of Calculus and Its Conceptual Development* (New York: Dover Publications, 1949), C. H. Edwards' *The Historical Development of the Calculus* (New York: Springer-Verlag, 1979), and Margaret E. Baron's *The Origins of the Infinitesimal Calculus* (New York: Dover Publications, 1969). Two books addressing the priority dispute specifically have proven useful. They are A. R. Hall's classic work, *Philosophers at War: The Quarrel Between Newton and Leibniz* (Cambridge: Cambridge University Press, 1980) and a newer book not so historically rigorous as Hall's but very readable, J.C. Bardi's *The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time* (New York: Thunder's Mouth Press, 2006).

Several edited collections offer fine articles on various aspects of the history of calculus. One is *MAA Notes Volume 5—Readings For Calculus* (The Mathematical Association of America, 1993), which includes essays such as "Background for Calculus" (Steven Galovich), "Anticipations of Calculus in Medieval Indian Mathematics" (Ranjan Roy), "Slicing it Thin" (Howard Eves), "Fermat" (George F. Simmons), "On the Seashore" (E. T. Bell), "The Creation of the Calculus" (Morris Kline), "Principia Mathematica" (Isaac Newton), "The World's First Calculus Textbook," and "Calculus Notation." Similar types of information may be found in *Historical Topics for the Mathematics Classroom*, 2nd ed., published by the National Council of Teachers of Mathematics, 1989. In addition to a very nice overview of the history of calculus by historian of mathematics Carl Boyer, this book offers short topics covering many of the main players in the history of calculus, including Archimedes, Kepler, Cavalieri, Fermat, Wallis, Barrow, Leibniz, and Newton. In addition it offers a few topics on non-European roots of calculus, such as developments in Japan and in India. A third volume in the same vein is *Learn from the Masters* (The Mathematical Association of America, 1995). Included are articles to help the mathematics teacher incorporate history into the classroom, including some specific suggestions on how to use history in the calculus classroom.

William Dunham has published several volumes with rich historical material written in an easy and engaging style. See for instance *Journey Through Genius: The Great Theorems of Mathematics* (New York: Wiley and Sons, 1990). This Dunham work offers interesting excerpts on "The Bernoullis and the Harmonic Series," "The Extraordinary Sums of Euler," and "A Gem from Isaac Newton." Also see Dunham's *Mathematical Universe: An Alphabetical Journey Through the Great Proofs, Problems, and Personalities* (New York: Wiley and

Sons, 1994). Articles include "Differential Calculus," "Knighted Newton," and "Lost Leibniz."

For primary source materials, two excellent resources are D. E. Smith's *A Source Book in Mathematics* (New York: Dover Publications, 1929) and Ronald Calinger's *Classics of Mathematics* (New Jersey: Prentice Hall, 1995). In Smith's book the reader finds primary source material from various branches of mathematics, including "Cavalieri on an approach to the calculus," "Fermat on maxima and minima," "Newton on fluxions," "Leibniz on the calculus," "Berkeley's 'Analyst,'" and "Cauchy on derivative and differentials." Calinger's work offers another source of primary material from various branches of mathematics, including work of Archimedes, Kepler, Leibniz, Newton, Jakob and Johann Bernoulli, Euler, and others.

In addition to the above monographs and edited collections, many mathematics journals publish historical articles. For instance, "The Evolution of Integration" (*The American Mathematical Monthly*, January 1994, Volume 101, Number 1, 66–72) by Shenitzer and Steprãns offers a very nice overview of the development of integration. The research journal *Historia Mathematica*, although targeting professional historians of mathematics, publishes many articles accessible to undergraduate students.

Finally, although I warn my students to be wary of information they find on the Internet, one site that does offer reliable and remarkably detailed historical information is the MacTutor History of Mathematics Website, http://www-groups.dcs.st-and.ac.uk/~history/index.html.

APPENDIX

Newton/Leibniz Questions
From the articles "Newton's DOT-age Versus Leibniz' D-ism" and "Newton and Leibniz" from Boyer's *The Calculus*, answer the following questions:

What is a fluxion?

What is a fluent?

What is a moment?

What does $\dot{x}$ represent?

What does o represent?

What does $\dot{x}o$ represent?

What does Newton call $\dot{x}o$?

What is $x + \dot{x}o$?

Use Newton's method to calculate the fluxion of $xy - a = 0$

Do you recognize what rule of derivatives you arrived at in the previous question?

Use Leibniz's method to differentiate.

Who discovered this method of derivatives first, Newton or Leibniz?

Who was most concerned with the theoretical basis of calculus, Newton or Leibniz?

Who was most concerned with good notation, Newton or Leibniz?

Who originated the term dot-age vs. d-ism?

Explain what you think dot-age vs. d-ism means.

What did Newton and Leibniz do differently than Descartes, Cavalieri, Fermat, Wallis and Barrow?

Show how Newton used expansion of an infinite series to show that (in modern notation):

$$\int \frac{1}{1+x}\, dx = 1n(1 + x)$$

(Hint: You will have to use long division to divide 1 by 1 + *x*.)

Why does Boyer credit Newton as being the first inventor of calculus?

Show Newton's method for finding q/p, or what we would call dy/dx, for: $x^3 - abx + a^3 - cy^2 = 0$

Why does Boyer say Newton's logical foundations for calculus were better than Leibniz's?

What does Boyer mean when he says "From a logical point of view the calculus of Leibniz was a failure, but heuristically it was a resounding success"?

When was the first calculus textbook published? What was it called? What is the title in English? Who wrote it?

List four reasons why Leibniz's calculus was more successful on the European continent than was Newton's?

Leibnizians

In the article by Evelyn Walker, there is an error on page 622. Find it!

There is also an error on page 625. Find it!

Explain what the first formula you see at the top on page 53 in the Morris Kline article means:

$ds = \sqrt{dx^2 + dy^2}$

Newtonians

In the article by Evelyn Walker on page 616, Newton finds a derivative one way (under explication), then turns around and finds the same derivative another way (under demonstration). Why two methods?

There is a mistake in the second method. Find it!

Explain case 1 on page 56 of the article titled "Principia Mathematica." Draw some diagrams to aid the explanation.