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## A GCD attack resistant CRTHACS for secure group communications

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### Abstract

*In this paper, we propose an improved CRTHACS scheme for secure group communications. The scheme resists several GCD attacks which exist in the original CRTHACS scheme [2] and were recently reported in [1].*

### 1. Introduction

Secure group communication (SGC) with hierarchical access control (HAC) refers to a scenario where a group of members is divided into different privileged subgroups located at different levels and a high-level subgroup can receive and decrypt messages within any of its descendant lower-level subgroups, but the converse is not allowed.

In [2], a Chinese Remainder Theorem based HAC scheme (CRTHACS) for SGC was proposed. The scheme was intended not only to enforce HAC but also to operate without disclosing the hierarchy and the receivers. However, some recent attacks, presented in [1], and based on computing certain *greatest common divisors* (GCDs), have been shown to disclose the hidden hierarchy. In this paper, we propose a solution to defeat these GCD attacks, thus keeping the hierarchy hidden.

In sections 2 and 3, we briefly summarize the CRTHACS scheme from [2] and three kinds of GCD based attacks from [1], respectively. We describe our solution for countering the attacks in section 4.

Throughout this paper,  $E_k(x)$  denotes public key encryption under key  $k$  and  $\{x\}_k$  secret key encryption under key  $k$ .

### 2. CRTHACS for SGC

In CRTHACS there is a Group Controller (GC) and subgroups  $G_i$ , as well as a subgroup controller for each  $G_i$ ,

which will also be denoted by  $G_i$ . The GC has a pair of public and private keys  $(P_{GC}, S_{GC})$ , and so does each subgroup  $G_i$ , denoted by  $(P_i, S_i)$ . The GC maintains the entire hierarchical structure of the group; generates a random set of pairwise relatively prime numbers  $N_0, N_1, N_2, \dots, N_r$ , where  $N_0$  is public, while the remaining  $N_i$  are secret. There are also positive integers  $\mathcal{N}_i$ , one for each subgroup  $G_i$ , defined by equation (2). The GC computes  $\mathcal{N}_i$  and  $COM\_CRT_i$  and sends  $\mathcal{N}_i$ ,  $COM\_CRT_i$  and  $N_i$  to  $G_i$  by means of a secure channel. Let  $\{G_{i_1}, G_{i_2}, \dots, G_{i_k}\}$  be the collection of all ancestral subgroups of  $G_i$ , and consider below the system of congruences (1) and equation (2) which define  $COM\_CRT_i$  and  $\mathcal{N}_i$  respectively.

$$\begin{aligned} COM\_CRT_i &\equiv E_{P_{i_1}}(K_i) \bmod N_{i_1} \\ COM\_CRT_i &\equiv E_{P_{i_2}}(K_i) \bmod N_{i_2} \\ &\vdots \\ COM\_CRT_i &\equiv E_{P_{i_k}}(K_i) \bmod N_{i_k} \end{aligned} \quad (1)$$

$$\mathcal{N}_i = N_{i_1} \cdot N_{i_2} \cdot \dots \cdot N_{i_k} \quad (2)$$

To every subgroup  $G_i$  a six-tuple  $(P_i, S_i, K_i, N_i, COM\_CRT_i, \mathcal{N}_i)$  is assigned, where  $K_i$  is the secret data communication key for  $G_i$ .  $K_i$  is sent securely to the GC by  $G_i$  and except for  $P_i$ , the remaining five elements are kept secret, known only by  $G_i$ . Besides knowing its subgroup's six elements, every participant  $j$  has its own public and private key  $(p_j, s_j)$ .

Whenever a participant  $j$  with identity  $ID_j$  in  $G_i$  sends a message  $M$ , it broadcasts the tuple  $(ID_j, CRT_i, \{M\}_{K_i})$  where  $CRT_i$  is computed as:

$$\begin{aligned} CRT_i &= COM\_CRT_i \bmod \mathcal{N}_i \\ CRT_i &= E_{s_j}(MAC_{K_i}(\{M\}_{K_i})) \bmod N_0 \end{aligned} \quad (3)$$

When a receiver  $m$  receives  $(ID_j, CRT_i, \{M\}_{K_i})$ , it can check whether the message is intended for itself, verify

both the sender and the message, and decrypt the message if the check and verification succeed or discard the message otherwise.

### 3. GCD based attacks

In [1], the authors put forth three possible attacks, all based on computing a number of GCD's, which make possible the disclosure of the hierarchy.

1. Note that  $CRT_i$  is dependent on messages, but  $COM\_CRT_i$  is *not*. In particular, from the first equation of (3) any two instances of  $CRT_i$  will differ by a multiple of  $N_i$ . Therefore from the  $CRT_i$ 's of more than two messages of the same subgroup  $G_i$ , an attacker (outside  $G_i$ ) can derive information about  $N_i$ , as  $CRT_{i_j} - CRT_{i_k} \equiv 0 \pmod{N_i}$ . Hence,  $gcd(CRT_{i_1} - CRT_{i_2}, CRT_{i_1} - CRT_{i_3})$  divides  $N_i$ . If the attacker has several  $CRT_{i_j}$  at his disposal he refines this information about  $N_i$ .

2. The subgroup  $G_i$  may figure out its ancestor  $G_j$  (i.e.,  $N_j$ ) by computing  $gcd(N_i, COM\_CRT_i - E_{P_j}(K_i))$  since  $G_i$  possesses  $N_i$  and  $COM\_CRT_i$  and can compute  $E_{P_j}(K_i)$ .

3. If two subgroup  $G_i$  and  $G_j$  collude, they may find their common ancestor  $G_k$  (i.e.,  $N_k$ ) by computing  $gcd(CRT_i - E_{P_k}(K_i), CRT_j - E_{P_k}(K_j))$ .

### 4. Improved CRTHACS

The solution to the three attacks can be summarized as three modifications to the original CRTHACS scheme:

1. Move the signed MAC out of  $CRT_i$  and send the signed MAC as a separate item. Thus,  $CRT_i$  will be independent of messages. This will beat the attack of the first kind.

2. Remove  $COM\_CRT_i$  and replace  $CRT_i$  with a new  $CRT_i$ . Moreover the new  $CRT_i$  will now be computed by the GC. Instead of sending  $COM\_CRT_i$  and  $N_i$  to  $G_i$ , the GC just sends  $CRT_i$  to  $G_i$ .  $G_i$  uses  $CRT_i$  directly but does not know  $N_i$ . Thus, the second attack is defeated.

3. Replace the public encryption of  $E_{P_{i_k}}(K_i)$ , corresponding to its ancestral subgroup  $G_{i_k}$ , with a secret key encryption  $\{K_i\}_{K_{i_k}}$  in the congruence system (1). As a result,  $G_i$  cannot compute  $\{K_i\}_{K_k}$  related to its ancestral subgroup  $G_k$  because  $G_i$  does not know  $K_k$ . This defeats the third type of attack. In fact, this modification also defeats the second attack since  $E_{P_j}(K_i)$  is not involved in  $COM\_CRT_i$ .

We describe the modifications in detail. The  $COM\_CRT_i$  and  $N_i$  are removed from the scheme and  $CRT_i$  is computed by the GC and sent to  $G_i$  directly. The system of equations (1) is replaced by the system (4) as follows:

$$\begin{aligned} CRT_i &\equiv \{K_i\}_{K_{i_1}} \pmod{N_{i_1}} \\ CRT_i &\equiv \{K_i\}_{K_{i_2}} \pmod{N_{i_2}} \\ &\vdots \\ CRT_i &\equiv \{K_i\}_{K_{i_k}} \pmod{N_{i_k}} \\ CRT_i &\equiv \{K_i\}_{K_i} \pmod{N_0} \end{aligned} \quad (4)$$

The  $N_0, N_1, \dots, N_r$  are as in the original CRTHACS. Every subgroup controller  $G_i$  will no longer have six but five elements:  $(P_i, S_i, K_i, N_i, CRT_i)$  and every participant  $k \in G_i$  will also have five elements  $(p_k, s_k, K_i, N_i, CRT_i)$ . Initially, the GC sends  $N_i$  and  $CRT_i$  to the subgroup controller  $G_i$  securely, and then  $G_i$  multicasts the two values to all participants in its subgroup.

Whenever participant  $k \in G_i$ , with identity  $ID_k$ , sends a message  $M$ , it computes and sends  $(ID_k, CRT_i, Signed\_MAC, \{M\}_{K_i})$ , where  $Signed\_MAC = E_{s_k}(MAC_{K_i}(\{M\}_{K_i}))$ . As indicated above, the  $Signed\_MAC$  is sent as a separate item.

Assume that sender  $k \in G_i$ . When a receiver  $m$  receives  $(ID_k, CRT_i, Signed\_MAC, \{M\}_{K_i})$ , it proceeds as follows: (1) If  $m \in G_i$ , then  $m$  has the same key  $K_i$  as sender  $k$ . If  $m \in G_j$  where  $G_j$  is an ancestral subgroup of  $G_i$ ,  $m$  computes  $t = CRT_i \pmod{N_j}$  (i.e.,  $\{K_i\}_{K_j}$ ) and decrypts  $t$  to get  $K_i$ , (2)  $m$  computes  $x = \{K_i\}_{K_i}$  and  $y = CRT_i \pmod{N_0}$ , (3)  $m$  compares  $x$  and  $y$ ; if  $x \neq y$ , the verification of the key fails (there are two possibilities: the  $CRT_i$  was modified during transmission or the receiver is not in the sender's subgroup or its ancestral subgroups). The receiver discards the message. Otherwise (i.e.,  $x = y$ ), the key is correct and the message is intended for  $m$ , (4) Decrypts the  $Signed\_MAC$  using  $k$ 's public key to get  $MAC_{K_i}(\{M\}_{K_i}) = E_{p_k}^{-1}(Signed\_MAC)$ , where  $E^{-1}$  stands for the decryption algorithm corresponding to  $E$ , (5) Computes  $MAC_{K_i}(\{M\}_{K_i})$  using  $K_i$  (which already passed the verification in (3)), (6) Compares the above two  $MAC$ s. If the two  $MAC$ s are equal, then both the sender and the message are authenticated. The receiver decrypts the message using  $K_i$ . Otherwise, the message was modified during transmission, and the receiver discards it.

It is worth pointing out that the improved CRTHACS has an extra advantage over the original one, viz., better efficiency because computing  $CRT_i$  and  $K_i$  will not involve public key encryption/decryption operations.

### 5. Conclusion

In this paper, we have proposed a solution which defeats a number of recent attacks, satisfies all original goals, and provides better performance.

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