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A Proposal for Improving the System of Financing Health Care in Singapore

Robert Keng Heong Lian* and Loi Soh Loi†

Abstract†

Like many other countries, including the United States, Singapore faces the dual problems of rising health care costs and an aging population. To cope with these problems, the Singapore government introduced the Medishield scheme in 1989 that provides low cost catastrophic medical insurance coverage. The scheme suffers from a serious deficiency, however: coverage ceases at age 70. This deficiency is exacerbated by Medishield's premium payment structure which is akin to the premium structure of a one year renewable term policy so no reserves are developed. As a result, coverage beyond age 70 requires exorbitant premiums that are beyond the reach of the average Singaporean.

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We propose a premium payment structure under which the annual premium payable remains level in real terms throughout the lifetime of the insured. This makes the premium structure similar to one that is a level percentage of salary. The model uses such key variables as the rate of return, the rates of inflation of general costs and of medical costs, and the rate of increase in the morbidity rate.

Key words and phrases: premiums, inflation, reserves, gains and losses, surplus

1 Introduction

Singapore soon will face the problem of rising health care costs as its population ages. To cope with this anticipated problem, the Singapore government introduced the Medisave scheme in 1984 under which every working person must contribute a certain percent of his or her income to meet personal or immediate family hospitalization expenses, particularly medical expenses incurred during old age.1

1.1 The Medisave Scheme

Medisave is compulsory and is administered by the Central Provident Funds (CPF) Board, a statutory board of the Singapore government. All employed persons are required to be members of the CPF and must contribute 40 percent of their income2 to meet their retirement, housing, education, investment, and health care needs. This 40 percent contribution is jointly and equally shared by the employer and the employee and is allocated as follows:

- 6 to 8 percent of the total contribution goes to the Medisave account;
- 4 percent of the total contribution goes to the special account;
- The balance (28 to 30 percent) of the total contribution remains in the ordinary account.

The operation of the Medisave special account and ordinary account are described below.

The Medisave Account: Funds in the Medisave accounts may be used to meet personal or immediate family's hospitalization expenses,

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1Medisave coverage ceases at age 70.
2Income is subject to a maximum monthly contribution of $2,400.
especially after retirement. For example, these funds can be used to pay for the hospital bills of the member's spouse, children, parents, or grandparents up to:

- $300 per day for daily hospital charges; and
- A fixed limit per table of surgical operation according to the complexity of the operation.

If the hospital bill exceeds Medisave limits, the member is obligated to pay the part of the hospital bill not covered by Medisave. In addition, Medisave can be used to pay for the hospital stay for the delivery of the first three children.

Employee contribution rates progressively increase with age as shown in Table 1: For self-employed workers, contribution rates are lower. Medisave contributions and savings earn interest at prevailing market rates and are tax-deductible. The Medisave balance can be accumulated up to $17,000; any amount in excess of this limit is automatically transferred into the ordinary account.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Singapore Medisave Contribution Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>Rate</td>
</tr>
<tr>
<td>Below 35</td>
<td>6%</td>
</tr>
<tr>
<td>35 to 44</td>
<td>7%</td>
</tr>
<tr>
<td>45 and above</td>
<td>8%</td>
</tr>
</tbody>
</table>

The Special Account: Funds in the special account can be used to finance the Minimum Sum scheme, which is a compulsory national retirement scheme to help members support a modest standard living during retirement. Starting from $40,000 in 1995, the minimum sum will be raised by $5,000 a year until it reaches $80,000 in 2003. At least half the minimum sum must be in cash, the other half may consist of tangible assets such as property. The cash portion ensures members of a monthly income in retirement. Prior to retirement, members have three options to invest their minimum sum: (i) buy a life annuity from an approved insurance company, (ii) keep it with an approved bank, or (iii) leave it with the CPF Board. If the income in the special account is less than
the minimum sum, the balance of the minimum sum is covered by the income in the ordinary account.

The Ordinary Account: Funds in the ordinary accounts can be used for several different purposes including: (i) retirement (together with the special account to meet the minimum sum requirement); (ii) housing (can be government or private houses, for the purpose of owner occupation or for investment in one or more houses); (iii) education (restricted to local tertiary education), and (iv) investment (in common stocks and bonds, government bonds, fixed deposit, unit trust, gold, as well as endowment insurance policy).

1.2 The Medishield Scheme

CPF members' Medisave accounts, however, may be strained in the event of a prolonged illness that requires long-term medical treatment. To protect the Medisave account, the Singapore government introduced the Medishield scheme in 1990, a low cost medical insurance that adds more value to Medisave. Medishield is major medical insurance with participation built in by way of deductible and co-insurance as measures to contain cost. All CPF members automatically are covered under Medishield unless they elect to opt out. The Medishield premium and any costs not covered by Medishield, such as deductibles, co-insurances, and amounts in excess of the maximum claimable amount, can be paid from the Medisave account. Table 2 shows the benefits and claim limits under Medishield. Those limits are per policy. Each of the family member will have his/her own policy. Therefore, for example the deductible $4,000 is for the particular family member and not the entire family. Likewise for all of the rest.

Let $AP(x, z)$ be the actual annual premium charged for one year health insurance coverage in calendar year $z$ payable under Medishield. The actual premium charged is determined from expected claims by applying the percentage loading according to the formula:

$$AP(x, z) = (1 + \theta)EC(x, z)$$  \hspace{1cm} (1)

where $EC(x, z)$ is the expected claim cost in the calendar year $z$ for insured age $x$ last birthday on 1/1/z, and $\theta$ is the expense and other loading applied to the net premium to get the actual annual premium charged.

Table 3 shows how these premiums increase with age.

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3 As these premiums are only for coverage for one year, they are similar to the premiums for yearly renewable term policy in life insurance.
Table 2
Medishield Benefits and Claim Limits (in Singapore $s)

<table>
<thead>
<tr>
<th>Benefits</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductible</td>
<td>$4,000 per policy year</td>
</tr>
<tr>
<td>Claim Limits</td>
<td>$70,000 per policy year; $200,000 lifetime</td>
</tr>
<tr>
<td>Hospital Stay*</td>
<td>$500 per day</td>
</tr>
<tr>
<td>Intensive Care Unit*</td>
<td>$800 per day</td>
</tr>
<tr>
<td>Surgical Operations</td>
<td>$400 to $5,500 per policy year</td>
</tr>
<tr>
<td>Implants</td>
<td>$3,500 per policy year</td>
</tr>
<tr>
<td>Outpatient Treatment</td>
<td>Limits vary according to treatment.</td>
</tr>
</tbody>
</table>

*Includes meals, prescriptions, investigations, and other miscellaneous charges.

The annual premiums shown in Table 1 are the 1995 published rates charged by the CPF's Medishield program on an annual basis for an insured of age $x$ last birthday. Because we do not have access to original data such as the morbidity rate and other assumptions used by Medishield in determining $AP(x, z)$, it is difficult to estimate the annual premiums for ages beyond age 70. A quick estimate of the projected premiums beyond age 70 can be obtained by assuming that health care costs continue to increase exponentially at advanced ages. The formula

$$AP(x, 1995) = 13.17(1.06)^x$$

(2)

for $x = 25, 35, 45, 55, 62.5, and 67.5$ gives a good least squares fit to the data in Table 1. Equation (1) is used to provide estimates for $AP(x, z)$ for higher ages as tabulated in Table 4. Table 4 shows that if coverage is extended beyond age 70, the annual premium for ages over 70 will be more than most Singaporeans can afford.

1.3 Objectives

The objective of this paper is to develop a model of the Medishield program without the restriction that coverage ceases after age 70, where it is needed most. To pay for this extended benefit, we develop a premium structure that remains level in real terms and is approximately a constant percentage of a worker's salary. A method for performing a detailed analysis of the annual gains and losses and the sources of surplus of the Medishield program is provided.
Table 3

Medishield

1995 Annual Premiums*

<table>
<thead>
<tr>
<th>Age ((x))</th>
<th>(AP(x, 1995))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 30</td>
<td>$ 60</td>
</tr>
<tr>
<td>31 - 40</td>
<td>$ 90</td>
</tr>
<tr>
<td>41 - 50</td>
<td>$180</td>
</tr>
<tr>
<td>51 - 60</td>
<td>$360</td>
</tr>
<tr>
<td>61 - 65</td>
<td>$480</td>
</tr>
<tr>
<td>66 - 70</td>
<td>$660</td>
</tr>
</tbody>
</table>

*In Singapore dollars.

2 Notation

The analysis presented in this paper is based on population information that may be most readily available at the start of each calendar year. If, however, information is gathered on a different basis, such as is the case for fiscal years, then our results can easily be modified. This requires changing 1/1/\(z\) to the date at the start of the fiscal year. All events are then defined over the fiscal year in an obvious manner.

The following notation is used throughout this paper:

- \(lbd\) = Last birthday;
- \(b\) = Annual morbidity rate of increase due to age;
- \(g\) = Annual rate of inflation (general cost of living);
- \(m\) = Annual rate of medical inflation;
- \(i\) = Annual valuation rate of interest;
- \(\nu\) = \((1 + i)^{-1}\);
- \(z_0\) = Calendar year of issue on the policy;
- \(x_0\) = Age \(lbd\) on 1/1/\(z_0\);
- \(z\) = Current calendar year;
- \(x\) = Current age \(lbd\) last birthday on 1/1/\(z\);
- \(k\)\(p_x^{(T)}\) = Probability that a Medishield insured age \(x\) \(lbd\) survives to age \(x + k\) \(lbd\);
- \(q_x^{(d)}\) = Probability that a Medishield insured age \(x\)
Table 4
Medishield Extrapolated
Annual Premium*

<table>
<thead>
<tr>
<th>Age</th>
<th>Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>$778</td>
</tr>
<tr>
<td>75</td>
<td>$1,041</td>
</tr>
<tr>
<td>80</td>
<td>$1,393</td>
</tr>
<tr>
<td>85</td>
<td>$1,865</td>
</tr>
<tr>
<td>90</td>
<td>$2,495</td>
</tr>
<tr>
<td>95</td>
<td>$3,339</td>
</tr>
</tbody>
</table>

*In Singapore dollars.

\[ q_x^{(w)} = \text{Probability that a Medishield insured age } x \text{ lbd dies before age } x + 1 \text{ lbd}; \]
\[ q_x^{(\tau)} = q_x^{(d)} + q_x^{(w)}; \]
\[ r_x^z = \text{Morbidity rate in calendar year } z \text{ for insured age } x \text{ lbd on } 1/1/z; \]
\[ ECS(x, z) = \text{Expected Medishield claim cost in calendar year } z \text{ for an insured age } x \text{ lbd on } 1/1/z \text{ who survives to } 1/1/z + 1; \]
\[ ECD(x, z) = \text{Expected Medishield claim cost in calendar year } z \text{ for an insured age } x \text{ lbd on } 1/1/z \text{ who dies before } 1/1/z + 1; \]
\[ ECW(x, z) = \text{Expected Medishield claim cost in calendar year } z \text{ for an insured age } x \text{ lbd on } 1/1/z \text{ who withdraws before } 1/1/z + 1; \]
\[ 1 + f_s = \text{Annual extra claim inflation factor for survivors}; \]
\[ 1 + f_d = \text{Annual extra claim inflation factor for deaths}; \]
\[ 1 + f_w = \text{Annual extra claim inflation factor for withdrawals}; \]
\[ LP(x_0, z_0) = \text{Level (in real terms) annual premium payable upto age 65 for Medishield coverage sold in calendar year } z_0 \text{ to a person age } x_0 \text{ lbd on } 1/1/z_0. \]
Note that for all calculations we will use the Singapore male mortality table for the minimum reserve calculation for annuity (MAS 309)\textsuperscript{4} with $q_{99} = 1$. This table is deemed suitable, as it is based on the a(90) table with improvement in mortality over time. There is no service table, with decrements for disability and withdrawals, available for use in Singapore's Medishield program as of May 1996.

3 The Individual Model

We develop a model in which the charged premium remains level in real terms throughout the working life of the member (Black & Skipper, 1994, Chapter 22). Level premium in real terms means that the premiums in subsequent years are adjusted by the rate of inflation of the general cost of living. Such level premiums also decrease the rate of growth of premiums at the older ages and tend to make lifetime coverage more affordable. Another advantage of this approach is that it makes financial planning easier in an environment where wages and salaries keep pace with the general cost of living increases; these premiums are approximately a constant percentage of wages and salaries.

A key feature of our model is that it removes the restriction that coverage ceases after age 70, as is the case with the Medishield program. Thus, coverage is available in the advanced ages where people may need it most.

3.1 Benefits

For increased flexibility, we develop three separate expected claim costs in any calendar year: (i) for those who continue their Medishield policy during the next calendar year ($ECS(x, z)$), (ii) for those who die within the calendar year ($ECD(x, z)$), and (iii) for those who withdraw from the Medishield program during the calendar year ($ECW(x, z)$). Using anti-selection arguments, one may expect those who withdraw to be healthier than those who remain, while those who die may tend to have larger than average costs. Thus anti-selection arguments suggest that we adjust the expected claim costs by the appropriate extra claim inflation factor $(1 + f)$ to reflect the different expected experiences among those who die, withdraw, or continue with the policy.

\textsuperscript{4}The MAS 309 is the notice No. 309 issued by the Monetary Authority of Singapore (equivalent to the Federal Reserve Board in the United States) to life insurers. In MAS 309, the mortality tables for calculating the minimum reserves for annuities are given.
We will assume that expected claim costs are paid at the end of the calendar year. As a result, we include the expected interest accrued to the end of the calendar year in our definition of the expected claims.

Our model assumes that the expected claim cost for an insured increases for two reasons: (i) increases in the underlying rate morbidity rate, and (ii) increases due to medical inflation. Motivated by equation (1), we model the morbidity rate due to age (within the a calendar year) as:

\[ r_{x+1}^z = (1 + b) r_x^z, \]

which gives

\[ ECS(x + 1, z) = (1 + b)ECS(x, z). \]

On the other hand, the impact of annual medical inflation on expected claim costs is assumed to affect only the size of claims not the morbidity rate. So, \( r_{x+1}^z = r_x^z \), and

\[ ECS(x, z + 1) = (1 + m)(1 + f_s)ECS(x, z). \]

The final factor to be applied is the extra claim inflation factor. The combined effect on expected claim costs is as follows:

\[
\begin{align*}
ECS(x + k, z + k) &= [(1 + b)(1 + m)(1 + f_s)]^kECS(x, z) \\
ECD(x + k, z + k) &= [(1 + b)(1 + m)(1 + f_d)]^kECD(x, z) \\
ECW(x + k, z + k) &= [(1 + b)(1 + m)(1 + f_w)]^kECW(x, z).
\end{align*}
\]

Note that we expect \( f_w \leq f_s \leq f_d \).

The actuarial present value of the future health claims on 1/1/\( z \) for individual age \( x \) on 1/1/\( z \) is denoted by \( AHC(x, z) \) where

\[
\begin{align*}
AHC(x, z) &= \sum_{k=0}^{\infty} \gamma^{(k+1)} p^{(\tau)}_{x+k} ECS(x + k, z + k) \\
&\quad + \sum_{k=0}^{\infty} \gamma^{(k+1)} p^{(\tau)}_{x+k} q^{(d)}_{x+k} ECD(x + k, z + k) \\
&\quad + \sum_{k=0}^{\infty} \gamma^{(k+1)} p^{(\tau)}_{x+k} q^{(w)}_{x+k} ECW(x + k, z + k).
\end{align*}
\]

Notice that equation (6) includes costs incurred after age 70.\(^5\) Using equations (3), (4), and (5) the expression for \( AHC(x, z) \) can be rewritten

\(^5\)Recall that the Medishield program ceases coverage at age 70 while the proposed Medishield program provides coverage beyond age 70.
as follows:

\[
AHC(x, z) = ECS(x, z) \sum_{k=0}^{\infty} (1 + i_1)^{-(k+1)} k_{x+k} p_x^{(\tau)} + ECD(x, z) \sum_{k=0}^{\infty} (1 + i_2)^{-(k+1)} k_{x+k} p_x^{(\tau)} q_{x+k}^{(d)} + ECW(x, z) \sum_{k=0}^{\infty} (1 + i_3)^{-(k+1)} k_{x+k} p_x^{(\tau)} q_{x+k}^{(w)}
\]

\[
= ECS(x, z) a_x + ECD(x, z) A_x^{(d)} + ECW(x, z) A_x^{(w)}
\]  

(7)

where

\[
a_x = \sum_{k=0}^{\infty} (1 + i_1)^{-(k+1)} k_{x+k} p_x^{(\tau)}
\]

(8)

\[
A_x^{(d)} = \sum_{k=0}^{\infty} (1 + i_2)^{-(k+1)} k_{x+k} p_x^{(\tau)} q_{x+k}^{(d)}
\]

(9)

\[
A_x^{(w)} = \sum_{k=0}^{\infty} (1 + i_3)^{-(k+1)} k_{x+k} p_x^{(\tau)} q_{x+k}^{(w)}
\]

(10)

\[
1 + i_1 = \frac{1 + i}{(1 + b)(1 + m)(1 + f_s)}
\]

(11)

\[
1 + i_2 = \frac{1 + i}{(1 + b)(1 + m)(1 + f_d)}
\]

(12)

\[
1 + i_3 = \frac{1 + i}{(1 + b)(1 + m)(1 + f_w)}
\]

(13)

3.2 Level Premiums in Real Terms

By assuming a constant rate of inflation, \( g \), we can determine the individual net premium (payable to age 65) that is level in real terms. That is, if \( LP(x, z) \) is the net level (in real terms) annual premium payable up to age 65 for health insurance coverage for a newly insured age \( x \) last birthday on 1/1/\( z \), then the premiums increase annually by a factor of \((1 + g)\). This implies that the premium for policy year \((k + 1)\) is \((1 + g)^k LP(x, z)\).
The actuarial present value of the future premiums on 1/1/20 on individual age $x_0$ on 1/1/20 is denoted by $AFP(x_0, z_0)$ where

$$AFP(x_0, z_0) = \sum_{k=0}^{65-x_0-1} \nu^k p^{(r)}(1+g)^k LP(x_0, z_0)$$

$$= LP(x_0, z_0) s \overline{\alpha}_{x_0;65-x_0}$$

(14)

where

$$s \overline{\alpha}_{x;65-x} = \sum_{k=0}^{65-x-1} (1+g)^k \nu^k p^{(r)}$$

$$= \sum_{k=0}^{65-x-1} (1+i_4)^{-k} p^{(r)}$$

$$= \overline{\alpha}_{x;65-x}$$ at $i_4$, (15)

and

$$i_4 = (i-g)/(1+g).$$

(16)

The net premium is then given by

$$LP(x_0, z_0) = [ECS(x_0, z_0)a_{x_0} + ECD(x_0, z_0)A^{(d)}_{x_0} + ECW(x_0, z_0)A^{(w)}_{x_0}] / s \overline{\alpha}_{x_0;65-x_0}. (17)$$

4 Analysis of Gains and Losses

Once expected claim costs have been determined and the premiums have been set, the actual experience of the fund must be evaluated annually to see how it compares with what was expected. Large surpluses or large deficits may signal problems inherent in the pricing or benefit structure. To get a clear picture of the year's experience, we need to develop a formal system for determining reserves, surpluses, and gains.

Let $V(x, z|x_0, z_0)$ be the actuarial reserve on 1/1/z for an insured active worker who is age $x$ last birthday on 1/1/z but was age $x_0$ last birthday on 1/1/z at which time the policy was issued.

$$V(x, z|x_0, z_0) = ECS(x, z)a_x + ECD(x, z)A^{(d)}_{x} + ECW(x, z)A^{(w)}_{x} - LP(x_0, z_0)(1+g)^{(z-z_0)} s \overline{\alpha}_{x;65-x}$$

(18)
for $x \geq x_0$. But as

$$a_x = \frac{p_x^{(r)}}{1 + t_1} (1 + a_{x+1}) \quad \text{and}$$

$$s \hat{a}_{x:65-x} = 1 + \frac{p_x^{(r)}}{1 + t_4} s \hat{a}_{x+1;64-x}$$

it follows that

$$V(x, z|x_0, z_0) = \nu E\{S(x, z)p_x^{(r)} + \nu E\{C(x, z)q_x^{(d)}

+ \nu E\{C(x, z)q_x^{(w)}

+ \nu p_x^{(r)}V(x + 1, z + 1|x_0, z_0)

- \nu p_x^{(r)}V(x + 1, z + 1|x_0, z_0)\} (1 + g)^{z-z_0}. \quad (19)$$

Equation (19) can be rearranged to give

$$V(x + 1, z + 1|x_0, z_0) = (1 + i)(V(x, z|x_0, z_0)

+ (1 + i)LP(x_0, z_0)(1 + g)^{z-z_0})

- E\{S(x, z)p_x^{(r)}

- E\{C(x, z)q_x^{(d)}

- E\{C(x, z)q_x^{(w)}

+ q_x^{(r)}V(x + 1, z + 1|x_0, z_0). \quad (20)$$

Given this recursive relationship between successive years' reserves, we can develop an analysis of gains and losses in a manner similar to Anderson (1990, Chapter 2). To this end we introduce the following notation:

$${\mathcal A}_z = \text{Set of the Medishield insureds alive on 1/1/z}$$

$${\mathcal W}_z = \text{Set of withdrawals from } {\mathcal A}_z \text{ during calendar year } z;$$

$${\mathcal D}_z = \text{Set of deaths from } {\mathcal A}_z \text{ during calendar year } z;$$

$${\mathcal N}_z = \text{Set of new Medishield insureds who joined the program during calendar year } z.$$

In addition, let $j$ denote a member of the set ${\mathcal A}_z$, and we append the subscript $j$ to our previous notation to refer to values that depend on the insured $j$'s policy experience.

We define $TV_z$ to be the total actual Medishield reserve as of 1/1/z, i.e.,

$$TV_z = \sum_{j \in {\mathcal A}_z} V_j(x, z|x_0, z_0) \quad (21)$$
where $V_j(x, z|x_0, z_0)$ is the reserve for insured $j$. Following Anderson, we decompose $TV_{z+1}$ to give

\[
TV_{z+1} = \sum_{j \in A_{z+1}} V_j(x + 1, z + 1|x_0, z_0)
\]

\[
= \sum_{j \in A_z} V_j(x + 1, z + 1|x_0, z_0)
- \sum_{j \in D_z} V_j(x + 1, z + 1|x_0, z_0)
- \sum_{j \in W_z} V_j(x + 1, z + 1|x_0, z_0)
+ \sum_{j \in N_z} V_j(x + 1, z + 1|x_0, z_0). \quad (22)
\]

In order to simplify our mathematical expressions and reduce clutter, we use the following abbreviation:

\[
V_j(z + 1) = V_j(x + 1, z + 1|x_0, z_0). \quad (23)
\]

Substituting equation (20) into equation (22) yields

\[
TV_{z+1} = (1 + i)TV_z + (1 + i)\sum_{j \in A_z} LP(x_0, z_0)(1 + g)^{z - z_0}
- \sum_{j \in A_z} ECS_j(x, z)p_x^{(\tau)}
- \sum_{j \in A_z} ECD(x, z)q_x^{(d)}
- \sum_{j \in A_z} ECW(x, z)q_x^{(w)}
- [\sum_{j \in D_z} V_j(z + 1) - \sum_{j \in A_z} q_x^{(d)}V_j(z + 1)]
- [\sum_{j \in W_z} V_j(z + 1) - \sum_{j \in A_z} q_x^{(w)}V_j(z + 1)]
+ \sum_{j \in N_z} V_j(z + 1). \quad (24)
\]

To develop an expression for the surplus, we must now consider the fund balance at the start of each calendar year. Let $F_z$ be the actual fund balance on 1/1/z.

\[
F_{z+1} = F_z + I_z + TP_z - TC_z \quad (25)
\]
where $I_z$ is the actual interest earned during calendar year $z$; $TP_z$ is the total premiums actually received during calendar year $z$; and $TC_z$ is the total claim costs actually paid during calendar year $z$. The surplus in hand on $1/1/z$, $SUR_z$ is defined to be

$$SUR_z = F_z - TV_z.$$  \hspace{1cm} (26)

Using equations (24) and (26) yields

$$SUR_{z+1} = (1+i)SUR_z +$$
$$+ [I_z - iF_z - ITP_z + ITC_z]$$
$$+ [(TP_z + ITP_z) - (1+i) \sum_{j \in A_z} LP_j(x_0, z_0)(1 + g)^{z-z_0}]$$
$$+ \left[ \sum_{j \in A_z} ECS_j(x, z) p_x^{(\tau)} - (TC_z^{(s)} + ITC_z^{(s)}) \right]$$
$$+ \left[ \sum_{j \in A_z} ECD(x, z) q_x^{(d)} - (TC_z^{(d)} + ITC_z^{(d)}) \right]$$
$$+ \left[ \sum_{j \in A_z} ECW(x, z) q_x^{(w)} - (TC_z^{(w)} + ITC_z^{(w)}) \right]$$
$$+ \left[ \sum_{j \in A_z} V_j(z+1) - \sum_{j \in A_z} q_x^{(d)} V_j(z+1) \right]$$
$$+ \left[ \sum_{j \in A_z} V_j(z+1) - \sum_{j \in A_z} q_x^{(w)} V_j(z+1) \right]$$
$$- \sum_{j \in A_z} V_j(z+1)$$  \hspace{1cm} (27)

where $ITP_z$ is the expected interest earned on $TP_z$; $TC_z^{(s)}$ is the total claims costs generated by those who survive to the end of calendar year $z$; $ITC_z^{(s)}$ is the expected interest earned on $TC_z^{(s)}$; $TC_z^{(d)}$ is the total claims costs generated by those who died during calendar year $z$; $ITC_z^{(d)}$ is the expected interest earned on $TC_z^{(d)}$; $TC_z^{(w)}$ is the total claims costs generated by those who withdrew during calendar year $z$; $ITC_z^{(w)}$ is the expected interest earned on $TC_z^{(w)}$;

$$TC_z = TC_z^{(s)} + TC_z^{(d)} + TC_z^{(w)}$$
$$ITC_z = ITC_z^{(s)} + ITC_z^{(d)} + ITC_z^{(w)}.$$

The actuarial gain for calendar year $z$ is defined to be

$$GAIN_z = SUR_{z+1} - (1+i)SUR_z.$$  \hspace{1cm} (28)

The various components of the gain can be identified from equation (27).
5 Closing Comments

The model we have developed is simply a first step in the process of developing a more comprehensive model of the Medishield program. We need actual Singapore data on mortality and morbidity to test the model.

There are several known limitations, however, inherent in our current model:

- We failed to separate morbidity by sex. Females, in general, have different morbidity patterns than males. They tend to have higher morbidity during their child-bearing years.

- We have simplified the morbidity pattern over calendar year as well as over age. A more appropriate model would use very general rates: $r_{x,z}^{(m)}$ for males and $r_{x,z}^{(f)}$ for females. These rates must be developed from population data.

- Insureds who die during a calendar year are likely to have very high medical expenses during the preceding five years of life. This should be explicitly included in the model. The current model accounts for these high end-of-life expenses through experience losses attributable to deaths.

- Persons who withdraw are likely to reenter if they expect to be ill or are ill. This poses severe anti-selection problems.

Our proposed model is only a first attempt at extending the current Medishield program. Much more work needs to be done before this proposal can be implemented in practice.

References


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