1997

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Fuzzy Underwriting: An Application of Fuzzy Logic to Medical Underwriting

Per-Johan Horgby, * Ralf Lohse, † and Nicola-Alexander Sittaro *

Abstract

One of the most difficult issues in the medical underwriting of life insurance applicants is diabetes mellitus. Compiling the prognosticating parameters for diabetic applicants results in a complex system of mutually interacting factors. In addition, neither the prognosticating factors themselves nor their impact on the mortality risk is clear cut.

We show how a fuzzy inference system can be used in underwriting diabetes mellitus. A fuzzy inference system can cope with the imprecise nature of medical parameters by converting them into fuzzy sets and aggregating them

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using mathematical techniques. The fuzzy underwriting system presented goes further than previous applications of fuzzy set theory in insurance, as it is a real life application with contributions from insurance economics, insurance medicine, and computer science.

Key words and phrases: *multiple risk factors, fuzzy inference, life insurance*

1 Introduction

An important challenge in competitive life insurance markets is the accurate underwriting of prospective policyholders. Underwriting in life insurance is designed to determine and evaluate the individual mortality risk of new applicants for insurance, and for current insureds who want to increase their amount of insurance. Underwriting quantifies the potential adverse deviations from “normal” mortality and converts them to higher premiums.

The resulting risk surcharge is justified by subjective factors such as certain recreational and sport activities, professional factors such as miners vs. white collar workers, and specific medical factors. More specifically, medical underwriting is aimed at quantifying the current and future mortality and morbidity risks arising from a health impairment and determining a premium commensurate with the overall risk.

1.1 Insurance Medicine

Since the early 1900s, insurance medicine has formed the scientific basis of medical underwriting in life insurance (Florschütz, 1914). It has established the life-shortening of many medical conditions including obesity and hypertensive diseases. No other medical discipline is involved with prognostic evaluations that span such long periods of time. Long-term prognosis is the most important feature distinguishing insurance medicine from other fields of medicine (Deutsch, 1938). Though this long-term approach is necessary because of the long-term nature of life insurance policies, it may adversely affect the accuracy of estimating a particular individual’s life expectancy. Few other scientific studies of human mortality, however, are designed to encompass decades.

The established selection criteria used in the insurance business are riddled with flaws. For example, the mortality and morbidity rates de-
termined decades ago are not applicable today.\(^1\) During the years be­
tween the application for a policy and its payout of the benefit, med­
ical advances may significantly influence any predictions. Moreover,
the problem is exacerbated by the fact that insurers rarely can iden­
tify whether death can be attributed to the disease for which the risk
surcharge was once levied. Because of these weaknesses, insurance
medicine has increasingly oriented its prognoses on studies developed

The disease-related prognostic findings are compiled in manuals for
reinsurance companies and provided to direct insurance companies. It
is the job of the underwriter to document the individual diseases of an
applicant and allocate them to a specific risk surcharge as defined by the
manuals. The problem with this task is that the information available on
a specific disease is usually not adequate for it to be accurately assigned
to a defined group with a known prognosis.

The basic problem can be illustrated with the diagnosis of chest pain.
This vague diagnosis applies to a large group. The sole risk surcharge
for a mention of the disease would be low, but it is unjustified for most
members of the affected group. If chest pain were subclassified further
as anterior myocardial infarction with moderate impairment of heart
pumping action, this diagnosis would apply to only a small portion
of the overall group. Hence, most of the applicants would be accepted
with a normal premium; the few with the anterior myocardial infarction
diagnosis would be rejected.

### 1.2 Common Problems in Underwriting

When the quality of information is poor, it is difficult to accurately
allocate diseases to rating classes. Obtaining detailed information cre­
ates a delay in processing time and an increase in costs. The costs are
imposing, when one considers the German experience: only 0.5 percent
to 1.0 percent of all life insurance applications are rejected, 2.0 percent
to 5.0 percent are accepted with a risk surcharge, and the 94 percent to
97 percent are accepted at the normal premium.\(^2\) To achieve this result
(and depending on the insurance company), 15 percent to 25 percent
of all applicants are assessed in the underwriting department for extra
mortality risks. Most underwriting is superfluous, i.e. the risk is under-

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\(^1\)An example is the Medical Impairment Ratings from 1932, edited by the Actuarial
Society of America and Association of Life Insurance Medical Directors.

\(^2\)The German experiences are compiled by the Federation of the German Life Insur­
ers, (Verband der Lebensversicherungsunternehmen E.V., Verbandrundschreiben Nr.
written and the policy issued without any surcharge levied. To reduce this superfluous underwriting, many German insurance companies are developing and installing computer-assisted underwriting systems (Ueberscher et al., 1996).

A problem of quality also exists, a problem that has not been tackled by computer-assisted expert systems. Table 1 shows the distribution of diseases for applicants for life insurance in Germany. Whereas some of the anomalies listed in Table 1 (such as hypertension and obesity) can be assessed automatically during application processing at the insurance company, other medical problems are too complex for immediate assessment. One disease that poses a key problem in underwriting is diabetes mellitus, especially when it is manifested as type I (IDDM). Diabetes mellitus usually affects persons up to 30 years of age. Onset of the disease is prior to the typical age at which most persons apply for life insurance. But the disease is characterized by a multitude of different clinical courses most of which are associated with a markedly lower life expectancy. There are unequivocal indicators for risk groups with a particularly poor prognosis. It is imperative that these indicators be surveyed and assessed within the scope of underwriting.

### Table 1

**Frequency of Abnormal Applications**  
**In Underwriting Life Insurance in Germany**

<table>
<thead>
<tr>
<th>Disease</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypertension</td>
<td>18%</td>
</tr>
<tr>
<td>Disorders of lipid metabolism (hypercholesterolemia)</td>
<td>15%</td>
</tr>
<tr>
<td>Alcohol-related organ changes</td>
<td>13%</td>
</tr>
<tr>
<td>Obesity</td>
<td>12%</td>
</tr>
<tr>
<td>Diabetes mellitus</td>
<td>10%</td>
</tr>
<tr>
<td>Heart disease</td>
<td>10%</td>
</tr>
<tr>
<td>Asthma</td>
<td>6%</td>
</tr>
<tr>
<td>Other</td>
<td>16%</td>
</tr>
</tbody>
</table>

*Source: Hannover Re, Karl-Wiechert-Allee 50, 30625 Hannover, Germany.*

1.3 **Outline of the Paper**

The objective of this paper is to show how a fuzzy inference system can be used in the underwriting of an applicant with diabetes mellitus
for a life insurance policy. Fuzzy inference provides mathematical tools for deriving a crisp (i.e., non-fuzzy) output from a multiple fuzzy input space. Fuzzy inference is useful in underwriting life insurance because the risk attributes of medical parameters are not "either/or" variables. An underwriting system based on fuzzy inference can cope with the imprecise nature of medical parameters by converting them into fuzzy sets and aggregating them. The fuzzy underwriting system differs from other risk assessment systems because it allows for gradual shifts in the input variables and allows for compensation between criteria.

In Section 2 we introduce a theoretical framework delineating how fuzzy inference can be used to analyze risks in general and to scrutinize multiple prognostic factors in diabetes mellitus in particular. The paper goes further than previous applications of the fuzzy set theory described in the insurance literature (see, for example, Lemaire, 1990; Cummins and Derrig, 1993; Ostaszewski, 1993; Derrig and Ostaszewski, 1995; and Young, 1996). The underwriting method is one of the first computer-based fuzzy underwriting system being implemented in insurance. In addition, the paper takes an interdisciplinary approach: It integrates the theory of fuzzy inference with the principles of insurance medicine and programming techniques in computer science.

Fuzzy underwriting provides powerful tools for the risk assessment of fuzzy and multiple prognostic factors. We believe that techniques of fuzzy underwriting will become standard tools for underwriters in the future.

2 Basics of Fuzzy Set Theory and Fuzzy Inference

2.1 Identification of Fuzzy Sets Over Membership Functions

To understand what a fuzzy set is, one must first understand what a classical set is. In classical set theory, a set has a crisp (well defined) boundary. For example, in a set of real numbers $A$, expressed as

$$A = \{x \mid x > 10\}, \quad (1)$$

a clear boundary point exists at 10, i.e., if $x$ is greater than 10 it belongs to set $A$; otherwise it does not. This membership in a classical subset $A$ of $X$ can also be viewed as a characteristic function $\mu_A$ from $X$ to $\{0, 1\}$, i.e.,

---

3DeWit (1982) is probably the first to consider underwriting to be a potential area of application of fuzzy set theory to insurance, but his analysis is not detailed.
The definition in equation (2) implies that a classical set only allows full membership or no membership. A fuzzy set is, on the other hand, a set without a crisp (well defined) boundary. The transition from belonging to a set and not belonging to a fuzzy set is gradual and not absolute. The membership function for a fuzzy set defines how each point in the input space is mapped to a membership value between 0 and 1. As a result, an element may belong to a set with a certain degree of membership, not necessarily 0 or 1. The closer the value of $\mu_A(x)$ is to 1, the more $x$ belongs to $A$. A common characterization of a fuzzy set $A$ is

$$A = \{(x, \mu_A) | x \in X \text{ and } \mu_A : X \rightarrow [0, 1]\},$$

where $x$ is the element of interest, $\mu_A$ is the membership function of $x$ in the subset $A$, and $X$ is the universe of discourse.

The only condition a membership function for a fuzzy set must satisfy is that it has to vary between 0 and 1. The function itself can assume an arbitrary shape and is defined from the point of view of simplicity, convenience, and efficiency. Most common are monotonic, triangular, trapezoidal, and bell-shaped membership functions; see Figure 1.

Due to their simplicity, both triangular and trapezoidal membership functions are used extensively. As the membership functions are composed of straight lines, however, they are not smooth at the transition points. The Gaussian and the generalized bell-shaped membership functions are smooth and nonzero at all points and are appropriate in cases where crisp transition points are misleading. To specify asymmetrical membership functions, the monotonic or sigmoidal membership functions can be used. An asymmetrical membership function is appropriate for expressing concepts that gradually increase or decrease, such as height or weight.\(^4\)

As the membership function is the essential component of a fuzzy set, it is logical to define operations with fuzzy sets by membership functions. Analogous to ordinary set operations, Zadeh (1965) defines extended operations valid on fuzzy sets. The most important connections of verbal fuzzy expressions are the logical operations and and or.

\(^4\)The assignment of membership function to the collection of objects $X$ is subjective. Therefore, there must be a rationale behind useful applications. Often the justification of an assignment relies on common sense, expertise, empirical knowledge, and so on. In the fuzzy underwriting system, a medical expert has assigned membership functions to corresponding fuzzy sets.
Consider the fuzzy subsets $A$ and $B$ of the universal set $X$. In fuzzy set theory, \textit{and} and \textit{or} operations are defined with respect to the operators $\land$ and $\lor$ respectively as follows:

\begin{align*}
\mu_{A \land B}(x) &= \mu_A(x) \land \mu_B(x) = \min\{\mu_A(x), \mu_B(x)\} \quad (4) \\
\mu_{A \lor B}(x) &= \mu_A(x) \lor \mu_B(x) = \max\{\mu_A(x), \mu_B(x)\}. \quad (5)
\end{align*}

The intersection of $A$ and $B$ refers to the largest fuzzy set that is contained in both $A$ and $B$. Analogously, the union of $A$ and $B$ refers to the smallest fuzzy set containing both $A$ and $B$.

The \textit{max} and \textit{min} operators have the disadvantage that the resulting membership value cannot assume a value between the maximal and
minimal value, i.e., extreme valuations cannot be offset by moderate ones. The \textit{max} and \textit{min} operators consider only one of the two membership functions. There are other operators with qualities for union and intersection that are different than those of \textit{max} and \textit{min}. These operators vary in their generality and justification of the connections to which they refer. Connectives consistent with the definitions for fuzzy \textit{and} and fuzzy \textit{or} have been proposed in the literature under the names T-norm and T-conorm operators, respectively (Dubois and Prade, 1980, p. 11). By following the basic requirements according to T-norms and T-conorms, the \textit{and/or} operators can be customized as desired. As it is beyond the scope of this paper to investigate T-norm and T-conorm, we refer the interested reader to Zimmermann (1991, pp. 28-43), in Böhme (1993, pp. 43-65), and in Klir and Yuan (1995, pp. 50-93).

2.2 Fuzzy Inference Rules By Generalized Modus Ponens

2.2.1 If-Then Rules

The basic rule of inference is \textit{modus ponens}.\footnote{Modus ponens means "demarcation inference" and belongs to the set of inference rules in the syllogism} Using an "if-then" rule and a premise, one can investigate the truth of a conclusion. Consider the following example:

\begin{align*}
\text{Rule:} & \quad \text{if } x \in A \text{ then } y \in B \\
\text{Premise:} & \quad x \in A \\
\text{Conclusion:} & \quad y \in B.
\end{align*}

In this case of binary logic, the "if-then" rules are easy to follow. If the premise is true, then the conclusion is true. We normally employ the modus ponens in an approximate manner. The premise does not correspond exactly with the antecedent in the "if-then" rule. To allow for statements that are characterized by fuzzy sets, the modus ponens must be extended for gradual numerical values.

Assume \( A \) and \( B \) are defined as fuzzy sets on the universes \( X \) and \( Y \), respectively, i.e., \( A = \{(x, \mu_A) \mid x \in X\} \) and \( B = \{(y, \mu_B) \mid y \in Y\} \). Now the modus ponens can be generalized as follows (Mizumoto and Zimmermann, 1982):

\footnote{Modus ponens means "demarcation inference" and belongs to the set of inference rules in the syllogism}
Rule: if \( x \in A \) then \( y \in B \)

Premise: \( x \in A' \)

Conclusion: \( y \in B' \)

where \( A' \) is a fuzzy set. The logic of this fuzzy example should be clear. If the premise is true to some degree of membership, then the conclusion is also true to that same degree. In order to perform this generalized modus ponens, Zadeh (1973) proposes inference methods based on fuzzy logic. In essence, fuzzy inference is based on two concepts: a fuzzy implication (or fuzzy rule) and a composition rule of inference.

The fuzzy rule "if \( x \in A \) then \( y \in B \)" expresses a relation between the objects \( A \) and \( B \). Without any loss of generality, we can define the fuzzy "if-then" rule as a binary fuzzy relation; a fuzzy rule is defined as the relation between the antecedent and the conclusion. For this purpose, let \( R_{xy} \) denote a fuzzy relation on the product space \( X \times Y \), then the fuzzy rule "if \( x \in A \) then \( y \in B \)" is specified by the following membership function:

\[
\mu_{R_{xy}}(x, y) = \mu_{A \times B}(x, y) = \mu_A(x) \land \mu_B(y)
\]

(6)

where \( \land \) refers to the intersection operator defined in equation (4) as the minimum connective. We can complete the inference method of the generalized modus ponens by applying the compositional rule of inference (Zadeh, 1973).

2.2.2 Compositional Rule

Next we define the compositional rule on inference to be based on \( \max \min \) composition. Let \( A, A', \) and \( B \) be fuzzy sets in the universes \( X, X, \) and \( Y, \) respectively. Further, let \( R_{xy} \) represent the fuzzy relation "if \( x \in A \) then \( y \in B \)". Therefore, we express the generalized modus ponens as

\[
\mu_{R_{xy}}(x, y) = \bigvee_{x \in A'} \{ \mu_{A'}(x) \land \mu_{R_{xy}}(x, y) \}
\]

\[
= \max_{x \in A'} \min \{ \mu_{A'}(x), \mu_{R_{xy}}(x, y) \}. \tag{7}
\]

---

6Throughout this paper, the prime notation is used to signify that the set is a fuzzy set. Thus \( A' \) is a fuzzy set, not the complement of \( A \).

7The binary fuzzy rule "if \( x \in A \) then \( y \in B \)" can be interpreted as \( A \) is coupled with \( B \). This rule is an extension of the classical Cartesian product, where each element \((x, y) \in X \times Y\) is identified with a membership grade denoted by \( \mu_{R_{xy}}(x, y) \).
By applying this inference procedure we assign the conclusion a degree of membership from the intersection of the premise and the fuzzy relation. Remember that max and min are just two of many other composition operators. It is possible to introduce other connectives: for example, an algebraic product or more generally T-norms as and operators; and an algebraic sum or more generally T-conorm operators as or operators.\(^8\)

As a general form of fuzzy inference, consider \(n\) multiple rules with multiple antecedents combined with "else":

- **Rule 1:** if \(x \in A_1\) or \(y \in B_1\), then \(z \in C_1\) else
- **Rule 2:** if \(x \in A_2\) or \(y \in B_2\), then \(z \in C_2\) else
- ... : ...
- **Rule \(n\):** if \(x \in A_n\) or \(y \in B_n\), then \(z \in C_n\)

**Premise:** \(x \in A'\) and \(y \in B'\)

**Conclusion:** \(z \in C'\)

When dealing with multiple rules we are faced with a problem: more than one rule can fire (take effect) simultaneously. To decide which consequence should be taken as the result of the simultaneous firing of several rules, we apply the process of conflict resolution (Berenji, 1992).

If \(A\) and \(B\) are the premise part, i.e., the inputs in a fuzzy inference system, then their corresponding membership functions are represented by \(\mu_{A_i}(x)\) and \(\mu_{B_i}(y)\) for the \(i\)-th rule \(i = 1, 2, \ldots\). The firing strength, \(\alpha_i\), of the \(i\)-th rule can be calculated by

\[
\alpha_i = \mu_{A_i}(x) \land \mu_{B_i}(y). \tag{8}
\]

The \(\alpha_i\) expresses the matching strength of the antecedents for each rule. By applying this strength on respective conclusions, we obtain the inferred fuzzy sets for each rule,

\[
\mu_{C'_i}(z) = \alpha_i \land \mu_{C_i}(z). \tag{9}
\]

As a result of the inputs \(A'\) and \(B'\), the inference of Rule 1 generates the conclusion \(\mu_{C'_1}(z)\), Rule 2 generates \(\mu_{C'_2}(z)\), and so on. Thus, each

---

\(^8\)When a fuzzy rule takes the form "if \(x \in A\) or \(y \in B\) then \(z \in C\)," the degree of fulfillment of this fuzzy rule is given as the maximum degree of a match with the antecedent part.
rule suggests a different output. To resolve this dilemma, the conflict-resolution process recommends that the conclusions of respective rules be aggregated by the union operator. We can derive the aggregate output \( C' \) as

\[
\mu_{C'}(z) = [\alpha_1 \land \mu_{C_1}(z)] \lor [\alpha_2 \land \mu_{C_2}(z)] \lor \cdots \lor [\alpha_n \land \mu_{C_n}(z)] \\
= \mu_{C_1}(z) \lor \mu_{C_2}(z) \lor \cdots \lor \mu_{C_n}(z).
\]  

(10)

The connective "else" is interpreted as the logical or. The or is interpreted as the \( \max \) operator. Hence, the final output is calculated by aggregating results from each rule using the \( \max \) operator.

2.3 Defuzzification Strategies

The implication of equation (10) is characterized by a membership function, i.e., the output of the fuzzy inference is a fuzzy set as well. Often it is necessary to receive an output in crisp terms. Therefore, the membership function of the final output must be translated, i.e., defuzzified, a single crisp value. A defuzzification strategy refers to the way a crisp value is extracted from a fuzzy output set. Several defuzzification strategies have been suggested in the literature (see Jager et al., 1994, pp. 179-185). We describe the most popular method called the center of area (coa) method. This defuzzification strategy returns the center of area under the membership curve as

\[
Z_{\text{coa}} = \frac{\sum_{j=1}^{q} z_j \mu_{C'}(z_j)}{\sum_{j=1}^{q} \mu_{C'}(z_j)}
\]

(11)

where \( Z_{\text{coa}} \) is the defuzzified output, \( q \) is the number of quantification levels of the output, \( z_j \) is the amount of output at the quantification level \( j \), and \( \mu_{C'}(z_j) \) is the aggregated output membership function. This defuzzification strategy is simply a weighted average of the \( z_j \)'s (similar to the expected value of probability theory). A common feature of this method and the computation of expected values is the nondiscrimination of extreme values. The center of area calculation is made on the basis of all aggregated outputs without eliminating endpoints. Other defuzzification strategies (such as mean of maximum, largest of maximum, and smallest of maximum) do not consider the parts of a fuzzy output, the membership values of which are below the maximum. Defuzzification can be performed in several arbitrary ways. Different strategies arise for specific applications. There is no accurate way to analyze them except through experimental studies.
3 A Computer-Based Fuzzy Underwriting System

We provide one description of how expert knowledge about underwriting diabetes mellitus in life insurance is processed for a fuzzy inference system. The system was developed and programmed in MS Excel 5.0 using Visual Basic. The rationale of the system relies on medical knowledge concerning the etiology of diabetes mellitus and underwriting principles in insurance economics.

3.1 Prognosticating Diabetes Mellitus

The list of prognostic parameters for diabetes mellitus is long. There are primary and secondary medical parameters, and an accurate assessment of the prognosis can be made taking into account a limited number of parameters.

Diabetes mellitus is characterized by an elevation in blood sugar values. In type I diabetes mellitus, this blood sugar elevation is caused when the pancreas secretes no insulin. Type II diabetes mellitus, which chiefly affects persons over age 30, has an underlying pathological mechanism, whereby, despite the fact that the pancreas secretes insulin, the activity is suppressed. If not treated successfully either by drugs or insulin replacement, life-threatening conditions will occur within a few days. Renal impairment occurs in type I diabetics, which often leads to kidney failure as early as 10 years to 20 years after onset. In general, the blood vessels in diabetics are damaged; heart attack, stroke, and neural and eye impairment are common complications (Mehnert et al., 1994, pp. 76–78). The prognosis in diabetes mellitus can be based on three primary factors (Rossing et al., 1996; Nathan, 1993): (i) the time factor; (ii) the therapy (adjustment) factor; and (iii) the complication factor.

If complications such as kidney failure, eye disorders, or heart attack are manifest, the underwriting normally ends in rejection of the applicant. While in the past, the insurer chiefly applied the time factor when underwriting a risk, new medical research increasingly has shown the importance of the therapy factor. The time factor ultimately reveals that the insurance company is only willing to accept an application for life insurance with a risk surcharge if the duration of the diabetes plus the applied term insurance do not exceed a specified period of time. In such a case, staggered risk surcharges are assigned for a period of 15 years to 35 years. Numerous case studies have shown, however, that the better the diabetes mellitus can be treated with insulin so that blood sugar levels approximate the level and course of a healthy per-
son, the lower the organ-related complication rate will be. These special forms of therapy cannot be given to every diabetic. Underwriting thus consists of evaluating as accurately as possible the quality of this therapy in terms of the adjustment parameters and excluding any possible complications by achieving a high quality of information (Mehnert et al. 1994, pp. 93, 131).

The quality of therapy can be established by current blood sugar values and the HbA1-values (glycolysated hemoglobin). The HbA1-value can be determined easily in the blood and reflects the blood sugar level over a period of around 90 days. These two parameters of insulin therapy—or other treatment strategies in type II diabetics—define the adjustment by medication or therapy efficiency.

Another important aspect to consider in patients with diabetes mellitus is that the more cardiovascular risk factors are present, the worse is the mortality risk. These factors include elevated blood lipids, high blood pressure, or smoking. These risk factors also must be reviewed within the scope of any prognostic assessment. In addition to these main parameters, several other prognostic factors are important for an adequate risk evaluation of diabetes mellitus.

Table 2 lists the prognostic factors that form the input space in our underwriting system. These prognosticating factors for diabetes mellitus result in a complex system of interdependent variables that mutually interact. All changes can be identified with regard to their effect on the overall prognosis for increased mortality. The prognosticating factors and their impact on the mortality risk is not clear cut.

3.2 Design of the Fuzzy Underwriting System

To depict the knowledge concerning the etiology of diabetes mellitus, the major areas were processed in chronological order:

1. Hierarchical structure of the prognosticating variables;

2. Membership functions of the terms of the prognosticating variables; then

3. Rule base.
Table 2
Prognostic Factors in Diabetes Mellitus

<table>
<thead>
<tr>
<th>27 Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of the patient</td>
</tr>
<tr>
<td>Age at onset of the disease</td>
</tr>
<tr>
<td>Duration of the disease</td>
</tr>
<tr>
<td>Type I or type II diabetes</td>
</tr>
<tr>
<td>Quality of the therapy (with medication)</td>
</tr>
<tr>
<td>Blood sugar level</td>
</tr>
<tr>
<td>Blood sugar profile</td>
</tr>
<tr>
<td>HbA1</td>
</tr>
<tr>
<td>Fructose amine concentration in the blood</td>
</tr>
<tr>
<td>Sugar detected in the urine</td>
</tr>
<tr>
<td>Compliance with dietary recommendations</td>
</tr>
<tr>
<td>Compliance in taking medicine</td>
</tr>
<tr>
<td>Insulin dose</td>
</tr>
<tr>
<td>Frequency of daily blood sugar checks</td>
</tr>
<tr>
<td>Intensified insulin therapy</td>
</tr>
<tr>
<td>Intercurrent complications in diabetes</td>
</tr>
<tr>
<td>Myocardial infarction</td>
</tr>
<tr>
<td>Coronary heart disease</td>
</tr>
<tr>
<td>Peripheral vascular disease</td>
</tr>
<tr>
<td>Eye disorders</td>
</tr>
<tr>
<td>Renal function</td>
</tr>
<tr>
<td>Blood pressure</td>
</tr>
<tr>
<td>Body weight</td>
</tr>
<tr>
<td>Frequency of hospitalization because of coma</td>
</tr>
<tr>
<td>Extent of blood sugar fluctuations</td>
</tr>
<tr>
<td>Profession</td>
</tr>
<tr>
<td>Education</td>
</tr>
</tbody>
</table>

First, the three primary factors attributed to diabetes mellitus (therapy factor, time factor, and complication factor) are subdivided into influencing factors on the subordinate level. This process of hierarchical top-down classification was repeated until input factors were present on the first level that either

- showed a continuous dimension (e.g., blood sugar level in milligrams per deciliter) or if such a dimension were lacking; or were

- subject to discrete evaluation by the medical expert (e.g., classification of vascular complications into minor, moderate, marked, or severe).

If a continuous dimension exists, the medical expert determines the values for which terms of language have no membership, i.e., the membership equals 0, and those have complete membership, i.e., the membership equals 1. A linear course of the membership function was defined between the mathematical items for no membership and complete membership defined in this way. If a discrete natural dimension existed for a variable, only complete membership values relating to one of the terms of the linguistic variables could be present. For example, retinopathy can only be present in either stage 1, 2, 3, or 4. In this way, the structure of the fuzzy inference system and all the system's elements are defined.

The next step is connecting these membership values according to a given structure. For this purpose, the expert is required to define rule sets \( \{R_1, R_2, \ldots, R_n\} \) for all allocations within the fuzzy inference system. The rule sets must account for all possible combinations from the terms of subordinate variables. For example, the variable "blood sugar level" and "HbA1-value" are defined by five terms each; in other words, 25 rules must be defined.

Each of the individual rules consisted of an antecedent and a conclusion. The antecedent includes the terms of the subordinate variables

\[9\] The expert knowledge is often referred to as a knowledge base of a fuzzy inference system. Most often the knowledge base also contains a set of rules that specifies the output as a function of a fuzzy input space. In general, there are four methods of rule generation (Sugeno, 1985):

i) Experience and knowledge of an expert;
ii) Modeling the operator's control actions;
iii) Qualitative modeling of a system; and
iv) Self-organization.

The first method is the most widely used, and it is the rule base used in this application. For a review of the other methods, see Sugeno (1985) or Klir and Yuan (1995, pp. 327-356).
and the conclusion includes the terms of the superior variable. In gen-
eral, a rule takes of the form of: “If \{variable 1\} is \{term set 1\} and
\{variable 2\} is \{term set 2\}, then \{consequence 1\} is \{term set 3\}”.

An example of a rule is: If the blood sugar level is very low and
the HbA1-value is normal, then the blood sugar profile is to be rated
as medium. The structure of the total system is illustrated in Figure
2. On the left in Figure 2 we see the final output: the risk-adjusted
premium. The lower risk factors extra mortality and age represent ac-
tuaria factors to calculate the extra premium for substandard risks.
While age is an original input factor, extra mortality is inferred by the
three primary factors attributed to diabetes mellitus: therapy factor,
complication factor, and time factor. All other medical risk factors
are regarded as fuzzy subfactors. To explain the whole fuzzy infer-
ence system would not make any sense in this limited space. We make,
therefore, an arbitrary demarcation in the presentation and consider
only how the therapy factor is inferred.

3.3 Inferring the Therapy Factor

From Figure 2, we see that the therapy factor is inferred by three
original input factors (blood sugar value, HbA1-value, and insulin in-
jections) connected in two places. Let us show in more detail how the
therapy factor is determined by considering an applicant who has the
profile described in Table 3.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Applicant Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors</td>
<td>Level</td>
</tr>
<tr>
<td>Blood sugar value</td>
<td>130 mg/dl</td>
</tr>
<tr>
<td>HbA1</td>
<td>13.5%</td>
</tr>
<tr>
<td>Insulin injections</td>
<td>10 per week</td>
</tr>
</tbody>
</table>

Figure 3 demonstrates how blood sugar value, HbA1, and number
of insulin injections are allocated to the terms of the variables (in other
words, how the inputs are fuzzified). The blood sugar value of 130
mg/dl has the membership values 0.67 as normal and 0.33 as high. No
other terms fire for 130 mg/dl. The HbA1-value of 13.5 percentage has
the membership values 0.75 as high and 0.25 as very high. The number
of 10 insulin injections per week has the membership values 0.57 as
low and 0.43 as medium.
Figure 2
A Fuzzy Inference System for Underwriting Diabetes Mellitus
Figure 3
A Pictorial Representation of the Inference Rules

Blood Sugar Value

HbA1-Value

Insulin Injections
In inferring the therapy factor, we must make two inferences. The first inference is to connect the blood sugar value with the HbA1-value and thereby infer the blood sugar profile. In the second inference, we connect blood sugar profile and insulin injections. This yields the therapy factor, which in turn has to be defuzzified into a premium surcharge. These two inference steps are illustrated in Figure 4.

The rule sets of inference 1 and inference 2 are given in Tables 4 and 5. In our example, we only consider rules that have positive membership values. In rules in which at least one term in the antecedent has a membership value of zero, then a membership of zero results for the conclusion. These rules have no effect on the further processing of information and, therefore, are not represented by numerals in the rule sets.

The operators in the rule base are defined to be the $\max$ and $\min$ functions, respectively, similar to the inference system proposed by Mamdami (1976). Such an inference method is called $\max$-$\min$ inference, because the membership function of the aggregated output is the union ($\max$) of the fuzzy sets assigned to that output after cutting their degree of membership values at the degree for the corresponding antecedents by the intersection $\min$ operator.

After the inference of blood sugar profile and insulin injections we receive the inferred output therapy factor. The last step is to translate, or defuzzify, the therapy factor into a crisp premium surcharge. In Figure 4, we see that the firing strength of the medium premium surcharge rule is 0.57 and 0.25 for the high premium surcharge rule, which means that the membership functions of the medium premium surcharge and high premium surcharge are cut at 0.57 and 0.25, respectively. This is illustrated in Panel B in Figure 5. Thereafter, a total function is produced from both firing rules.

From the resulting fuzzy output set (Panel C in Figure 5) we use the center of area method defined in equation (11) to extract a crisp premium surcharge. This applicant must pay a premium surcharge of 207 percent on top of the class rate. The underwriting is now complete, and the gradual risk of diabetes has been translated into a premium surcharge using fuzzy set theory and fuzzy inference. Allowing for gradual shifts in the input space makes this system flexible and gives a better mapping of individual risk profiles than classical expert systems.
Blood Sugar Value
Natural Value 130 mg/dL

Fuzzification 1
MSV to Normal 0.67
MSV to High 0.33

HbA1
Natural Value 13.5%

Fuzzification 2
MSV to High 0.75
MSV to Very 0.25

Blood Sugar Profile
MSV to Medium 0.67
MSV to Bad 0.25

Inference 1

MSV to Low 0.57
MSV to Medium 0.43

Insulin Injections
Natural Value 10 per week

Fuzzification 3

Inference 2

Therapy Factor
MSV to Medium 0.57
MSV to Bad 0.25

Defuzzification

Premium Surcharge 207%

MSV = Membership Value
### Table 4

**Fuzzy Inference of Blood Sugar Values and HbA1**

<table>
<thead>
<tr>
<th>Rule</th>
<th>BSV</th>
<th>MV</th>
<th>HBA1</th>
<th>MV</th>
<th>BSP</th>
<th>MV</th>
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<tr>
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<td></td>
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</tr>
<tr>
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</tr>
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<td></td>
</tr>
<tr>
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</tr>
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<td></td>
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<td></td>
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<td>medium</td>
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<td>very high</td>
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</tr>
</tbody>
</table>

BSV = Blood Sugar Value; BSP = Blood Sugar Profile; and MV = Membership Value.
Summary and Concluding Remarks

We introduce a new inference technique, called fuzzy inference, for underwriting in life insurance. Fuzzy inference systems are well suited for compiling medical facts and can help underwriters cope with the complexity of prognostic decision making. Fuzzy logic presents medical information more realistically than do the classical methods. Medical practice has had to rely on auxiliary constructions for prognostic parameters by forming intervals of demarcation to increase practicability. These intervals in effect convert continuous functions into discontinuous ones.

<table>
<thead>
<tr>
<th>Rule</th>
<th>BSP</th>
<th>MV</th>
<th>INIJ</th>
<th>MV</th>
<th>THEF</th>
<th>MV</th>
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</thead>
<tbody>
<tr>
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<td>normal</td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>normal</td>
<td>medium</td>
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<td></td>
<td>good</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>normal</td>
<td>high</td>
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<td></td>
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<tr>
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<tr>
<td>8</td>
<td>medium</td>
<td>very high</td>
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<td>good</td>
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</tr>
<tr>
<td>9</td>
<td>bad</td>
<td>0.25</td>
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<td>0.57</td>
<td>bad</td>
<td>0.25</td>
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<tr>
<td>10</td>
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<td>0.43</td>
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<td>0.25</td>
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<tr>
<td>12</td>
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<td>very high</td>
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<td></td>
<td>medium</td>
<td></td>
</tr>
</tbody>
</table>

BSP = Blood Sugar Profile; INIJ = Insulin Injections; THEF = Therapy Factor; and MV = Membership Value.

Another feature of medical descriptions is their fuzziness. It is easy to reach a consensus among physicians that a disease or a symptom is mild, moderate, or severe. A quantitative expression of these fuzzy terms is not normal practice in medicine, but is necessary to be able to make precise prognostic statements. Fuzzy inference systems provide an excellent approach to reaching such solutions.
Figure 5
Defuzzification of the Aggregated Premium Surcharge
The fuzzy underwriting system presented here has been used in practical applications. It shows that the fuzzy language of physicians combined with fuzzy prognostic parameters can be expressed as fuzzy inference rules and, thus, be implemented as crisp decisions. Using fuzzy underwriting, more prognostic factors can be taken into account than are currently possible in underwriting practices of direct insurance companies. The prognosis of disease is generally not determined by one factor alone, but by a combination of factors.

Even in the early versions of the fuzzy underwriting system, practical cases from everyday insurance could be used to show that correct decisions are possible in about 80 percent of all cases. The remaining 20 percent does not result from weakness of the system, but from deficiencies in the information available. The PC-supported system presented also makes decisions when the information available is sparse. Such decisions are naturally less reliable. Prognosis structures can be constructed for many diseases analogously to our example of diabetes mellitus. Fuzzy inference systems can be devised for most diseases. For direct insurance companies, such fuzzy systems would make decision-making process more precise, more transparent, and more free of the subjective errors that have hindered accurate underwriting assessments in the past.

References


*MEDICAL IMPAIRMENT RATING*. Actuarial Society of America and Association of Life Insurance Medical Directors, 1932.


