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Multi-structural fast nonlinear model-based predictive control of a hydronic heating system

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ABSTRACT

Buildings consume a significant amount of energy to maintain the indoor thermal comfort. One way to reduce the energy consumption in buildings is to improve the overall energy efficiency through integrated advanced controls. It is undoubted that incorporating a model and utilizing future information in real-time building operation offers great energy saving potentials. However, there are many barriers preventing this from happening. Building systems are nonlinear and multiple-input-multiple-output (MIMO) in nature. Conventional approach with a global search solver and a detailed model simulator or direct nonlinear model predictive control (MPC) incurs prohibitive computational cost. This paper proposes a multi-structural fast nonlinear model predictive control (MPC) for handling nonlinear building systems and applied it on a hydronic heating system. The methodology remains the advantages of linear classical MPC and solves the nonlinearity issues involved in thermal comfort and energy conservation oriented control. The simulation shows that the controllers can achieve the optimal solutions in less than five minutes for five days simulation in a full look-ahead scenario with a Duo T6400 2.0 GHz computer. The fastest scenario takes only thirty more seconds to accomplish, which makes the approach feasible for online implementation. The techniques of using MPC for achieving smooth day-night switch, band control, dynamic constraints, and dynamic weighting are discussed. The energy saving potential of applying the proposed MPCs is found to be between six to forty two percent. In addition, the techniques decouple the building system from the mechanical system and are applicable to other space conditioning systems as well.

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1. Introduction

Buildings in the U.S. consume 70 percent of the electricity, about 50 percent of which is generated from the combustion of fossil fuels [1]. Although the efficiency of individual equipment has increased considerably with better design, manufacturing, and engineering, the energy use in buildings has continued to rise because of increased use of energy intensive devices and evolved demand on indoor thermal comfort. The amount of energy Americans use doubles every 20 years [2]. In the meantime, according to the U.S. Environmental Protection Agency (EPA), about 30 percent of energy in buildings is used inefficiently or unnecessarily [3]. Significant improvement of building energy efficiency is needed in an attempt to achieving energy and environment sustainability.

An advanced building automation system, which interconnects and automates the various systems and devices, is typically deployed in commercial buildings to maintain the required thermal comfort with minimum energy consumption. Functions of building control are typically fulfilled in a two-level structure: a supervisory level and a local level. The supervisory-level controller comprises a group of logics and typically resides in a central station. It takes actions based on preset conditions and/or rules, or the commands from the operators, such as heating/cooling switch-over, components sequencing, event scheduling, etc. However, this type of rule-based responsive control, with local On/Off and Proportional-Integral-Differential (PID) controllers, is expensive in the long run since they operate at a non-optimal efficiency [4]. In addition, neither of the two control objectives, thermal comfort and energy savings, can be explicitly expressed in the conventional control laws [5].

An advanced building control system needs to possess some kind of predictive capability with building dynamics and
Research on MPC in building systems has just recently started to thrive [9–19]. Upon the literature review, we found the majority of these studies are in the area of general MPCs, which usually couple energy sources and thermal storage. They found that while significant improvements in system operation are possible, a search for and not suitable for online implementation. For example, Zhang and Hanby applied genetic algorithm to a system with different tools, such as the linear quadratic regulator theory, Kalman filtering theory, etc., to explore the potentials. Study of utilizing the good features of linear MPC on multivariate HVAC systems is gradually gaining more interest recently [15–19]. For instance, Kolokotsa optimization, etc., are applied to identify the optimums. This approach is generally associated with very high computational cost and not suitable for online implementation. For example, Zhang and Hanby applied genetic algorithm to a system with different energy sources and thermal storage. They found that while significant improvements in system operation are possible, a search for and not suitable for online implementation. For example, Zhang and Hanby applied genetic algorithm to a system with different tools, such as the linear quadratic regulator theory, Kalman filtering theory, etc., to explore the potentials. Study of utilizing the good features of linear MPC on multivariate HVAC systems is gradually gaining more interest recently [15–19]. For instance, Kolokotsa
et al. used a simplified bilinear-model based predictive controller to predict and control the indoor environmental conditions of a single zone laboratory [15]. Morosan et al. studied a distributed predictive control structure for thermal regulation in buildings to avoid the exponentially growing computational demand. The centralized and distributed MPC showed an improvement about 36 percent on thermal comfort meeting hours with 13 percent of energy savings [16]. Freire et al. utilized an MPC for thermal comfort control in a single zone one-actuator air-conditioning building. Two case studies in terms of an MPC and different metabolic rate and clothing index were simulated to validate the proposed method [17]. Hazuyk et al. applied linear programming method in the canonical form to the predictive control of intermittent heated buildings. The mathematic summation of energy consumption and thermal comfort were treated as the cost function and constraints, respectively. Heat flux from the radiator was used as the control input [18]. Yuan and Perez applied MPCs to a variable air volume system to achieve an acceptable indoor air quality. The ventilation condition in the six zones with MPC was found improved compared to the traditional control [19].

Despite the findings from those studies, very few publications considered both thermal comfort and energy savings explicitly in the formulation of MPCs. The method and application of a linear classical MPC for MIMO nonlinear buildings systems have not been fully explored. The objective of this study is to investigate the methodologies and potentials of utilizing linear classical MPCs for nonlinear systems, with both thermal comfort and energy cost considered. In this article, we first briefly reviewed the methodology of MPC in terms of the formulation, model structure, and solving method. Then, Wiener, Hammerstein, and Hammerstein–Weiner structures were introduced, which can be used to handle the nonlinearity in a building system. Thereafter, they were applied to a hydronic heating system to evaluate the performance. The results were presented and the paper concludes with some discussions.

2. Background on MPC

Model based optimal control technology provides us with opportunities for improving the building energy system performance by incorporating the system models and utilizing future information. With over 2000 industrial installations, MPC is currently the most widely implemented advanced control technology for process plants [8]. A lot of literature summarized the MPC theory and recent studies in both industry and academia [6,8,20,21]. Although MPC paradigm includes many different variants with all kinds of features, all MPC systems involve the basic elements to generate the control trajectory for a process: an internal model, an optimization solver, and a moving horizon feature. Fig. 1 illustrates how an MPC functions. Different MPCs have the varieties of the model format, disturbance estimation method, objective, solver, etc. Many techniques are utilized in addition to improve the performance of MPC for given circumstances, for instance, moving block, reference look-ahead, time-variant weighting, time variant constraints.

2.1. MPC in broad definition

From a broad point of view, any control strategy with the aforementioned three elements can be considered as an MPC. For every move, an open loop optimal control problem is formulated and solved over a finite horizon. The solver evaluates the cost function based on the system simulation and the given constraints. The controller picks the first set of optimal control variables for implementation and the whole process repeats.

2.1.1. Controller formulation

The formulation can be put as:

\[
\begin{align*}
\min J &= f(u, x, y) \\
\dot{x} &= f(x, u, w) \\
0 &= h(x, u) \\
x^l \leq x \leq x^U \\
y^l \leq y \leq y^U \\
u^l \leq u \leq u^U
\end{align*}
\]

For a general MPC, the objective function as given in Eq. (1) can be loosely defined as almost anything, as long as it has a clear mathematical expression to represent the cost of the system. The system model is also a general model, which can be, for example, a group of differential algebraic equations, neural network, or model bank. The system is subject to a set of constraints on the states, outputs, or control variables.

2.1.2. Solving methods

The dynamic control problem defined by Eqs. (1)–(3) is very often casted as a numerical optimization programming problem. The control inputs, or both the control inputs and the states in the modeling equations are parameterized along the prediction horizon. When only control inputs are parameterized, it’s referred as a sequential approach. The system model is remained as a simulator to obtain the cost and evaluate the constraints for different control actions. The second approach is termed collocation or simultaneous approach, where both the control and system model are converted into polynomial form that satisfies the system differential algebraic equations. With the later approach, models in general need to be expressed in a set of explicit equations and follow some strict syntax in order to perform automatic numeric processing. After that, many methods and optimization packages can be applied to solve the optimization programming problems. The solution approaches vary a lot in terms of involved mathematic techniques/principles and computational cost. The selection of a proper solver can be difficult, depending on the problem features and the required performance of the control.

In general, a numerical solver is usually designed for a particular problem type. No optimization solver is universally applicable, or it works at an extra cost. Meanwhile, the solution of a general optimization problem (e.g. constrained, nonlinear, mixed integer etc.) can be computationally expensive and no global optimization is guaranteed. The satisfaction of conditions is only ensured at the selected evaluation points. Findiezen categorized three principle approaches to solving optimal control problems: Hamilton–Jacobi–Bellman equations, Euler–Lagrange differential equations, and finite parameterization on the control inputs and constraints [22]. A few literature available discussed the related issues involved in general MPC from different points of view (e.g. Refs. [23–25]).

2.2. Linear quadratic MPC

Linear classical MPC is one of the most mature optimal control strategies. It refers to a group of optimization problems where: 1) the internal dynamic model is linear and time-invariant (LTI); 2) the objective function is a quadratic function in terms of the control inputs, measurable outputs, and rates of control inputs; and 3) the constraints are on the control inputs, measurable outputs, and rates of control inputs.
2.2.1. Controller formulation

A classical MPC has the following quadratic programming formulation, including cost function, linear model, and boxed constraints:

$$\min \sum_{i=1}^{k} \|e^i\|_Q^2 + \|\Delta u\|_S^2 + \|u\|_R^2$$  \hspace{1cm} (4)

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (5)

$x^l \leq x \leq x^u$

$y^l \leq y \leq y^u$

$u^l \leq u \leq u^u$

$\Delta u^l \leq \Delta u \leq \Delta u^u$ \hspace{1cm} (6)

The common model types are finite impulse and step response models, auto-regressive moving average with exogenous input (ARMAX)/controlled auto-regressive integrated moving average (CARIMA) models, transfer function models, and more general, state-space models.

A finite impulse response (FIR) or step response (SR) model is widely used in system control since it can fit arbitrarily complex stable linear dynamics based on experimental data. No or less advance knowledge about the system is required to develop the model. Methods, such as ordinary least-square or optimization based on minimization of prediction and measured errors, can be used to process the data and obtain the model parameters. Eqs. (7) and (8) give the general expression of an FIR and a SR model, respectively.

$$y(k) = \sum_{i=1}^{n} b_i u(k - i)$$ \hspace{1cm} (7)

$$y(k) = \sum_{i=1}^{n} s_i \Delta u(k - i) + s_n u(k - n - 1)$$ \hspace{1cm} (8)

For a MIMO system, the coefficients become a matrix instead of a vector to represent the mapping relationship between the multiple inputs and multiple outputs. As seen obviously from the expression, a large number of data are generally needed to identify the parameters used in an FIR or SR model for a complex system. The parameters can be up to hundreds for a complex system. For a MIMO system, the disadvantage of great number of parameters becomes troublesome. Since the model does not include any structure information from the system, data over-fitting and high modeling uncertainty may exist.

Eqs. (9) and (10) show the structure of an ARMAX and a CARIMA model, respectively. In Eq. (9), from left to right, the terms are auto regressive part, moving average part, and exogenous inputs. In Eq. (10), the moving average term includes an integral part.

$$Ay(k) = Ce(k) + Bu(k)$$ \hspace{1cm} (9)
Both models can be deduced based on measurements and in an adaptive (predictive) manner. With CARIMA model, good output predictions and future control sequence can be obtained alternatively to minimize the cost function. The derivation of optimal prediction of CARIMA model can be obtained by recursion of Diophantine equation [26].

A transfer function model is applicable to both stable and unstable plants. It can be deduced based on regression or from first-principles. As the term implied, the model maps the direct relationship between inputs and outputs. The advantage of using a transfer function model also includes its compact expression, which requires less parameter than in an FIR model. Eq. (11) gives the structure for a transfer function model. Like an FIR model, a transfer function is considered less effective for MIMO plants.

\[ Ay(k) = Bu(k) \] (11)

Any aforementioned linear dynamic model for classical MPC can be transformed to a state space model. A state space model provides a nice structure for MPC, and it can easily handle MIMO systems. It has uniform treatment of stable, integrating, and unstable processes.

2.2.2. Solving methods
Since the model and the constraints are linear, and the cost function is formed in quadratic, classical linear MPC can be eventually casted as a quadratic programming problem. A quadratic programming problem is a convex optimization problem with a unique optimal solution. It can generally be solved with low computational cost, which makes it suitable for online implementation. To offset nonzero values on the variables, an integral term of manipulated variables based on augmented system is generally used in MPC. For most plants in building systems, there are hard constraints and/or soft constraints on the inputs, outputs, and/or changing rates. For instance, the total input to a fan cannot be more than the maximum capacity. The supply air temperature needs to stay in an acceptable range. A constraint imposed on compressor speed changing rate can prevent the compressor from frequent and large scale speed modulation. A wider constraints during unoccupied hours and narrower constraints during occupied hours on the indoor air temperature can lead to more energy conservative operation. Constraints can also be utilized to guide MPC toward the desired operation direction. Techniques, such as Lagrange multipliers, Kuhn–Tucker conditions, active set methods, primal-dual method, interior-points, etc.; are the solution candidates for constrained quadratic programming problems.

3. Handling of nonlinear elements in MPC
While a classical linear MPC runs fast, it cannot be directly handle nonlinear problems. Most building energy systems are nonlinear in nature. Additional considerations are needed in order to utilize it. For conventional buildings, there are generally single mechanical heating or cooling sources deployed in the space; the controlled variable is typically the space air temperature. For instance, in a VAV conditioned building, the supply air temperature and the air flow delivered into the individual room are the manipulated variables. In a fan coil conditioned space, the supply water temperature and the air flow through each fan coil are the manipulated variables. In a residential building, the power input to the air-conditioner or furnace can be considered as the manipulated variable. Under these circumstances, it is beneficial to design an MPC with the building treated as a linear time-invariant system. Fig. 2 illustrates the layout for a linear MPC structure with heat flux as the system inputs and direct measurable variables as outputs.

This approach has the following benefits: 1) by using a linear model and quadratic cost function, the problem becomes a highly structured convex problem and can be solved fast. There are mature solution and analysis methodologies; 2) the space system can be decoupled from the mechanical system. The space model can be re-utilized for other study purposes; and 3) the manipulated variables can be set as the heat flux and the cost function can directly reflect the overall energy consumption.

While the building system dynamics can be reasonably simplified into a linear time-invariant model as a Resistance-Capacitance (RC) thermal network, the nonlinearity still exists. Fig. 3 illustrates the typical model structure in reality. The two ends of the linear time-invariant (LTI) system are static nonlinear mappings. A system with only the static nonlinearity before the LTI is termed a Hammerstein nonlinear system. A system with only the static nonlinearity after the LTI is a Wiener nonlinear system. A system with the nonlinearity at both ends is termed Hammerstein–Wiener (HW) nonlinear system [27]. The nonlinearity in a Hammerstein system comes from the fact that the heat flux into the air space is usually not the direct manipulated variable. For example, with a VAV box, we control the damper position instead of the amount of heat into the space. In a Wiener system, the direct measurable variables are not the target control variables. For instance, we want to control thermal comfort but we usually only have measurements on the air temperature and humidity.

To utilize the advantages of linear MPC for online implementation and solve the nonlinear system with static nonlinearity, the system inputs and outputs need to be processed accordingly. For a Wiener model, as shown in Fig. 4, the outputs from the LTI are mapped into the desired outputs by using the inverse function of the mapping. The signal is then compared to the inverse reference set point. The errors are fed into the linear MPC to form a close loop feedback control. In a thermal comfort oriented control, the direct map is from the room air temperature, room air humidity, air velocity, metabolic rate, external work, clothing insulation to the thermal comfort index. In a Hammerstein model, as illustrated in Fig. 5, the control action generated by the linear MPC first goes through a static nonlinear inverse function, e.g. from the heat flux to the water flow rate. The actual signal is then obtained and fed into the system after the inverse map.

Nonlinear Hammerstein–Wiener (HW) model consists of a linear dynamic block and two static output non-linearity functions on both sides. This structure is more realistic in representing space conditioning for thermal comfort in most building systems. The nonlinear HW system can be described by the following equations:

\[ x(k + 1) = Ax(k) + Bu(k) \] (12)

\[ y(k) = Cx(k) \]

\[ z(k) = h(y(k)) \] (13)

\[ v(k) = f(u(k)) \] (14)

For the hydronic heating and cooling system in this study, the HW model and the linear MPC-based control loop is illustrated in Fig. 6.
4. System description, modeling, and MPC formulation

4.1. System description

The Intelligent Workplace north integrated building energy system is a living laboratory for the research on high performance buildings and conditioning techniques as shown in Fig. 7. It is located on the campus of Carnegie Mellon University, Pittsburgh, USA. The building features a reconfigurable open space and accommodates offices for some thirty graduate students and faculty members. Approximately 600 m² area is conditioned.

Each set of water mullion consists of four vertical embedded pipes and is equipped with a modulating valve on the inlet pipe. A surface temperature sensor is also installed on the outlet point of each mullion. The water is supplied from the campus grid via a tertiary pump. A heat exchanger is installed in the basement to covert the steam from the campus grid into hot water and feed the space in winter. In the heating season, the primary supply water temperature is determined by the campus grid and not adjustable locally. To avoid condensation on the surface of the water-based conditioning terminals and also provide addition control flexibility, a mixing valve is mounted after the tertiary pump. The mixing valve controls the supply water temperature circulated to the space. The other water-based terminals for local environment control include suspended radiant ceilings and ductless radiant chilled beams. When additional or individual conditioning is needed, these terminals can be controlled individually to provide extra heating and cooling.

As a living and lived in laboratory, the space has been constantly used for experimental research projects. The control algorithm and logic of the subsystems and equipment are not always the same. The basic structure of the control architecture includes a layer of rule-based supervisory control and the local SISO based On/Off and PID control. The space is designed for easy reconfiguration and accommodates different experiments. For this study, the space is divided into two zones: one along the east facade, and the other along the west facade. Fig. 8 illustrates the space layout.

4.2. Model development

4.2.1. Space model

The thermal dynamics of a building is determined mainly by the thermal mass of the building materials and the inside air. The time constant of a heavy building can be up to hours. To predict the future evolution of indoor air condition for acceptable thermal comfort, the dynamics need to be represented accurately in the model. Calculating instantaneous space load is critical in dynamic modeling of building thermal systems. Building envelope includes the external walls, roof, and floor. They are the boundary between the indoor and the outdoor environment. For energy consumption and thermal comfort evaluation, the dynamics and interaction of the indoor air and the envelope are of great importance since they determine the actual thermal load and evolution of indoor air temperature and influence the thermal comfort. The time constant of a model tells how fast the building system can respond to the change in the mechanical system while the model provides some insights on what corresponding actions shall be taken for a given indoor temperature goal.

The deduction of the transient space model is based on the energy conservation, the first law of thermodynamics. The thermal capacitance and resistance involved in the heat balance of a building thermal system is analogue to the capacitance and resistance as in an electric network. Similar to the electric network, the thermal network has the linearity and can be put in the state space format. This approach is also used in the whole building system simulation tools (EnergyPlus, Trnsys, etc). Fig. 9 depicts such an analogue of a wall divided into two layers with two capacitances and three resistances. A heat flux, such as radiation, can be regarded as a current source acting on the inside or outside capacitance.

Heat transfer through an opaque wall as shown in Fig. 9 can be modeled as:

\[
C_{w,e}\frac{dT_{w,e}}{dt} = \frac{(T_{oa} - T_{w,e})}{R_{w,e}} - \frac{(T_{w,e} - T_{w,i})}{R_{w,m}} \tag{15}
\]

\[
C_{w,i}\frac{dT_{w,i}}{dt} = q_{r} + \frac{(T_{w,e} - T_{w,i})}{R_{w,m}} - \frac{(T_{w,i} - T_{ra})}{R_{w,i}} \tag{16}
\]

Fig. 10 illustrates the thermal network of one construction with an opaque wall and a transparent window for a building with four rooms.

Convection is the main heat transfer mechanism between the air node, the internal heat gains, and the enclosures. Based on the heat balance, a first-order equation can be obtained to model the evolution of room air temperature:

\[
\rho V_{ra} c_p \frac{dT_{ra,j}}{dt} = q_{plant,j} + q_{g,j} + q_{w,j} + q_{gain,j} + q_{inf,j} \tag{17}
\]
The thermal components in Eq. (17) can be further decomposed to the driven factors, e.g. temperature differences. For simplicity, some of the subscripts for enumeration are omitted in the following text.

\[ q_{g_i} = \sum A_{g \cdot U_g}(T_{ra,i} - T_{e}) \]  
(18)

\[ q_{w_j} = \sum A_{w \cdot U_w}(T_{ra,j} - T_{w,j}) \]  
(19)

\[ q_{\text{inf,j}} = ACH \cdot V_{ra,j}(T_{ra,j} - T_{oa})/3 \]  
(20)

The heat gain from solar radiation through fenestrations is treated differently due to the two-step heat transfer mechanism. The heat flux of solar radiation first penetrates fenestrations, hits the floor, and then is reflected to all other surfaces. The surfaces are warmed up by the heat flux then gradually transfer the heat into the air node mainly via convection. The algorithm proposed by Skartveit & Olesen [28,29] based on the sky cleanness index is used to process the hourly horizontal solar radiation. Normal solar radiation is then corrected into the plane of building surface at any azimuth and tilt angle with the method established by Liu & Jordan [30]. The model of solar radiation transmitted into the space through a fenestration has the following relationship:

\[ q_r = A_g \cdot \varepsilon \cdot q_{r \text{-surf}} \]  
(25)

The internal heat gains can be regarded as instant load and simplified as a function of total load intensity:

\[ q_s = f(I_s) \]  
(26)

\[ q_l = f(I_l) \]  
(27)

4.2.2. Thermal comfort model

Thermal comfort is defined as the “condition of mind which expresses satisfaction with the thermal environment and is assessed by subjective evaluation” [31]. As a subjective thing, an absolute satisfaction is almost impossible to achieve in a conditioned space with more than one occupant. However, a relatively acceptable thermal comfort to the occupants can be evaluated based on the existing findings. Predictive mean vote (PMV) and predicted percentage of dissatisfaction (PPD) are the indexes utilized to assess the thermal comfort condition of an indoor environment.

PMV gives the mean value of the votes of a large group of people. Values from –3 to 3 are utilized to express the conditions from very
cold to very hot. A value, between −0.5 and 0.5, indicates a good thermal comfort indoor environment. PPD is another index in the quantitative evaluation of thermal comfort. A PPD value higher than 10% indicates the indoor environment may not be acceptable. Models of PMV and PPD are quasi-steady-state and can be expressed as the following equations [31,32]:

\[
\begin{align*}
\text{PMV} & = (0.028 + 0.3033e^{-0.036M}) \\
& \left\{ (M - W) - 3.05 \cdot 10^{-3}[5733 - 6.99(M - W) - Pa] - 0.42[(M - W) - 58.15] - 1.7 \cdot 10^{-3}M(5867 - Pa) - 0.0014M(34 - T_a) - 3.96 \cdot 10^{-8}f_{cl}\left[ (T_{cl} + 273)^4 - (T_{mrt} + 273)^4 \right] - f_{cl}h_c(T_{cl} - T_a) \right\} \\
\end{align*}
\]

(28)

\[
\text{PPD} = 100 - 95 \cdot e^{-0.0353 \cdot \text{PMV}^4 - 0.2179 \cdot \text{PMV}^2}
\]

(29)

The surface temperature of clothing is given by:

\[
T_{cl} = 35.7 - 0.028(M - W) - 0.155L_{cl} \left\{ 3.96 \cdot 10^{-8}f_{cl}\left[ (T_{cl} + 273)^4 - (T_{mrt} + 273)^4 \right] - f_{cl}h_c(T_{cl} - T_a) \right\}
\]

(30)

\[
h_c = \begin{cases} 
2.38(T_{cl} - T_a)^{0.25} & \text{for } 2.38(T_{cl} - T_a)^{0.25} > 12.1\sqrt{V_a} \\
12.1\sqrt{V_a} & \text{for } 2.38(T_{cl} - T_a)^{0.25} \leq 12.1\sqrt{V_a}
\end{cases}
\]

(31)

4.2.3. Hydronic thermal system

The test bed space is mainly conditioned by a group of radiant pipes deployed along the envelope. Additional radiant panels and cool beams are installed in the offices to provide extra thermal conditioning capacity during extreme weather. Lumped parameter models are adopted in this study to simulate the heat transfer of the water based terminals. The average representative temperature of the terminal mass can be found by solving:

\[
M_{cl}c_{pt}\frac{dT_{t}}{dt} = q_{in} - q_{out}
\]

(32)

The total heat getting into the fluid residing in the terminals is simply:

\[
q_{in} = m_{cl}(T_{w,s} - T_i)
\]

(33)

The heat transferred from the terminals to the air node can be modeled by:

\[
q_{out} = U_fA_t(T_t - T_i)^n
\]

(34)

The order of the model expressed with Eqs. (32)–(34) can be increased to achieve a higher accuracy for the study of the terminals.

4.3. MPC formulation

An MPC works to achieve a minimum cost based on existing knowledge and future prediction. With the above first-principle model for the building system, a cost function needs to be determined. For a classical MPC, this cost function needs to be defined as a quadratic one. It may include reference tracking errors, control efforts, and actuator movement efforts, etc. Air temperature, thermal comfort, and even monetary cost can be wrapped into it as well. Also, a band of thermal comfort value may be imposed as constraints to differentiate the operation variations due to occupancy changes.

As shown in Fig. 11, two heat fluxes from the terminals are the direct inputs to the air nodes. The outdoor air temperature, boundary surface temperature, and solar radiation heat flux are the external inputs that affect the space temperature through the construction surfaces. In addition, the sensible heat gain, latent heat gain, infiltration air flow rates, etc., are the disturbances to the system. The indoor air temperature and average air humidity are the direct outputs from the LTI space model. The mathematic expression of the LTI system for the two room layout 2R1C space can be put as:

\[
x(k + 1) = Ax(k) + Bu(k)
\]

(35)

\[
x = \begin{bmatrix} T_{ra,i-1,2} & T_{m,i-1,11} \end{bmatrix}^T_{14 \times 1}
\]

(36)

\[
u = \begin{bmatrix} Q_{i-1,2} \end{bmatrix}^T_{2 \times 1}
\]

(37)
\[ d = \begin{bmatrix} \dot{Q}_{\text{sen.},i-1.2} & \dot{Q}_{\text{lat.}} & \dot{Q}_{\text{sol.},i-1.2} \end{bmatrix}^T_{9\times1} \]  
\[ w = \begin{bmatrix} T_{\text{oa},i-1.6} & T_{\text{og}} & m_{\text{oa}} & g_{\text{oa}} \end{bmatrix}^T_{9\times1} \]  

In the nonlinear HW-MPC used in this study, water flow rates to the space terminals are considered as the manipulated variables for the space thermal conditioning. Instead, the supply water temperature may be regarded as one manipulated variable if the whole space is treated as one zone. In the prediction horizon, e.g. 100 min, twenty variables in total needs to be optimized by the MPC controller. Constraints on the inputs are the minimum and maximum water flow rates. With 10-20 min set as one control interval, which is long enough to achieve a stable water distribution among the terminals, the water flow changing rates are not constrained. Fig. 12 illustrates the model built in Matlab/Simulink. The nonlinear map from the air temperature and humidity to thermal comfort is calculated by a function programmed in Matlab.

5. Experimental results and analysis

The experiments were conducted for five days. Fig. 13 plots the outdoor air temperature, relative humidity, solar radiation to the two rooms for the five days. The outdoor air fluctuates between 2 °C and 18 °C during the simulation period, which represents a typical winter.

A conventional space thermal conditioning is to track a room air temperature reference. It is typically a fixed set point, e.g. 24 °C. A base case is set as a 24/7 occupied schedule with the fixed room air temperature set point at 24 °C. Fig. 14 plots the room air temperature, relative humidity, PMV value, and heat inputs. The MPC modulates the heat inputs (heat flux from the terminals as in Eq. (37)) properly and maintained well the room air temperature close to the reference. The overall energy consumption for the 24/7 base case in the five days is 3091 kWh. The PMV calculation shows that, with a fixed room air temperature, the PMV value fluctuates around 0.2. Energy savings can be achieved if the thermal comfort can be used as the reference instead of the air temperature.

An office occupancy based air temperature control is also simulated for the five day period serving as an additional base case. The results are collected in Fig. 15. The system is configured to have a morning warm-up operation based on rule-of-thumb to ensure a smooth transition in the space from unoccupied status in the night to occupied status in the day. The overall energy consumption for this base case is 1941 kWh, reduced by about 37% from SC-1.

Scenario SC-2 is constructed to investigate the thermal comfort based MPC control. The nonlinearity on the energy consumption side is ignored in this control. With a Wiener structure for the thermal comfort mapping, the internal classical MPC works well with the overall hourly summation on square PMV equal to 0.07.
The PMV tracking output is plotted in Fig. 16, which is really close to the set point 0 for neutral thermal comfort. The manipulated variables in this scenario are the heat fluxes to the space air. Compared to Scenario SC-1, the room air temperature drops to be below 24 °C and fluctuates along with the room air humidity. The overall energy consumption is reduced by around 6% from SC-1 to 2895 kWh. With different room air temperature, the indoor air relative humidity increases slightly.

While setting PMV to 0 in MPC can maintain a very good indoor thermal comfort, a slightly cool indoor air condition in winter can bring in more energy savings. Less than 10% occupants vote dissatisfaction when PMV drops to −0.5 according to the existing study in [31]. In Scenario SC-3, the MPC is configured to minimize the energy consumption with PMV constrained between −0.5 and 0.5. As shown in Fig. 17, the MPC successfully drives the system to the lower limit since it causes the lowest energy consumption in winter. Compared to Scenario SC-2, with PMV equal to −0.5 and the corresponding room air humidity, the indoor air temperature is reduced by about 2 °C. The overall energy consumption decreases to 1977 kWh, which is about 36% less than the base condition.

From Scenario SC-2 and SC-3, we see that the proposed Wiener nonlinear MPC can make full use of the advantages of classical linear MPC. It turns the room air temperature based space air conditioning into a thermal comfort oriented control while achieving additional energy savings. The overall run time for 5 days...
takes only 76–176 s to identify the optimums. The nonlinearity is successfully handled with the Wiener MPC approach. In reality, a heat flux is usually not directly controllable from the controller. The corresponding nonlinearity from the actual manipulated variables, e.g., water flow rates, to the heat fluxes needs to be considered. Scenarios SC-4 to SC-7 are designed to investigate the performance of HW nonlinear MPC, including the aspects of reference tracking, disturbance rejection, cost function minimization, and computational cost.

In Scenario SC-4, the water flow rates are used as the manipulated variables in the HW nonlinear MPC. The controller is designed to minimize the discomfort from the occupants by letting PMV track neutral comfort value. From Fig. 18, it can be seen that, compared to Scenario SC-2, the HW MPC handles the system well and the PMV tracking error is close to that of a Wiener nonlinear MPC where the heat fluxes are used as the manipulated variables. With this control system layout, the computational cost is increased by only 2 s than Scenario SC-2. Similarly, Scenario SC-5 is included to evaluate the performance of HW nonlinear MPC with thermal comfort constrained. The controller brings the system to the lowest acceptable PMV in order to minimize the energy consumption. The system performance in Fig. 19 is very similar to Scenario SC-3 as plotted in Fig. 17 in terms of PMV value, except that the driving force becomes the hot water flow rates in SC-5. In the following scenarios, HW nonlinear MPC is utilized to respect the reality of nonlinearity.

Commercial office buildings may have variable occupancy. The change of set point is also a disturbance to the controller in addition to the unmeasured solar heat flux and internal heat gains.

### Table 1

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Problem description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-1</td>
<td>Classical MPC, direct heat flux inputs, temperature oriented control. 24/7 schedule.</td>
</tr>
<tr>
<td>SC-1’</td>
<td>Classical MPC, direct heat flux inputs, temperature oriented control. Office schedule.</td>
</tr>
<tr>
<td>SC-2</td>
<td>Wiener nonlinear MPC, thermal comfort oriented control.</td>
</tr>
<tr>
<td>SC-3</td>
<td>Wiener nonlinear MPC, energy consumption and thermal comfort oriented control. Thermal comfort serves as constraints, PMV = [-0.5, 0.5].</td>
</tr>
<tr>
<td>SC-4</td>
<td>HW nonlinear MPC, thermal comfort oriented control.</td>
</tr>
<tr>
<td>SC-5</td>
<td>HW nonlinear MPC, energy consumption and thermal comfort oriented control. Thermal comfort serves as constraints, PMV = [-0.5, 0.5].</td>
</tr>
<tr>
<td>SC-6-1</td>
<td>As case 5, with variable occupancy schedule. When unoccupied, PMV = [-3, 3]; when occupied, PMV = [-0.5, 0.5]. Band control.</td>
</tr>
<tr>
<td>SC-6-2</td>
<td>As case 6-1, dynamic constraints based on outdoor air temperature.</td>
</tr>
<tr>
<td>SC-7</td>
<td>As case 4, with variable occupancy schedule and full look-ahead ability. When unoccupied, PMV = [-3, 3]; when occupied, PMV = 0.</td>
</tr>
</tbody>
</table>
Fig. 13. Outdoor air condition and solar radiation.

Fig. 14. Simulation results, SC-1.
In Scenario SC-6-1, a variable occupancy schedule is studied and a band control is imposed. When the space is unoccupied, the constraint on PMV is loosened to \([-3, 3]\). They are reduced to \([-0.5, 0.5]\) when the space is occupied to maintain acceptable thermal comfort. Under those conditions, the controller is set to minimize the energy consumption during the 5 days. As plotted in Fig. 20, the controller drives the system between the two different constraints with the occupancy switched between occupied status and unoccupied status. The simulation takes 187 s to obtain the optimums.
Fig. 17. Simulation results, SC-3.

Fig. 18. Simulation results, SC-4.
However, it can also be seen from Fig. 20 that the PMV value during the occupied hours can be lower than $-0.5$ and may cause discomfort. It is due to the various factors of the system, including heating capacity limitation, outdoor air condition, and inertia of building structure, etc. With the energy consumption reduced by 56.7% compared to SC-1, the summation of squared PMV during the occupied hours increases to 39. Additional consideration should be taken to improve the performance.

In Scenario SC-6-2, the constraints are adjusted dynamically based on the outdoor air temperature prediction. The modified MPC drives the system to be within the acceptable PMV constraints with minimum energy consumption. Fig. 21 illustrates the simulation
results of SC-6-2. Compared to Scenario SC-6-1, the overall energy consumption is increased by 293 kWh but the summation of squared PMV value is reduced to 13.44. While energy consumption is of our concern, the thermal comfort shall not be compromised. Compared to base case SC-1 with the office schedule, the energy saving in SC-6-2 is about 41.6%. The strategy adopted in Scenario SC-6-2 is regarded proper to improve the overall performance with both energy and thermal comfort considered.

When a space is switched from unoccupied hours to occupied hours, a smooth transition of the indoor air thermal condition is needed to ensure the space thermal comfort when people come back to the building in the morning. It is termed morning warm-up in winter or cool-down in summer. The strategy is also to minimize the energy consumption by greatly relaxing the room air condition when the space is unoccupied. Fig. 22 plots the desired set point regulation in a receding horizon manner. The set point guides the system gradually toward the comfort condition with PMV set to 0 during the day time.

Unless a heuristic and/or iterative search is conducted, we don’t know what a proper profile shall be used to guarantee the smooth transition with the changing internal and external conditions. With MPC, the desire is fulfilled by penalizing the reference tracking errors and utilizing look-ahead measure on the references and constraints in this configuration. The look-ahead technique is similar to a feed-forward strategy used in conventional SISO PID control. Scenario SC-7 is designed to evaluate the performance with the techniques. The control horizon is set as 1 time interval and the prediction horizon is set as 10. Fig. 23 plots the simulation results. As can be seen in the plot, the controller successfully brings the system back to the desired comfort with PMV equal 0 when the space just starts to be occupied. The summation of squared PMV during the occupied hours is reduced to 0.45. The overall energy consumption with SC-7 compared to the base condition SC-1 decreases by about 7%. The modification on the HW nonlinear MPC increases the computational time to about 460 s for a 5-day simulation due to the increased complexity.

The simulation results in terms of the energy consumption, thermal comfort, and simulation duration for the scenarios are collected in Table 2. In addition to the simulation duration and energy savings, an additional index based on PMV value in the occupied hours is also included. We assume −0.5 should be adopted in winter to ensure thermal comfort. The results show that the higher the mean PMV value, the lower the energy savings. With the office hour schedule, Scenario SC-6-1 stands out since cool morning hours happen which is not acceptable. Overall, with a 24/7 schedule, comfort oriented MPC control in the test bed can bring in from 6 to 36 percent energy savings; with an office schedule, comfort oriented MPC control can offer from 7 to 41.6 percent energy savings.

6. Discussions and conclusions

In this study, we investigated the strategy of using a model-based predictive control (MPC) for a nonlinear hydronic heating model.
system. The methodology utilizes the techniques including Hammerstein, Wiener, and Hammerstein-Wiener (HW) modeling structures to convert a nonlinear system into a combination of a linear dynamic building system and nonlinear static mappings. A linear MPC can therefore be applied for online implementation to ensure much lower computational cost. Several scenarios with various MPCs are designed based on the proposed approach. The many aspects often run in predictive optimal control, including reference tracking, disturbance rejection, computational cost, energy savings, and so forth, are compared.

With the proposed approach, all the optimums for five-day operations are successfully identified in less than eight minutes. The nonlinear multiple-input-multiple-output (MIMO) system is controlled either toward a minimum energy consumption condition, tracking a desired thermal comfort condition, or combined targets. The rejection capability on the unmeasured disturbances from the infiltration, internal heat gains, and so forth, is acceptable. The controller can proactively guide the system transit from unoccupied condition to occupied condition in an energy conservative manner. Furthermore, dynamic look-ahead on both the set points and the weights improves the overall performance since the future information is utilized for the preparation of a building system, where the time scale is of hours level. It can be very useful when there are energy storage capacitance, space use difference, and multiple thermal sources.

The simulation results show that thermal comfort imposed as a band constraint offers more energy savings than a fixed thermal comfort reference approach. By applying thermal comfort oriented control, in a 24/7 schedule condition, an MPC helps acquire about 6–36 percent of energy savings. In an office schedule condition, about 7–42 percent of energy may be saved by using the MPC empowered thermal comfort control.

Future studies can be conducted to expand the variables involved in the thermal comfort oriented control. For the personal comfort control, the occupants’ preference, activity, clothing, and so forth, might be embedded. The research of HW-MPC can also be applied to various thermal conditioning terminals, such as variable air volume terminal boxes, fan coils, and so forth, and be experimentally compared.

Acknowledgments

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References


Table 2

Comparison of energy consumption, thermal comfort, and simulation duration.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Heating energy consumption (kWh)</th>
<th>Mean (PMV + 0.5) in occupied hours</th>
<th>Energy savings (%)</th>
<th>Simulation duration (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-1</td>
<td>3091.7</td>
<td>0.72</td>
<td>0</td>
<td>36.1</td>
</tr>
<tr>
<td>SC-2, 4</td>
<td>2895</td>
<td>0.56</td>
<td>6</td>
<td>76.9–78.1</td>
</tr>
<tr>
<td>SC-3, 5</td>
<td>1977</td>
<td>0.00</td>
<td>36.1</td>
<td>175.8–198.1</td>
</tr>
<tr>
<td>SC-1’</td>
<td>1941</td>
<td>0.69</td>
<td>40.6</td>
<td></td>
</tr>
<tr>
<td>SC-6-1</td>
<td>840.8</td>
<td>–0.25</td>
<td>56.7</td>
<td>187.4</td>
</tr>
<tr>
<td>SC-6-2</td>
<td>1134</td>
<td>0.01</td>
<td>41.6</td>
<td>205.3</td>
</tr>
<tr>
<td>SC-7</td>
<td>1800</td>
<td>0.51</td>
<td>7</td>
<td>460.0</td>
</tr>
</tbody>
</table>

Fig. 23. Simulation results, SC-7.


