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Estimating Griliches' *k*-shifts

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Estimating Griliches' *k*-shifts

Abstract. Griliches' *k*-shift, a crucial parameter in the welfare evaluation of technological change, is shown to be equal to the radial rate of technological change plus a vector of commodity bias parameters obtained from the distance function. The analysis permits decomposition of sectoral productivity growth into productivity growth by commodity. The *k*-shifts estimated for wheat, corn, soybeans and beef in U.S. agriculture indicate a decrease in the marginal cost of production of corn, soybeans and wheat during the 1950-1993 years.

Keywords : distance function, *k*-shift, rate of technical change, output biases, rate of commodity progress.

JEL classification: D24, Q16, O47.

I. Introduction

Productivity growth is defined as an increase in output per unit of inputs. It can be represented by an upward shift of the production function, or a downward shift of the marginal cost of production. When productivity[1] change is measured in input-output space by a production function shift, it is usually described by technology parameters indicating the rate and input biases of that change. When productivity change is measured in output-price space as a shift in the marginal cost of the commodity, it is usually described by what has been referred to in the literature as Griliches' *k*-shift. This concept was introduced in Griliches' seminal 1958 article on social returns to hybrid corn, to describe the shift in the supply curve (page 423) and later has been used by many others.[2] Even though changes in productivity can equivalently be assessed by estimating the relevant technology parameters or by

investigating the nature of the changes in the commodity's industry-level supply function, it has been customary in the literature to study one or the other without explicit recognition of their exact relationship.

In this study Grilches' *k*-shift, a supply curve shift, is decomposed into an overall rate of technical change and the commodity specific output biases of technical change. Previous studies have used 'ad-hoc' estimates of the *k*-shift in single commodity markets to study the welfare impacts of R&D. Other studies have estimated total factor productivity indexes (TFP) at a sectoral or highly aggregated level. None has attempted to make these two measures consistent, nor has any study decomposed sectoral productivity growth into productivity growth by commodities within a simple and theoretically consistent framework.

Section two summarizes the literature. Section three introduces the output distance function, characterizes productivity change in terms of the distance function and shows its relationship with the *k*-shift. Section four presents an example where the *k*-shift for wheat, corn, soybeans and beef in U.S. agriculture is estimated. Section five is a summary and conclusions.

II. Other Productivity Studies.

A review of the literature shows two types of approaches used to measure the economic consequences of technical change. The first one includes economy-wide and sectoral studies that use indexes, production, cost and profit functions to estimate the rate and bias of technical change. Studies that measured technical change as a shifter of the production function or as an output over input index start with Tinberger's 1942 effort and include the early works of Schmookler, Fabricant, Kendrick, Abramovitz, Solow, Griliches (1960, 1963), Jorgenson and Griliches and many others that followed.[3]

Introduction of duality theory has provided studies where productivity change is captured as a shifter of the cost, revenue or profit functions. Along these lines we found the studies by Christensen, Jorgenson, and Lau, Berndt and Khaled, and others.[4] These studies are done at a high level of aggregation, providing estimates of the rate of technical change and the input biases at the industry, sector, or country level.

The alternative to the approach above goes beyond the production technology to look at the productivity impact on the firm and industry supply functions. In view of the fact that the 'output' of innovative activity does not present itself in countable units, it has proven useful to define a quantifiable dimension for innovations in value terms, that is, in terms of their impact on social welfare. In other words, these studies seek an answer to the question of how much additional consumer and producer surplus was generated by technical change in a particular commodity market during a period of time. The so called economic surplus approach has been used extensively to evaluate the benefits from a productivity induced supply shift starting with Griliches' (1958) study on the social returns to hybrid corn research. Early work includes the evaluation of agricultural research by Peterson and by Schmitz and Seckler, of industrial innovations by Mansfield et al., of mainframe computers by Bresnahan, and more recently the study by Trajtenberg of computed tomography scanners.[5] While the last two studies presented hedonic analyses of the impact of changes in *product* qualities on consumers' welfare, all of the others analyzed the welfare impact of a *process* innovation as a supply shift and compute the benefits from the implied price reductions. These studies assume an exogenously determined shift of the marginal cost due to innovative activities and calculate the returns to these investments as changes in economic surplus. Critical assumptions in these models include the supply and demand elasticities and the nature of the productivity-induced supply shift. This supply shift is

what Griliches called *k*-shift. Researchers agree on the importance of such a parameter, but all studies have used ad-hoc approximations.

This paper, first, shows that Griliches *k*-shift can be decomposed into an overall rate of technical change plus corresponding output biases. Second, it shows how to use the distance function to derive these concepts. Third, it illustrates by estimating *k*-shifts, the commodity specific rates of technical change for wheat, corn, soybeans and beef in U.S. agriculture.

III. The Distance Function Approach to Productivity Measurement

A change in productivity is defined as a *technology-driven* divergence in the sizes of the output bundles obtainable from given inputs. To measure it we need a representation of the technological possibilities of the firm. If there are many outputs a useful representation is provided by the distance function. The output distance function is particularly fit to the study of productivity growth. This is because, by definition, it allows representation of the *maximum* amount by which outputs could be expanded *given* available inputs.

The first references to the distance function are Wold who uses it to define a utility function, Debreu who uses it to define the 'coefficient of resource utilization', Malmquist who develops a series of index numbers based on it and Shephard, who extensively discusses it in the context of production theory. More recent publications in the production area that use and describe the properties of this function can be found in Fuss and McFadden, Blackorby, Lovell, and Thursby, and Färe and Primont.[6] Much of what follows benefits from one or more of the contributions listed above and from the general exposition in Cornes. These references do not provide a systematic treatment of the distance function in the context of productivity measurement. Färe et al. (1997) shows a decomposition of the Malmquist

productivity index into a technical change and input and output biases. Although most applications have used non-parametric techniques, Fuentes et al. implement the ideas in Färe et al. using econometric techniques.

Formally, for given output and input vectors $\boldsymbol{y}$ and $\boldsymbol{x}$, and output set $P(\boldsymbol{x})$ defined as the set of all output bundles that can be produced from input bundle $\boldsymbol{x}$, the output distance function[7] is defined as

$$\theta^* \equiv \mathrm{D}_O(\boldsymbol{x},\mathbf{y}) \equiv \inf_{\theta}\{\theta \mid (1/\theta)\mathbf{y} \in P(\boldsymbol{x})\}. \tag{1}$$

For a firm using a *(nx1)* vector of inputs $\boldsymbol{x}$ with prices $\boldsymbol{W}$ to produce a *(mx1)* vector of outputs $\boldsymbol{y}$ with prices $\boldsymbol{P}$ subject to output set $P(\boldsymbol{x})$, assuming revenue maximization, the revenue function dual to the output distance function $D_O(\boldsymbol{x}, \boldsymbol{y})$ solves the following problem:

$$R(\boldsymbol{P},\boldsymbol{x}) \equiv \max_{\boldsymbol{y}}\{\boldsymbol{P}.\boldsymbol{y} \mid D_O(\mathbf{x},\mathbf{y}) = 1\}, \tag{2}$$

that provides the revenue-maximizing bundle $\boldsymbol{y}^*$ when output prices are $\boldsymbol{P}$ and inputs are $\boldsymbol{x}$. We normalize output prices ($p_i = P_i/R$) so that the maximum revenue obtained when producing the target vector of outputs is unity, that is $R(\boldsymbol{p}, \boldsymbol{x}) = 1$.

A useful property of the output distance function is its derivative property. The derivative of $D_O(\boldsymbol{x}, \boldsymbol{y})$ with respect to the mth output, which we write $\psi_m(\boldsymbol{x}, \boldsymbol{y})$, is the marginal cost for output m,

$$\partial D_O(\mathbf{x},\mathbf{y})/\partial y_m \equiv \psi_m(\mathbf{x},\mathbf{y}) = p_m \equiv \frac{P_m}{R} \text{ for m=1,...,M.} \tag{3}$$

The derivative of $D_O(\boldsymbol{x}, \boldsymbol{y})$ with respect to the *nth* input, which we write $\varphi_n(\boldsymbol{x}, \boldsymbol{y})$, is the marginal revenue product of input n,

$$\partial D_O(\mathbf{x},\mathbf{y})/\partial x_n \equiv \phi_n(\mathbf{x},\mathbf{y}) = w_n \equiv \frac{W_n}{R} \text{ for n=1,...,N.} \tag{4}$$

Using the envelope theorem, price differentiation of the normalized revenue function $R(\boldsymbol{p}, \boldsymbol{x})$ gives (compensated) supply functions $y_m(\boldsymbol{p}, \boldsymbol{x})$.

The Radial Rate of Technical Change.

In the presence of technical change, each observation through time is possibly associated with a different technology. The technology index A_t is used as a local representation of technical progress that shifts the production frontier across observations. Modifying (1) to include a technology index, the output distance function can be written

$$D_O^t(\mathbf{x}_t,\mathbf{y}_t,A_t) = \min_{\theta}\{\theta > 0 : (\mathbf{y}_t/\theta) \in P(\mathbf{x},A_t)\}. \tag{5}$$

where the output set $P(\boldsymbol{x}, A_t)$ is defined as the set of all output bundles that can be produced from input bundle $\boldsymbol{x}$ and technology A_t. Technological change is progressive if for $A_{t+1} > A_t$, it expands the output set and allows output bundles formerly infeasible with inputs $\boldsymbol{x}$ to be in the new feasible set, or $P(\boldsymbol{x}, A_t) \subseteq P(\boldsymbol{x}, A_{t+1})$. It is regressive if for $A_{t+1} > A_t$, $P(\boldsymbol{x}, A_t) \supseteq P(\boldsymbol{x}, A_{t+1})$, it shrinks the output set by eliminating feasible output bundles. Locally, the behavior of $D_O(\boldsymbol{x}, \boldsymbol{y}, A_t)$ in A_t is easy to categorize. If technical change is *progressive* the output distance function is *non-increasing* in A_t, if it is *regressive* it is *non-decreasing* in A_t. This is because technical change expands the production set so the minimum achieved on the expanded set cannot be larger than the minimum achieved on the original output set since the original output bundle remains feasible. The same argument establishes the relationship between regressive technical change and the output distance function.

Figure 1 illustrates the case where the output set is enlarged from $P(\boldsymbol{x}^0, A_t)$ to $P(\boldsymbol{x}^0, A_{t+1})$ as a result of technical change. Let the bundle $\boldsymbol{y}^0$ be attainable with period t technology and inputs $\boldsymbol{x}^0$ but not on the production frontier $F(\boldsymbol{x}^0, A_t)$ of the feasible set $P(\boldsymbol{x}^0, A_t)$. Let $\boldsymbol{h}^0$ be the output bundle just attainable in this period, then $\boldsymbol{h}^0 = \boldsymbol{y}^0/\theta^0$, where θ^0 is the smallest scalar by which all outputs are expanded to reach the frontier. After technical change, the output set is enlarged and the new frontier is $F(\boldsymbol{x}^0, A_{t+1})$, with the output bundle $\boldsymbol{h}^1$ just attainable with the new technology. In this case $\boldsymbol{h}^1 = \boldsymbol{y}^0/\theta^1$, the scalar θ^1 by which all outputs should be expanded in order to reach the new frontier, is smaller than the one before the expansion ($\theta^1 < \theta^0$). So progressive technical change represented by $P(\boldsymbol{x}, A_t) \subseteq P(\boldsymbol{x}, A_{t+1})$ as $A_{t+1} > A_t$, implies $\theta^1 < \theta^0$, or $\partial D / \partial A_t \leq 0$.

For observations on the frontier:

$$D_o^t(\mathbf{x}_t^*, \mathbf{y}_t^*, A_t) = 1 \tag{6}$$

we define the rate of technological change

$$\delta_t(\mathbf{x_t}, \mathbf{y_t}, A_t) = \frac{-\partial \ln D_o^t(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\partial A_t} = \frac{-\partial D_o^t(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\partial A_t}. \tag{7}$$

Following Atkinson et al. and totally differentiating equation (6); considering that by definition the output distance function maintains inputs constant ($dx = 0$); that it is radial in output space so that $dlny_1/dA_t = dlny_2/dA_t = ... = dlny_M/dA_t$ and equal to the common scalar $dlny_m/dA_t$; and that in addition it is linear homogeneous in outputs so that $\sum_{m=1}^{M} (\partial \ln D_o^t / \partial \ln y_{mt}) = 1$, we are able to derive

$$\delta_t(\mathbf{x_t}, \mathbf{y_t}, A_t) = \frac{-\partial \ln D_o^t(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\partial A_t} = \frac{d \ln y_{mt}}{dA_t}. \tag{8}$$

Equation (8) indicates that δ, the rate of technical change obtained from an output distance function, equals the common rate of expansion of outputs along a ray through the origin due to an increase in the

technology index A when inputs are not allowed to change. Once a particular parametric specification for the distance function is chosen, δ can be estimated.

Alternatively, and using the property of homogeneity of the output distance function in $\boldsymbol{y}$, this radial rate of technical change can be shown[8] to be a weighted average of the changes in marginal cost of the commodities due to technical change

$$\delta_t(\mathbf{x}_t, \mathbf{y}_t, A_t) = -\frac{\partial \ln D_o(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\partial A_t} = -\sum_m S^v_{mt} \frac{\partial \ln \psi_{mt}}{\partial A_t}, \tag{9}$$

where δ is the radial rate of technological change, S^v_{mt} is the shadow (or virtual) share of commodity m, and $\partial ln\psi_{mt}/\partial A_t$ is the change in the logarithm of the marginal cost of commodity m due to technical change.

Notice the relationship between the primal rate in (9) and a dual rate of technical change obtained from the normalized revenue function. Figure 2 illustrates these concepts in price space. For progressive technical change, represented by $A_{t+1} > A_t$, the price frontier moves inward and the radial revenue function is non-decreasing in A as seen by the segment OA/OB being smaller than OA/OC or $\partial R^t / \partial A_t \geq 0$. For observations on the price frontier, following the same procedure as in (8), we can derive the dual rate of technical change as

$$\frac{\partial \ln R^t(\boldsymbol{x_t}, \boldsymbol{p_t}, A_t)}{\partial A_t} = -\frac{d \ln p_{mt}}{dA_t} = \mu_t(\boldsymbol{x_t}, \boldsymbol{p_t}, A_t) \tag{10}$$

where we have used the definition of the normalized revenue function and its property of linear homogeneity in prices. This equation indicates that the dual rate of technical change μ equals the common rate of change of output prices along a ray through the origin in price space, when inputs are not allowed to change. Once a parametric revenue function is specified, μ can be estimated.

From problem (2) and using the envelope theorem we see that

$$\frac{\partial R^t(\mathbf{x}_t, \mathbf{p_t}, A_t)}{\partial A_t} = -\lambda(\mathbf{x}_t, \mathbf{p}_t, A_t)\frac{\partial D_o^t(\mathbf{x_t}, \mathbf{y_t}, A_t)}{\partial A_t}. \tag{11}$$

In addition the first order conditions of problem (2) are $p_m = \lambda\, \partial D_o(.)/\partial y_m$, for all $m = 1,...,M$, and $D_o(.) = 1$. Multiplying the first order conditions by $\boldsymbol{y(p, x,} A)$ and using the linear homogeneity property of the distance function, we see that λ is the normalized revenue which is equal to *1* at the optimum.

Then

$$\mu_t = \frac{\partial \ln R^t(\mathbf{p}_t, \mathbf{x}_t, A_t)}{\partial A_t} = -\frac{\partial \ln D_O^t(\mathbf{y}_t, \mathbf{x}_t, A_t)}{\partial A_t} = \delta_t \tag{12}$$

which establishes the equivalence between the *radial* primal and dual rates of technical change when inputs are constant. So, if inputs are held constant[9], we define the radial rate of technical change as the rate of contraction of the output distance function or equivalently as the rate of expansion of the normalized revenue function. Due to their radial nature, these rates only allow measurement of neutral[10] technical change, as shown next.

Hicksian and Overall Biases of Technical Change.

In a multiple-output production process, technological change may privilege some outputs resulting in some outputs growing faster than others. Hicks introduced the definition of neutral and biased technological change for input pairs. He suggested that inventions could be classified in terms of their effects on the marginal product of one factor relative to another, or on the marginal rate of substitution between two factors. I use Blackorby, Lovell, and Thursby 's interpretation of Hicks neutrality as the invariance of the marginal rate of technical substitution at different points on the firm's expansion path. The radial primal rate of technical change δ captures Hicks neutral technical change.[11] When technical change is not Hicks neutral the distance function is also an useful concept.

Consider extending the concept of Hicksian biases from input to output space

$$\mathrm{B}_{mj}(\mathbf{y},\mathbf{x},A_t) \equiv \frac{\partial \ln(\mathrm{MRT}_{mj})}{\partial A_t} = \frac{\partial \ln(\psi_m / \psi_j)}{\partial A_t}$$

$$\mathrm{m,j=1,...,M,\ m \neq j} \tag{13}$$

which uses the fact that the MRT_{mj} is the relative cost of producing additional units of commodity m in terms of units of commodity j given up. This bias concept measures the rotation of the production possibilities frontier at a point in output space in response to technical change. As illustrated in Figure 1, the firm is producing at $\boldsymbol{h^0}$ on the initial expansion path. After technological change has occurred, the firm produces at $\boldsymbol{h^2}$, on a new expansion path. This movement can be decomposed into a Hicks neutral change from $\boldsymbol{h^0}$ to $\boldsymbol{h^1}$ and a substitution change from $\boldsymbol{h^1}$ to $\boldsymbol{h^2}$. B_{mj} measures the change in slope of the production frontiers through $\boldsymbol{h^1}$ on the initial expansion path. Hicks neutrality is captured by $B_{mj} = 0$, for all m, j, when technical change does not change the expansion path. If $B_{mj} > 0$ the opportunity cost of output j in terms of output m for given inputs has decreased, and the technological change is biased toward the production of output j relative to output m. $B_{mj} < 0$ when as a result of technical change, production of one more unit of output j with the same inputs requires the firm to give up more units of output m than before the technological change, so that it is j^{th} output reducing relative to the mth output.

Hicks defined factor biases in terms of a two-input production function. This definition is not very useful in the multiple-output, multiple-input framework described by the output distance function of this paper because it provides $(m^2-m)/2$ potential forms of relative bias. For example, technical change could enhance the production of corn relative to that of wheat, while diminishing the production of corn relative to soybeans. This definition does not give a clear interpretation as to whether technical change is expanding or contracting in each output.

An overall measure of bias, in the manner of Antle and Capalbo, defined in product space with the use of the distance function is

$$B_m(\mathbf{x}_t, \mathbf{y}_t, A_t) \equiv \sum_{j \neq m=1}^{M} S_j^{\mathrm{v}} B_{mj}(\mathbf{x}_t, \mathbf{y}_t, A_t) = \frac{\partial \ln \psi_m}{\partial A_t} - \sum_{j=1}^{M} S_{\mathrm{j}}^{\mathrm{v}} \frac{\partial \ln \psi_j}{\partial A_t} \tag{14}$$

where $S_{\mathrm{j}}^{\mathrm{v}} = \psi_j y_j / D_O$ is the virtual or shadow share of output *j*. Equivalently, using (8) and (9) and for given outputs,

$$B_m(\mathbf{x_t}, \mathbf{y_t}, A_t) = \frac{\partial \ln \psi_m}{\partial A_t} - \frac{\partial \ln D_o}{\partial A_t} = \left. \frac{\partial \ln S_{mt}^{v}}{\partial A_t} \right|_y \tag{15}$$

which provides a convenient taxonomy of effects associated with technical change. Equation (14) and (15) indicate that if the marginal input requirement of output *m* is increasing relative to all others, then $B_m > 0$ and the technological change is output-*m* reducing overall or bias against the production of this output (*anti-output m biased)* as its virtual share increases due to increases in its opportunity cost of production. If $B_m = 0$ then technical change is Hicks neutral. $B_m < 0$ indicates that an additional unit of output *m* requires less inputs than other outputs after the technical change has taken place, therefore the technological change has been output-*m* augmenting and its virtual share decreases as its cost of production has done so. More of the mth output can be produced now with the same inputs and technological change is *pro-output m biased*.

If technological change were completely unbiased (all B_{mj}*'s* are 0 and all B_m*'s* are *0*), from (15) and (10) we have that the radial rate of technical change is

$$\delta = -\frac{\partial \ln \psi_m}{\partial A_t}, \forall m. \tag{16}$$

and all marginal costs change at the same rate.

Other results of interest derived from the symmetry of the Hessian of the distance function are as follows. Let $\partial^2 ln\ D(.)/\partial A_t \partial ln\ y_m = -\partial\delta/\partial ln\ y_m$ be the impact of changes in outputs on the radial rate of technical change. One can then easily show that

$$\frac{\partial^2 \ln D(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\partial A_t \partial \ln y_m} = \frac{\partial S_m^v}{\partial A_t} = S_m^v \left[\frac{\partial \ln \psi_m}{\partial A_t} - \frac{\partial \ln D(\mathbf{x_t}, \mathbf{y_t}, A_t)}{\partial A_t} \right] = S_m^v B_m \tag{17}$$

Thus, the share-weighted technological bias index captures the impact of technological progress on output composition. It results from the linear homogeneity of the distance function in outputs and from the definition of B_m that

$$-\sum_i \frac{\partial \delta}{\partial \ln y_m} = \sum_m S_m^v B_m = \sum_m \frac{\partial^2 \ln D}{\partial A_t \partial \ln y_m} = \sum_m \frac{\partial S_m^v}{\partial A_t} = 0 \tag{18}$$

that is, the rate of technical change is homogeneous of degree 0 in output quantities.[12] Equation (18) indicates that if technical change is biased at least one B_m must be positive and one must be negative.

Biases are also obtained in dual space from the normalized revenue function $R(\mathbf{p}, \mathbf{x}, A)$. Pairwise biases are defined in terms of changes in the ratio of two outputs

$$B_{mj}^r(\mathbf{p}_t, \mathbf{x}_t, A_t) = \frac{\partial \ln(y_{mt} / y_{jt})}{\partial A_t} = \frac{\partial \ln R_m(\mathbf{p}_t, \mathbf{x}_t, A_t)}{\partial A_t} - \frac{\partial \ln R_j(\mathbf{p}_t, \mathbf{x}_t, A_t)}{\partial A_t} \tag{19}$$

$m, j = 1,...,M,\ m \neq j$, where the revenue function subscripts indicate first derivatives while overall biases are

$$B_m^r(\mathbf{p}_t, \mathbf{x}_t, A_t) = \sum_{j \neq m=1}^{M} S_j B_{mj}^r(\mathbf{p_t}, \mathbf{x_t}, A_t) = \frac{\partial \ln y_m}{\partial A_t} - \sum_{j \neq m=1}^{M} S_j \frac{\partial \ln y_j}{\partial A_t} \tag{20}$$

where $S_j = y_j\, p_j / R$ is the actual revenue share of output j. Alternatively

$$B_m^r(\mathbf{p_t}, \mathbf{x_t}, A_t) = \left. \frac{\partial \ln S_m}{\partial A_t} \right|_p \tag{21}$$

which should not be confused with the definition in equation (15). If the optimal output mix is produced, the primal and dual output shares are equal although their behavior with respect to technical change need not be. Recall that the primal output share keeps all other outputs fixed, while the dual share keeps all other output prices fixed. $B_m^r = 0$ indicates dual Hicks neutrality, [13] while $B_m^r > 0$ indicates that technical change has been *pro-output m biased* while the opposite is true for $B_m^r < 0$.

Once a parametric specification of the distance function or the revenue function is chosen, pairwise and overall biases can be estimated.

Griliches' k-shift.

We have shown above how the output distance function provides information about the rate and biases in technological change. The task now is to relate these concepts to Griliches' *k*-shift. In his 1958 paper "Research Costs and Social Returns: Hybrid Corn and Related Innovations," Griliches defines the parameter *k* used in the welfare analysis there as ' . . . the relative shift in the supply curve, . . . ' (p. 423) due to the introduction of the new varieties, and calculates it casually as the percentage change in yields.[14] In Figure 3, this is a shift in the marginal cost curve of commodity *m* and can be represented by the percentage change in the virtual price of that commodity measured at the original equilibrium value of *y*.[15] Griliches' *k*-shift is then

$$k_m = \left. \frac{\partial \ln \psi_m}{\partial A_t} \right|_y \tag{22}$$

We know from equations (14) and (15) that the overall bias is

$$B_m(\mathbf{x_t}, \mathbf{y_t}, A_t) = \left.\frac{\partial \ln S_m^v}{\partial A_t}\right|_y = \left[\frac{\partial \ln \psi_m}{\partial A_t} - \frac{\partial \ln D_o(\mathbf{x_t}, \mathbf{y_t}, A_t)}{\partial A_t}\right] \tag{23}$$

and from here we obtain that

$$k_m(\mathbf{x_t}, \mathbf{y_t}, A_t) = \frac{\partial \ln \psi_m(\mathbf{x_t}, \mathbf{y_t}, A_t)}{\partial A_t} = \left.\frac{\partial \ln S_m^v}{\partial A_t}\right|_y + \frac{\partial \ln D_o(\mathbf{x_t}, \mathbf{y_t}, A_t)}{\partial A_t} = B_m(\mathbf{x_t}, \mathbf{y_t}, A_t) - \delta_t(\mathbf{x_t}, \mathbf{y_t}, A_t). \tag{24}$$

In (24) we have decomposed Griliches' *k*-shift for output *m* into the output bias and the radial rate of technological change. If we estimate these two technology parameters we are able to estimate Griliches' *k*-shift which could alternatively be interpreted as an output specific rate of technical change.

An equivalent set of relationships can be obtained in dual space. They describe the *horizontal* shift of the marginal cost curve, as opposed to the *vertical* shift. This is referred to in the literature as Griliches' *K*-shift. Given output prices and factor endowments, this *K*-shift is defined in terms of the dual radial rate of technical change, μ, as [16]

$$K_m(\mathbf{p}_t, \mathbf{x}_t, A_t) = \frac{\partial \ln y_m(\mathbf{p}_t, \mathbf{x}_t, A_t)}{\partial A_t} = \left.\frac{\partial \ln S_m}{\partial A_t}\right|_p + \frac{\partial \ln r(\mathbf{p}_t, \mathbf{x}_t, A_t)}{\partial A_t} = B_m(\mathbf{p}_t, \mathbf{x}_t, A_t) + \mu\ (\mathbf{p}_t, \mathbf{x}_t, A_t). \tag{25}$$

Both the *k*-shift and the *K*-shift are estimable once an output distance function or a revenue function is specified.

IV. An Application: Griliches' *k*-shift in U.S. Agriculture.

In this section we illustrate and use the theory to estimate Griliches' *k*-shift for wheat, corn, soybeans and beef in U.S. agriculture.

There have been numerous studies of productivity growth at the aggregate, sectoral and industry level for the U.S. These studies have used a number of different approaches to productivity

measurement, including parametric and non-parametric, stochastic and deterministic. Productivity studies that focus on the agricultural sector include Gollop and Jorgenson, Capalbo and Vo, Ball et al., Pardey et al., Huffman and Evenson, Cox and Chavas, Lim and Shumway, and Gopinath and Roe, among others. These studies estimate productivity growth using index numbers, estimates of production, cost and profit functions, and other non-parametric approaches. Most of these studies obtain estimates of the rate of agricultural productivity growth for the sector as a whole. They are consistent in estimating positive rates of productivity growth in U.S. agriculture during the last half of the 20th century. Few are able to differentiate productivity growth in the crops and livestock subsectors. None of these studies have obtained estimates of productivity growth at the commodity level or have estimated Griliches' *k*-shifts.

The Data.

The five commodities chosen in this study constitute 100 percent of the value of all U.S. agricultural production. The data used in the analysis consists of annual observations on quantity (produced and used) and price indexes (paid and received) from 1950 to 1993 obtained from a number of sources. I estimate a structure with five outputs (corn, wheat, soybeans, beef cattle and all other commodities), one input (all production inputs), and time as a proxy for technological change. The choice to aggregate inputs into one index reflects my interest in illustrating the concepts in output space. Estimates of input biases in U.S. agriculture have been more common in the literature. The variables used in this analysis are described in Table 1.

The Quadratic Specification.

A flexible representation of the technology that embodies the regularity conditions required by theory is desirable for implementation of this model. The translog functional form has been used in a number of studies (Lovell et al., Grosskopf et al., Coelli and Perelman, Morrison Paul et al. (2000)) in the distance function context. Here, a generalized quadratic form is used (for simplicity the *t* subscript is dropped).

In general,

$$\begin{aligned} &D_O^* = \alpha_0 + \alpha'\mathbf{d}^* + \frac{1}{2}\mathbf{d}^{*\prime}\Gamma\mathbf{d}^*, \\ &\text{where} \\ &D_O^* = D_O / y_1 \\ &\text{and} \\ &\mathbf{d}^* = \begin{bmatrix} \mathbf{y}^* / y_1 \\ \mathbf{x} \\ \mathbf{z} \end{bmatrix} \end{aligned} \tag{26}$$

where $\boldsymbol{y}$ is an M output vector, $\boldsymbol{y}^*$ is a vector of M-1 outputs (one output is used for normalization), $\boldsymbol{x}$ is an N vector of inputs and $\boldsymbol{z}$ is a K vector of exogenous variables such as A_t (technical change), and α_0, $\boldsymbol{\alpha}'$, and $\boldsymbol{\Gamma}$ are parameters to be estimated (a scalar, a vector and a matrix, respectively). A convenient partition consists of $\boldsymbol{\alpha}' = (\boldsymbol{\alpha}_{y^*}, \boldsymbol{\alpha}_x, \boldsymbol{\alpha}_z)'$, and

$$\Gamma = \begin{bmatrix} \Gamma_{\mathbf{yy}} & \Gamma_{\mathbf{yx}} & \Gamma_{\mathbf{yz}} \\ \Gamma_{\mathbf{xy}} & \Gamma_{\mathbf{xx}} & \Gamma_{\mathbf{xz}} \\ \Gamma_{\mathbf{zy}} & \Gamma_{\mathbf{zx}} & \Gamma_{\mathbf{zz}} \end{bmatrix}. \tag{27}$$

Theoretically required regularity conditions for this function include homogeneity of degree one in outputs and symmetry. The normalized quadratic maintains linear homogeneity of the output distance function. Symmetry requires the constraints:

$$\gamma_{ij} = \gamma_{ji} \;\; \forall i \neq j \tag{28}$$

for all outputs, inputs and other exogenous variables and their cross products.

First order differentiation of the normalized output distance function with respect to outputs yields a system of marginal cost, shadow (or virtual) prices that are linear in normalized output quantities, in input quantities and in other exogenous variables

$$\psi = \alpha_y + \Gamma_{\mathbf{y}^*\mathbf{y}^*}\mathbf{y}^* + \Gamma_{\mathbf{y}^*\mathbf{x}}\mathbf{x} + \gamma_{\mathbf{y}^*\mathbf{z}}\mathbf{z}, \tag{29}$$

where $\boldsymbol{\psi}$ is an *m x 1* column vector consisting of marginal valuations ($\partial D_O/\partial y_m^* = \psi_m(.)$).

Using equation (4), the marginal revenue product of inputs is

$$\phi = -(\alpha_x + \Gamma_{\mathbf{y}^*\mathbf{x}}\mathbf{y}^* + \Gamma_{\mathbf{xx}}\mathbf{x} + \gamma_{\mathbf{xz}}\mathbf{z}), \tag{30}$$

where $\boldsymbol{\phi}$ is a *nx1* vector of marginal input revenues. Note that $\boldsymbol{\Gamma}_{zz}$, which is needed to evaluate technical change, cannot be estimated from equations (28) and (29). The output distance function in equation (25) must be estimated either alone or jointly with these equations.

By the envelope theorem, $\boldsymbol{\psi}$ is a vector of inverse supply functions and $\boldsymbol{\phi}$ is a vector of inverse derived demand functions. Convexity in output quantities implies a positive semi-definitive matrix of second order derivatives of the output distance function with respect to outputs, $\boldsymbol{\Gamma}_{yy}$.[17] If in addition, concavity in inputs is desired the Hessian implied by the estimated parameters in inputs $\boldsymbol{\Gamma}_{xx}$, must be negative semi-definite. These properties are maintained in estimation of this system. Monotonicity is satisfied if the predicted valuations are positive. This property is not maintained but evaluated after estimation.

Equation (9) indicates how the output distance function provides a measure of the rate of technical change. If $z = A$, this rate is obtained as

$$\delta = -(\alpha_z + \Gamma_{\mathbf{yz}}\mathbf{y} + \Gamma_{\mathbf{xz}}\mathbf{x} + \Gamma_{zz}z).\frac{z}{D_O(\mathbf{x},\mathbf{y},z)}, \tag{31}$$

where δ is the radial rate of technical change and all outputs, including the numeraire, are contained in the vector $\boldsymbol{y}$ and the parameter matrix $\boldsymbol{\Gamma}_{yz}$. Equation (30) can be evaluated for given values of outputs, inputs and other exogenous variables once the coefficients are estimated. The coefficients for the numeraire output are retrieved from the homogeneity condition.

Hicksian pairwise bias measures for outputs are obtained using equation (13) which gives

$$B_{mj} = \left[\frac{\gamma_{mz}}{\psi_m} - \frac{\gamma_{jz}}{\psi_j} \right] z \tag{32}$$

for all outputs $m,j = 1,...,M,$ and when $z = A$. If $B_{mj} = 0$, then technical change does not bias the optimal mix between outputs, while $B_{mj} > 0$ implies a bias toward the production of the jth relative to the mth output, and $B_{mj} < 0$ implies a bias toward the production of the mth output relative to the jth output. Overall biases are obtained using the pairwise biases and equation (14). In terms of the parameters of the normalized quadratic output distance function

$$B_m = \sum_j S_j B_{mj} = \left[\frac{\gamma_{mz}}{\psi_m} z + \delta \right] \tag{33}$$

for all $m, j = 1,..., M,$ and $z = A$. Once the radial rate of technical change and the overall biases per commodity are obtained using equations (30) and (32), they are combined to obtain the commodity specific rate of productivity growth giving, according to equation (22), the respective k-shifts

$$k_m = B_m - \delta = \frac{\gamma_{mz}}{\psi_m} z \tag{34}$$

for all $m = 1,...,M,$ and $z = A$.

Econometric Estimation.

Equations (25), (28) and (29) are modified slightly for estimation purposes. It is maintained that all observations are efficient, so for each year the production bundle is not only feasible but it is on the frontier. This amounts to assuming that $D_O(\boldsymbol{x}, \boldsymbol{y}) = 1$ in every period, and as a result, equation (25) regresses $(y_1)^{-1}$, the numeraire output, on all other normalized outputs, input quantities, and other exogenous variables. Random disturbances are added to the normalized distance and normalized price equations. These disturbances represent the effect of random weather conditions and approximation error; they are assumed homoscedastic and uncorrelated within equations. Contemporaneous cross-equation correlation of the disturbance terms is permitted.

If in addition to the above assumptions, the vector of disturbances is multi-normally distributed, maximum likelihood estimation can be performed. Under the stated stochastic assumptions, the maximum likelihood estimators are consistent, asymptotically normal, and asymptotically efficient.

Using the data described in the previous section, equations (25), (28) and (29) are estimated by the method of maximum likelihood, using the IML procedure in SAS. Cross-equation symmetry and identity restrictions are imposed on the parameters at estimation. Linear homogeneity in outputs is imposed by normalizing outputs by the index of 'all other' outputs. Convexity in outputs must be imposed on this system. The output distance function will be convex in outputs if $\boldsymbol{\Gamma}_{yy}$ is a positive semi-definite matrix, implying that the diagonal elements of this matrix are nonnegative. Convexity is imposed by estimating the system subject to nonegativity constraints on these parameters. This is done using the NLPQM (Dual Quasi Newton Method) optimization subroutine in the IML procedure in SAS. This approach allows estimation of the parameters in the system by maximizing the likelihood function subject to equality and inequality, linear and nonlinear constraints on the parameters. Once these

parameters are estimated, their standard errors are obtained from running one iteration of the SUR option of the MODEL procedure in SAS, with all parameter values restricted to the values estimated by the previous approach.

The system has six equations, the dependent variables being the inverse of the numeraire output ('others'), the normalized prices of corn, wheat, soybeans, beef cattle and the negative of the input price index. The stacked model has 264 observations and 28 estimated parameters.

Collinearity diagnostics developed by Belsley, Kuh and Welsch indicate an absence of strong multi-collinearity. Because time-series data are used, the presence of auto-correlation in the residuals is possible. Simple Durbin-Watson statistics for each of the equations in the system fall in the inconclusive range. Guilkey's likelihood ratio test statistic for a system of simultaneous equations that do not contain lagged endogenous variables as regressors does not lead to rejection of the hypothesis that the matrix of first order vector auto-regressive coefficients is zero. Estimation proceeds under the assumption of serially independent errors. Table 2 presents the parameter estimates of the restricted model. The table contains a total of twenty eight estimated parameters, eight of which are significant at the 1 percent level, five at the 5 percent level, and four at the 10 percent level. The signs of the estimated parameters are in general consistent with the theoretical model. The own responses of the output supply equations are positive, while the own response of the input demand is negative. Monotonicity is satisfied at the mean of the data, but violated at 36 of the 264 data points.

Among the most significant estimated parameters are those of the time variable, indicating a strong autonomous component in the trend of the supply and demand equations. In all cases but one this trend is associated with a decrease in normalized prices of outputs, suggesting the presence of technical change.

Estimates of Technical Change, Bias and Griliches' k shifts.

Equation (30) is used to estimate the radial rate of technical change. Table 3 shows the estimates of this rate by decades and compares them with estimates from well-known studies. The results indicate that during the 1950-93 period, U.S. agriculture grew at an average rate of 1.77 percent per year. This rate is lower than recent estimates of 1.85 percent to 2.24 percent by Ball, et al., Pardey et al., and Huffman and Evenson, who used an index number approach covering slightly different time periods.

Figure 4 shows the evolution of this rate as well as those from these other studies. The econometrically estimated rate of technical change in this study is consistent with the non-parametric, non-stochastic rates obtained from indexes.

Pairwise biases are obtained from equation (31). The results, shown in Table 4, indicate that technical change, on average, has not been Hicks neutral. In fact, it has been biased in favor of corn relative to soybeans and wheat, and soybeans relative to wheat, with the rest of the pairwise biases being insignificant. The average bias measures, B_m, calculated according to equation (32), indicate that on average, technological change has been biased in favor of corn, soybeans and wheat. The overall bias measure is insignificant for beef and 'others'.

Finally, the commodity specific rate of technical change, the *k*-shift, is estimated using equation (32) as the sum of the radial rate of technical change and the overall bias per commodity. These results are presented in Table 5.[18] On average, the marginal cost of corn, soybeans and wheat has decreased during the 1950-1993 period. The *k*-shift or downward shift of the supply due to technical change is estimated at 5.8 percent for corn, 3.5 percent for soybeans and 1.3 percent for wheat. The study by Gopinath and Roe, based on the estimation of a revenue function with aggregate U.S. data, finds a 2.9

percent and a 2.8 percent downward shift in the supply of crops and grains and a 2.29 percent downward shift in the supply of meat and dairy over the 1949-1991 period. Huffman and Evenson's index numbers show an average regional productivity growth range of 0.27-3.15 percent for crops and a 0.58-3.09 percent for livestock during the 1950-1982 period. A study by Morrison Paul et al. (2001), done with the objective of evaluating the impact of infrastructure on U.S. agriculture at the state level during 1960-1996, used a cost function with two outputs, animals and crops, and estimated supply shifts of 0.2 percent for each one.

These estimates indicate that the percentage reduction in the marginal cost of corn has been more than in soybeans, while the reduction in the marginal cost of soybeans has been more than in wheat. In other words, during these fifty years U.S. agriculture became more productive in the production of corn relative to soybeans and wheat, and in the production of soybeans relative to wheat. It is clear from the estimates that this study has not been useful at understanding the characteristics of technical change in the animal sector.

Summary and Conclusions

I have developed a theoretical decomposition of Griliches' *k*-shift into the rate and output bias components of technological change. I have shown how the output distance function and the inverse supplies or virtual prices obtained from it may be used to specify the *k*-shift, the radial rate of technical change and the output biases. This information is important because it allows productivity measurement by commodity within the context of overall technical change, and enables estimation of the downward shift of the marginal cost per commodity. Griliches' *k*-shifts, a crucial parameter in the welfare evaluation of technological change usually chosen in an ad-hoc manner, can be econometrically estimated based on theory.

I have also discussed the dual relationship between the output distance function and the normalized revenue function in this context, establishing the similarities and differences between the radial dual and primal rates of technical change and the respective output biases. It is possible to define a dual rate of commodity specific technical change which describes the horizontal shift of the marginal cost for each commodity in contrast to the vertical *k*-shift. This is usually referred to in the literature as Griliches' *K*-shift.

I used the approach to estimate the *k*-shift or commodity specific rates of technical progress for corn, wheat, soybeans and beef in U.S. agriculture. The radial rate of technical change is estimated at about 1.77 percent per year, lower than that estimated by others using very different approaches and for slightly different time periods. The *k*-shift for corn is about 5.8 percent, with 3.5 percent for soybeans and 1.3 percent for wheat. This shows that U.S. agriculture has become more competitive in the production of corn, soybeans and wheat. This study is inconclusive with respect to the characteristics of technical change in the beef sector.

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Table 1. Variables Describing the Agricultural Sector

$D_o(\boldsymbol{x},\boldsymbol{y},A)$	output distance function: value of one as efficiency is assumed in estimation.
$\boldsymbol{y}$	vector of outputs: Tornquist-Theil index of production for corn, wheat, soybeans, beef livestock, and aggregate of all other products. Prices are corresponding implicit prices. Original source of data: *Agricultural Statistics, Crop Production, Agricultural Prices, Livestock and Poultry Situation and Outlook.*
$\boldsymbol{x}$	vector of inputs: Tornquist-Theil index of all inputs used. Implicit price index from the same source. Source: Ball, et al.
A	time trend used as proxy for technical change, 1950-1993.

Table 2. Parameter Estimates (t-ratios in parentheses), Symmetry, Homogeneity and Convexity Imposed, 1950-1993, U.S. Agriculture.

Prices	First Order Coefficients	Second Order Coefficients Quantities					
		Corn	Soybeans	Wheat	Beef	Input	Time
Corn	-3.526 (-3.91)	0.030 (2.95)	-0.017 (-7.01)	0.027 (5.46)	-0.021 (-1.49)	-0.078 (-4.39)	-0.002 (-3.96)
Soybeans	5.265 (6.36)		0.031 (5.36)	-0.016 (-4.87)	-0.038 (-4.38)	-0.006 (-0.42)	-0.003 (-6.09)
Wheat	-0.955 (-0.77)			0.030 (4.67)	-0.027 (-1.58)	-0.048 (-1.93)	-0.0005 (-0.85)
Beef	16.453 (6.78)				0.377 (3.31)	0.419 (10.44)	0.009 (6.87)
Inputs	-23.355 (-10.69)					-0.999 (-7.88)	0.013 (11.80)
Time	-0.1783 (-14.26)						-0.00009 (-14.32)

Table 3. Rate of Technical Change, U.S. Agriculture, 1950-1993.

Years	Delta	Ball et al	Alston and Pardey	Huffman and Evenson
1950-59	0.31	1.93	2.33	1.75
1960-60	0.90	2.10	1.39	2.04
1970-79	1.94	1.76	1.38	1.67
1980-89	3.08	3.39	2.65*	3.34**
1990-93	3.92	1.24		
1950-93	1.77	2.21	1.85#	2.24##

*** 1980-85 #1950-85**
****1980-1990 ##1950-90**

Table 4. Pairwise and Overall Output Biases, U.S. Agriculture, 1950-1993 (standard errors in parentheses).

i\j	Soybeans	Wheat	Beef	Others	Overall
Corn	-0.023	-0.046	-0.093	-0.051	-0.040
	(0.002)	(0.009)	(0.093)	(0.032)	(0.014)
Soybeans		-0.023	-0.070	-0.028	-0.018
		(0.006)	(0.072)	(0.098)	(0.007)
Wheat			-0.047	- 0.005	0.005
			(0.044)	(0.056)	(0.002)
Beef				0.042	0.052
				(0.043)	(0.097)
Others					0.010
					(0.080)

Table 5. Griliches' *k*-shift, Commodity Specific Technical Change, U.S. 1950-1993 (standard errors in parentheses)

Outputs	*k*-shift (%)[18]
Corn	5.82
	(0.99)
Soybeans	3.54
	(0.63)
Wheat	1.26
	(0.56)
Beef	-3.47
	(4.7)
Others	0.01
	(1.9)

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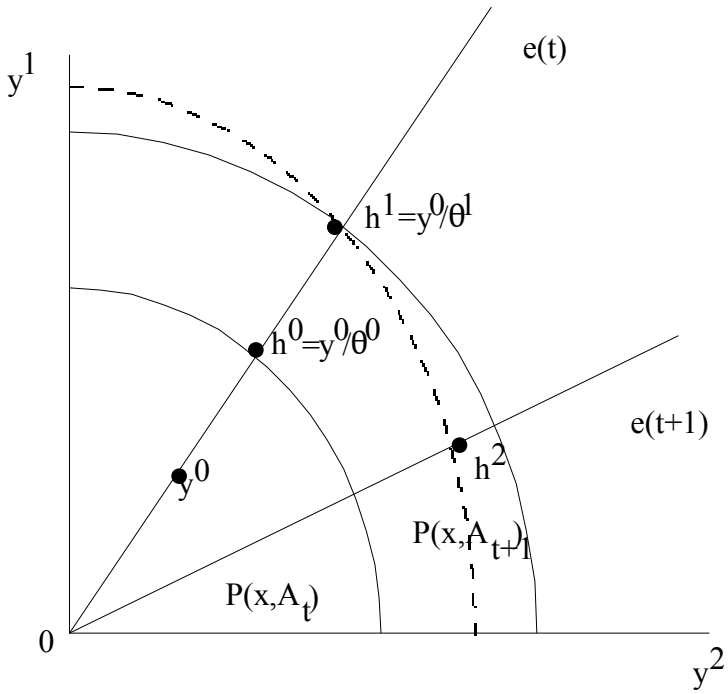

Figure 1. The radial primal rate of technical change and output biases.

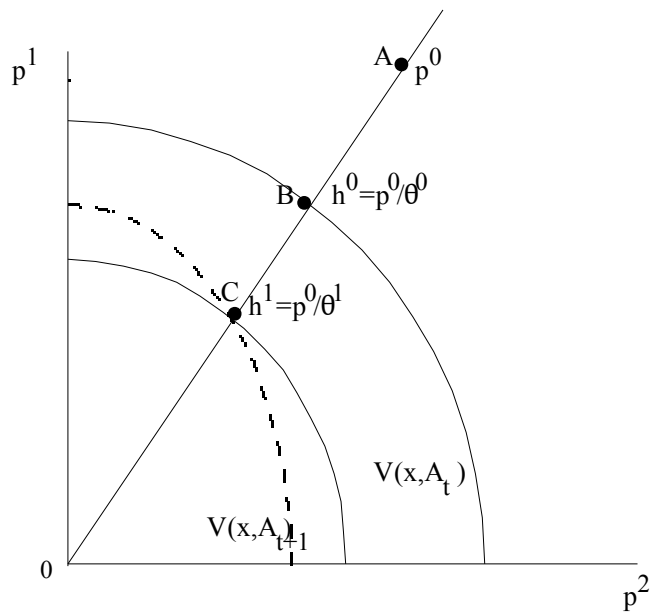

Figure 2. The normalized revenue function and the radial dual rate of technical change.

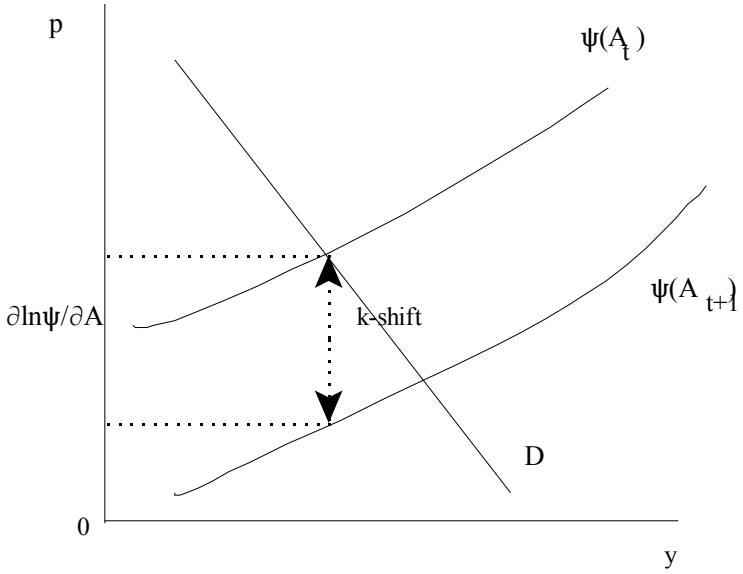

Figure 3. Griliches' k-shift

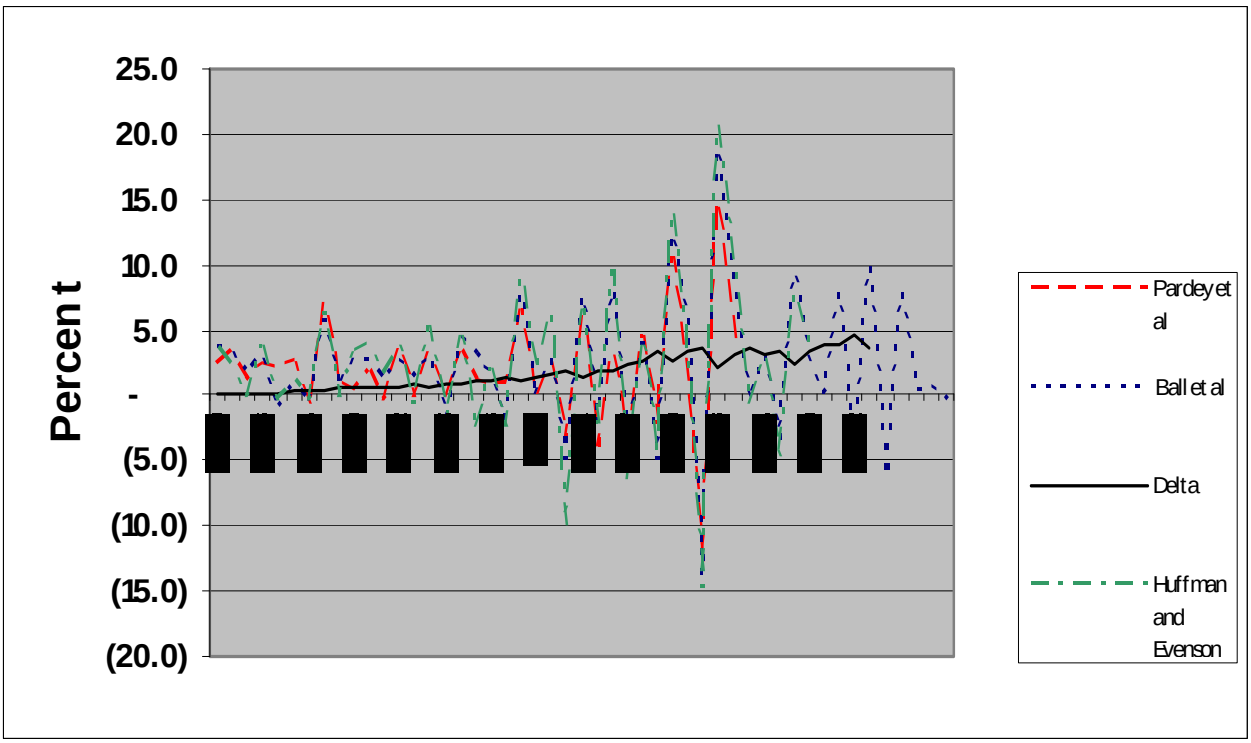

Figure 4. Evolution of the rate of technical change, U.S. Agriculture

Endnotes

[1] Productivity change and technical change are used interchangeably in this paper.

[2] For a review of this literature see Alston et al.

[3] A summary of the early work can be found in Griliches (1996). For more recent attempts see Barro and Xala-i-Martin.

[4] A summary of the efforts along these lines can be found in Berndt, Morrison-Paul (1999) and Alston et al.

[5] See Bresnahan and Gordon for papers along this line.

[6] Uses of the distance function in demand theory are found in Gorman (1976), and a seminal article by Deaton (1978.) A good general exposition is in Cornes.

[7] The output distance function is non-increasing in each input level, non-decreasing in each output level, homogeneous of degree one and convex in outputs. For a complete description and proofs see Shephard (1970).

[8] Linear homogeneity in ***y*** implies

$$\frac{\partial D(\mathbf{x_t}, y_t, A_t)}{\partial A_t} = \sum_m \frac{\partial^2 D(\mathbf{x_t}, \mathbf{y_t}, A_t)}{\partial A_t \partial y_{mt}} y_{mt} = \sum_i \frac{\partial \psi_{mt}(\mathbf{x_t}, \mathbf{y_t}, A_t)}{\partial A_t} y_{mt}$$

[9] If inputs are allowed to change (i.e. $dx \neq 0$) the equivalence between the primal and the dual rate includes an adjustment for returns to scale.

[10] Hicks neutrality is defined in the next section following Blackbory, Lovell and Thursby

[11] By definition of Hicks neutrality $\partial MRT_{ij}/\partial A=0$, or $\partial lnD_{oi}/\partial A=\partial lnD_{oj}/\partial A=\delta$ for all i, j.

[12] Follows from $\delta = \frac{\partial \ln D}{\partial A} = -\frac{\partial D / \partial A}{D}$ where numerator and denominator are linear homogeneous in outputs.

[13] Dual Hicks neutrality is defined as $\partial ln(y_i/y_j)/\partial A=0$ so that $\partial ln\ y_i/\partial A=\partial ln\ y_j/\partial A=\mu$ and the dual rate of technical change measures Hicks neutrality. Also from (22) $B_{ij}=0$ and $\partial ln\ R_i/\partial A=\partial ln\ R_j/\partial A=\mu$.

[14] Griliches' 1958 article says on page 421 " I assume that the superiority of hybrid over open-pollinated varieties is 15 per cent..." In footnote 8 he explains that "Plant breeders conservatively estimate increases in yield of 15 to 20 per cent from using hybrid seed..." and in footnote 10 he says "Assuming k = 13, i.e., 15/115..." Since then many other studies have used this concept in a similar way in welfare evaluation of research in particular markets. These studies are summarized in Alston et al.

[15] An alternative way of measuring the shift of the marginal cost would be to represent the proportional rightward shift of this curve from the original equilibrium value of *p*. This is sometimes referred to in the literature as the *K*-shift (*J*-shift in Alston et al) and measures the proportional change in quantities given prices. In his original study, Griliches does not use this concept.

[16] Other relationships of interest are

$$\mu = \frac{\partial \ln R}{\partial A_t} = \sum_i S_i \frac{\partial \ln y_i}{\partial A_t}$$

$$\frac{\partial \mu}{\partial \ln p_i} = S_i B_i^r = \frac{\partial S_i}{\partial A_t}$$

$$\sum_i \frac{\partial \mu}{\partial \ln p_i} = 0.$$

[17] This matrix is the supply side equivalent to the Antonelli matrix used in demand analysis.

[18] The sum of averages in Tables 3 and 4 do not add up to the average k-shifts in Table 5 due to rounding approximations. In Table 5 calculations use more decimal places. Delta is 1.772951 and the overall biases are- 4.056 for corn, -1.767 for soybeans, 0.509 for wheat, 5.2515 for beef, and 1.0176 for others.