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The Economics of Tiered Pricing and Cost Functions: Are Equity, Cost Recovery, and Economic Efficiency Compatible Goals?

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Abstract

The paper develops a framework to analyze equity and economic efficiency of increasing block rates (IBR) for regulated products such as electricity or water. The analytical model assumes that consumers are heterogeneous in their demand characteristics. Conditions are identified under which economic efficiency and cost recovery can be achieved in a manner that also reduces inequality, which is measured through changes in the Gini coefficient of consumer surplus. Under IBR, a utility with significant variability in its marginal costs has a greater ability to improve equity while still remaining revenue neutral and maintaining economic efficiency. Under marginal cost pricing, the Gini coefficient is primarily affected by parameters of the demand function, but with increasing block rate pricing both demand and supply parameters impact this measure. The results are illustrated through the use of a numerical example.

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1 Introduction

Due to limited competition, suppliers of water and energy frequently have their pricing regulated by governmental agencies. Pricing schemes are evaluated by the efficiency of the resource allocation they lead to, the capacity of the utilities to capture their costs, and the distributional effects of the policies, in particular, impacts on the poor. One pricing approach has been average cost pricing, which guarantees cost recovery and allows utilities to provide their product at relatively low prices (Bonbright, Danielsen & Kamerschen 1988). However, average cost pricing leads to economically inefficient consumption levels. For this reason, economists have often argued to price resources at their long run marginal cost. However, this can lead to positive profits for a regulated industry. An alternative approach is increasing block rates (hereafter, IBR or tiered pricing), where individuals pay a low rate for an initial consumption block and a higher rate as they increase use beyond that block. Increasing block rates are frequently used by regulated utilities in the United States and worldwide. For example, Borenstein (2008) describes the adoption of IBR pricing by California electrical utilities during the 1980s. An OECD study of water rates in developed countries shows frequent use of increasing block rates (OECD 1999). Concerns about conservation have led to a widespread shift in pricing patterns; while only 4 percent of public water suppliers in the United States used IBR in 1982, over 30 percent did by 1997 (OECD 1999). Over the same period, the use of decreasing block rates fell from 60 to 34 percent of public water suppliers. Advocates of IBR argue that it can improve equity by offering the poor a subsidized rate on consumption (Agthe & Billings 1987). Others argue that tiered pricing will encourage overconsumption if the subsidized block is too large. For example, a study of water utilities by the Asian Development Bank (1993) found that the average size of the subsidized block is almost 300% of basic needs. Thus, if IBR pricing is not properly designed, it could lead to consumption in excess of basic needs or economically inefficiently levels.

In this paper, we answer the question “to what extent can tiered pricing be used to improve equity while maintaining economic efficiency and revenue neutrality?”. We find that it depends on the variability of supply and that greater inequality in the marginal input cost enables a larger improvement in equity. We show that under certain conditions, a regulated utility can achieve all of these goals. The feasibility depends critically on the underlying cost structure for the resource and the parameters of the demand function. Specifically, utilities with a variable marginal cost of provision of the product they use (which may reflect the diversity of suppliers) and without extremely poor customers are best able to achieve these joint goals.

To answer this question, we develop an analytical model of a regulated utility with heterogeneous customers. We demonstrate how shifts in parameters of the benefit or marginal cost functions affect the design of a tiered pricing rate structure. We develop a measure for the Gini coefficient of consumer surplus and compare the impacts of a single rate structure with increasing block rates on equity. We include a numerical example to illustrate how differences in the demand or supply function affect the equilibrium under marginal cost and tiered pricing. We calculate the Gini coefficient of consumer surplus for each type of pricing, and compare the equity gain associated with tiered pricing over marginal cost pricing.

Much of the previous literature on the economics of tiered pricing has examined the consumer response to a tiered pricing rate structure. Most of the work in this field has been empirical (Hewitt & Hanemann 1995, Castro-Rodríguez, Da-Rocha & Delicado 2002, Rietveld, Rouwendal & Zwart 2000, Bar-Shira, Finkelshtain & Simhon 2006, Reiss & White 2005, Borenstein 2008). There has been a paucity of theoretical work examining the feasibility and implications of tiered pricing. Wilson (1993) briefly discusses the issue, but only in the context of decreasing block rates and a profit-maximizing monopolist. Bar-Shira and Finkelshtain (2000) find that increasing block rates (IBR) affect the long-run incentives for entry and exit into an industry, thus affecting the long-run optimal outcome.

However, they find that when the number of firms is fixed, the social optimum may be achieved through IBR. Certain limitations of IBR in developing countries include the feasibility of every family unit having its own meter (Whittington 1992) or unintended impacts on large families (Whittington 1992, Dahan & Nisan 2007), although suggestions exist to remedy this problem such as customer specific block prices and quantities (Pashardes & Hajispyrou 2002). None of these papers has developed a theory of how to design tiered pricing to achieve equity goals. A related area of literature exists on two-part pricing with a monopolist (Oi 1971, Spence 1977, Cassou & Hause 1999), however this literature examines how a monopolist can achieve a goal of profit maximization.

There are several major differences between previous literature and this paper. We jointly examine the three primary goals of equity, economic efficiency, and cost recovery, and relate these goals to parameters of the demand and supply functions. We combine previous research on consumer demand under IBR with production costs. This is important because costs of production can vary greatly, and can impact the feasibility of providing equitable access to services.

One of the broader implications of our results is what we refer to as the “inequality in leads to equity out” result. By this, we mean specifically that when two utilities have the same marginal cost of inputs, the utility with more variability in marginal input costs has a greater capacity to offer tiered pricing, and hence, improve equity, while still achieving economic efficiency. This is because the utility with more variability in input costs can utilize the producer surplus gained from low-cost inputs to improve equity. This result is important, because it implies that those utilities with diverse sources of inputs (i.e., electricity providers that utilize coal, natural gas, and hydropower; or water providers that have sources from multiple rivers and groundwater aquifers) are better able to use tiered pricing to improve equity than those that rely on a single input source.

2 General Model

This section develops the economic model of demand and supply under IBR. We first develop a general model of increasing block rate pricing, and then move to the model of heterogeneous consumer demand. We then outline the supply model with the marginal cost function for producers.

2.1 Model of Increasing Block Rates

We first develop a general model of tiered pricing, and use it to derive conditions under which tiered pricing can achieve economic efficiency. For simplicity we assume that tiered pricing is characterized by two parameters, with the higher price equal to the long-term marginal cost. In practice, tiered pricing is sometimes designed with many blocks. For example, a redesign of California electricity rates included up to five different price levels (Reiss & White 2005). However, recent research supports the assumption that consumers respond to the marginal block price (Nataraj & Hanemann 2008).

The subsidized price or ‘lifeline price’ that individuals pay is denoted by w_L , and is below marginal cost. The size of the block or ‘lifeline quantity’ (the maximum quantity individuals can purchase at the subsidized price) is denoted by q_L . We impose the restriction that q_L is non-negative. In theory, w_L could be negative, implying that consumers receive a per-unit payment for consumption below some minimum quantity. However, in our analysis we focus on nonnegative values of w_L as net rebates to consumers would pose potential problems to resource providers. Figure 1 shows an example of this, where w^M denotes the long run marginal cost. The figure allows heterogeneity by including two demand curves for types θ_L and θ_H . This diagram shows that consumption under tiered pricing may not be economically efficient; and depends on the chosen parameters. Type θ_H consumes at the economically efficient level, since at the margin he/she faces a price equal to the long run

marginal cost. However, type θ_L overconsumes with consumption equal to $q_{\theta_L}^{TP}$ instead of $q_{\theta_L}^*$.

<< Insert figure 1 >>

The rate structure described results in an individual cost function $c(q)$ equal to the following:

$$c(q) = \begin{cases} (q - q_L)w^M + w_L q_L & \text{if } q > q_L \\ q_L w_L & \text{if } q = q_L \\ q w_L & \text{if } q < q_L \end{cases} \quad (1)$$

2.2 Demand

We model a utility maximizing individual, where consumers are heterogeneous, and utility is an additive function of monetized benefits of the amount of the good consumed minus costs and heterogeneity parameter θ . Heterogeneity could be due to differences in family or operation size, or due to income or wealth. In the following framework, we use θ to represent income heterogeneity. We assume that θ is distributed over a finite interval $[\theta_L, \theta_H]$ with pdf $f(\theta)$. Denoting $U(\theta, q)$ as the net utility of an individual of type θ consuming quantity q and the benefits $B(\theta, q)$ be the benefits from consumption, we let the utility function be additively separable as follows. As shown in Olmstead, Hanemann, and Stavins (2007), an increasing block rate mechanism creates a point of non-differentiability in the budget constraint (i.e., a kinked budget constraint).

An individual will choose to maximize their utility:

$$\max_q U(\theta, q) = B(\theta, q) - c(q) \quad (2)$$

where $c(q)$ is defined as in Equation 1.

We assume that the benefit of higher consumption increases at a decreasing rate, or that $B_q > 0$ and $B_{qq} < 0$, where subscripts denote partial derivatives. We also assume that the

benefits of a certain consumption level are larger for a higher value of θ ($B_\theta > 0$) and that the marginal benefits of additional consumption are greater at higher values of theta ($B_{\theta q} > 0$). As Equation 2 is piecewise differentiable, we can solve for the first order conditions when $q \neq q_L$, and we can determine the corner solution outcomes when $q = q_L$. Thus, the following are the conditions for benefit maximization:

$$\text{For } \theta \text{ s.t. } \begin{cases} B_q(\theta, q_L) > w^M & \Rightarrow B_q = w^M \\ w_L < B_q(\theta, q_L) < w^M & \Rightarrow q = q_L \\ B_q(\theta, q_L) < w_L & \Rightarrow B_q = w_L \end{cases} \quad (3)$$

For any pair $\{q_L, w_L\}$, there are three potential groups of individuals that are formed, based on the first order conditions in Equation 3. We define $\theta_1(q_L, w_L)$ and $\theta_2(q_L, w^M)$ as the two values of θ that separate the types of individuals. We also define the $q(\theta, w)$ as the quantity demanded by type θ at a marginal price of w . These are based on the appropriate marginal conditions as follows:

$$\text{For } \begin{cases} \theta > \theta_2(q_L, w^M) & \Rightarrow B_q(\theta, q_L) > w^M \text{ and } q = q(\theta, w^M) \\ \theta \in [\theta_1(q_L, w_L), \theta_2(q_L, w^M)] & \Rightarrow w_L < B_q(\theta, q_L) < w^M \text{ and } q = q_L \\ \theta < \theta_1(q_L, w_L) & \Rightarrow B_q(\theta, q_L) < w_L \text{ and } q = q(\theta, w_L) \end{cases} \quad (4)$$

2.3 Marginal Cost of Supply

Until this point, we have focused on the analysis of the demand model. However, in many cases a revenue neutrality condition is required. Therefore, the marginal cost function is critical in determining if a particular rate structure allows a utility to cover total costs without cross-subsidization from another revenue source. Revenue neutral pricing is one reason that average cost pricing is frequently used by regulated utilities (Bonbright et al. 1988). However, the choice of rate structure will affect the quantity demanded. For most utilities the fixed

costs of providing resources can be large due to the expense of building infrastructure. However, those fixed costs can be covered through a consumer fee that does not depend on consumption. Thus, in our analysis revenue neutrality refers to a utility's ability to cover its marginal costs of supply. In this section we consider the feasibility of cost recovery under a tiered pricing rate structure.

The long run variable cost function is denoted by $VC(Q)$ where Q is the total quantity demanded by all individuals and $VC'(Q) = MC(Q)$ is the long run marginal cost function. We assume that $MC'(Q) \geq 0$. One of the primary reasons cited for using tiered pricing is to try to improve equity in access to services (Agthe & Billings 1987, OECD 1999). Tiered pricing is designed to assure that all consumers get a minimum benefit from water or electricity. We model this as a minimum level of utility, or well-being that is socially desirable, and we denote this level by \underline{u} . This could be based on some standard such as a minimum amount necessary to meet basic living standards. However, this general standard is not defined by a specific level of consumption as some substitution is possible between the regulated good and all other goods (i.e., income). If a social goal is to guarantee a minimum level of utility, the parameters of the cost function will determine the choices for \underline{u} that are *potentially revenue neutral*.

Definition We define a *potentially revenue neutral* choice of \underline{u} as one that can be achieved without subsidization from the government or other sectors of the economy.

2.4 Combining Demand and Supply under IBR

The total quantity demanded depends on the distribution of individuals, the long run marginal cost, and the choice of lifeline price and quantity. We assume that the long run marginal cost is constant over the range of interest. Using the blocks of consumer types

defined in Equation 4, we can determine the average quantity demanded by the following:

$$\begin{aligned}
AQ &= \int_{\theta_L}^{\theta_1(q_L, w_L)} q(\theta, w_L) f(\theta) d\theta + \int_{\theta_1(q_L, w_L)}^{\theta_2(q_L, w^M)} q_L f(\theta) d\theta \\
&\quad + \int_{\theta_2(q_L, w^M)}^{\theta_H} q(\theta, w^M) f(\theta) d\theta \\
&= AQ(w_L, q_L, w^M, f(\theta))
\end{aligned} \tag{5}$$

Equation 5 includes the integration of demand across three segments of the population. The first component is the total quantity demanded by all individuals who consume below the tier. For these individuals, their demand level is determined by the lifeline price, w_L . The second component is the total demand by all individuals who consume exactly at the tier, q_L . The third segment includes all individuals who consume above the tier. In this segment of the distribution, the quantity demanded is determined by the marginal cost.

Average revenue is determined by the following:

$$\begin{aligned}
AR(Q) &= \int_{\theta_L}^{\theta_1(q_L, w_L)} w_L q(\theta, w_L) f(\theta) d\theta + \int_{\theta_1(q_L, w_L)}^{\theta_2(q_L, w^M)} w_L q_L f(\theta) d\theta \\
&\quad + \int_{\theta_2(q_L, w^M)}^{\theta_H} (w_L q_L + w^M (q(\theta, w^M) - q_L)) f(\theta) d\theta \\
&= AR(Q(w_L, q_L, w^M, f(\theta)))
\end{aligned} \tag{6}$$

We denote the size of the population by N , which we assume is exogenous. Thus $Q(w_L, q_L, w^M, f(\theta), N) = N \cdot AQ(\cdot)$ and $TR(w_L, q_L, w^M, f(\theta), N) = N \cdot AR(Q(\cdot))$ are the total quantity demanded and total revenue, respectively.

Definition For two utilities that produce at $Q = Q^*$, we define the one with lower input costs as the utility with the lower value of $\int_0^{Q^*} MC(q) dq$.

Proposition 1 *For two marginal cost functions that result in the same equilibrium output quantity Q^* , the one with lower input costs can support a higher level of \underline{u} while still*

maintaining revenue neutrality.

Proof: See Appendix A.

This result is particularly important for comparing the feasibility of using tiered pricing in different locations. Those producers or locations with a range of low-cost inputs have a greater capacity to subsidize low-income consumers than those relying on a single input source or a range of high cost sources.

3 Characteristics of Increasing Block Rate Equilibria

In evaluating tiered pricing rates, there are three characteristics we are particularly interested in: *cost recovery*, *economic efficiency*, and *equity improvement*. An economically efficient outcome is defined as an outcome where the value of the marginal unit equals the long run marginal cost for all individuals. We can measure the distributional cost by the total subsidy level, and the efficiency cost as the deadweight loss associated with inefficient pricing.

3.1 Revenue Neutrality Outcomes

While theory cannot predict the exact shape of the isocost and isorevenue curves, we can predict that there will be a locus of intersection points where the total cost equals the total revenue. These points are the set of revenue neutral combinations. We first discuss some of the important implications of this, and then provide more discussion of the characteristics of the isocost and isorevenue curves.

Proposition 2 *There is a maximum level of \underline{u} , denoted by u_{Max} such that any social goal where $\underline{u} < u_{Max}$ results in a potentially revenue neutral outcome. A social goal of $\underline{u} > u_{Max}$ requires subsidization.*

Proof: See Appendix A

The intuition behind this result is that any subsidy needs to be funded using producer surplus. We use a benchmark of total producer surplus under a competitive market, as this is where social welfare is maximized. Figure 2 shows one example of this result graphically. It compares the isoprofit curves with isoutility curves for type θ_L . An individual has a higher utility level when receiving a larger lifeline quantity and a lower lifeline price. There is a maximum level of utility that intersects the $\pi = 0$ curve. This is the maximum social goal \underline{u} that can be achieved while still maintaining revenue neutrality. This social goal will exhaust the surplus earned by the company, and providing a higher level of \underline{u} will result in a deficit.

<< Insert figure 2 >>

Remark There is a maximum price for w_L (denoted by \hat{w}_L), and any $w_L > \hat{w}_L$ leads to positive profits for the regulated utility.

Proof: See Appendix A

This result is important, as it shows that there is a limit on the the lifeline price when a utility is required to achieve revenue neutrality. Figure 3 shows one example of this result, and also illustrates the effect on total revenue of choosing a q_L, w_L combination that is not in the revenue neutral locus of points. This result is important, as regulations that require revenue neutrality have often led to rates based on average-cost pricing. This proposition shows that using a lifeline price of w_L can achieve revenue neutrality, but may not lead to the economic inefficiencies of average-cost pricing.

<< insert figure 3 >> To find the pairs of $\{q_L, w_L\}$ that result in revenue neutrality, we consider the isocost curves and isorevenue curves for a utility. Each isocost and isorevenue curve correspond to a single value of $Q(q_L, w_L, w^M, f(\theta), N)$. Since $Q_{q_L} > 0$ and $Q_{w_L} \leq 0$, the isocost curves are upward sloping and increase in value as they move away from the w_L axis. However, the direction of the isorevenue curve is ambiguous and depends on how shifts in the lifeline price and quantity affect marginal revenue. We calculate the signs of TR_{q_L} and

TR_{w_L} , where the subscripts denote the partial derivatives. Taking the derivative of Equation 6 with respect to q_L and w_L , and letting ϵ_d be the price elasticity of demand, we find the following:

$$TR_{w_L} = N \left(\int_{\theta_L}^{\theta_1(\cdot)} \left(q(\theta, w_L) + w_L \frac{\partial q(\theta, w_L)}{\partial w_L} \right) f(\theta) d\theta + \int_{\theta_1(\cdot)}^{\theta_H} q_L f(\theta) d\theta \right) \quad (7)$$

The second component of Equation 7 is positive. Thus, the sign of TR_{w_L} depends on the first component, and the sign of the first component depends on whether the price elasticity of demand is elastic or inelastic, as is shown below:

$$\begin{aligned} N \int_{\theta_L}^{\theta_1(\cdot)} \left(q(\theta, w_L) + w_L \frac{\partial q(\theta, w_L)}{\partial w_L} \right) f(\theta) d\theta &= N \int_{\theta_L}^{\theta_1(\cdot)} \left(1 + \frac{w_L}{q(\cdot)} \frac{\partial q(\theta, w_L)}{\partial w_L} \right) q(\cdot) f(\theta) d\theta \quad (8) \\ &= N \int_{\theta_L}^{\theta_1(\cdot)} (1 + \epsilon_d) q(\cdot) f(\theta) d\theta \end{aligned}$$

When demand is price elastic ($|\epsilon_d| > 1$), an increase in the price leads to a decrease in total revenue from the first tier, while the opposite effect holds when demand is price inelastic ($|\epsilon_d| < 1$). In the following diagrams, we assume that demand is price inelastic. Previous empirical research has shown this to be the case for regulated products like electricity and water. For example, Reiss and White (2005) finds price elasticity estimates of -0.39 for residential electricity demand while a metaanalysis of the price elasticity of urban water demand studies finds a mean of -0.41 and a median of -0.35 (Dalhuisen, Florax, de Groot & Nijkamp 2003).

The slope of the isorevenue curve in $\{q_L, w_L\}$ space also depends on the sign of TR_{q_L} . As the lifeline quantity increases, the total quantity consumed will either increase or stay constant. However, the impact on total revenue is ambiguous, as some consumers will buy more, but others will pay less for what they were already consuming. In calculating the

impact of changes in q_L on total revenue, we find the following:

$$TR_{q_L} = Nq_L((1 - F(\theta_1(q_L, w_L)))w_L - (1 - F(\theta_2(q_L, w^M)))w^M) \quad (9)$$

Using this, we find the following condition:

$$\text{If } \begin{cases} \frac{1-F(\theta_1(\cdot))}{1-F(\theta_2(\cdot))} > \frac{w^M}{w_L} \Rightarrow TR_{q_L} > 0 \\ \frac{1-F(\theta_1(\cdot))}{1-F(\theta_2(\cdot))} < \frac{w^M}{w_L} \Rightarrow TR_{q_L} < 0 \end{cases} \quad (10)$$

To provide some insight into the implications of Equation 10, we consider a couple of extreme scenarios. If all individuals are consuming at the socially optimal level where the marginal benefit equals the long run marginal cost, the lower two groups collapse, and $\theta_1(\cdot) = \theta_2(\cdot)$. In this case, the term on the left equals one, and any increase in the lifeline quantity decreases revenue. This occurs because individuals are already consuming above the tier, and an increase in the subsidized block reduces the amount paid. Thus, in cases when consumption levels are economically efficient, $TR_{q_L} < 0$ and both the isocost and isorevenue curves will be upward sloping in $\{q_L, w_L\}$ space. In another scenario, rates could be designed so that most individuals consume at the tier, or that $F(\theta_1(\cdot)) \sim 0$ while $F(\theta_2(\cdot)) \sim 1$. In this case, the term on the left approaches a limit of infinity, and an increase in the lifeline quantity increases total revenue to the utility, as most individuals will increase consumption accordingly.

3.2 Combining Revenue Neutrality and Economic Efficiency

In this section we show how various lifeline price and quantity combinations can be chosen to achieve revenue neutrality, economic efficiency or both. The previous analysis considered how the choice of lifeline price and quantity affect revenue neutrality. However, we are also concerned with achieving a second goal, economic efficiency. This concern is particularly

relevant due to the reliance of many regulated industries on limited natural resources. If IBR pricing encourages consumption levels above an economically efficient outcome, it will lead to excessive depletion of natural resources such as water and coal.

Proposition 3 *For any non-negative w_L there is a maximum level of \underline{u} , denoted by \hat{u} such that any social goal where $\underline{u} > \hat{u}$ results in an economically inefficient outcome.*

Proof: See Appendix A

We define all $\underline{u} \leq \tilde{U}$ as *potentially economically efficient*, meaning that there exists at least one set $\{q_L, w_L\}$ that achieves the social goal of \tilde{U} and results in economically efficient consumption by all individuals. This result is important, as it shows that there is a limit to the level of equity in consumption that can be achieved through IBR while still achieving economically efficient outcomes. This result is important in deciding what minimum level of utility should be guaranteed by regulators. For example, regulators may want to ensure that all consumers get enough for basic needs, but this goal may not provide an economically efficient outcome. Equity in consumption may be improved beyond the level \tilde{U} , but it will require some acceptance of economically inefficient outcomes. When considering the use of a scarce resource such as water, this result needs is important, as consumption above economically efficient levels increases costs and reduces social welfare in the current and future periods.

As shown in the proof of Proposition 3, there is a maximum lifeline quantity that permits economic efficiency. Figure 4 shows two possible values for this quantity. When the maximum lifeline quantity is q_{LMax} , the combinations that achieve both goals are labeled. However, there may be parameter values that result in an empty set of $\{q_L, w_L\}$ pairs that satisfy revenue neutrality and economic efficiency. For example, if the maximum lifeline quantity is q_{L2} , any economically efficient outcome will result in some positive level of surplus earned by the utility.

<< insert figure 4 >>

3.3 Equity Outcomes

Equity is frequently cited as a reason to use tiered pricing, as it can improve access to services when it is properly designed (Agthe & Billings 1987, OECD 1999). Tiered pricing is designed to assure that all consumers get a minimum benefit from water or electricity. We model this as a minimum level of utility, or well-being that is socially desirable.

Proposition 4 *For any social goal of minimum utility \underline{u} , there exists at least one set $\{q_L, w_L\}$ that can achieve this goal.*

Proof: See Appendix A

Proposition 4 is important because it shows that a minimum level of equity can always be achieved when cross-sector subsidization is permitted. Depending on the costs of providing service, it is not always possible to provide a minimum level of service with revenue neutrality. However, if a government is willing to subsidize the utility, minimum service goals are achievable.

There are a variety of measures that can be used to measure equity improvements under IBR. These include: 1) the Gini coefficient of consumption, 2) a *maxi-min* or Rawlsian social welfare measure, which is a social goal of maximizing the minimum level of consumption, 3) the percentage of total consumption by the lower population percentiles, or 4) the Gini coefficient of consumer surplus. The Lorenz curve of consumption levels can be a useful tool to calculate a variety of equity measures, as it can be used to calculate any of the first three measures. The Gini coefficient of consumption measures the actual distribution of consumption relative to an even distribution, while the second and third indicators are useful if regulators are concerned about ensuring a minimum level of consumption for low-income groups. For example, the media frequently print statistics that compare the proportion of

income earned by the highest versus the lowest percentiles of the population. The fourth indicator (the Gini coefficient of consumer surplus) is highly relevant to economic welfare measurements. While most of the economic literature uses income or wealth to estimate Gini coefficients, other indicators have been used. For example, Castelló and Doménech (2002) use information on educational attainment to measure a Gini index of human capital formation while Alesina and Rodrik (1994) use the Gini coefficient of land holdings as a measure of wealth inequality.

Each of these indicators has some advantages and disadvantages in measuring equity. First, it is important to recognize that any nominal measure is less useful than calculating how the indicator changes under alternative rate structures. Due to differences in demand functions, it is not desirable to have an equal distribution of consumption, as it results in economic inefficiency. For example, a Gini coefficient of consumption levels may decrease with higher rates, as low levels of demand that are the most price inelastic. However, this may not correspond to an increase in equity, since the welfare of all individuals is decreased. However, a comparison of consumption with a constant marginal price and changes in a subsidized block could be a useful measure.

The third measure, which looks solely at consumption in the lowest and highest percentiles, is most useful as a comparison between different rate structures. A social goal of equal consumption for all individuals is unlikely to be desirable, but a minimum level of consumption (i.e., a lifeline quantity) is more relevant. For example, regulators may be concerned with ensuring a sufficient level of consumption for the bottom 10 percent of the population. However, a disadvantage of this measure, or any measure that specifically targets low income levels, is that it requires regulators to know which customers fall in this category. Therefore, when this information is not available to regulators, other measures are necessary.

Finally, the last option of measuring the Gini coefficient of consumer surplus is highly relevant as an economic measure. It measures real changes in welfare. Therefore, we choose to use this measure in the following section and the numerical illustration.

3.3.1 Impacts of Rate Structure on Equity Measurements

Equations 11 - 14 develop the Gini coefficient of consumer surplus. We assume that the marginal utility function is bounded, and therefore consumer surplus is finite. This implies that there is a maximum price (i.e., a choke price) that consumers are willing to pay for the good. This reflects the availability of an outside option such as relying exclusively on bottled water, or a backstop technology such as using solar panels to produce electricity.

For an individual of type θ and rate structure $c(q)$, consumer surplus is defined by the following, where $\hat{q}(\theta, c'(q))$ is defined by the appropriate marginal conditions:

$$CS(\theta, c'(q)) = \int_0^{\hat{q}(\theta, c'(q))} \left(\frac{\partial B(\theta, q)}{\partial q} - c'(q) \right) dq \quad (11)$$

To determine the total social welfare to consumers, we integrate over the distribution $f(\theta)$, giving the following:¹

$$CS_{tot} = \int_{\theta_L}^{\theta_H} CS(\theta, c'(q)) f(\theta) d\theta \quad (12)$$

An equal distribution implies that the total consumer surplus CS_{tot} is distributed based on the population density $f(\theta)$, or that the cumulative distribution of consumer surplus is $F(\theta)CS_{tot}$. The Gini coefficient measures how far the actual distribution of consumer surplus is from an equal distribution. We define the actual level of aggregate surplus for $\theta \leq \tilde{\theta}$ as

¹To be precise, the measures presented should be multiplied by the population size N . However, this is irrelevant to the measures of distribution and equity. Thus, we choose not to include it for a more transparent exposition.

$\overline{CS}(\tilde{\theta})$, which is calculated by the following:

$$\overline{CS}(\tilde{\theta}) = \int_{\theta_L}^{\tilde{\theta}} CS(\theta, c'(q))f(\theta)d\theta \quad (13)$$

For a given set of parameters, the Gini coefficient of consumer surplus is defined as the following:

$$GINI = \frac{\int_{\theta_L}^{\theta_H} (F(\theta)CS_{tot} - \overline{CS}(\theta))f(\theta)d\theta}{CS_{tot}} \quad (14)$$

The numerator measures the difference between a completely equal distribution of consumer surplus and the actual distribution.

3.3.2 Impacts of Changing a Fixed Rate

So far, we have not put an explicit form on the shape of $c'(q)$. Either marginal or average cost pricing correspond to a constant value of $c'(q)$. Rewriting Equation 11 with a constant marginal cost (equal to w) gives the following:

$$CS(\theta, w) = \int_0^{\hat{q}(\theta, w)} \left(\frac{\partial B(\theta, q)}{\partial q} - w \right) dq \quad (15)$$

Proposition 5 *When all units are priced at a single rate, a change in the rate can either increase or decrease equity. When consumer surplus levels are relatively stable at high levels of θ , an increase in rates unambiguously reduces equity. When consumer surplus levels are relatively stable at low levels of θ , a increase in rates can either improve or reduce equity.*

Proof: See Appendix A

The proof of Proposition 5 shows that the impact of changing the price of the resource on the size of the Gini coefficient depends critically on the relative shift in consumer surplus at low and high levels of θ . The intuition for this result is that the relative shift represents the regressiveness of pricing on consumer surplus. When an increase in the price has a regressive

impact (i.e., the proportional change is larger at low levels of θ than at high levels of θ), then the expression $\frac{\partial GINI}{\partial w}$ is unambiguously positive and an increase in rates reduces equity. Empirical estimates from urban residential customers have found this situation with water rates (Agthe & Billings 1987).

If the opposite holds and pricing is progressive instead of regressive (i.e., the proportional change is larger at high levels of θ than at low levels of θ), the impact of a change in rates on the Gini coefficient is ambiguous. This is because there are two conflicting effects - first, higher rates will decrease aggregate consumer surplus. However, the impact of that reduction is not equal, and is greater on high levels of θ , which will reduce the Gini coefficient. Thus, the net impact is unclear, and the sign of $\frac{\partial GINI}{\partial w}$ could be either positive or negative.

3.3.3 Impacts of Demand Shifts

In addition to examining the effect of a change in rates, other forces could lead to shifts in consumer demand functions. For example, an increase in economic activity could lead to greater demand for services that require water or electricity inputs. These shifts in demand, if not accompanied by changes in the rate structure, could either improve or reduce equity in access. To determine this, we consider a change in the marginal benefit of consumption. Using a similar analysis as in Section 3.3.2, we measure the impact of a shift in demand on consumer surplus at each level of θ , aggregate consumer surplus, and on the Gini coefficient of consumer surplus.

To determine the effect of a shift in demand on individual consumer surplus, we differentiate Equation 11 with respect to a shift in the marginal benefit function. Denoting $\frac{\partial B(\theta, q)}{\partial q} = B_q$ and using Leibniz's rule, we find the following:

$$\frac{\partial CS(\theta, c'(q))}{\partial B_q} = (B_q(\theta, \hat{q}) - c'(\hat{q})) \frac{\partial \hat{q}(\theta, c'(q))}{\partial B_q} + \hat{q} > 0 \quad (16)$$

Using this result, we look at the impact of a shift in the marginal benefit function on the Gini coefficient of consumer surplus. Using the notation from Section 3.3.2, a shift in the demand function affects Gini coefficient as follows:

$$\frac{\partial GINI}{\partial B_q} = \frac{1}{CS_{tot}} \left\{ \frac{\partial INT}{\partial B_q} - \underbrace{\frac{INT}{CS_{tot}} \frac{\partial CS_{tot}}{\partial B_q}}_{<0} \right\} \quad (17)$$

If a demand shift is fairly equal for all values of θ , then $\frac{\partial INT}{\partial B_q} \approx 0$, and $\frac{\partial GINI}{\partial B_q} < 0$. This means that a shift to a more elastic benefit function improves equity. This implication of this is that an exogenous demand shift that impacts all consumers could either lead to an increase or decrease in equity. For example, a reduction in economic activity that reduces marginal benefit for everyone will exacerbate inequity, while an expansion will improve equity.

3.3.4 Impacts of Adding IBR

When all consumers are above the lifeline quantity, using an IBR pricing structure is equivalent to giving all consumers a fixed rebate. In this section we examine the impact of a fixed rebate on the Gini coefficient of consumer surplus.

Proposition 6 *An increase in the subsidy or rebate block unambiguously increases equity by reducing the Gini coefficient of consumer surplus.*

Proof: See Appendix A

Proposition 5 and 6 show that while a change in the rate has ambiguous effects on equity, an increase in a rebate block will improve equity. This has important implications for setting rates. For example, a new technology such as desalination of sea water can increase the marginal cost of production. If a water service provider is able to use that cost to increase rates, it will allow them a greater level of surplus that can be redistributed back to customers via rebate block. Recognizing these tradeoffs call allow rate-setting regulators using IBR to

determine the pair (w_L, q_L) that minimizes the Gini coefficient while still considering the social goals of revenue neutrality and economic efficiency.

4 Numerical Illustration

We consider the case where $f(\theta)$ is distributed uniformly over the $[0,1]$ interval. We use a linear marginal utility function, as is frequently used in the literature (Mussa & Rosen 1978, Caswell & Zilberman 1986, Castro-Rodríguez et al. 2002). There are two primary reasons that we decide to use a linear function. First, it implies there is a maximum price (i.e., a choke price) that consumers are willing to pay for the good. The maximum price or choke price is denoted by w^P and is the same for all individuals, reflecting the availability of an outside option for the good. The second reason for using a linear function is that it implies there is a satiation level for the good. We assume the marginal utility function is denoted by the following:

$$B'(\theta, q) = w^P - \frac{1}{\theta + a}q \text{ where } a > 0 \quad (18)$$

While the choke price (w^P) does not depend on an individual's type; the level of consumption where demand is satiated does depend on θ and a , and occurs at $q(\cdot) = w^P(\theta + a)$. The assumption that the satiation level depends on the type is also made in other literature on tiered pricing (Castro-Rodríguez et al. 2002). This can be used to calculate the aggregate demand at any price by the following:

$$\begin{aligned} Q(\cdot) &= N \int_0^1 (\theta + a)(w^P - p)f(\theta)d\theta \\ &= N \int_0^1 (\theta + a)(w^P - p)d\theta \\ &= N(a + \frac{1}{2})(w^P - p) \end{aligned}$$

Most natural resource providers have a variety of sources, each associated with different

marginal costs. While the marginal cost from one source may increase as resource extraction becomes more expensive, a shift to a new source typically increases the marginal cost by a sizeable margin. For example, a water utility that uses groundwater as its primary source will have increasing marginal costs as the groundwater table falls. However, there will be a large increase in the marginal cost as the utility needs to expand and also use desalinization to supply its customer. Thus, we make a minor simplification to this and model the marginal cost function as a step function, where each source has a limited amount of the resource. Indexing the number of sources by $s = 1, \dots, S$, we assume that each source has a limited quantity available. These quantities are denoted by the vector x , where x_s is the quantity available from source s . The cost vector is denoted by b , where b_s is the marginal cost for source s . Thus, the total cost function is as follows:

$$VC(Q) = \sum_{s=1}^S b_s x_s \text{ s.t. } \sum_{s=1}^{S-1} x_s < Q \leq \sum_{s=1}^S x_s \quad (19)$$

For the numerical example, we assume that the utility uses two sources, so $VC(Q) = b_1 x_1 + b_2(Q - x_1)$ and the marginal cost is b_2 . This allows us to examine shifts in the total cost function by looking at different values of b_1 , x_1 , and $b_2 - b_1$.

4.1 Economically Efficient Outcomes

A economically efficient outcome requires that every type of individual pays the marginal cost for the **last** unit consumed. Calculating these gives the following, where P^* is the optimal marginal price:

$$P^* = b_2 \quad (20)$$

$$Q^* = N(a + \frac{1}{2})(w^P - b_2) \quad (21)$$

We calculate the level of producer surplus, which gives a measure of the total surplus that can be distributed using subsidized pricing.

$$PS = (b_2 - b_1)x_1 \tag{22}$$

The measure of producer surplus is what sets a limit on the level of rebates that can be used under tiered pricing. For a revenue neutral outcome, all rebates must be funded through this surplus measure. As shown in Proposition 2, there is a maximum level of utility that can be distributed to consumers while still maintaining revenue neutrality.

4.2 Finding Tiered Pricing Parameters

With the lifeline price denoted by w_L and the lifeline quantity denoted by q_L , an efficient outcome with tiered pricing requires that the total subsidy to all individuals equal the available producer surplus and that the lifeline price and quantity be set so that type $\theta = 0$ uses water efficiently. These requirements are summarized in the following two conditions:

$$N(P^* - w_L)q_L = (b_2 - b_1)q_1 \tag{23}$$

$$q_L \leq a(w^P - b_2) \tag{24}$$

Equation 23 gives the revenue neutrality constraint, while Equation 24 gives the efficiency compatibility constraint. The set of $\{q_L, w_L\}$ that satisfies both of these constraints is the feasible set for policy makers implementing tiered pricing who wish to maintain economic efficiency.

Solving this set of equations we find that an outcome that is both economically efficient and revenue neutral is only feasible when $a \geq \frac{(b_2 - b_1)q_1}{N(w^P - b_2)(b_2 - w_L)}$. A higher value of a implies that increased consumption is necessary to satiate demand. This result is important, as

it shows that if there are customers with very low levels of demand, setting economically efficient tiered pricing rates may not be feasible.

These constraints can be analyzed graphically as shown in Figure 5, which illustrates the set of $\{q_L, w_L\}$ that is feasible for a particular set of exogenous values of N , w^P , a , x , and b . The feasible set contains the locus of points which satisfy the revenue neutral constraint and have a lifeline quantity below that defined by the efficiency compatibility constraint. For different parameter values, the feasible set of $\{q_L, w_L\}$ will shift to reflect the different conditions.

<< insert figure 5 >>

Using the previous results we can show the effects of changes in the key parameters of the model both analytically and graphically.

4.2.1 Changes in the Marginal Benefit Function

Figure 6 shows the results of a change in the demand function. The change examined is an expansion of demand, and the results show that as total demand expands, the feasible set of lifeline prices and quantities expands in size. In this particular example, the change in the marginal benefit function does not affect the revenue neutrality curve, it just affects the efficiency compatibility constraint. This is due to the assumption of a constant marginal cost over the range of interest. In the more realistic case where changes in the total quantity demanded result in small increases in the long run marginal cost, the revenue neutrality constraint will shift to the left, as higher costs result in less surplus that can be distributed. The specific numerical values used are less important than the direction of the change based on underlying parameter values.

<< insert figure 6 >>

4.2.2 Changes in the Marginal Cost Function

We are also interested in how shifts in the marginal cost function affect the feasibility of tiered pricing. Figure 7 shows the impact of a change in the marginal cost curve on the feasible choice set of lifeline price and lifeline quantity. A steeper marginal cost curve limits the ability to subsidize consumption, due to higher costs. This result is in part due to the assumption of a linear marginal cost curve, which has a constant rate of change. With other functional forms, the total amount of producer surplus will be the primary indicator of a utility's ability to subsidize consumption.

<< insert figure 7 >>

4.3 Measuring the Equity Implications of Tiered Pricing

The previous results allow us to examine the equity implications of a shift from marginal cost pricing to tiered pricing. We use the same analytical model, and continue to assume that θ is distributed uniformly between 0 and 1. We also continue to assume that a utility chooses $\{q_L, w_L\}$ to satisfy both the revenue neutral and the economic efficiency constraints.

As shown in Proposition 5, the direction of the change in equity (based on consumer surplus) of a rate change from average to marginal price is ambiguous. It depends on the relative shifts in consumption and welfare levels at the different ends of the population distribution, and thus a shift from marginal to average cost pricing could either increase or decrease the Gini coefficient of consumer surplus. In this numerical example, a shift in a single rate does not change the Gini coefficient. This result occurs because with the functional form used for a linear marginal benefit function, all individuals increase or decrease their consumption proportionally to initial levels with a rate change.

Unlike the ambiguous effect of a shift in a single rate, based on Proposition 6 we know that a shift from marginal cost pricing to IBR will reduce the Gini coefficient of surplus, leading

to an unambiguous improvement in equity. Based on the functional forms in the numerical illustration, we find the following analytical functions for the Gini coefficient under marginal cost pricing and IBR:

<< insert table 1 >>

One important result is that it only parameters of the benefit function affect the equity measure under marginal cost pricing. The parameter a measures the demand response to increased prices, and also corresponds to various levels of satiation. A higher value of a implies that demand is satiated at a greater consumption level. This decreases the Gini coefficient, corresponding to a more equitable outcome. This result is due to the fact that all individuals consume more as a increases.

The results also show that moving from marginal cost pricing to efficient tiered pricing leads to a reduction in the Gini coefficient. The level of the reduction depends on a , b_1 , and w^P . Under an economically efficient outcome, the total level of producer surplus is constant and the lifeline price and quantity measures do not affect the Gini coefficient. This result is especially important, since the parameters of the supply function are only important in determining equity under tiered pricing, and not under marginal or average cost pricing.

Figure 8 shows the impact of changes in either the demand or cost parameters on the Gini coefficient under IBR. A range of parameters for a and b_2 are considered, with the parameter w^P held constant. The results show that reductions in either of these parameters reduce equity. However, the impacts from the two parameters are not symmetric.

<< insert figure 8 >>

The results from the numerical illustration are consistent with the analytical model. Proposition 1 shows that a more inelastic supply function allows a greater increase in equity, due to a larger amount of surplus that can be distributed to consumers. Figure 8 shows that the result from the numerical simulation is consistent with this result, as higher levels of the b_2 parameter result in a lower Gini coefficient of consumer surplus. Holding other

parameters constant, a larger number for b_2 indicates a larger level of producer surplus that can be redistributed back to consumers. This result has important policy implications, as increases in the marginal cost function that allow a company to increase the marginal price to consumers can be used to facilitate greater levels of equity improvement.

Figure 8 also shows the impact of a shift in the parameters of the demand function. An increase in the level of a means that consumers are satiated at a higher level of consumption. Section 3.3.3 shows this result in the analytical model. When the marginal benefit shifts by the same proportion for all levels of θ , a more elastic benefit function improves equity. As shown in Table 1, a larger satiation requirement will reduce the Gini coefficient under both marginal cost pricing and IBR.

5 Conclusion

The choice of rate structure for a regulated natural resource affects aggregate consumption, economic efficiency, and the distribution of the benefits from natural resource use. There are several social goals that can be targeted through the rate structure choice. For example, marginal cost pricing promotes economically efficient consumption levels, average cost pricing leads to revenue neutrality, and subsidized rates benefit the poor.

Existing literature has argued that tiered pricing can improve equity in the rate structure for regulated utilities. Agthe and Billings (1987) find evidence that rate increases disproportionately affect low-income consumers and use their results to argue for steep increases in block rates. In the current paper, we find that in some circumstances tiered pricing can allow the full repayment of costs to meet financial obligations, economically efficient consumption levels, and the redistribution of resources to support equity goals. However, the capacity of tiered pricing to achieve these outcomes is limited and depends on exogenous underlying parameters. This result indicates that a tiered pricing rate structure needs to be designed

carefully, with consideration given to a firm’s cost structure and customer distribution. Ad-hoc choices for the lifeline price and lifeline quantity, or the simple duplication of a successful rate structure from another location are unlikely to be successful.

The feasibility of economic efficiency under IBR primarily depends on the marginal consumer (i.e., the consumer with the lowest quantity demanded at a certain price). In cases where the marginal consumer has a very low marginal benefit, it limits the feasibility of compensating them while still achieving economically efficient consumption. In some cases where satiation levels are very low, achieving economic efficiency under IBR is impossible. As a result, a consumer distribution with very poor customers is likely to result in economically inefficient consumption. However, the social cost of this economic inefficiency may be small enough that doing so is acceptable. In addition, if the lifeline quantity is set too high, consumption levels will not be economically efficient. This will result in an excessive transfer from those consumers with high levels of demand to those with low levels of demand.

Achieving improved equity with tiered pricing is particularly effective when there are various sources of low-cost inputs for a utility. We refer to this as the “inequality in leads to equity out” result. Since tiered pricing is a mechanism to redistribute producer surplus to consumers it is most effective with high levels of producer surplus, a direct result of low input costs. Results comparing the Gini coefficient under marginal cost and tiered pricing show that there is an improvement in equity in a transition from marginal cost pricing to tiered pricing. However, there is a limit to the extent of redistribution that is possible under efficient consumption levels.

In addition to the issues discussed above, it is critical to recognize that feasible tiered pricing formulas cannot be set once and left unchanged. Changes in the demand function, marginal source of production, availability of new technology, or in the underlying distribution of customers will may all result in differences in the optimality of tiered pricing. In this paper we have made the assumption that demand is constant, and have not adjusted for

seasonality. Tiered pricing could be paired with another form of pricing such as peak load pricing in cases where regulators want to discourage consumption at certain times of the day or season. When regulators really learn how to use tiered pricing effectively, it can be used in combination with other pricing mechanisms, leading to multiple dimensions of efficiency and equity considerations.

References

- Agthe, D. E. & Billings, R. B. (1987), 'Equity, price elasticity and household income under increasing block rates for water', *American Journal of Economics and Sociology* **46**(3), 273–286.
- Alesina, A. & Rodrik, D. (1994), 'Distributive politics and economic growth', *Quarterly Journal of Economics* **109**(2), 465–490.
- Bank, A. D. (1993), Water utilities handbook: Asian and pacific region, Technical report, Asian Development Bank, Manila, Philippines.
- Bar-Shira, Z., Finkelshtain, I. & Simhon, A. (2006), 'Block-rate versus uniform water pricing in agriculture: An empirical analysis', *American Journal of Agricultural Economics* **88**(4), 986–999.
- Bonbright, J. C., Danielsen, A. L. & Kamerschen, D. R. (1988), *Principles of Public Utility Rates*, second edn, Public Utilities Reports.
- Borenstein, S. (2008), Equity effects of increasing-block electricity pricing, Working Paper 180, Center for the Study of Energy Markets.
- Cassou, S. P. & Hause, J. C. (1999), 'Uniform two-part tariffs and below marginal cost prices: Disneyland revisited', *Economic Inquiry* **37**(1), 74–85.
- Castelló, A. & Doménech, R. (2002), 'Human capital inequality and economic growth: Some new evidence', *The Economic Journal* **112**, C187–C200.
- Castro-Rodríguez, F., Da-Rocha, J. M. & Delicado, P. (2002), 'Desperately seeking θ 's: Estimating the distribution of consumers under increasing block rates', *Journal of Regulatory Economics* **22**(1), 29–58.
- Caswell, M. & Zilberman, D. (1986), 'The effect of well depth and land quality on the choice of irrigation technology', *American Journal of Agricultural Economics* **68**(4), 798–811.
- Dahan, M. & Nisan, U. (2007), 'Unintended consequences of increasing block tariffs pricing policy in urban water', *Water Resources Research* **43**.
- Dalhuisen, J. M., Florax, R. J. G. M., de Groot, H. L. F. & Nijkamp, P. (2003), 'Price and income elasticities of residential water demand: a meta-analysis', *Land Economics* **79**(2), 292–308.
- Hewitt, J. A. & Hanemann, W. M. (1995), 'A discrete/continuous approach to residential water demand under block rate pricing', *Land Economics* **71**(2), 173–192.
- Mussa, M. & Rosen, S. (1978), 'Monopoly and product quality', *Journal of Economic Theory* **18**, 301–317.

- Nataraj, S. & Hanemann, W. M. (2008), Does marginal price matter? a regression discontinuity approach to estimating water demand, Working Paper 1077, UC Berkeley: Department of Agricultural and Resource Economics.
- OECD (1999), Household water pricing in oecd countries, Technical report, Organisation for Economic Cooperation and Development, Paris.
- Oi, W. Y. (1971), 'A disneyland dilemma: Two-part tariffs for a mickey mouse monopoly', *The Quarterly Journal of Economics* **85**(1), 77–96.
- Olmstead, S. M., Hanemann, W. M. & Stavins, R. N. (2007), 'Water demand under alternative price structures', *Journal of Environmental Economics and Management* **54**, 181–198.
- Pashardes, P. & Hajispyrou, S. (2002), Consumer demand and pricing under increased block pricing. University of Cyprus, Discussion Paper 2002-07.
- Reiss, P. C. & White, M. W. (2005), 'Household electricity demand, revisited', *Review of Economic Studies* **72**(3), 853–883.
- Rietveld, P., Rouwendal, J. & Zwart, B. (2000), 'Block rate pricing of water in indonesia: An analysis of welfare effects', *Bulletin of Indonesian Economic Studies* **36**(3), 73–92.
- Spence, M. (1977), 'Nonlinear prices and welfare', *Journal of Public Economics* **8**, 1–18.
- Whittington, D. (1992), 'Possible adverse effects of increasing block water tariffs in developing countries', *Economic Development and Cultural Change* **41**(1), 75–87.
- Wilson, R. B. (1993), *Nonlinear Pricing*, Oxford University Press, New York, NY.

APPENDIX A

Proof of Proposition 1:

Proof Consider two marginal cost function $MC_1(Q)$ and $MC_2(Q)$ where marginal cost pricing results in the same equilibrium quantity in both cases, and that quantity is denoted by Q^* . Suppose that $MC_1(Q^*) = MC_2(Q^*)$, and that $MC_1(Q^* - \epsilon) < MC_2(Q^* - \epsilon) \forall \epsilon > 0$. The supply function is determined by the marginal cost curve, and since $MC_2'(Q) < MC_1'(Q)$, $MC_2(\cdot)$ is a more price elastic supply function. The total revenue that can be distributed for subsidies via tiered pricing is determined by the total producer surplus. A supply function with a higher level of producer surplus will increase the feasible subsidy.

The difference in producer surplus is given by the following:

$$\begin{aligned} PS_1 - PS_2 &= (MC_1(Q^*)Q^* - \int_0^{Q^*} MC_1(q)dq) \\ &\quad - (MC_2(Q^*)Q^* - \int_0^{Q^*} MC_2(q)dq) \\ &= \int_0^{Q^*} (MC_2(q) - MC_1(q))dq > 0 \end{aligned} \quad (25)$$

Supply function MC_1 is more price inelastic, but has a greater level of producer surplus. Therefore, a higher level of \tilde{u} that can be supported while maintaining revenue neutrality. ■

Proof of Proposition 2:

Proof Let $\tilde{u} = U(\theta_L, q(\theta_L, w^M))$, where w^M is the long-run marginal price. Potential subsidy levels under revenue neutrality depend on the level of producer surplus available, which is maximized when marginal revenue equals the marginal cost $V(Q)$ and results in a producer surplus level $\bar{P}S$. With equal distribution, let α denote each individual's share of the total surplus. Thus, type θ_L will earn a total utility level of $U(\theta_L, q(\theta_L, w^M)) + \alpha\bar{P}S$. Setting $u_{Max} = U(\theta_L, q(\theta_L, w^M)) + \alpha\bar{P}S$ provides the highest possible level of guaranteed utility under revenue neutrality. Achieving a greater level of guaranteed utility will require cross-subsidization from other sectors or individual specific rebates. ■

Proof of Remark:

Proof Let $q_L = \tilde{q}_L$ s.t. $\frac{\partial B(\theta_L, \tilde{q}_L)}{\partial q} = w^M$. First, we consider $w_L = w^M$, the long run marginal cost. Since the long run marginal cost is greater than the average cost, the total profit is positive ($\pi(w^M, \tilde{q}_L) > 0$). Now, let $q_L = \tilde{q}_L$, but set $w_L = 0$. Setting a price of w^M for consumption over \tilde{q}_L , with the lifeline quantity available for free leads to negative profits ($\pi(0, \tilde{q}_L) < 0$). By the Intermediate Value Theorem, if $\pi(w^M, \tilde{q}_L) > 0$ and $\pi(0, \tilde{q}_L) < 0$, $\exists \tilde{w}_L \in [0, w^M]$ s.t. $\pi(\tilde{w}_L, \tilde{q}_L) = 0$.

Let $q_L = \tilde{q}_L - \epsilon$ for any $\epsilon > 0$. Since the total subsidy is distributed over a smaller quantity, the maximum level of $w_L < \tilde{w}_L$. Now let $q_L = \tilde{q}_L + \epsilon$ for any $\epsilon > 0$ (this is an economically inefficient outcome). The same proof applies as with \tilde{q}_L . The total profit for $w_L = w^M$ is positive ($\pi(w^M, \tilde{q}_L + \epsilon) > 0$). The total profit for $w_L = 0$ is negative ($\pi(0, \tilde{q}_L + \epsilon) < 0$). Again, by the Intermediate Value Theorem, $\exists \tilde{w}_L \in [0, w^M]$ s.t. $\pi(\tilde{w}_L, \tilde{q}_L + \epsilon) = 0$. ■

Proof of Proposition 3:

Proof We denote w^M as the long run marginal cost, and define \tilde{q} s.t. $\frac{\partial B(\theta_L, q)}{\partial q} = w^M$ at $q = \tilde{q}$. Setting $w_L = 0$, we also define $U(\theta_L, \tilde{q}) = \tilde{U}$. For any $\epsilon > 0$, $\frac{\partial B(\theta_L, q)}{\partial q} < w$ at $q = \tilde{q} + \epsilon$. At this point the marginal benefit of consumption is less than the long run marginal cost, resulting in economic inefficiency. Therefore, setting $q_L > \tilde{q}$ results in an economically inefficient outcome and $q_{LMax} = \tilde{q}$ is the largest lifeline quantity that can be offered while still maintaining economic efficiency. Any social goal $\underline{u} > \tilde{U}$ cannot be achieved without some inefficiency in consumption. Therefore, any $\underline{u} < \tilde{U}$ is *potentially economically efficient*. ■

Proof of Proposition 4:

Proof Consider the lowest type θ_L where $B(\theta_L, 0)$ is normalized to 0. Since $\frac{\partial B}{\partial q} > 0 \forall q$, there exists \tilde{q} s.t. $B(\theta_L, \tilde{q}) = \underline{u}$. If the quantity and price pair are set by $\tilde{q} = q_L$ and $w_L = 0$, then the utility level of the lowest type is $U(\theta_L, q_L) = \underline{u}$. Since the price w_L is zero, the lowest type can afford this quantity. And, since $\frac{\partial B}{\partial \theta} > 0$, if the social goal \underline{u} is achieved for type θ_L , then it is achieved for all types. ■

Proof of Proposition 5:

Proof Our first objective is to determine the sign of $\frac{\partial GINI}{\partial w}$. Total consumption and consumer surplus increase at higher levels of θ , but the proportional change from a change in price is not clear.

$$\frac{\partial GINI}{\partial w} = \frac{1}{CS_{tot}} \left\{ \frac{\partial \int_{\theta_L}^{\theta_H} (F(\theta)CS_{tot} - \overline{CS}(\theta))f(\theta)d\theta}{\partial w} - \frac{\partial \int_{\theta_L}^{\theta_H} (F(\theta)CS_{tot} - \overline{CS}(\theta))f(\theta)d\theta}{CS_{tot}} \frac{\partial CS_{tot}}{\partial w} \right\} \quad (26)$$

Defining $INT = \int_{\theta_L}^{\theta_H} (F(\theta)CS_{tot} - \overline{CS}(\theta))f(\theta)d\theta$, Equation 26 can be rewritten. The term INT measures the nominal difference between an equal distribution and the actual distribution of consumer surplus, while the denominator CS_{tot} normalizes that value by the actual surplus measure, guaranteeing that the value for the Gini coefficient is between 0 and 1.

$$\frac{\partial GINI}{\partial w} = \frac{1}{CS_{tot}} \left\{ \frac{\partial INT}{\partial w} - \frac{INT}{CS_{tot}} \frac{\partial CS_{tot}}{\partial w} \right\} \quad (27)$$

To determine the sign of this expression, we first need to determine the sign of $\frac{\partial CS(\theta, w)}{\partial w}$. The sign of this expression shows how consumer surplus changes with under different rates, and can be used to sign the different components of Equation 27. Using Leibniz's formula, we calculate the following:

$$\frac{\partial CS(\theta, w)}{\partial w} = \left(\frac{\partial B(\theta, q)}{\partial q} - w \right) \frac{\partial \hat{q}(\theta, w)}{\partial w} - \int_0^{\hat{q}(\theta, w)} 1dq \quad (28)$$

$$\begin{aligned}
&= \left(\frac{\partial B(\theta, q)}{\partial q} - w \right) \frac{\partial \hat{q}(\theta, w)}{\partial w} - \hat{q}(\theta, w) \\
&= -\hat{q}(\theta, w) < 0
\end{aligned}$$

From the result in Equation 28, it follows directly that if $\frac{\partial CS(\theta, w)}{\partial w} < 0 \forall \theta$, then $\frac{\partial CS_{tot}}{\partial w} < 0$.

$$\frac{\partial GINI}{\partial w} = \frac{1}{CS_{tot}} \left\{ \underbrace{\frac{\partial INT}{\partial w}}_{+/-} - \underbrace{\frac{INT}{CS_{tot}} \frac{\partial CS_{tot}}{\partial w}}_{+} \right\} \quad (29)$$

■

Proof of Proposition 6:

Proof We assume that the outcome is economically efficient (i.e., all consumers are above the lifeline quantity). We first modify the equation for individual consumer surplus to adjust for a fixed rebate, denoted by \bar{R} . Rewriting Equation 11, we have the following measure of individual consumer surplus:

$$CS(\theta, c'(q)) = \int_0^{\hat{q}(\theta, c'(q))} \left(\frac{\partial B(\theta, q)}{\partial q} - c'(q) \right) dq + \bar{R} \quad (30)$$

This measure of individual consumer surplus will change both the cumulative measures CS_{tot} and $\overline{CS}(\theta)$. However, both of these measures will change by the exact same amount, since the extra surplus is distributed equally to all individuals. Thus, the numerator of the Gini coefficient (shown in Equation 14) will not change.

$$\begin{aligned}
\frac{\partial GINI}{\partial \bar{R}} &= -\frac{\partial CS_{tot}}{\partial \bar{R}} \frac{INT}{CS_{tot}^2} \\
&= -\bar{R} \frac{INT}{CS_{tot}^2} < 0
\end{aligned} \quad (31)$$

■

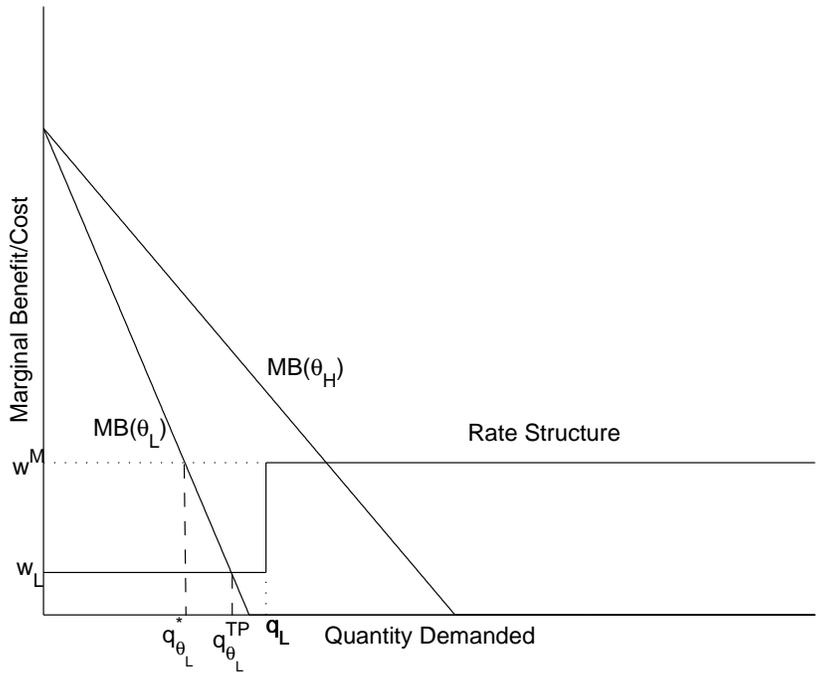


Figure 1: Basic Tiered Pricing Rate Structure

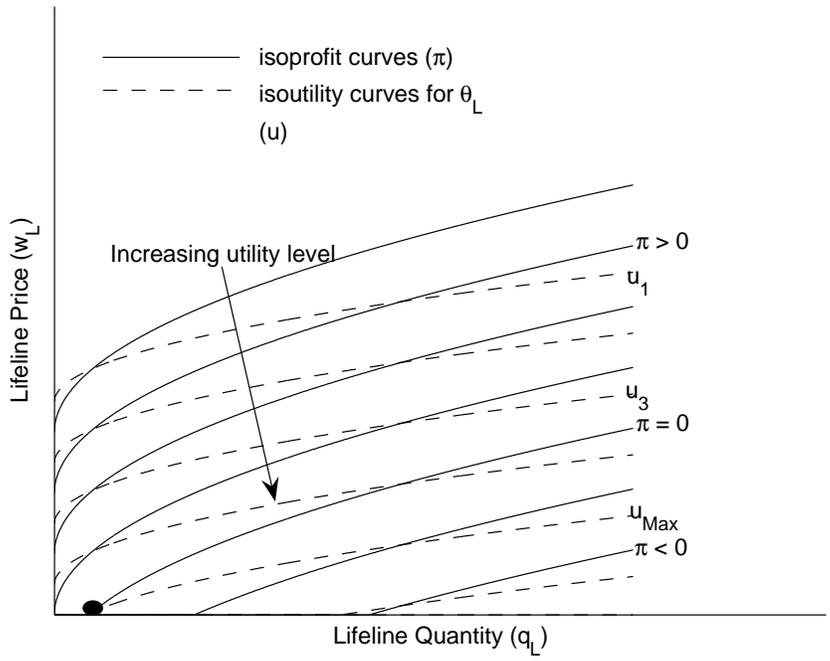


Figure 2: Maximum Utility under Revenue Neutrality

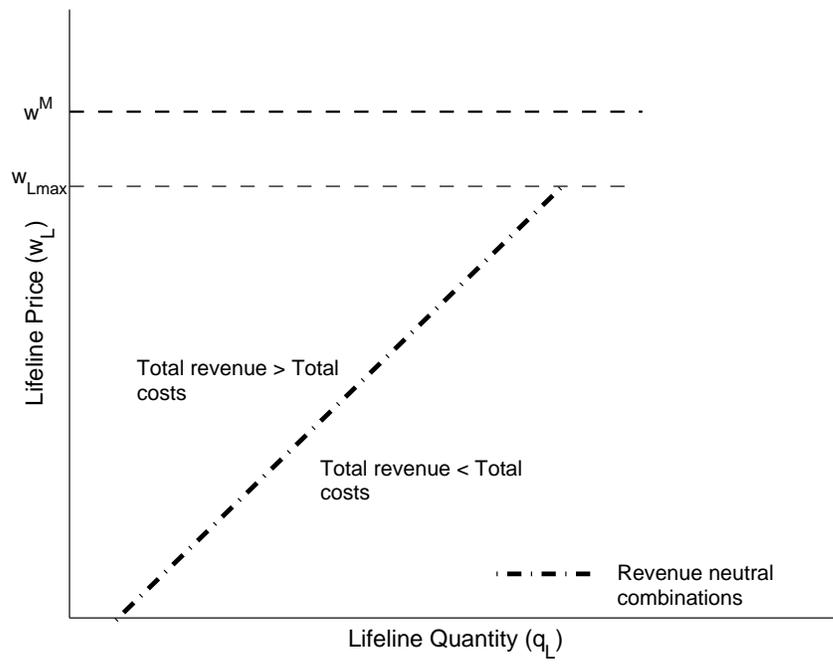


Figure 3: Total Profit and Revenue Neutral Combinations

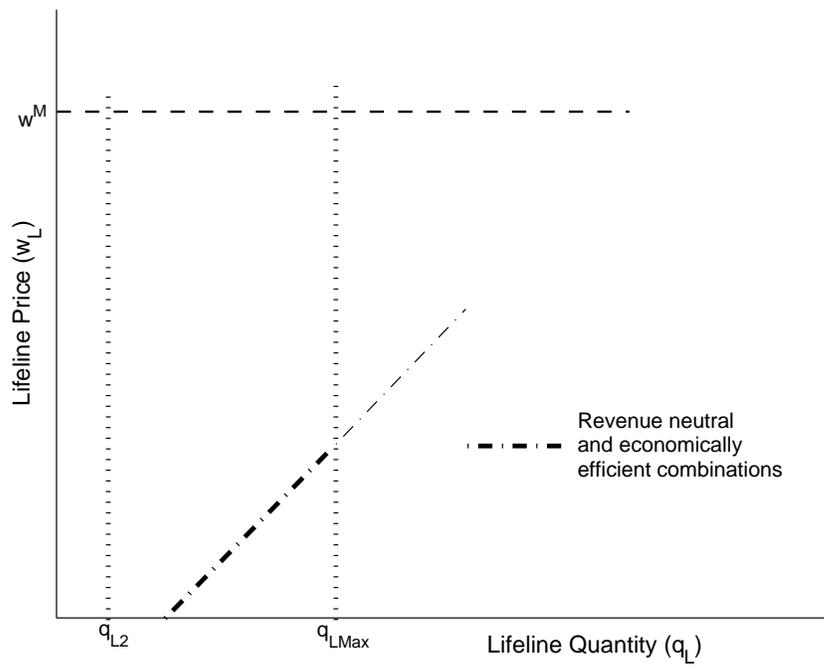


Figure 4: Satisfying Revenue and Efficiency Goals with Tiered Pricing

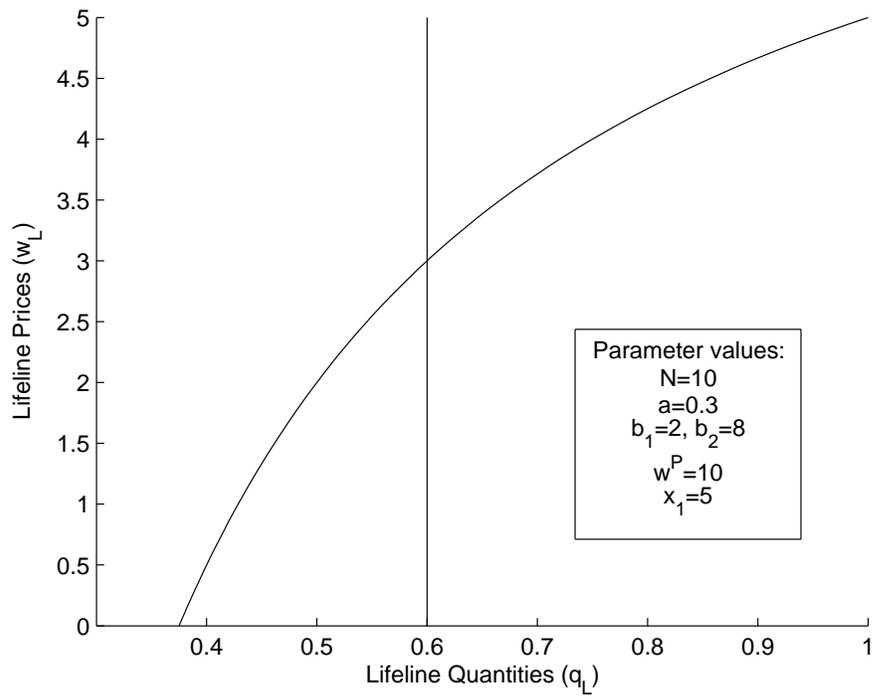


Figure 5: Feasible Set of Lifeline Prices and Quantities

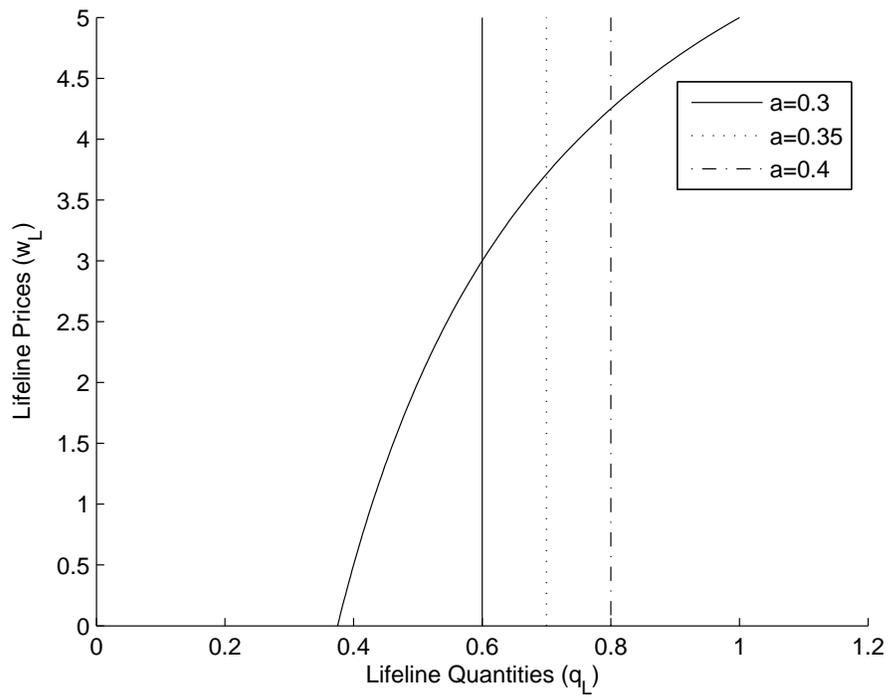


Figure 6: Effects of a Shift in Demand on Feasible Set Choices

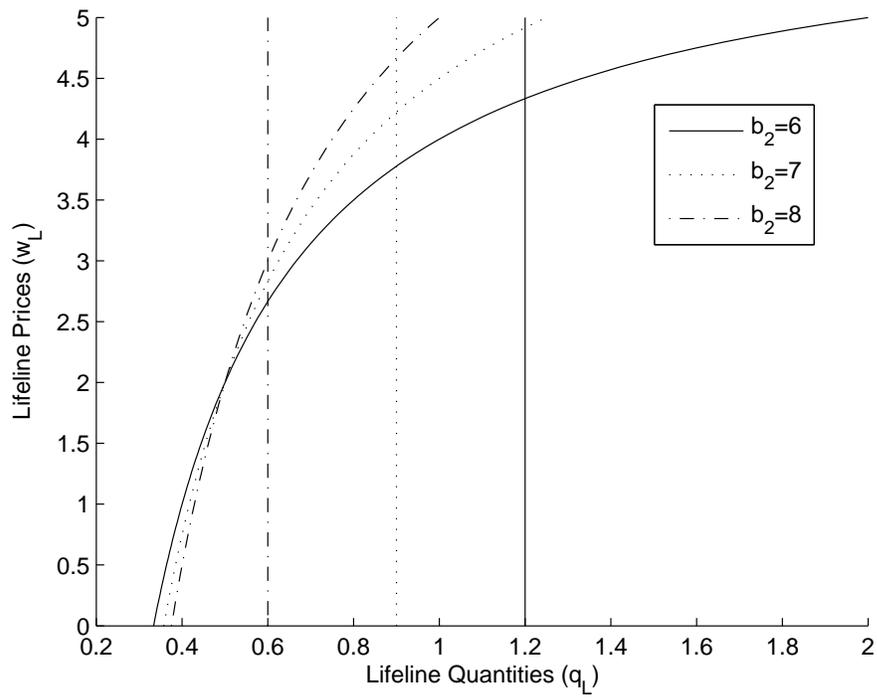


Figure 7: Effects of a Shift in Marginal Cost on Feasible Set Choices

	Marginal Cost Pricing	Efficient Tiered Pricing
Gini coefficient	$\frac{1}{6a+3}$	$\frac{1}{6a+3} \frac{(w^P - b_2)^2}{(w^P - b_2)^2 + 2(b_2 - b_1)}$

Table 1: Comparison of Gini Coefficient of Consumer Surplus under Marginal Cost and Tiered Pricing

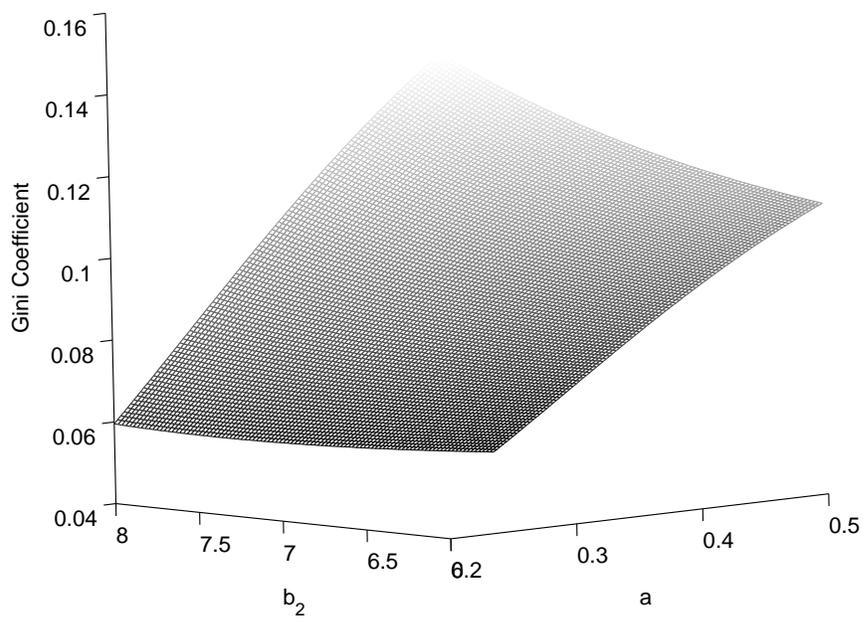


Figure 8: Gini Coefficient Measures with Varying Parameters