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Simplified RBE-Dose Calculations for Mixed Radiation Fields

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A problem for radiobiology, radiation protection and treatment planning is the calculation of RBE as a function of the delivered dose when this dose is a result of a mixture of low and high LET radiations, as for neutron and pion beams. Track structure theory (Katz and Sharma, 1975) has recently been used to produce an algorithm for calculating survival, RBE, and OER as a function of dose in such environments. This simple formulation is based on the concept of irradiation equivalence (Katz *et al.*, 1972) through which the effect of a mixed radiation environment is calculated as if it were due to a combination of equivalent track segment bombardments representing the low LET component (e.g. X-rays, electrons) and one or more high LET components. The equivalent irradiation for any one component of a mixed environment is the track segment bombardment which yields the same survival at a given dose as the original component. Additionally, the values of Π_i and Π_γ (the probabilities for survival at the same dose in the ion-kill and gamma-kill modes) for the component must be equal to those of the track segment bombardment. The agreement of theoretical calculations with experimental observation (Railton *et al.*, 1974 & 1975) for CHO cells when sequentially radiated with neutrons and gamma rays (Katz and Sharma, 1974) and the recent calculations of RBE-dose relations for neutrons and pions lend support to the use of this algorithm.

The surviving fraction of a cellular population irradiated with a mixed radiation field may be calculated from the equation (Katz and Sharma, 1975):

$$N/N_0 = e^{-(\sum f_i P_i / L_i) \sigma_0 D} \times \{1 - [1 - e^{-(\sum (1 - P_i) f_i) D / E_0}]^m\}, \quad (1)$$

so that the dose, D_x , of X-rays leading to the same survival is

$$D_x = -E_0 \ln [1 - (1 - N/N_0)^{1/m}], \quad (2)$$

and the RBE is

$$\text{RBE} = D_x / D. \quad (3)$$

Here f_i is the fraction of the total absorbed dose D delivered by a particular component whose equivalent irradiation is characterized by P_i (the ion-kill probability of the track segment) and L_i (the stopping power of the track segment). It should be evident that $\sum f_i = 1$. Radiosensitivity parameters of the cellular system are E_0 , σ_0 , and m . All of the parameters necessary for calculations for mammalian cells, irradiated with neutrons of selected spectra and pions, are listed in Table 1.

For the case of a mixed radiation environment involving only two components, such as neutron beams with gamma-ray contamination, any predetermined value of f_i also specifies the value of $f_2 = 1 - f_1$. Stopped pions introduce additional complications since there are contributions to the delivered dose from pion stars, neutrons, and a complex of "low LET radiations." The neutrons resulting from a pion irradiation have a wide range of energies up to about 100 MeV (Guthrie *et al.*, 1968). As an approximation, the calculation of secondary charged particle spectra from these neutrons (Dennis, private

Table 1. Cellular radiosensitivity parameters for mammalian cells and equivalent irradiations for neutrons and pions.

Cell		Hamster	HeLa	Kidney	Leukemia	Bone Marrow				
Environment		O ₂	N ₂	O ₂	N ₂	O ₂				
Parameters	m	2.5	3.0	3.0	2.5	2.5				
	$E_0 \times 10^{-4}$ (ergs cm ⁻³)	1.9	3.7	1.4	4.6	1.7				
	$\sigma_0 \times 10^7$ (cm ²)	4.6	5.6	5.6	6.7	6.7				
Neutron Energy (MeV)	0.10	L [†]	3.34	3.36	3.38	3.35	3.37	3.26	3.30	3.38
		P	0.910	0.934	0.953	0.921	0.944	0.814	0.866	0.957
	0.20	L [†]	3.98	4.03	4.10	4.00	4.07	3.82	3.90	4.13
		P	0.847	0.892	0.952	0.867	0.922	0.705	0.775	0.974
	0.50	L [†]	4.83	4.90	5.08	4.87	5.00	4.58	4.70	5.20
		P	6.634	0.689	0.822	0.662	0.756	0.477	0.547	0.928
	1.00	L [†]	5.58	5.60	5.64	5.59	5.62	5.54	5.55	5.61
		P	0.509	0.545	0.670	0.531	0.612	0.399	0.446	0.820
	2.00	L [†]	5.65	5.69	5.51	5.63	5.51	5.80	5.73	5.06
		P	0.294	-0.318	0.424	0.312	0.379	0.213	0.247	0.594
	14.00	L [†]	15.3	15.4	12.6	14.8	12.9	18.2	16.9	8.18
		P	0.235	0.242	0.273	0.240	0.261	0.207	0.219	0.325
MRC	L [†]	10.5	10.5	8.48	10.0	8.69	13.1	11.8	5.82	
	P	0.196	0.205	0.249	0.204	0.232	0.163	0.177	0.332	
Californium	L [†]	5.31	5.46	5.12	5.26	5.06	5.52	5.44	4.32	
	P	0.130	0.144	0.220	0.142	0.190	0.076	0.098	0.358	
Pions	Stars	L [†]	13.4	13.5	11.2	12.9	11.4	16.0	14.8	7.99
		P	0.146	0.158	0.206	0.154	0.185	0.106	0.123	0.278
	Neutrons ≤ 20 MeV	L [†]	10.4	10.3	8.56	10.0	8.79	12.7	11.6	6.15
		P	0.222	0.233	0.283	0.230	0.263	0.183	0.199	0.372
	Neutrons > 20 MeV	L [†]	12.9	17.4	14.7	16.9	15.0	20.4	19.1	10.1
		P	0.282	0.292	0.328	0.289	0.313	0.248	0.263	0.380

L[†] stands for LET_∞ × 10⁻² (MeV g⁻¹ cm²).

communication) is divided into two parts: the first for neutrons up to 20 MeV; and the second for neutrons of energies greater than 20 MeV, for which there is no nuclear cross-section information available. The secondary particle spectrum for the second part is estimated by treating this fraction of the neutron spectrum as if it were made up of monoenergetic neutrons of 19.5 MeV. For this purpose the portion of the neutron dose below 20 MeV is taken to be twice as great as the portion above 20 MeV.

Once the equivalent irradiations have been evaluated, the preceding algorithm enables us to calculate the synergistic effects of mixed radiation environments without recourse to complex computer programs. Using Table 1 and equations (1)–(3), the survival, D_x , and RBE at a given dose can be evaluated with a slide rule. A more practical and convenient approach is to implement

this algorithm as a program for a pocket programmable calculator. Such a program has been written for the Hewlett-Packard HP-65 and has been accepted by their User Library (Fullerton, 1975) as program No. 02955A.

The HP-65 has nine storage registers and four temporary memory registers arranged in a "stack," which allows the program to handle a mixed radiation field of up to and including four components, one of which must be a low LET part. The cellular parameters and the values of the variables f_i , L_i , and P_i for each high LET component are entered into the nine storage registers. For the low LET component P is taken to be zero. Therefore, L is not needed and f is calculated internally. The desired total dose, D , is entered into the first position of the stack. By pushing three buttons in succession, one obtains the surviving fraction, the dose of X-rays yielding the same survival, D_x , and the RBE at the dose D .

To illustrate, the surviving fraction, D_x , and RBE are calculated for several different doses, cell lines and radiation environments.

Case I

Kidney cells are aerobically irradiated with a contaminated beam of pions, in a region where 60% of the dose is delivered as pion stars, 20% of the dose is delivered as neutrons, and 20% as "low LET radiation." We have, therefore, f_1 (pion stars) = 0.6, f_2 (neutrons up to 19.5 MeV) = 0.13, f_3 (neutrons greater than 19.5 MeV) = 0.07, and f_4 (low LET radiation) = 0.2. We find

Dose (rads)	N/N_0	D_x (rads)	RBE
100	0.482	249	2.49
150	0.312	336	2.24

Case II

Kidney cells are aerobically irradiated with a contaminated beam of pions, in a region where 80% of the dose is delivered as pion stars and 20% of the dose is delivered as "low LET radiation." We have f_1 (pion stars) = 0.8 and f_2 (low LET radiation) = 0.2, and find

Dose (rads)	N/N_0	D_x (rads)	RBE
100	0.524	2319	2.31

Case III

Kidney cells are aerobically irradiated with a contaminated beam of 14 MeV neutrons where 80% of the dose is delivered by the neutrons and 20% of the dose is delivered by gamma rays. Therefore, f_1 (neutrons) = 0.8 and f_2 (gamma rays) = 0.2, and

Dose (rads)	N/N_0	D_x (rads)	RBE
100	0.465	256	2.56

Case IV

Kidney cells are aerobically irradiated with a contaminated beam of 14 MeV neutrons where 60% of the dose is delivered by the neutrons and 40% of the dose is delivered by gamma rays. Therefore, f_1 (neutrons) = 0.6 and f_2 (gamma rays) = 0.4, so that

Dose (rads)	N/N_0	D_x (rads)	RBE
100	0.544	223	2.23

Case V

HeLa cells are aerobically irradiated with a contaminated beam of pions, in a region where 60% of the dose is delivered as pion stars, 20% of the dose is delivered as neutrons, and 20% as "low LET radiation." We have f_1 (pion stars) = 0.6, f_2 (neutrons up to 19.5 MeV) = 0.13, f_3 (neutrons greater than 19.5 MeV) = 0.07, and f_4 (low LET radiation) = 0.2. Then

Dose (rads)	N/N_0	D_x (rads)	RBE
100	0.506	219	2.19
150	0.330	291	1.94

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