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Physics, Chapter 9: Hydrodynamics (Fluids in Motion)

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Hydrodynamics (Fluids in Motion)

9-1 Steady Flow of a Liquid

When a liquid flows through a pipe in such a way that it completely fills the pipe, and as much liquid enters one end of the pipe as leaves the other end of the pipe in the same time, then *the liquid is said to flow at a steady rate.* At any point of the pipe, the flow of the liquid does not change with time. The path of any particle of liquid as it moves through the pipe is

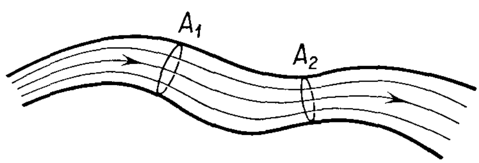

Fig. 9-1 Streamlines of a liquid flowing through a pipe at a steady rate.

called a *streamline.* We can map the flow of liquid through the pipe by drawing a series of streamlines following the paths of the particles of liquid, as shown in Figure 9-1. The instantaneous velocity of a liquid particle is always tangent to the streamline. The rate of flow may be represented by the density of streamlines, or the number of streamlines passing through a surface of unit area perpendicular to the direction of flow. Thus the streamlines will be close together where the liquid is moving rapidly and farther apart in regions of the pipe where the liquid is moving slowly. Similar conventions were used to represent the direction and magnitude of the gravitational field intensity.

Since the liquid is incompressible and there are no places in the pipe where the liquid can be stored, the volume of liquid which flows through any plane perpendicular to the streamlines in any interval of time must be the same everywhere in the pipe. Consider two typical planes whose intersections with the pipe have areas A_1 and A_2 perpendicular to the streamlines. The volume of liquid Q passing through area A_1 in unit time is

$$Q = A_1 v_1, \tag{9-1}$$

where v_1 is the velocity of the liquid at this point. Similarly, the volume of liquid passing through A_2 in unit time is

$$Q = A_2 v_2.$$

Since these two quantities must be equal for steady flow, we have

$$A_1 v_1 = A_2 v_2, \tag{9-2}$$

or

$$\frac{v_1}{v_2} = \frac{A_2}{A_1}.$$

Thus the velocity of the liquid at any point in the pipe is inversely proportional to the cross-sectional area of the pipe. The liquid will be moving slowly where the area is large and will be moving rapidly where the area is small.

Illustrative Example. Water flows out of a horizontal pipe at the steady rate of 2 ft^3/min. Determine the velocity of the water at a point where the diameter of the pipe is 1 in.

The area A of the 1-in. portion of the pipe is

$$A = \frac{\pi \times 1}{4 \times 144}\ \mathrm{ft}^2 = 0.0055\ \mathrm{ft}^2.$$

When we substitute the values for Q and A in Equation (9-1),

we obtain

$$\frac{2}{60}\frac{\mathrm{ft}^3}{\mathrm{sec}} = 0.0055\ \mathrm{ft}^2 \times v,$$

from which

$$v = 6.10\,\frac{\mathrm{ft}}{\mathrm{sec}}.$$

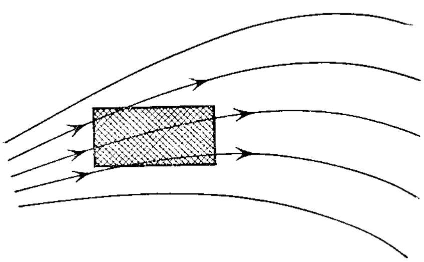

Fig. 9-2 The number of streamlines entering a volume element is equal to the number leaving the volume element when there is steady flow.

In the steady or streamline flow of a liquid, the total quantity of liquid flowing into any imaginary volume element of the pipe must be equal to the quantity of liquid leaving that volume element. If we represent the flow by streamlines, this implies that the streamlines are continuous and do not pile up anywhere within the liquid. The same number of streamlines enter a volume element as the number which leave it, as shown in Figure 9-2. Another characteristic of streamline flow is that it is layerlike, or *lamellar*. There is no circulation of the liquid about any point in the pipe. We might imagine a small paddle wheel, as shown in Figure 9-3, placed in the pipe. When the flow is lamellar, no rotation of the paddle wheel is produced by the flow of liquid.

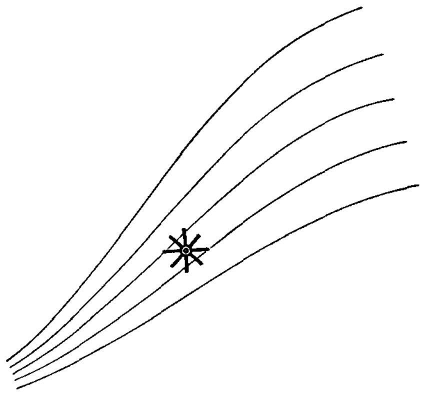

Fig. 9-3 A small paddle wheel placed in a flowing liquid will not rotate when the liquid is in steady or lamellar flow.

9-2 Bernoulli's Theorem

The fundamental theorem regarding the motion of fluids is due to Daniel Bernoulli (1700–1782), a Swiss physicist and mathematician. Bernoulli's theorem is essentially a formulation of the mechanical concept that the work done on a body is equal to the change in its mechanical energy, in the case that mechanical energy is conserved; that is, where there is no loss of mechanical energy due to friction.

Let us consider the motion of an incompressible fluid of density ρ along an imaginary tube bounded by streamlines, as shown in Figure 9-4. We shall call such a tube a *streamtube*. Since each streamline represents the direction of motion of a particle of liquid in steady flow, no particle of liquid may cross a streamtube. At the left-hand end of the tube, the liquid has a velocity v_1, the tube has cross-sectional area A_1, the pressure is P_1, and the tube is at a height h_1 above some reference level. At the right-hand end of the tube, the velocity is v_2, the cross-sectional area is A_2, the pressure is P_2, and the height is h_2. When a small quantity of fluid of volume V is moved into the tube through the action of the external fluid, an equal volume of fluid must emerge from the streamtube against the force exerted by the pressure P_2 of the fluid outside the tube at the right-hand end of the streamtube. Let us imagine that the flow of fluid in the streamtube takes place as the result of the displacement s_1 of a piston of area A_1 which just fits the streamtube at its left-hand end such that the volume swept out by the motion of the piston is $A_1s_1 = V$, and of a corresponding displacement s_2 of a piston of area A_2 which just fits the streamtube at its right-hand end such that the volume swept out by the second piston is $A_2s_2 = V$; that is, a quantity of fluid of volume V has just passed through the streamtube. The piston at position 1 has done work *on* the fluid equal to the product of the force exerted by the displacement of that force, hence the work done on

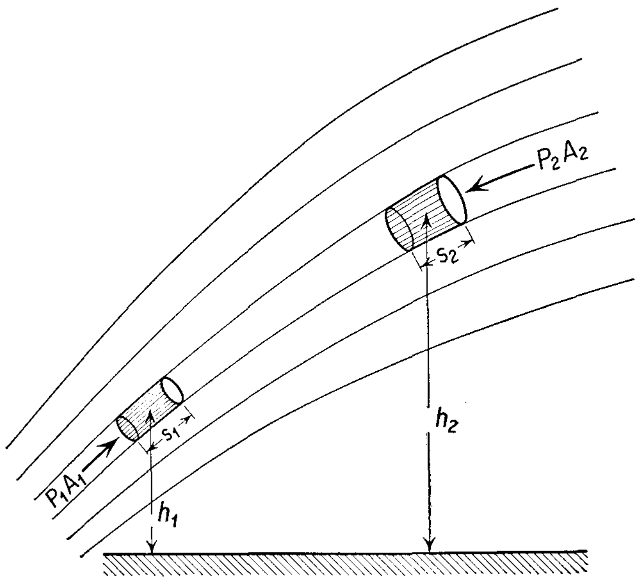

Fig. 9-4

the fluid is $P_1A_1s_1$, while the fluid has pushed back the piston at position 2 so that the fluid has done work on the second piston of amount $P_2A_2s_2$. The net work done on the fluid is therefore equal to

$$P_1A_1s_1 - P_2A_2s_2.$$

Since we have assumed the fluid to be incompressible,

we have $$A_1s_1 = A_2s_2 = V.$$

From the above equations, we find that the net work done on the fluid within the streamtube is

$$P_1V - P_2V.$$

If there was no loss of energy of the fluid due to frictional forces, this work done on the fluid in the streamtube must have resulted in a change of mechanical energy of the liquid which flowed into the tube at position 1 and out of the tube at position 2. The sum of the potential and kinetic energies of the liquid flowing into the tube at position 1 is $\rho Vgh_1 + \frac{1}{2}\rho Vv_1^2$, while the sum of these energies of the liquid flowing out of the tube at position 2 is $\rho Vgh_2 + \frac{1}{2}\rho Vv_2^2$. Equating the work done on the fluid to the difference in its energy, we find

$$P_1V - P_2V = \rho Vgh_2 + \tfrac{1}{2}\rho Vv_2^2 - (\rho Vgh_1 + \tfrac{1}{2}\rho Vv_1^2). \qquad \text{(9-3a)}$$

Dividing the equation through by V and transposing all quantities with subscript 1 to the left-hand side and all quantities with subscript 2 to the right-hand side of the equation, we find

$$P_1 + \rho gh_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \tfrac{1}{2}\rho v_2^2. \qquad \text{(9-3b)}$$

This equation expresses *Bernoulli's theorem,* which states that *at any two points along a streamline the sum of the pressure, the potential energy of a unit volume of fluid, and the kinetic energy of a unit volume of fluid has the same value.* As indicated in the derivation, Bernoulli's theorem holds rigorously only for frictionless, incompressible, streamline flow. Bernoulli's theorem is a statement of the principle of conservation of energy expressed in a form suited to the description of fluids in steady frictionless flow.

Although Bernoulli's theorem holds rigorously only for an incompressible fluid, experience indicates that it is valid for air in streamline flow at speeds up to about half the speed of sound. (The speed of sound is about 740 mi/hr.) Actual fluids such as water and air have internal fluid friction, or *viscosity,* so that, to be strictly true, the equal sign of Equation (9-3a) should be replaced by a greater than or equal sign ($\geqq$), meaning that the work done on the fluid in the streamtube is greater than, or at least equal to, the increase in mechanical energy. Some of the work done will be converted to heat energy through the action of internal friction. In examples and problems in the application of Bernoulli's theorem, we shall neglect the effects of viscosity.

9-3 Torricelli's Theorem

Let us apply Bernoulli's theorem to the flow of a liquid out of an orifice C at the base of a tank, as shown in Figure 9-5. We shall choose as the reference level for the measurement of potential energy the altitude of the emergent stream at D, where the cross-sectional area of the stream is A, and the velocity of the stream is v. At both positions B at the top of the tank and D at the emergent stream, the liquid is in free contact with the air and is therefore at atmospheric pressure P_{atm}. If the tank is sufficiently large so that the flow does not appreciably change the level of liquid in the tank, we may assume that the liquid at B is very nearly at rest; that is, the velocity of the liquid at B is zero. Applying Bernoulli's theorem in the form of

Equation (9-3b) to the flow of the fluid,

$$P_{\text{atm}} + \rho gh + 0 = P_{\text{atm}} + 0 + \tfrac{1}{2}\rho v^2,$$

from which

$$v^2 = 2gh,$$

or

$$v = \sqrt{2gh}. \tag{9-4}$$

If the orifice at the base of the tank has sharp edges, it may be observed that the stream narrows as it emerges from the tank at C. The portion of the stream with parallel sides is called the *vena contracta.* The narrowing of

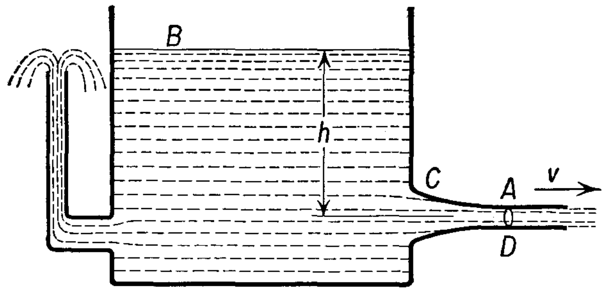

Fig. 9-5

the stream is due to the fact that the liquid is being accelerated and has not yet reached its final velocity. In accordance with Equation (9-2), the cross-sectional area of the stream must be larger where the velocity of flow is smaller.

If we wish to calculate the quantity of liquid flowing out of the tank per second, we may apply Equation (9-1) to the result obtained in Equation (9-4), obtaining

$$Q = Av,$$

so that

$$Q = A\sqrt{2gh}.$$

The speed with which a liquid emerges from a tank is the same as it would have had if it had been dropped over the top of the tank, for, in dropping through a height h, a unit volume of liquid would have lost potential energy ρgh and gained kinetic energy in equal amount. Thus

$$\rho gh = \tfrac{1}{2}\rho v^2,$$

so that

$$v = \sqrt{2gh}.$$

9-4 Venturi Tube

Let us consider a horizontal tube containing a constriction in which a fluid is flowing, as shown in Figure 9-6. We will refer to the wide section of the tube by the subscript 1 and the narrow portion of the tube by the subscript 2. Applying Equation (9-3b) to the fluid flow, we find

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2.$$

Since the tube is horizontal, $h_1 = h_2$, so that

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2. \tag{9-5}$$

From Equation (9-1) the velocity of the liquid must be greater in the constricted portion of the tube, so that the pressure in the constriction must be lower than the pressure in the wider portion of the tube.

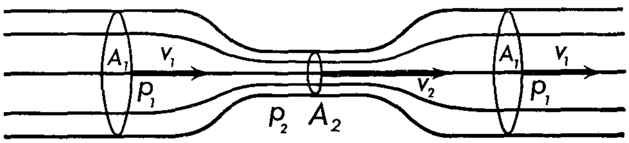

Fig. 9-6 The Venturi tube.

A Venturi tube finds application as a *flowmeter*. Referring to Equations (9-5) and (9-1), we have

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2,$$

and

$$A_1 v_1 = A_2 v_2.$$

If the quantity $P_1 - P_2$ is measured by a manometer or other pressure gauge, the two equations may be solved for v_1 and v_2, and this knowledge of the speed of flow determined by a pressure measurement, together with the known areas of the tube and the constriction, suffices to determine the rate of flow of fluid through the tube.

Although we have derived the relationship between pressure and velocity for a Venturi tube, the same relationship is true for any case of streamline flow. Thus, as air is entrained by a passing railroad train, the air close to the train is moving at considerably higher speed than the air some distance away. The pressure in the immediate vicinity of the train is low, and passengers on the station platform are cautioned to stand back from the edge of the platform when a rapidly moving train passes, lest the pressure differential created by the motion of the train force the passenger into the side of the train. Two passing ships or stunting airplanes which get too close together may be forced to collide through the low-pressure area between them which is created by their own motion.

There are many applications of the Bernoulli theorem utilizing variations of the Venturi tube. For example, the aspirator, sketched in Figure 9-7, is used as a vacuum pump. Water from a faucet flows through the horizontal tube and comes out of the constriction with a high velocity, thus reducing the pressure P in this region below atmospheric pressure. The tube T may be connected to a chamber to be evacuated, and the pressure in it may be reduced to about the vapor pressure of water, usually about 2 or 3 cm of mercury. The same type of design is used for a chemical filter pump. A jet pump used in raising water from a well drives water through a

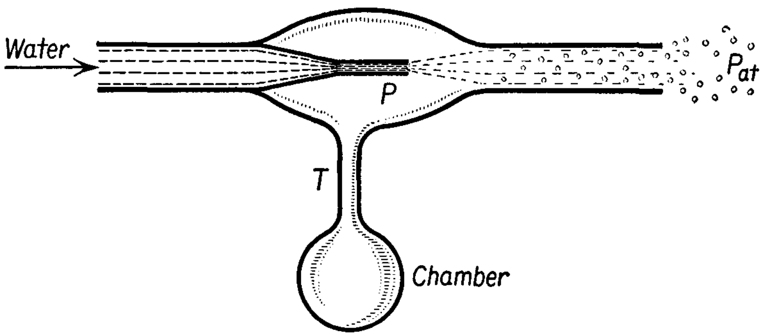

Fig. 9-7 An aspirator.

restricted orifice similar to that in Figure 9-7, and the pressure of the outside air forces the well water into the tube T and into the moving water stream. A similar design of tube, except that air is blown through the horizontal tube and constriction of Figure 9-7, is used in pneumatic conveyers of sand, grain, and other granular materials.

Illustrative Example. Water is flowing through a horizontal Venturi tube at the rate of 100 ft^3/min. The pressure in the wide portion of the tube is 15 lb/in.^{2}, and its diameter is 6 in. Determine the pressure in the narrow portion of the tube, called the throat, whose diameter is 3 in.

The area of the tube A_1 is

$$A_1 = \frac{\pi}{16}\,\text{ft}^2,$$

while the area of the throat A_2 is

$$A_2 = \frac{\pi}{64}\,\text{ft}^2.$$

From Equation (9-2), we have

$$Q = A_1 v_1 = A_2 v_2.$$

Substituting numerical values, we get

$$\frac{100}{60}\,\frac{\text{ft}^3}{\text{sec}} = \frac{\pi}{16}\,\text{ft}^2 \times v_1 = \frac{\pi}{64}\,\text{ft}^2 \times v_2,$$

from which
$$v_1 = 8.5\,\frac{\text{ft}}{\text{sec}},$$

and $$v_2 = 34\,\frac{\text{ft}}{\text{sec}}\,.$$

In applying Equation (9-5), as in every other equation, it is essential that a consistent set of units be used. Now

$$P_1 = 15\,\frac{\text{lb}}{\text{in.}^2} = 15 \times 144\,\frac{\text{lb}}{\text{ft}^2} = 2{,}160\,\frac{\text{lb}}{\text{ft}^2},$$

and water, which weighs 62.4 lb/ft^3, has a density of

$$\rho = \frac{62.4}{32}\,\frac{\text{slugs}}{\text{ft}^3}\,.$$

Since the tube is horizontal, $h_1 = h_2$, and we may write Equation (9-5) as

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2,$$

and, substituting numerical values, we get

$$P_2 = 2{,}160\,\frac{\text{lb}}{\text{ft}^2} + \frac{1}{2}\,\frac{62.4}{32} \times (8.5)^2\,\frac{\text{ft lb}}{\text{ft}^3} - \frac{1}{2}\,\frac{62.4}{32} \times (34)^2\,\frac{\text{ft lb}}{\text{ft}^3}$$

$$= 1{,}100\,\frac{\text{lb}}{\text{ft}^2},$$

or $$P_2 = \frac{1{,}100}{144}\,\frac{\text{lb}}{\text{in.}^2} = 7.64\,\frac{\text{lb}}{\text{in.}^2}\,.$$

9-5 Lift of an Airfoil

The lift of an airplane wing or the action of a propeller of an airplane or a ship may be analyzed in terms of Bernoulli's theorem. The cross section of such an airfoil generally has a rather blunt nose and thin trailing edge to permit the fluid medium to flow around it in streamline flow. Ideally, an element of fluid volume parted by the airfoil will be reconstructed when the fluid passes the trailing edge of the airfoil, as shown in Figure 9-8. This

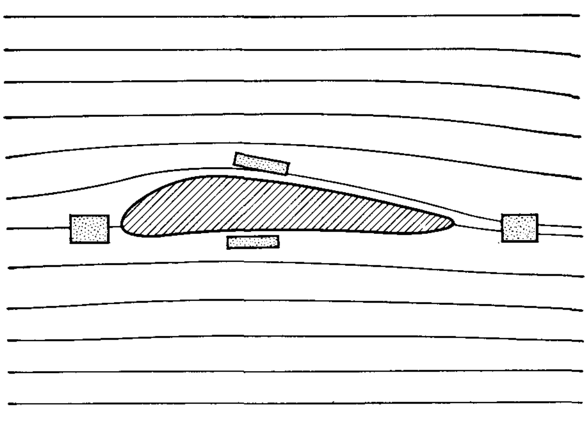

Fig. 9-8

implies that the fluid on top of the airfoil is moving more rapidly than the fluid at the bottom of the airfoil, so that the pressure at the top surface is lower than the pressure at the bottom surface, creating an upward force on the airfoil. It is for this reason that the greatest lift is often associated with an airfoil of rather marked asymmetry. A perfectly symmetrical wing had no lift unless it is turned at an angle with respect to the flow of fluid, termed the *angle of attack*, so that the fluid moving over the top surface travels a greater distance than the fluid moving over the bottom surface of the airfoil. The motion of air past a symmetrical airfoil for several angles of attack is shown in Figure 9-9.

Surfaces not normally considered to be airfoils may also experience lift. For example, the lift on a bridge in a high wind may be several times the drag force tending to blow the bridge off its piers.

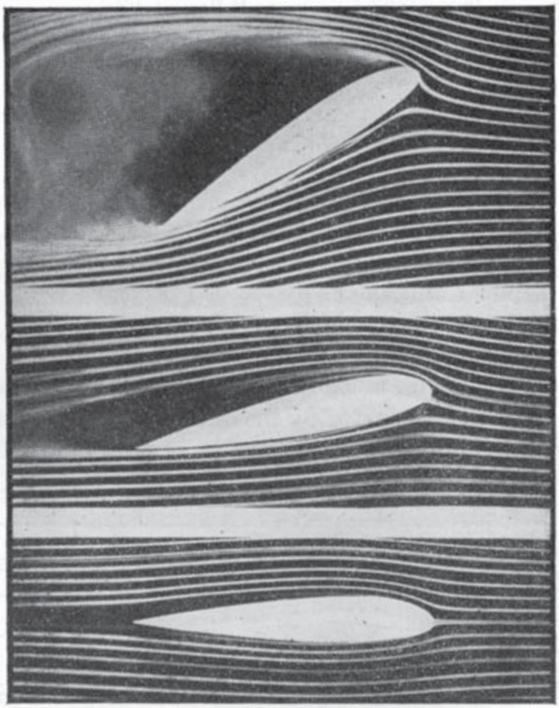

Fig. 9-9 Smoke is used in the NACA smoke tunnel at Langley Field, Virginia, to make the flow of air visible, as illustrated in these photographs. Note the smoothness of the air flow in the lowest picture. When the angle of attack has been increased to 10°, the air flow begins to separate from the upper surface of the airfoil (center view); and when the angle is increased to 30°, the flow separates completely from the upper surface. Turbulence behind the trailing edge of the airfoil may be observed in this picture. (Reproduced from the *Journal of Applied Physics*, August, 1943, with permission of the National Advisory Committee for Aeronautics.)

9-6 The Siphon

A siphon is a bent tube used for transferring a liquid from one vessel to another one at a lower level. If the bent tube shown in Figure 9-10 is filled with liquid so that streamline flow can take place, liquid will emerge from the orifice with a speed given by Torricelli's theorem. By applying Bernoulli's theorem, we can determine the maximum height h of the bend

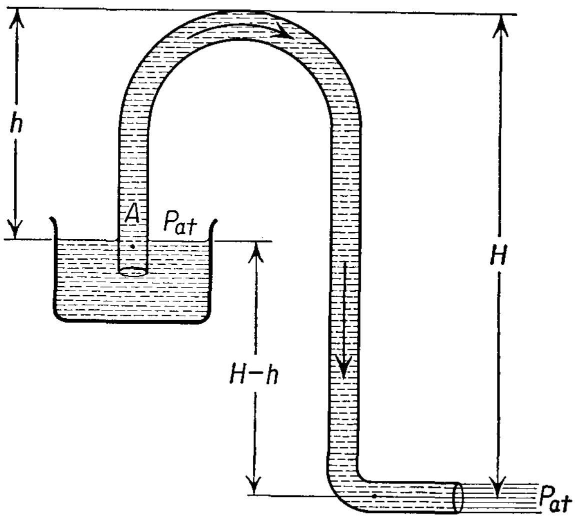

Fig. 9-10 Operation of a siphon.

in the tube above the level of the free liquid in the higher vessel, for a liquid cannot support a negative pressure, that is, a state of internal tension, except for extremely short time intervals. If the pressure of the liquid at its highest point is zero (or more accurately, if it is below the vapor pressure of the liquid), the liquid will pull apart, and bubbles will form, destroying the continuity of the liquid flow, and thus interrupting the siphon. The limiting height of the siphon may be found by setting P_2 equal to zero in Bernoulli's equation. Thus

$$P_{\text{atm}} = 0 + \rho g h + \tfrac{1}{2}\rho v^2,$$

from which

$$h = \frac{P_{\text{atm}} - \frac{1}{2}\rho v^2}{g}.$$

Thus the faster the fluid flows in the siphon, the lower the bend in the tube must be to maintain siphon flow.

9-7 Fluid Friction; Viscosity

When a fluid, either a liquid or a gas, is set in motion, different parts of the fluid move with different velocities. For example, if a jar of water is tilted so that the water starts flowing out, the top layer of the water moves over the lower part of the water. Just as there is friction when one surface of a solid slides over another, so there is friction when one layer of a fluid slides over another. This friction in fluids is called *viscosity*. When a fluid flows through a cylindrical pipe, the part of the fluid in contact with the pipe adheres to it and remains at rest. We may think of the rest of the fluid as divided into concentric cylindrical layers, the velocity of each succeeding inner layer increasing as we go to the center. A difference in pressure between the two ends of the pipe is needed to maintain a steady flow through it and oppose the force due to the viscosity of the fluid.

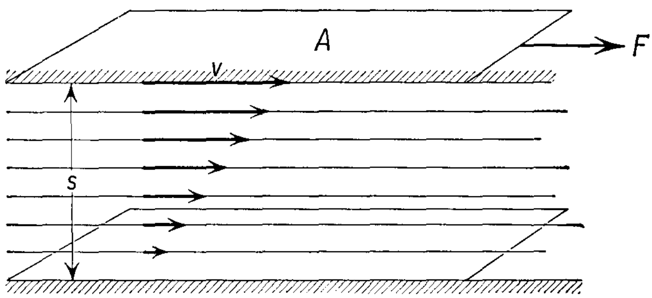

Fig. 9-11

The resistance experienced by a solid moving through a fluid is due essentially to the viscosity of the fluid. A certain amount of fluid adheres to the surface of the solid and moves with it, and this layer drags along an adjacent layer and so on, until, at sufficiently large distance from the solid, the fluid is at rest. Some of the momentum of the solid has been given up to setting the fluid in motion, and if no external force is applied, the solid will come to rest. An additional cause of the resistance experienced by objects moving through fluids is the turbulence set up in the fluid. When turbulence occurs, the fluid flow is no longer streamline and cannot be considered as taking place in layers, for the layers are sometimes broken up into eddies in which rotational motion takes place, and sometimes into waves. Such eddies are seen in the wake of the oars of a rowboat and occur in the air about the wingtips or the propeller tips of an airplane. If the paddle wheel of Figure 9-3 is placed at the center of an eddy, the paddle wheel will turn.

To formalize the concept of viscosity, we imagine a layer of fluid between two flat plates, one of which is at rest and the other moving at velocity v, as shown in Figure 9-11. The fluid in contact with the stationary plate is at rest, while the fluid in contact with the moving plate is moving with velocity v, and the fluid in the space between the two plates is moving with intermediate velocities. To maintain the moving plate at a constant speed, it is found experimentally that a force F is required which is directly proportional to the velocity, inversely proportional to the separation of the two plates s, and directly proportional to the area of the moving plate A. Calling the constant of proportionality η (eta), the *coefficient of viscosity,* we find

$$F = \eta \frac{Av}{s} \cdot \qquad (9\text{-}6)$$

From Equation (9-6) the unit of coefficient of viscosity is force times distance divided by area times velocity. By replacing the unit of force by its equivalent from Newton's second law, we see that the unit of viscosity is that of mass divided by the product of the unit of length by the unit of time. Thus in mks units, the coefficient of viscosity is expressed in kilograms per meter seconds, while in British gravitational units it is expressed in slugs per foot seconds. In the cgs system of units a coefficient of viscosity of 1 gm/cm-sec is called a *poise.*

The coefficient of viscosity varies considerably with temperature. For liquids the viscosity generally decreases with increasing temperature, while the viscosity of gases increases with increasing temperature. The viscosity of gases does not depend upon the pressure.

To achieve an experimental condition in which the coefficient of viscosity can be measured, the relative motion of two plane surfaces is approximated by two concentric cylinders, one of which is held fixed and the other is caused to rotate by the application of a constant torque. From a measurement of the angular speed, the dimensions of the apparatus, and the applied torque, the viscosity of the liquid between the two cylinders can be determined.

9-8 Reynolds' Number

To discuss problems in fluid flow, it is convenient to group the variables determining the motion into dimensionless parameters. One such parameter, called *Reynolds' number,* is especially useful in distinguishing between streamline or lamellar flow and turbulent flow. Reynolds' number R is given by the formula

$$R = \frac{\rho v d}{\eta},$$

where ρ is the density of the fluid, v is its speed relative to a pipe or to some obstacle to the flow, d is some dimension characteristic of the flow, and η is the coefficient of viscosity. In the flow of a fluid through a pipe, d

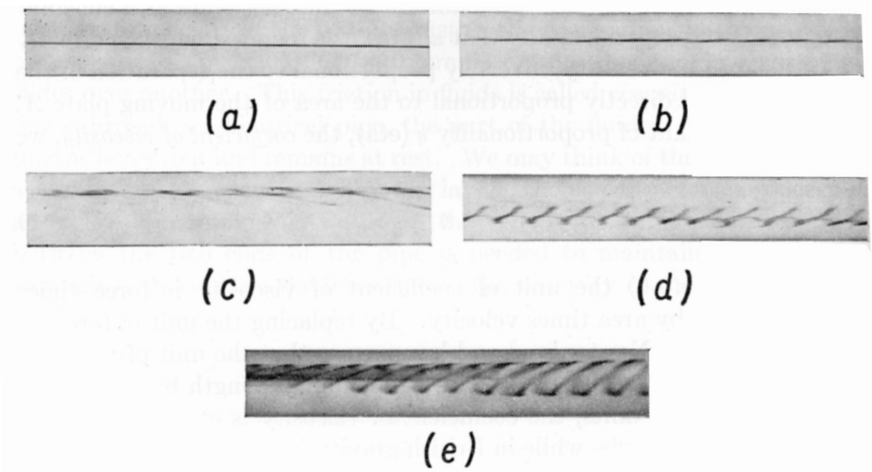

Fig. 9-12 Transition from lamellar to turbulent flow for water in a glass tube, with increasing value of Reynolds' number. Velocity lowest for (a), which shows lamellar flow, and highest for (e), which shows turbulent flow. (Reproduced by permission from *Fluid Mechanics*, 3rd ed., by R. C. Binder, p. 108. Copyright, 1943, 1949, 1955, by Prentice-Hall, Inc., Englewood Cliffs, N.J.)

represents the diameter of the pipe, while in the flow of a fluid past a cylinder, d is the diameter of the cylinder.

Experiments have shown that the stable flow of a fluid through a pipe is normally streamline or lamellar for values of R less than 2,000, whatever the density or viscosity of the fluid. The flow is generally turbulent if R is greater than 3,000, and the type of flow obtained in the transition region between these two values of R depends on such factors as surface roughness, the way the flow was started, and so on. The flow of water through a glass tube at increasing speeds, that is, at increasing values of R, is shown in Figure 9-12. The flow has been made visible by immersing a source of dye in the water stream.

The resisting force opposing the motion of an immersed body, such as an airfoil, a sphere, a cylinder, or a bridge, is also related to Reynolds' number. For example, eddies break off alternately on either side of a cylinder in a periodic manner, as shown in Figure 9-13, when R is greater than 20. Below this value of R, the flow is lamellar. The release of eddies from alternate sides of a cylinder exerts a periodic force on the cylinder. This is responsible for the generation of the vibrations of chimneys in a high wind, the vibrations of the periscopes of submarines, the vibrations of towlines, the singing of telephone wires, and so on. Similar phenomena are responsible for the flutter of the wings and propellers of an airplane, and

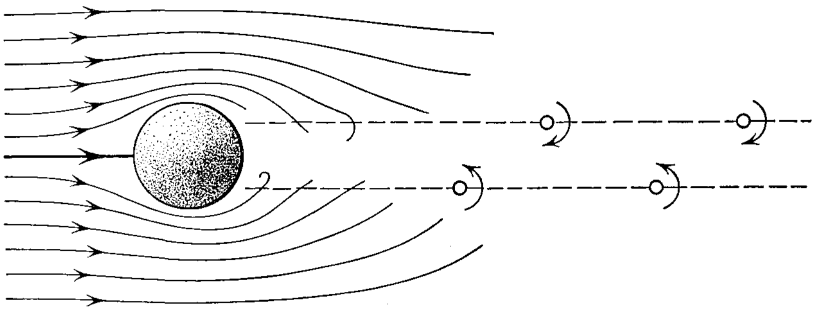

Fig. 9-13

for the failure of the Tacoma Narrows bridge in November, 1940. When the frequency of eddy formation approaches the natural frequency of vibration of the structure, excessively large vibrations may be excited, with resulting destructive failure.

Problems

9-1. Water flows through a horizontal pipe of varying cross section at the rate of 4 ft^3/min. Determine the velocity of the water at a point where the diameter of the pipe is (a) 1.5 in. and (b) 2 in.

9-2. Oil flows through a 12-in.-diameter pipeline with a speed of 3 mi/hr. How many gallons of oil are delivered each day by this pipeline? (One gallon = 231 $in.^3$.)

9-3. At a place in a pipeline where the diameter is 6 in., the speed of a steady stream of water is 12 ft/sec. (a) What will be the speed of the water in that portion of the pipeline where the diameter is 4 in.? (b) At what rate, in cubic feet per minute, is water being delivered by this pipeline?

9-4. A water storage tank is filled to a height of 16 ft. (a) With what speed will water come out of a valve at the bottom of the tank if friction is negligible? (b) To what height will this water rise if the opening is directed upward? (c) What quantity of water will emerge from the tank in each second? The area of the valve is $\frac{1}{3}$ $in.^2$.

9-5. A cylindrical water storage tank of diameter 10 ft is filled to a height of 16 ft. At the bottom of the tank, there is an opening 1 in. in diameter. How long will it take for the tank to drain itself empty? [Hint: Use the methods of the calculus].

9-6. Water flows steadily through a Venturi tube at the rate of 40 ft^3/min. At a place where the diameter of the tube is 4 in., the gauge pressure is 15 lb/$in.^2$. Determine the gauge pressure in the throat of the tube where the diameter is 2 in.

9-7. Gauges attached to a vertical tube in which water is flowing steadily show an absolute pressure of 25 $lb/in.^2$ where the diameter of the tube is 4 in. and a pressure of 15 $lb/in.^2$ where the diameter of the tube is 3 in., at a point 1 ft below the first gauge. Determine (a) the velocity of the liquid in the wider portion of the tube and (b) the quantity of water per second flowing through the tube.

9-8. Oil of density 0.9 gm/cm^3 flows through a horizontal tube 3 cm in diameter at a pressure of 1.5×10^6 $dynes/cm^2$. At one portion the tube narrows down to 2 cm in diameter, and the pressure drops to 10^6 $dynes/cm^2$. (a) Determine the velocity of the oil in the wider portion of the tube. (b) Determine the rate at which oil flows through this tube.

9-9. In a wind-tunnel experiment the pressure on the upper surface of a wing was 13.05 $lb/in.^2$, while the pressure on the lower surface was 13.15 $lb/in.^2$. Determine the lifting force of a wing of this design if it has a spread of 40 ft and a width of 9 ft.

9-10. A monoplane weighing 14,000 lb has a wing area of 600 ft^2. (a) What difference in pressure on the two sides of the wing surface is required to maintain this plane in level flight? (b) If the plane is flying at a level of 13,000 ft and the pressure on the lower wing surface is 9.0 $lb/in.^2$, determine the pressure on the upper wing surface.

9-11. The lower end of a siphon is 8 ft below the level of the water surface in the tank. (a) Determine the speed with which the water flows out of the open end of the siphon. (b) If the cross-sectional area of the siphon tube is 3.0 $in.^2$, determine the rate at which water is siphoned out. (c) If the bend in the siphon is 3 ft above the surface of the water in the tank, determine the pressure of the water in the bend of the siphon.

9-12. The pressure in the cylinder of a water pump is 45 $lb/in.^2$. Determine the height to which water may be lifted by this pump.

9-13. The level of water in a tank is 15 m above the ground. Water flows out of this tank in a horizontal direction through a valve located 5 m below the surface. Determine (a) the velocity with which the water escapes, neglecting friction, (b) the distance from the valve where the water strikes the ground, and (c) the velocity of the water when it reaches the ground.

9-14. Water falls from a height of 60 ft and drives a water turbine. If the rate of flow of water is 480 ft^3/min, determine the maximum power that can be developed by this turbine.

9-15. In a viscosimeter constructed of two concentric cylinders with an annular space between them, the outer cylinder is fixed, and the inner cylinder is made to rotate by the application of a torque. The length of the cylinders is 5 in., their mean radius is 3 in., and the space between them is 0.05 in. It is found that the inner cylinder will rotate with an angular speed of 50 rpm when a torque of 0.10 ft lb is applied. Using British gravitational units, determine the viscosity of the oil in the annular space between the two cylinders.

9-16. Find the conversion factor by which the poise may be converted from cgs to British gravitational units of viscosity.

9-17. Sea water, which weighs 64 lb/ft^3, is stored in an open tank. The water is piped to ground level by a vertical pipe of diameter 1 in., and emerges through a bend in the pipe in a stream of 1 in. diameter. The surface of the water in the tank is 100 ft above the opening in the pipe. (a) Find the quantity of water which flows from the tank per second. (b) Find the pressure of the water in the pipe 10 ft above the ground.

9-18. An open tank is filled with sea water, which weighs 64 lb/ft^3. The water flows through a siphon tube of cross-sectional area 0.02 ft^2 which has a nozzle of area 0.01 ft^2 at its end. The open end of the nozzle is 5 ft below the free surface of the water in the tank. (a) What is the speed with which water flows from the nozzle? (b) What is the volume of flow from the nozzle, in ft^3/sec? (c) What is the speed of flow in the siphon tube at a point 3 ft above the nozzle? (d) What is the gauge pressure in the siphon tube at a point 3 ft above the nozzle?