

University of Nebraska - Lincoln

DigitalCommons@University of Nebraska - Lincoln

Papers in the Earth and Atmospheric Sciences

Earth and Atmospheric Sciences, Department of

6-2004

A Concept of Maximum Stream Depletion Rate for Leaky Aquifers in Alluvial Valleys

Vitaly A. Zlotnik

University of Nebraska-Lincoln, vzlotnik1@unl.edu

Follow this and additional works at: <http://digitalcommons.unl.edu/geosciencefacpub>



Part of the [Earth Sciences Commons](#)

Zlotnik, Vitaly A., "A Concept of Maximum Stream Depletion Rate for Leaky Aquifers in Alluvial Valleys" (2004). *Papers in the Earth and Atmospheric Sciences*. 146.

<http://digitalcommons.unl.edu/geosciencefacpub/146>

This Article is brought to you for free and open access by the Earth and Atmospheric Sciences, Department of at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Papers in the Earth and Atmospheric Sciences by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.

A concept of maximum stream depletion rate for leaky aquifers in alluvial valleys

Vitaly A. Zlotnik

Department of Geosciences, University of Nebraska at Lincoln, Lincoln, Nebraska, USA

Received 4 December 2003; revised 31 March 2004; accepted 21 April 2004; published 24 June 2004.

[1] Existing analytical models for evaluating stream depletion by wells in alluvial aquifers are based on the assumption that stream depletion supplies 100% of groundwater withdrawals. Analysis of specific hydrostratigraphic conditions in leaky aquifers indicates that stream depletion may range from 0 to 100%. A new concept of maximum stream depletion rate (MSDR) is introduced and defined as a maximum fraction of the pumping rate contributed by the stream depletion. Several new analytical solutions indicate that the MSDR is determined by aquifer hydrostratigraphic conditions, geometry of recharge and discharge zones, and locations of pumping wells. *INDEX TERMS*: 1829 Hydrology: Groundwater hydrology; 1836 Hydrology: Hydrologic budget (1655); 1842 Hydrology: Irrigation; 1860 Hydrology: Runoff and streamflow; *KEYWORDS*: aquitard, groundwater, hydraulic conductivity, leaky aquifer, streams, stream depletion rate

Citation: Zlotnik, V. A. (2004), A concept of maximum stream depletion rate for leaky aquifers in alluvial valleys, *Water Resour. Res.*, 40, W06507, doi:10.1029/2003WR002932.

1. Introduction

1.1. Analytical Methods for Evaluation of Stream Depletion

[2] Stream depletion (SD) is one of the most widely used hydrogeological concepts developed in the twentieth century for water resources management. Recent droughts and the proliferation of large-capacity wells for irrigation have renewed interest in the SD concept. In the United States, tens of thousands of wells capable of pumping over 1000 m³/d are located in alluvial valleys. Vast water withdrawals have dramatically changed local and regional water budgets of aquifers and streams. For example, maps comparing perennial streams in Kansas in the 1960s with those of the 1990s show a marked decrease in the length of streamflow [Sophocleous, 1997].

[3] Wells upset the dynamic equilibrium of the water budget that existed in predevelopment conditions. A decrease of groundwater drainage into a stream or increase of stream water losses into the aquifer are examples of changes in natural discharge or discharge [Theis, 1940, 1941; Sophocleous, 1997; Bredehoeft, 1997, 2002]. The sum of these two terms required for a transition to a new dynamic equilibrium under groundwater pumping is sometimes referred as “capture.” Hantush [1965] introduced the term “stream depletion” as synonymous with “capture” to characterize changes in natural groundwater discharge to the streams. This term is sometimes applied to direct water losses (fluxes) from streams that are hydraulically connected with the pumped aquifers [e.g., Wilson, 1993].

[4] Direct measurements of stream depletion rate (SDR) using stream discharge data are difficult and rare [Sophocleous et al., 1988; Hunt et al., 2001; Nyholm et al., 2002, 2003; Kollet and Zlotnik, 2003]. Their results are fraught with uncertainties due to the runoff variability

and available accuracy of discharge measurements. Therefore mathematical modeling is commonly used for SDR evaluation. In cases where appropriate information is available, numerical modeling allows one to determine the SDR and other water budget items using the draw-down and runoff characteristics for calibration [e.g., Nyholm et al., 2002, 2003]. However, this information is often limited, so more simple analytical models are generally used for SDR evaluation.

[5] There are four analytical solutions available for estimating SDR that differ in their descriptions of streambed properties and degree of penetration. The dimensionless SDR function D can be defined as a fraction of pumping rate Q ,

$$\frac{q_s(t)}{Q} = D, \quad (1)$$

where q_s is stream depletion rate, which depends on time t , well characteristics, including distance from stream bank d , and aquifer parameters (hydraulic conductivity K , thickness b , storativity S , etc.). Jenkins [1968] introduced the characteristic timescale t_a , which sometimes is called the “stream depletion factor,”

$$t_a = \frac{S d^2}{T}, \quad (2)$$

where $T = Kb$ is transmissivity. Only a short summary of the equations is presented below, since hydrogeological conditions for applications of these solutions have been summarized elsewhere [e.g., Barlow and Moench, 1998; Zlotnik and Huang, 1999; Hunt, 1999; Zlotnik et al., 1999; Butler et al., 2001].

[6] 1. A stream fully penetrates a uniform aquifer with an impermeable horizontal base. Theis [1941] and Glover and Balmer [1954] derived the SDR function D_{TGB} for a well

with a pumping rate Q at the distance d from the stream as follows:

$$\frac{q_S(t)}{Q} = D_{TGB}\left(\frac{t}{t_a}\right); \quad D_{TGB}(u) = \operatorname{erfc}\left(\frac{1}{2\sqrt{u}}\right) \quad (3)$$

After $t > 100 t_a$, more than 94% of groundwater withdrawal Q is supplied by the SDR; eventually this fraction reaches 100%. A short review of this solution is given by Wallace *et al.* [1990].

[7] 2. A stream fully penetrates a uniform aquifer; the aquifer and streambed have contrasting hydraulic conductivity. Hantush [1965] accounted for a partial stream penetration and properties of streambed sediments by introducing a fictitious thin incompressible “vertical” layer of reduced hydraulic conductivity K_S ($K_S < K$) and thickness m_S . The SDR function (D_H) utilizes retardation coefficient B_S , which accounts for streambed properties:

$$\frac{q_S(t)}{Q} = D_H\left(\frac{t}{t_a}, \frac{d}{B_S}\right), \quad B_S = \frac{K m_S}{K_S} \quad (4)$$

$$D_H(u, v) = \operatorname{erfc}\left(\frac{1}{2\sqrt{u}}\right) - e^{v^2 u + v} \operatorname{erfc}\left(\frac{1}{2\sqrt{u}} + v\sqrt{u}\right) \quad (5)$$

Compared with the Theis-Glover-Balmer solution, the pace of the SDR increase over time is slower, and the term “retardation” properly describes the later onset of the 100%. However, to highlight the hydrogeological context of this coefficient, the term “streambed leakage coefficient” for B_S is more appropriate. A short review of this solution is given by Darma [2001].

[8] 3. A stream with a streambed of finite thickness negligibly penetrates the aquifer; aquifer and streambed have contrasting hydraulic conductivities. Zlotnik *et al.* [1999], Hunt [1999], and Butler *et al.* [2001] obtained the SDR for this realistic streambed geometry. In the particular case of a very shallow stream of finite width W , the SDR function can be presented in a closed form [Zlotnik *et al.*, 1999, equations (11) and (12)]:

$$\frac{q_S(t)}{Q} = D_H\left(\frac{t}{t_a}, \frac{d}{B_S}\right), \quad (6)$$

$$B = \sqrt{\frac{m_S T}{K_S}}, \quad B_S = B \coth \frac{W}{2B} \quad (7)$$

Note that equation (6) is identical to the Hantush solution D_H in equation (5), but a different form of streambed leakage coefficient B_S more realistically represents the streambed and the water fluxes. In the case of a small stream width ($W \ll 2B$) the expression for B_S simplifies to

$$B_S \approx \frac{2 m_S T}{W K_S}, \quad (8)$$

and the problem reduces to the Hunt [1999, equation (20)] solution.

[9] Butler *et al.* [2001] extended this approach to include the effects of large-scale heterogeneity and finite alluvial

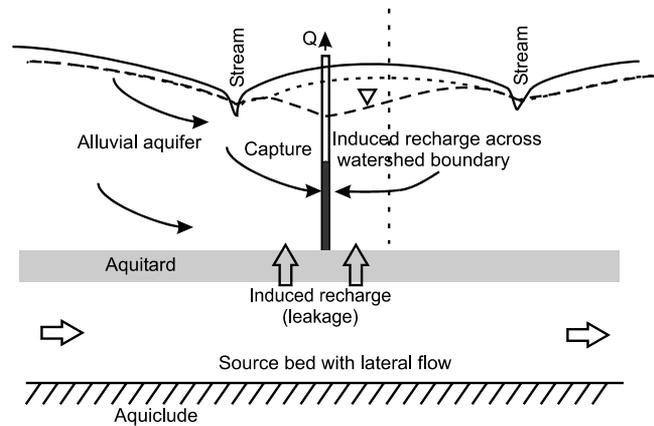


Figure 1. Induced recharge. The diagram shows both capture directly affecting the runoff and induced aquifer recharge by leakage from the adjacent aquifer.

valley width on estimates of SDR and drawdown. It was shown numerically that this semianalytical method and an accompanying code are accurate for many cases having anisotropic conditions and varying degrees of aquifer penetration.

[10] 4. A stream fully penetrates a uniform unconfined aquifer and partially penetrates an aquitard beneath; water is pumped from a well in an adjacent semiconfined aquifer with an impermeable horizontal base. Hunt [2003] developed a two-term solution:

$$\frac{q_S(t)}{Q} = D_H\left(\frac{t}{t_a}, \frac{d}{B_S}\right) + \Delta, \quad (9)$$

where t_a is calculated using the parameters of the semiconfined aquifer (instead of the unconfined aquifer) and Δ incorporates the aquitard parameters.

[11] All of these approaches share one essential trait: They predict that the stream will supply 100% of the groundwater withdrawal after a sufficiently long pumping period. Application of these solutions for water resources management without considering hydrogeological conditions may lead to overestimation of stream depletion.

1.2. Leaky Aquifers and the Concept of Maximum Stream Depletion Rate (MSDR)

[12] In some cases, groundwater withdrawals in alluvial aquifers can be partially supplied by leakage from adjacent aquifers. Indeed, the base of many alluvial aquifers consists of low permeability bedrock that can be considered an aquitard (Figure 1). In these instances, the absence of water budget data for alluvial aquifers evolves into the assumption of negligible flow from the aquifer base for practical purposes. However, there are many situations where alluvial aquifers are in hydraulic connection with adjacent aquifers. “This occurs in rivers of the Gulf Coast and in the High Plains. In this case, there may be a significant contribution from the underlying sediments to the baseflow of the stream” [Larkin and Sharp, 1992, p. 1609]. Sharp [1988, p. 278] noticed that recharge from bedrock aquifers may be an important factor in the water budget of alluvial aquifers, “. . .but because alluvium is usually more permeable, the effects are less pronounced. The major evidence for this

type of recharge shows in water chemistry.” Zones of anomalous water chemistry were found in the alluvium of the Missouri River, the Ohio River, and the Arkansas River, but in smaller alluvial systems recharge from the bedrock was found to be proportionately more important, especially in carbonate bedrock terrain. Low aquitard permeability causes difficulties in quantifying this recharge [e.g., *Neuzil, 1994*].

[13] Such conditions are common in alluvial valleys in the midwestern United States. For example, in the Platte River (Missouri River basin), fine sediments of eolian origin separate large areas of the alluvial aquifer from the sand and gravel High Plains Aquifer [e.g., *McGuire and Kilpatrick, 1998*]. Similar hydrostratigraphic conditions exist in various geologic environments (e.g., Florida [*Motz, 1998*], Netherlands [*Heij, 1989*], Hong Kong [*Jiao and Tang, 1999*]).

[14] To investigate the SDR in such hydrogeological conditions, I consider a stream in an alluvial valley, which is separated from the lower aquifer by a unit of thickness m_A and low hydraulic conductivity K_A (Figure 2). Sometimes, the hydraulic conductivity of the underlying aquifer can be comparable to the hydraulic conductivity of the alluvial aquifer K , and the underlying aquifer serves as a source bed. This concept of the source bed will be discussed in light of the groundwater budget later.

[15] These hydrogeological conditions introduce a new paradigm in analytical SDR assessment. The emphasis shifts from consideration of the streambed properties and stream partial penetration to the effect of induced recharge of the alluvial aquifer from a source bed (leakage). Therefore the major purpose of this paper is to investigate the effects of induced recharge on stream depletion. I will demonstrate that the maximum SDR may vary in the range from 0 to 100% of the pumping rate of an individual well. I define maximum stream depletion rate (MSDR) as a maximum fraction of the pumping rate supplied by stream depletion.

[16] The objectives of this paper are as follows: (1) to derive an estimate of the transient SDR induced by an individual well for streams in leaky aquifers; (2) to estimate the maximum SDR (MSDR), which is reached in steady state conditions; (3) to develop estimates of the induced recharge (the leakage from the lower source-bed aquifer); and (4) to assess the effects of the alluvial valley width on the MSDR.

2. Transient Stream Depletion Rate in Alluvial Aquifers

[17] Recent SD studies focused on the effects of partial penetration and reduced streambed conductivity, but they concluded that MSDR ultimately reached 100%. In this study the mitigating effects of partial penetration and reduced streambed conductivity are neglected (Figure 2). Thereby this estimate is the maximum possible SDR that is induced by a well. Below, I outline the mathematical problem and focus on analyzing the various factors that affect the SDR. However, the major steps of the derivations are relegated to Appendix A.

2.1. Problem Statement

[18] The linearized equation of groundwater flow toward a well located at distance d from a stream in a laterally semi-

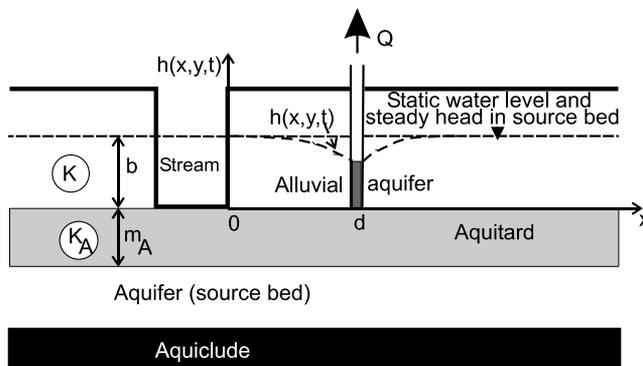


Figure 2. Stream depletion induced by pumping in a wide alluvial valley; the alluvial aquifer is connected with the lower aquifer (source bed) through the aquitard.

infinite aquifer with a leaky incompressible aquitard below is as follows:

$$S \frac{\partial h}{\partial t} = T \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) + \frac{K_A}{m_A} (h_0 - h) - Q \delta(x - d) \delta(y), \quad 0 < x < \infty \quad (10)$$

This approach considers the equilibrium between the alluvium and a source bed with constant head h_0 in prepumping conditions. Boundary and initial conditions are as follows:

$$h(0, y, t) = h(\infty, y, t) = h_0, \quad -\infty < y < \infty, \quad 0 < t < \infty \quad (11)$$

$$h(x, y, 0) = h_0, \quad 0 < x < \infty, \quad -\infty < y < \infty \quad (12)$$

The stream depletion rate $q_S(t)$ is calculated from a solution of this problem as follows:

$$q_S(t) = - \int_{-\infty}^{\infty} T \frac{\partial h(0, y, t)}{\partial x} dy \quad (13)$$

2.2. Method of Solution for Transient Stream Depletion Rate

[19] The drawdown for the problem of equations (10)–(12) can be based on the *Hantush* [1964, equation (62)] solution for drawdown in the leaky aquifer of infinite lateral extent. This solution can be extended for a semi-infinite aquifer using the superposition principle. After that, the SDR can be evaluated using equation (13).

[20] However, the SDR is often the only variable of interest in many hydrogeological problems. *Zlotnik et al.* [1999] showed that the geometry of the problem permits one to focus on SDR, $q_S(t)$, without solving for the drawdown. This technique is systematically applied below.

[21] For this purpose, I introduce a new variable $H(x, t)$:

$$H(x, t) = \int_{-\infty}^{\infty} [h_0 - h(x, y, t)] dy \quad (14)$$

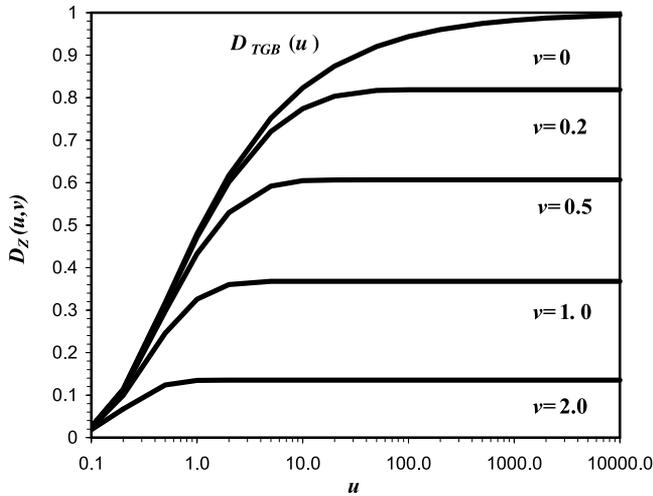


Figure 3. Stream depletion rate function $D_Z(u, v)$ for water withdrawal near a stream in an alluvial leaky aquifer.

which is integrated over the y -line drawdown. The boundary value problem (10)–(12) can be rewritten in a one-dimensional form for this integrated drawdown H :

$$S \frac{\partial H}{\partial t} = T \frac{\partial^2 H}{\partial x^2} - \frac{K_A}{m_A} H + Q\delta(x-d), \quad 0 < x < \infty \quad (15)$$

$$H(0, t) = H(\infty, t) = 0, \quad 0 < t < \infty, \quad (16)$$

$$H(x, 0) = 0, \quad 0 < x < \infty \quad (17)$$

Integrated drawdown yields the total discharge across the y -line, $q(x, t)$, at any x :

$$q(x, t) = T \frac{\partial H(x, t)}{\partial x} \quad (18)$$

and SDR is defined as the integral discharge at $x = 0$:

$$q_S(t) = q(0, t) = T \frac{\partial H(0, t)}{\partial x} \quad (19)$$

Details of solution of the problem (15)–(17) are relegated to Appendix A.

2.3. Solution for the Transient Stream Depletion Rate

[22] The aquitard leakage coefficient B_A after *Hantush* [1964, p. 307] is as follows:

$$B_A = \sqrt{\frac{m_A T}{K_A}} \quad (20)$$

This parameter, with units of length, has quite a different meaning as compared with the streambed leakage coefficient B_S in equation (4) or (6). A new solution for the SDR by equation (18) from the equations (15)–(17) is as follows (see Appendix A):

$$\frac{q_S(t)}{Q} = D_Z\left(\frac{t}{t_a}, \frac{d}{B_A}\right), \quad (21)$$

where $D_Z(u, v)$ has two equivalent forms:

$$D_Z(u, v) = \frac{2}{\sqrt{\pi}} \int_{\frac{1}{2\sqrt{u}}}^{\infty} \exp\left(-\xi^2 - \frac{v^2}{4\xi^2}\right) d\xi \quad (22a)$$

$$D_Z(u, v) = \frac{e^v}{2} \operatorname{erfc}\left(\frac{1}{2\sqrt{u}} + v\sqrt{u}\right) + \frac{e^{-v}}{2} \operatorname{erfc}\left(\frac{1}{2\sqrt{u}} - v\sqrt{u}\right) \quad (22b)$$

$D_Z(u, v)$ is shown in Figure 3. The following properties of this function

$$\lim_{v \rightarrow 0} D_Z(u, v) = D_{TGB}(u), \quad (23a)$$

$$\lim_{u \rightarrow \infty} D_Z(u, v) = e^{-v} \quad (23b)$$

$$\lim_{v \rightarrow \infty} D_Z(u, v) = 0 \quad (23c)$$

result in the following SDR properties, respectively: (1) For an impermeable aquifer base ($v = d/B_A \ll 1$), this solution is identical to that of *Theis* [1941]; (2) for large times ($u = t/t_a \gg 1$), the SDR is less than 1, because $e^{-v} < 1$; and (3) SDR reduces to zero for large distances between the well and the stream.

[23] Note that the SDR in all equations does not depend on streambed properties, but rather on the aquifer and aquitard parameters only, because we assumed a perfect connection between the stream and alluvial aquifer.

3. Effect of Alluvial Valley Width on Maximum Stream Depletion Rate

[24] The MSDR for an individual well in a given hydrogeological setting is defined as the SDR that is reached at steady state conditions (large time). In some cases, the limited lateral extent of the alluvial valley (valley width L) may be an important factor affecting the MSDR (Figure 4). This effect can be essential when the distance from the well to the valley wall or to the stream is comparable to the aquitard leakage coefficient B_A . In this section, we analyze the effect of alluvial valley width on the water budget.

3.1. Maximum Stream Depletion in a Wide Alluvial Valley

[25] To make a comparison with previously published solutions, we start from the case of a very wide alluvial valley. The MSDR for large times can be obtained from equation (21) using equation (23b):

$$MSDR = \lim_{t \rightarrow \infty} \frac{q_S(t)}{Q} = \lim_{t \rightarrow \infty} D_Z\left(\frac{t}{t_a}, \frac{d}{B_A}\right) = \exp\left(-\frac{d}{B_A}\right). \quad (24)$$

The MSDR decays exponentially with distance between the well and the stream. The aquitard leakage coefficient B_A is linear scale for the decay distance.

[26] The width of an alluvial valley (L) can be considered large when $L \gg B_A$. Note that the MSDR value can vary between 0 and 1, and consideration of the streambed properties and partial penetration will further reduce this parameter. Only when a well is located very close to the stream ($d \ll B_A$) does the MSDR reach 100%. In this case, groundwater withdrawals at the steady state are supported by

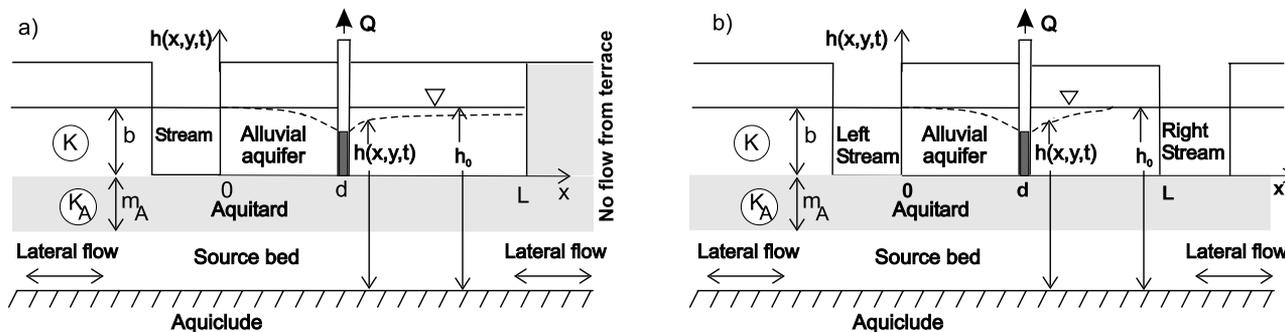


Figure 4. Diagram for calculating the stream depletion rate induced by pumping in an alluvial valley of finite width; alluvial aquifer is recharged from the lower aquifer (source bed) through the aquitard: (a) valley without impermeable boundary; (b) valley between two streams.

two complementary sources: SD and the induced recharge from leakage across the aquitard beneath the alluvial valley. Induced recharge across the aquitard, expressed as a fraction of the pumping rate (AR), is the balance of the 100% value:

$$AR = \lim_{t \rightarrow \infty} \frac{1}{Q} \int_0^{\infty} \frac{K_A}{m_A} H(x, t) dx = 1 - \exp(-d/B_A) \quad (25)$$

The well water budget is as follows: Pumping rate = SDR + induced recharge, or

$$MSDR + AR = 1 \quad (26)$$

The induced recharge plays the dominant role when the depression cone develops far from the stream; in this case, the underlying aquifer is the only source of water to the well. Therefore the MSDR must be estimated for each particular well location in differing hydrogeological conditions.

3.2. Maximum Stream Depletion Rate in an Alluvial Valley Without Lateral Recharge

[27] Consider the case of an alluvial valley with an impermeable boundary at a distance L (Figure 4a). The magnitude of the MSDR will be affected by the vertical cross flow from the lower aquifer to the alluvium based on the aquitard properties and the distance from the stream in the SDR when valley width L is comparable with leakage parameter B_A .

[28] To obtain the MSDR values only, one starts from the steady state form of the groundwater flow equation (10) for drawdown $h(x, y)$. Boundary conditions (11) are modified to take into account the steady state, impermeable valley wall, and the finite valley width:

$$h(0, y) = h_0; \frac{\partial h(L, y)}{\partial x} = 0, -\infty < y < \infty, \quad (27)$$

Using equation (14) as a definition of the integrated along the y -axis steady state drawdown, $H(x)$, the problem is reduced to the one-dimensional steady state analogue of equations (15)–(16) that easily lends itself to a simple solution for $H(x)$. Omitting the straightforward derivations, we provide the final equation for the MSDR:

$$MSDR = \lim_{t \rightarrow \infty} \frac{q_S(t)}{Q} = \cosh \left[\frac{L}{B_A} \left(1 - \frac{d}{L} \right) \right] / \cosh \left(\frac{L}{B_A} \right), \quad (28)$$

Note that the MSDR can vary between 0 and 1. Only when a well is located very close to the stream ($d \ll L$) does the MSDR reach 1.

[29] The hydrological explanation of this case is similar to that of an alluvial valley with a very large width. Well pumping at steady state is supported by two complementary sources, SD and induced recharge (leakage across the aquitard beneath the alluvial valley). Induced recharge, as a fraction of the pumping rate at steady state (AR), is as follows:

$$AR = \frac{1}{Q} \int_0^L \frac{K_A}{m_A} H(x) dx = 1 - \cosh \left[\frac{L}{B_A} \left(1 - \frac{d}{L} \right) \right] / \cosh \left(\frac{L}{B_A} \right). \quad (29)$$

The well water budget indicates that “pumping rate = SDR + induced recharge,” similar to equation (25). The induced recharge from the lower aquifer plays the dominant role for well supply when the depression cone develops far from the stream.

3.3. Maximum Stream Depletion in an Alluvial Valley Between Two Streams

[30] Consider an alluvial valley between two streams (Figure 4b) that are represented by constant head boundaries. The magnitude of MSDR in the case of the lateral recharge boundary will be determined by the well location between the streams, in addition to the aquitard properties and valley width.

[31] To simplify derivations, one can start directly from the steady state form of the groundwater flow equation (10) for $h(x, y)$. Boundary conditions (11) are modified to account for finite valley width and a constant head at each stream:

$$h(0, y) = h(L, y) = h_0, \quad -\infty < y < \infty \quad (30)$$

Using equation (14) as a definition of the integrated over y -axis steady state drawdown, $H(x)$, the problem is reduced to a one-dimensional steady state analogue of equations (15)–(16) that easily lends itself to a simple solution. Omitting the straightforward derivations, we provide the final equation for the MSDR:

$$MSDR = \lim_{t \rightarrow \infty} \frac{q_S(t)}{Q} = \sinh \left[\frac{L}{B_A} \left(1 - \frac{d}{L} \right) \right] / \sinh \left(\frac{L}{B_A} \right), \quad (31)$$

Only when a well is located very close to the “left” stream ($d \ll L$) can the MSDR reach 1. However, the left stream does not contribute to the well pumping rate when the well is located close to the “right” stream ($L - d \ll L$), and the right stream supplies a major fraction of the MSDR.

[32] In general, the fraction of the pumping rate attributed to the presence of the other constant head boundary (in general, lateral recharge LR) is

$$LR = \frac{1}{Q} \int_{-\infty}^{\infty} T \frac{\partial h(L, y, t)}{\partial x} dy = \sinh\left(\frac{d}{B_A}\right) / \sinh\left(\frac{L}{B_A}\right), \quad (32)$$

The fraction of the pumping rate that is supplied by leakage from underlying aquifer (AR) is

$$AR = \frac{1}{Q} \int_0^L dx \frac{K_A}{m_A} H(x) = 1 - \left\{ \sinh\left[\frac{L}{B_A} \left(1 - \frac{d}{L}\right)\right] + \sinh\left(\frac{d}{B_A}\right) \right\} / \sinh\left(\frac{L}{B_A}\right). \quad (33)$$

This fraction AR vanishes only when a well is located very close to the left stream ($d \ll L$) or very close to the right stream ($L - d \ll L$), and the leakage coefficient serves as a scaling factor for distance

[33] The well water budget can be expressed as follows: Pumping rate = SDR + induced recharge + lateral recharge, or

$$MSDR + AR + LR = 1 \quad (34)$$

where the sources other than the stream depletion supply water to the well when the depression cone develops far from streams.

4. Discussion

4.1. Examples

[34] Consider a well at a site similar to the Management System Evaluation Area on the Platte River watershed [see Zlotnik *et al.*, 1993; McGuire and Kilpatrick, 1998]. The site is located about 3 km north of the north channel of the Platte River and 5 km from the main channel of the Platte River. The primary aquifer in the study area consists of three units: the shallow alluvial aquifer, a silt and clay unit, and a part of the High Plains aquifer (mostly Tertiary Ogallala Group with slightly cemented sand). The High Plains aquifer in the study area is underlain by the Pierre Shale, which is considered to be an aquiclude.

[35] Data on the unconfined alluvial aquifer are available from various programs of aquifer characterization including pumping tests of various durations [e.g., Zlotnik *et al.*, 1993; Zlotnik and McGuire, 1998; Chen and Ayers, 1998]. In our example, we use a hydraulic conductivity of alluvium $K = 100$ m/d, $S = 0.1$, and saturated thickness $b = 10$ m.

[36] The aquitard thickness at different sites varies from 5 to 10 m [Zlotnik *et al.*, 1993; McGuire and Kilpatrick, 1998]. The presence of this layer is pretty consistent across the area. Hydraulic conductivity and specific storage data for the aquitard are very rare, and $K_A = 10^{-2}$ m/d is used as an estimate, which is characteristic of silt and loess; $m_A = 10$ m, and we assume incompressible conditions. (For comparison, we also will consider $K_A = 10^{-4}$ m/d that is characteristic of clay.)

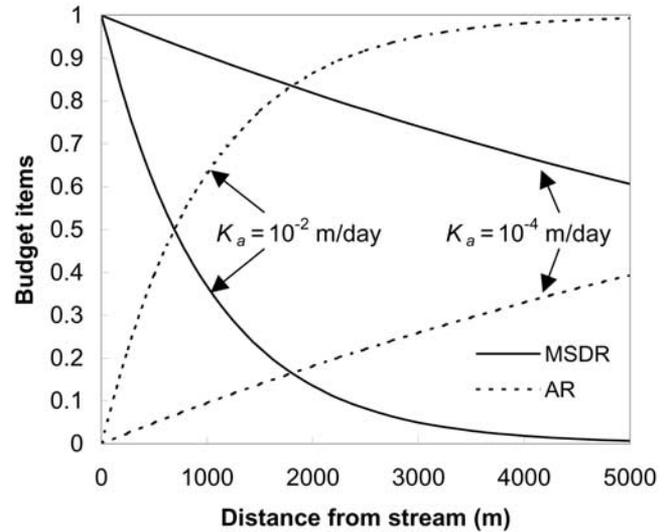


Figure 5. Components of well water budget as functions of distance from the stream in a wide alluvial valley with induced recharge from the lower aquifer ($K_a = 10^{-2}$ m/d and $K_a = 10^{-4}$ m/d).

[37] The proposed methods do not include the hydraulic conductivity of the Ogallala Group explicitly (it is relatively high, of the order of 10 m/d). The major characteristics of this portion of the High Plains Aquifer are a large saturated thickness (60–360 m), stable water levels in spite of new well construction, high porosity, abundance of water, and relatively consistent eastward regional groundwater flow [Miller and Appel, 1997, p. D13]. This essentially confirms the assumption of constant head in the lower source bed.

[38] The aquitard leakage coefficient (equation (20)) establishes a linear scale for the distance from streams: $B_A = \sqrt{m_A T / K_A} = \sqrt{10 \times 100 \times 10 / 10^{-2}} = 1000$ m. For clay with $K_A = 10^{-4}$ m/d, one obtains $B_A = 10^4$ m. The following MSDR estimates for two aquifer geometries are based on these parameters.

4.1.1. Alluvial Valley of Large Width

[39] The MSDR as a function of distance between the stream and the well is given by equation (24): $MSDR = e^{-0.001d}$. This model indicates that $MSDR = 0.37$ of the pumping rate for a well at a distance $d = 1000$ m, while the MSDR for a well at a distance $d = 2000$ m is 0.14. The balance of the pumping rate comes from the induced recharge: $AR = 0.63$ and $AR = 0.86$, respectively, according to equation (25). The results summarized in Figure 5 indicate that leakage may be a major source of the groundwater withdrawals for some wells in spite of the contrast in hydraulic conductivity between aquitard and aquifer by a factor of 10^4 .

[40] For a clay aquitard with $K_A = 10^{-4}$ m/d, $MSDR = 0.90$ and 0.82 for wells at distances 1000 m and 2000 m, respectively. This indicates that for wells at distances less than 2000 m from the stream, the aquitard acts as an aquiclude if the contrast in the hydraulic conductivity between the alluvial aquifer and the underlying bed is 10^6 (Figure 5). However, wells at larger distances from the stream still receive significant recharge from the lower aquifer. This comparison between clay and silt aquitards shows the importance of obtaining information about aquitard characteristics [van der Kamp, 2001].

[41] The SDR as a function of time for a well at a distance $d = 2000$ m from the stream can be shown using the transient model, equation (21). In the case of the silt aquitard, $K_A = 10^{-2}$ m/d, $\nu = 2000/1000 = 2$, the corresponding curve in Figure 3 shows that the system arrives at steady state with a $MSDR = 0.14$ after $u = t/t_a \approx 1$. For a clay aquitard, $K_A = 10^{-4}$ m/d, $\nu = 2000/10000 = 0.2$, the $MSDR = 0.82$, and stabilization time for SDR is $u = t/t_a \approx 10$.

[42] From equation (2), the stabilization period is $t_a = 0.1 \times 2000^2 / (100 \times 10) = 400$ days, or about a year. The SDR function after the Theis-Glover-Balmer model for a fully penetrating stream with perfect connection to the aquifer ($\nu = 0$) arrives at steady state much later ($u = t/t_a > 100$). These results indicate a dramatic difference between the commonly used and new model.

4.1.2. Alluvial Valley of Finite Width Between Two Streams

[43] Consider wells that are located in an alluvial valley of width $L = 5000$ m. In this case, the $MSDR$ (induced flow from left stream) will be determined based on equation (31). Results for a silt aquitard are summarized in Figure 6. This figure also displays the induced recharge rate from the lower aquifer (LR) and the increase in lateral recharge from the right stream (AR), using equations (32) and (33), respectively. A clay aquitard would have the effects on the well water budget that are similar to those presented in Figure 5.

[44] It is apparent that close to the stream ($d = 500$ m), the well water is supplied mainly by the stream depletion ($MSDR = 0.61$, $AR = 0.38$, $LR = 0.03$). A well closer to the right stream ($d = 4500$ m) induces significant recharge from this stream ($MSDR = 0.03$, $AR = 0.38$, $LR = 0.61$). A well between the streams ($d = 2500$ m) will draw the largest contribution from the lower aquifer by induced recharge ($MSDR = 0.08$, $AR = 0.84$, $LR = 0.08$).

4.2. On Simplifying Assumptions in the Estimation of Induced Recharge

4.2.1. The Presence of a Prolific Source Bed

[45] This *Hantush* [1964] concept implies a significant lateral inflow into the adjacent underlying aquifer (Figure 1). In our examples above, this inflow can originate from eastward regional groundwater flow in Nebraska from the Rocky Mountains toward the Missouri River. However, this assumption needs to be verified in every specific case. In some cases, the relative stability of observed heads in the underlying aquifer may be sufficient.

4.2.2. The Incompressibility of the Aquitard

[46] Hydraulic analysis indicates that this assumption reduces the stabilization time for the SDR but does not change the value of the $MSDR$. In practice, the actual SDR will reach the $MSDR$ later than predicted by our model, but will not exceed it.

4.2.3. Multilayered Leaky Aquifers

[47] In some cases, hydrostratigraphic conditions may require the appropriate multilayered models, and the $MSDR$ concept can be extended to these conditions. The ultimate criteria for choosing the model will depend on availability of hydrogeological information.

4.2.4. Continuity of the Aquitard

[48] The presence of fractures, lithological windows, and other discontinuities may lead to further localization of the pumping influence in an unconfined alluvial aquifer near

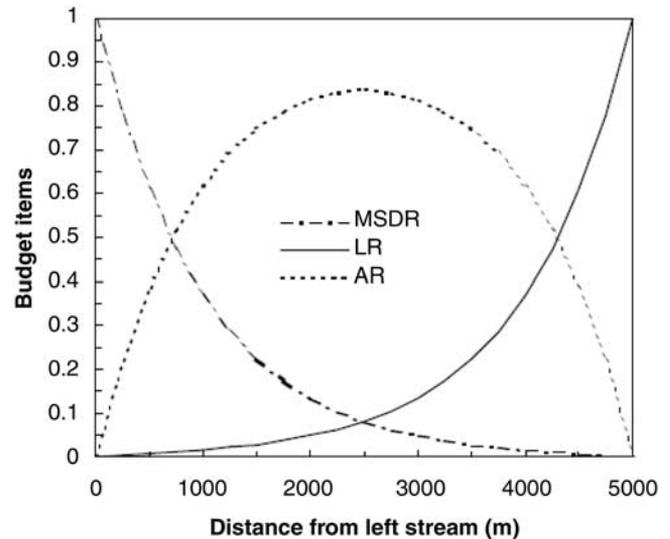


Figure 6. Components of well water budget as functions of distance from the stream in an alluvial valley of finite width with induced recharge from the lower aquifer ($K_a = 10^{-2}$ m/d).

the pumping well if the lower aquifer serves as a source bed. However, the presence of aquitard discontinuities is difficult to quantify at large spatial scale [e.g., *Neuzil*, 1994] that may affect the assessment of the $MSDR$.

4.2.5. Intermittent Pumping

[49] Some wells have a nonuniform, prolonged pumping schedule. In these cases, the annual average pumping rate can be used for the parameter Q [*Wallace et al.*, 1990; *Darama*, 2001].

4.3. Implications of the MSDR Concept for Water Resources Management

[50] The reduction of the discharge from the aquifer to the streams is not the only source of the water supplying the pumping wells [*Sophocleous*, 1997; *Bredehoeft*, 1997, 2002]. Other sources such as the induced recharge from the adjacent aquifer can supply the groundwater withdrawals. This recharge “shields” streams and springs in an alluvial aquifer from the pumping impact. Instead of streams, the groundwater withdrawals from the alluvial aquifer can be manifested in groundwater regime changes within the adjacent aquifer.

[51] The adjacent aquifer shifts the effects of groundwater withdrawal (e.g., the SD and the reduction of the length of perennial streams) to the more remote locations in the hydrological system. This process is determined by spatial and temporal characteristics of groundwater withdrawals, aquifer hydrostratigraphy, and geometry of recharge and discharge. Presented models show that the stream reach nearest to the individual pumping well is not necessary the only supplier of water to sustain the groundwater withdrawal, and low values of the $MSDR$ indicate the significance of the impact of groundwater withdrawals on adjacent aquifers. The concept of a $MSDR$ requires explicit assessment of the induced recharge in realistic hydrogeological conditions.

[52] This study indicates that both the pace of the SDR and the magnitude of the $MSDR$ are affected by well location. The assumption of a $MSDR$ at 100% can be used

only for the most restrictive schemes of groundwater use in water resources management.

5. Summary

[53] Stream depletion may only partially support groundwater withdrawal from a pumping well in leaky aquifers. Therefore the maximal stream depletion rate (MSDR) was defined as a fraction of the pumping rate supplied by stream depletion. This rate is achieved after the hydrologic system arrives at a new equilibrium after the start of pumping. The MSDR can be assessed only with full consideration of hydrogeological conditions that include the hydrostratigraphy, geometry of recharge and discharge zones, and location of pumping well.

[54] In general, the MSDR may range from 0 to 100%. The balance of groundwater withdrawals that is not supported by the stream depletion can be supplied from other sources. For example, these sources include the induced recharge from the adjacent aquifers or streams to the pumped aquifer. Well location determines the pace of the stream depletion rate, and the MSDR is also strongly affected by the proximity of the well to the sources of recharge and discharge.

[55] It is shown that a large contrast of hydraulic conductivity between the pumped aquifer and underlying bed (10^4 – 10^6) may not be a sufficient criterion for considering the aquifer base as an aquiclude. Long-term continuous, or intermittent pumping induces the appreciable cross flow through the aquitard.

[56] The proposed methods can be used for approximate assessment of stream depletion rates and the MSDR in alluvial valleys with different sources of recharge and discharge. Such methods will be a useful complement to numerical techniques that are applied for detailed assessment of aquifer water budgets.

Appendix A: Derivations

[57] Equations (15)–(17) in dimensionless variables

$$\bar{t} = \frac{Tt}{Sd^2} = \frac{t}{t_a}, \quad \bar{x} = \frac{x}{d}, \quad (\text{A1})$$

are as follows:

$$\frac{\partial H}{\partial \bar{t}} = \frac{\partial^2 H}{\partial \bar{x}^2} - \left(\frac{d}{B}\right)^2 H + \frac{Qd}{T} \delta(\bar{x} - 1), \quad 0 < \bar{x} < \infty \quad (\text{A2})$$

$$H(0, \bar{t}) = H(\infty, \bar{t}) = 0, \quad 0 < \bar{t} < \infty, \quad (\text{A3})$$

$$H(\bar{x}, 0) = 0, \quad 0 < \bar{x} < \infty \quad (\text{A4})$$

The SDR can be obtained from $H(\bar{x}, \bar{t})$ using equation (19):

$$q_s(\bar{t}) = \frac{T}{d} \frac{\partial H(0, \bar{t})}{\partial \bar{x}} \quad (\text{A5})$$

Application of the Laplace transform $\bar{H}(\bar{x}, p) = \int_0^\infty H(\bar{x}, \bar{t}) e^{-p\bar{t}} d\bar{t}$ with respect to dimensionless time \bar{t} to equations (A2)–(A4) yields [e.g., *Carslaw and Jaeger*, 1986]:

$$\frac{\partial^2 \bar{H}}{\partial \bar{x}^2} - c^2 \bar{H} = \frac{Qd}{Tp} \delta(\bar{x} - 1), \quad 0 < \bar{x} < \infty, \quad c^2 = p + \left(\frac{d}{B_A}\right)^2 \quad (\text{A6})$$

$$\bar{H}(0, p) = \bar{H}(\infty, p) = 0 \quad (\text{A7})$$

The Laplace transform for the SDR in equation (A5) is as follows:

$$\bar{q}_s(p) = \frac{T}{d} \frac{\partial \bar{H}(0, p)}{\partial \bar{x}} \quad (\text{A8})$$

Standard techniques for the boundary value problem in equations (A6)–(A7) yield

$$\bar{H}(\bar{x}, p) = \frac{Qd}{pcT} \cdot \begin{cases} e^{-c} \sinh(c\bar{x}), & 0 < \bar{x} < 1 \\ \sinh(c) e^{-c\bar{x}}, & 1 < \bar{x} < \infty \end{cases} \quad (\text{A9})$$

The Laplace transform for the SDR is obtained by substituting equation (A9) into (A8):

$$\bar{q}_s(p) = Qe^{-c}/p \quad (\text{A10})$$

Applying the shifting theorem to the Laplace transform

$$e^{-v\bar{t}} \leftrightarrow \frac{1}{2(\pi\bar{t}^3)^{1/2}} \exp(-1/4\bar{t}) \quad (\text{A11})$$

[e.g., *Carslaw and Jaeger*, 1960, p. 495, formula 6], one obtains

$$\frac{q_s(\bar{t})}{Q} = \int_0^{\bar{t}} \frac{\exp(-1/4w) - w(d/B_A)^2}{2(\pi w^3)^{1/2}} dw \quad (\text{A12})$$

Substitution $z = 1/(2\sqrt{w})$ and introduction of function of two variables $Z(w, v)$ yields

$$\frac{q_s(\bar{t})}{Q} = Z\left(\frac{1}{2\bar{t}^{1/2}}, \frac{d}{B_A}\right) \quad (\text{A13})$$

$$Z(w, v) = \frac{2}{\sqrt{\pi}} \int_w^\infty \exp\left(-z^2 - \frac{v^2}{4z^2}\right) dz \quad (\text{A14})$$

Function Z lends itself to a further simplification [Abramowitz and Stegun, 1964, equation 7.4.33]:

$$Z(w, v) = \frac{e^v}{2} \operatorname{erfc}\left(w + \frac{v}{2w}\right) + \frac{e^{-v}}{2} \operatorname{erfc}\left(w - \frac{v}{2w}\right) \quad (\text{A15})$$

Introducing variable $w = 1/(2\sqrt{u})$, one arrives at a final expression for the SDR function:

$$D_Z(u, v) = Z\left(\frac{1}{2\sqrt{u}}, v\right) \quad (\text{A16})$$

[58] **Acknowledgments.** I am grateful to J. Barker, University College London, UK, for discussion of special functions; to J. Bredehoeft, Hydrodynamics Group, California, and M. Sophocleous, University of Kansas, for inputs on the MSDR concept; and to J. Butler, University of Kansas, and F. E. Harvey and C. Flowerday, both of UNL, for technical and editorial reviews.

References

Abramowitz, M., and I. A. Stegun (1964), *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, Natl. Inst. of Stand. and Technol., Gaithersburg, Md.

- Barlow, P. M., and A. F. Moench (1998), Analytical solutions and computer programs for hydraulic interaction of stream-aquifer systems, *U.S. Geol. Surv. Open File Rep.*, 98-415A, 85 pp.
- Bredehoeft, J. (1997), Safe yield and the water budget myth, *Ground Water*, 35(6), 929.
- Bredehoeft, J. D. (2002), The water budget myth revisited: Why hydrogeologists model, *Ground Water*, 40(4), 340–345.
- Butler, J. J., Jr., V. A. Zlotnik, and M.-S. Tsou (2001), Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream, *Ground Water*, 39(5), 651–659.
- Carlaw, H. S., and J. C. Jaeger (1960), *Conduction of Heat in Solids*, Clarendon, Oxford, UK.
- Chen, X. H., and J. F. Ayers (1998), Hydraulic parameters determined from two analytical solutions, *Ground Water*, 36(5), 783–791.
- Darama, Y. (2001), An analytical solution for stream depletion by cyclic pumping of wells near streams with semipervious beds, *Ground Water*, 39(1), 79–86.
- Glover, R. E., and G. G. Balmer (1954), River depletion resulting from pumping a well near a river, *Eos Trans. AGU*, 35(3), 468–470.
- Hantush, M. S. (1964), Hydraulics of wells, in *Advances in Hydroscience*, edited by V. Chow, pp. 281–432, Academic, San Diego, Calif.
- Hantush, M. S. (1965), Wells near streams with semipervious beds, *J. Geophys. Res.*, 70(12), 2829–2838.
- Heij, G. J. (1989), River-groundwater relationships in the lower parts of the Netherlands, *J. Hydrol.*, 108, 35–62.
- Hunt, B. (1999), Unsteady stream depletion from ground-water pumping, *Ground Water*, 37(1), 98–102.
- Hunt, B. (2003), Unsteady stream depletion when pumping from semi-confined aquifer, *J. Hydrol. Eng.*, 8(12), 12–19, doi:10.1061/(ASCE).
- Hunt, B., J. Weir, and B. Clausen (2001), A stream depletion field experiment, *Ground Water*, 39(2), 283–289.
- Jenkins, C. T. (1968), Techniques for computing rate and volume of stream depletion by wells, *Ground Water*, 6(2), 37–46.
- Jiao, J. J., and Z. Tang (1999), An analytical solution of groundwater response to tidal fluctuation in a leaky confined aquifer, *Water Resour. Res.*, 35(3), 747–751.
- Kollet, S., and V. A. Zlotnik (2003), Stream depletion predictions using data of pumping tests in a heterogeneous stream-aquifer system in the Great Plains, USA, *J. Hydrol.*, 281(1–2), 96–114.
- Larkin, R. G., and J. S. Sharp Jr. (1992), On the relationship between river-basin geomorphology, aquifer hydraulics, and ground-water flow direction in alluvial aquifers, *Geol. Soc. Am. Bull.*, 104, 1608–1620.
- McGuire, V. M., and J. M. Kilpatrick (1998), Hydrogeology in the vicinity of the Nebraska management systems evaluation area site, central Nebraska, *U.S. Geol. Surv. Water Resour. Invest. Rep.*, 97-4266, 25 pp.
- Miller, J., and C. L. Appel (1997), Ground water atlas of the United States: Segment 3, Kansas, Missouri, Nebraska, *Hydrol. Atlas 730-D*, 24 pp., U.S. Geol. Surv., Reston, Va.
- Motz, L. H. (1998), Vertical leakage and vertical conductance for karst lakes in Florida, *Water Resour. Res.*, 34(2), 159–167.
- Neuzil, C. E. (1994), How permeable are clays and shales?, *Water Resour. Res.*, 30(2), 145–150.
- Nyholm, T., S. Christensen, and K. R. Rasmussen (2002), Flow depletion in a small stream caused by ground water abstraction from wells, *Ground Water*, 40(4), 425–437.
- Nyholm, T., K. R. Rasmussen, and S. Christensen (2003), Estimation of stream depletion and uncertainty from discharge measurements in a small alluvial stream, *J. Hydrol.*, 274, 129–144.
- Sharp, J. M., Jr. (1988), Alluvial aquifers along the major rivers, in *The Geology of North America*, vol. O-2, edited by W. Back, J. S. Rosenshein, and P. R. Seaber, pp. 272–282, Geol. Soc. of Am., Boulder, Colo.
- Sophocleous, M. (1997), Managing water resources systems: Why “safe yield” is not sustainable, *Ground Water*, 35(4), 561.
- Sophocleous, M., M. A. Townsend, L. D. Vogler, T. J. McLain, E. T. Marks, and G. R. Goble (1988), Experimental studies in stream-aquifer interactions along the Arkansas River in central Kansas—Field testing and analysis, *J. Hydrol.*, 98, 249–273.
- Theis, C. V. (1940), The source of water to wells: Essential factors controlling the response of an aquifer to development, *Civ. Eng.*, 10, 277–280.
- Theis, C. V. (1941), The effect of a well on the flow of a nearby stream, *Eos Trans. AGU*, 22, 734–738.
- van der Kamp, G. (2001), Methods for determining the in situ hydraulic conductivity of shallow aquitards—An overview, *Hydrogeol. J.*, 9(1), 5–16.
- Wallace, R. B., Y. Darama, and M. D. Annable (1990), Stream depletion by cyclic pumping of wells, *Water Resour. Res.*, 26(6), 1263–1270.
- Wilson, J. L. (1993), Induced infiltration in aquifers with ambient flow, *Water Resour. Res.*, 29(10), 3503–3512.
- Zlotnik, V. A., and H. Huang (1999), Effect of partial penetration and streambed sediments on aquifer response to stream stage fluctuations, *Ground Water*, 37(4), 599–605.
- Zlotnik, V. A., and V. L. McGuire (1998), Multi-level slug tests in highly permeable formations: 2. Hydraulic conductivity identification, method verification, and field applications, *J. Hydrol.*, 204, 283–296.
- Zlotnik, V. A., R. F. Spalding, M. E. Exner, and M. E. Burbach (1993), Sampling of non-point source contamination in high-capacity wells, *Water Sci. Technol.*, 28(3–5), 409–413.
- Zlotnik, V. A., H. Huang, and J. J. Butler Jr. (1999), Evaluation of stream depletion considering finite stream width, shallow penetration, and properties of streambed sediments, in *Proceedings of Water 99, Joint Congress*, pp. 221–226, Inst. of Eng., Barton, ACT, Australia.

V. A. Zlotnik, Department of Geosciences, University of Nebraska at Lincoln, 214 Bessey Hall, Lincoln, NE 68588-0340, USA. (vzlotnik@unl.edu)