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Aquitard effect on drawdown in water table aquifers

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[1] The effects of an aquitard on drawdown in an overlying unconfined aquifer can be represented by a drainage-type term at the aquitard-aquifer interface. The functional form of this boundary condition is similar to the Boulton-Neuman boundary condition used for water table aquifers except the kernel contains an inverse square root of time instead of a negative exponential. Type curves using the new boundary condition were obtained in semianalytical form. Examples for several representative conditions show that the effect of the underlying aquitard can contribute to the type curve at early and intermediate times, but the effect becomes negligible at late times.

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1. Introduction

[2] Interpretation of pumping tests in unconfined aquifers remains a matter of ongoing studies due to the complexity of the multiparametric problem. *Moench* [1995, 1997, 2004] and *Moench et al.* [2001] investigated effects of the unsaturated zone on the interpretation of unconfined aquifer pumping tests. *Chen and Ayers* [1998] compared existing models for field data interpretations. *Grimstad* [2002] explored the unaccounted sources of the aquifer recharge. *Kollet and Zlotnik* [2005] focused on effects of heterogeneity and return flow. After multiple interpretations of two detailed data sets (Borden and Cape Cod sites) as benchmark cases, it is apparent that physical processes are too complex to be represented by a simple physical model [*Gilham and Roy*, 2004].

[3] One factor that was not investigated in all cited studies previously is effect of water release from an underlying aquitard. *Neuman and Witherspoon* [1968] studied flow in an aquitard of finite or infinite thickness adjacent to a leaky confined aquifer. They postulated that the aquitard did not play an important role in the aquifer drawdown, but the aquitard drawdown is fully defined by flow in the aquifer. Contrast between values of hydraulic conductivity implies a minor flux across the aquifer-aquitard interface. However, if the aquitard diffusivity is comparable to the diffusivity of the aquifer, the cross flow at the aquifer-aquitard interface may affect the head in the aquifer. The purpose of this note is to investigate the aquitard effect on the unconfined aquifer-aquitard system.

2. An Aquifer-Aquitard Problem

2.1. Problem Statement

[4] Consider a point sink and a partially penetrating well in an anisotropic unconfined aquifer above an infinitely

thick aquitard (Figure 1). The origin of the Cartesian x , y , and z coordinates is at the aquifer/aquitard interface. The semi-infinite aquitard extends downward from $z = 0$ to $-\infty$. An aquifer has an initial saturated thickness d and extends laterally to infinity. The decline of water table is assumed to be much smaller than d , thus a linearized free surface boundary condition will be used for the water table [e.g., *Neuman*, 1974].

[5] Consider a point sink at (x_0, y_0, z_0) in an unconfined aquifer. Transient groundwater flow obeys the following governing equation for unconfined aquifer drawdown $s(x, y, z, t)$:

$$S_s \frac{\partial s}{\partial t} = K_x \frac{\partial^2 s}{\partial x^2} + K_y \frac{\partial^2 s}{\partial y^2} + K_z \frac{\partial^2 s}{\partial z^2} + Q \delta(x - x_0) \delta(y - y_0) \delta(z - z_0),$$

$$0 < z_0, z < d \quad (1)$$

where S_s is the specific storage, t is time, K_x , K_y , and K_z are the principal hydraulic conductivities along the x , y , and z axes, respectively, Q is the point sink strength (positive for pumping), and $\delta(u)$ is the Dirac delta function.

[6] A one-dimensional vertical flow equation in the aquitard for drawdown $s_a(x, y, z, t)$ is as follows [*Neuman and Witherspoon*, 1968]:

$$S_a \frac{\partial s_a}{\partial t} = K_a \frac{\partial^2 s_a}{\partial z^2}, \quad -\infty < z < 0 \quad (2)$$

where S_a and K_a are the specific storage and vertical hydraulic conductivity of the aquitard, respectively.

[7] The initial condition is:

$$s(x, y, z, 0) = s_a(x, y, z, 0) = 0. \quad -\infty < x, y < \infty, -\infty < z < d \quad (3)$$

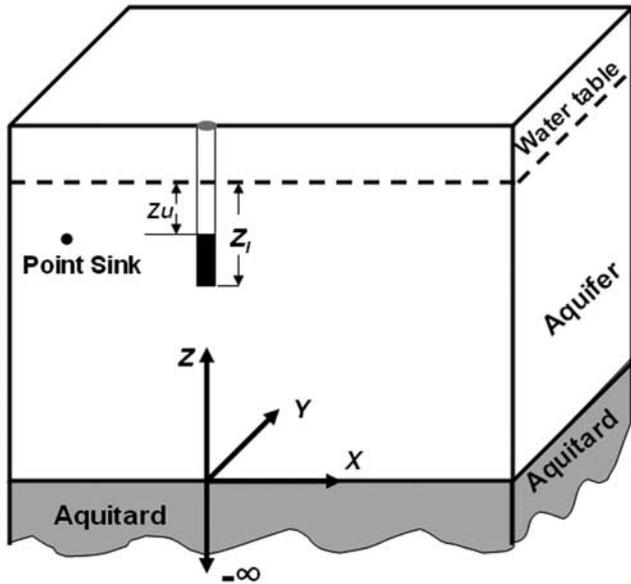


Figure 1. A point sink and a well in an unconfined aquifer-aquitard system.

The instantaneous drainage condition at the water table with specific yield S_y is as follows:

$$K_z \partial s(x, y, d, t) / \partial z + S_y \partial s(x, y, d, t) / \partial t = 0 \quad (4)$$

At large depths in the aquitard, the following boundary condition holds:

$$s_a(x, y, z \rightarrow -\infty, t) = 0, \quad (5)$$

and the lateral boundary conditions at the infinite horizontal distance from sink are

$$\begin{aligned} s(\pm\infty, y, z, t) &= s(x, \pm\infty, z, t) = s_a(\pm\infty, y, z, t) \\ &= s_a(x, \pm\infty, z, t) = 0. \end{aligned} \quad (6)$$

[8] Drawdown and flux are subject to the continuity conditions at the aquifer-aquitard interface:

$$s(x, y, 0, t) = s_a(x, y, 0, t), \quad K_z \frac{\partial s(x, y, 0, t)}{\partial z} = K_a \frac{\partial s_a(x, y, 0, t)}{\partial z}. \quad (7)$$

[9] If a delayed yield Boulton-Neuman model is used, equation (4) is replaced by

$$K_z \frac{\partial s(x, y, d, t)}{\partial z} = -\alpha_1 S_y \int_0^t \frac{\partial s(x, y, d, \tau)}{\partial \tau} \exp[-\alpha_1(t - \tau)] d\tau. \quad (8)$$

where α_1 is the empirical constant for drainage from the unsaturated zone. $1/\alpha_1$ is the so-called “delay index” [e.g., Moench, 1995]. If $\alpha_1 \rightarrow \infty$, this equation reduces to the instantaneous drainage condition (4).

2.2. Reduction of the Aquifer-Aquitard Problem to a Single-Aquifer Problem

[10] *Streltsova* [1988, p. 368] presented an analysis of cross flow at the aquifer-aquitard interface for delineation of the head in the aquitard. Instead, we use this approach for excluding the consideration of the aquitard head and focusing on the head in the aquifer.

[11] Applying a Laplace transform to equations (2), (5) and (7) results in

$$p' S_a \bar{s}_a = K_a \frac{d^2 \bar{s}_a}{dz^2}, \quad \bar{s}_a = 0 \text{ at } z = -\infty, \text{ and } \bar{s}_a = \bar{s} \text{ at } z = 0, \quad (9)$$

where \bar{s}_a and \bar{s} are the Laplace transforms of the aquitard and aquifer drawdowns, respectively, and p' is the Laplace transform parameter with respect to time t . The solution of this problem is as follows:

$$\bar{s}_a = \bar{s}|_{z=0} \exp\left(z \sqrt{\frac{p' S_a}{K_a}}\right), \quad z < 0. \quad (10)$$

[12] Using a Laplace transform of the flux continuity at the aquifer-aquitard interface (equation (7)), we obtain a condition at the base of the unconfined aquifer:

$$K_z \frac{\partial \bar{s}}{\partial z} = \sqrt{K_a S_a p'} \bar{s}. \quad (11)$$

[13] The inverse Laplace transform results in the boundary condition in the real time domain:

$$K_z \frac{\partial s}{\partial z} = \sqrt{K_a S_a} \int_0^t \frac{\partial s}{\partial \tau} \frac{d\tau}{\sqrt{\pi(t - \tau)}}. \quad (12)$$

2.3. Significance of the New Boundary Condition

[14] Boundary condition equation (12) was obtained as a replacement of equations that describe the head in the aquitard. Equations (8) and (12) have similar forms, namely

$$K_z \frac{\partial s}{\partial z} = \pm \int_0^t \frac{\partial s}{\partial \tau} G(t - \tau) d\tau, \quad (13)$$

where signs + or – are related to the boundaries orientation, and $G(t)$ is a kernel function of the integral on the right-hand side of the boundary condition:

$$G(t) \propto \begin{cases} \exp(-\alpha_1 t) & \text{for water table condition} \\ t^{-m} & \text{for aquifer – aquitard boundary condition; } m = 1/2 \end{cases} \quad (14)$$

[15] *Boulton* [1954] made a formal effort to account for noninstantaneous drainage from above the falling water table and assumed the exponential functional form of the kernel. Later, *Youngs* [1960] and *Gardner* [1962] showed that certain assumptions about aquifer properties could lead to the exponential drainage response [see also *Hillel*, 1998, p. 482]. Interestingly, *Horton* [1940] used exponential term

Table 1. Definitions of Dimensionless Terms

| | Definitions |
|---------------------------------|---|
| Drawdown and time variables | $s_D = \frac{4\pi K d s}{Q}, t_D = \frac{K}{S_y d^2} t, t_{Dy} = \frac{K}{S_y d} t$ |
| Aquifer and aquitard parameters | $K = (K_x K_y K_z)^{1/3}, \alpha_{1D} = \frac{S_y d^2 \alpha_1}{K}, \eta_d = \frac{K_x S_y}{S_y K_x}, \eta_K = \frac{K_x}{K_z}, \alpha_x = \left(\frac{K}{K_x}\right)^{1/2}, \alpha_y = \left(\frac{K}{K_y}\right)^{1/2}, \alpha_z = \left(\frac{K}{K_z}\right)^{1/2}, \sigma = \frac{S_y d}{S_y \alpha_z}$ |
| Spatial parameters | $x_D = \alpha_x \frac{x}{d}, y_D = \alpha_y \frac{y}{d}, z_D = \alpha_z \frac{z}{d}, z_{1D} = \alpha_z \frac{z_1}{d}, z_{uD} = \alpha_z \frac{z_u}{d}, x_{0D} = \alpha_x \frac{x_0}{d}, y_{0D} = \alpha_y \frac{y_0}{d}, z_{0D} = \alpha_z \frac{z_0}{d}, r_D^2 = x_D^2 + y_D^2$ |

for description of infiltration. This kernel is a useful concept in many practical applications [e.g., *Moench et al.*, 2001]. Although *Neuman* [1979] argued that this term could be explained by instantaneous drainage, *Moench* [2004] went further and advocated superposition of several exponential kernels with different constants. This approach yielded a good match between observed and simulated responses for the Cape Code data set.

[16] Our boundary condition equation (12) offers another functional form of kernel. This inverse square root time kernel ($m = 1/2$) is used widely in vadose zone hydrology. For example, it enters *Philip's* [1957] infiltration equation. In a more general case ($m \neq 1/2$), this function enters *Kostiakov's* [1932] infiltration equation or the formula for drainage by *Richards et al.* [1956] [see also *Hillel*, 1998, pp. 392, 465]. The function $1/\sqrt{t}$ has a longer tail than the exponential function $\exp(-\alpha_1 t)$, indicating a “longer lasting” drainage. Thus the empirical drainage term by *Boulton* [1954] can be regarded as an intermediate case between the instantaneous drainage and the case of $1/\sqrt{t_D}$.

3. Laplace Transform Solution

3.1. Solution for a Point Sink in an Unconfined Aquifer

[17] Using the dimensionless parameters defined in Table 1 and denoted by subscript “ D ,” one arrives at a dimensionless initial boundary value problem. Application of the Laplace transform to this problem results in:

$$p \bar{s}_D = \nabla^2 \bar{s}_D + \frac{\partial^2 \bar{s}_D}{\partial z_D^2} + \frac{4\pi \delta(\vec{r}_D - \vec{r}_{0D}) \delta(z_D - z_{0D})}{p}, \quad (15)$$

$$\sigma \partial \bar{s}_D(\vec{r}_D, \alpha_z, p) / \partial z_D + p \bar{s}_D(\vec{r}_D, \alpha_z, p) = 0, \quad (16)$$

$$\bar{s}_D(\vec{r}_D \rightarrow \infty, z_D, p) = 0, \quad (17)$$

$$\eta_K \frac{\partial \bar{s}_D(\vec{r}_D, 0, p)}{\partial z_D} + \sqrt{\eta_d p} \bar{s}_D(\vec{r}_D, 0, p) = 0, \quad (18)$$

where ∇^2 is a two-dimensional Laplace operator in cylindrical coordinates, p is the Laplace transform parameter that corresponds to the dimensionless time t_D , \bar{s}_D is the Laplace transform for the dimensional drawdown function s_D ; $\vec{r}_D = (x_D, y_D)$ and $\vec{r}_{0D} = (x_{0D}, y_{0D})$ are dimensionless vectors of the observation point and sink, respectively, in two-dimensional space, and

$$r_D = \sqrt{x_D^2 + y_D^2}.$$

[18] The solution of equation (15) subject to conditions in equations (16)–(18) can be written in a general form [*Dougherty and Babu*, 1984; *Moench*, 1997]:

$$\bar{s}_D(p) = \sum_{n=0}^{\infty} H_n(\vec{r}_D, \vec{r}_{0D}, p) \cos(\omega_n z_D + \mu_n). \quad (19)$$

Substitution of equation (19) into the water table condition equation (16) and into equation (18) results in:

$$\omega_n \tan(\omega_n \alpha_z + \mu_n) = p/\sigma, \quad \omega_n \tan(\mu_n) = -\sqrt{\eta_d p} / \eta_K, \quad n = 0, 1, \dots \quad (20)$$

Expansion of $\tan(\omega_n \alpha_z + \mu_n)$ into $\tan(\omega_n \alpha_z)$ and $\tan(\mu_n)$ terms results in an implicit expression for ω_n and an explicit equation for μ_n :

$$\left(\omega_n - \frac{p \sqrt{\eta_d p}}{\sigma \eta_K} \frac{1}{\omega_n} \right) \tan(\omega_n \alpha_z) = \frac{p}{\sigma} + \frac{\sqrt{\eta_d p}}{\eta_K},$$

$$\mu_n = -\tan^{-1} \left(\frac{\sqrt{\eta_d p}}{\eta_K \omega_n} \right), \quad n = 0, 1, \dots \quad (21)$$

[19] After ω_n and μ_n are known, one can substitute equation (19) into equation (15), multiply equation (15) by $\cos(\omega_n \alpha_z + \mu_n)$, and integrate from 0 to α_z in the z direction to obtain an equation for H_n . The details of computation are given by *Moench* [1997], *Zhan et al.* [2001], and *Zhan and Zlotnik* [2002, Appendix A]. The final solution for a point sink in the Laplace domain becomes

$$\bar{s}_D = \sum_{n=0}^{\infty} \frac{8 \cos(\omega_n z_{0D} + \mu_n) \cos(\omega_n z_D + \mu_n)}{p b(\omega_n, \mu_n)} K_0(\Omega_n |r_D - r_{0D}|), \quad (22)$$

where

$$b(\omega_n, \mu_n) = 2\alpha_z + \frac{\sin(2\omega_n \alpha_z + \mu_n) - \sin(\mu_n)}{\omega_n}, \quad \Omega_n = \sqrt{p + \omega_n^2} \quad (23)$$

and $K_0(u)$ is the second kind, zero-order modified Bessel function.

[20] After finding the Laplace transform solution $\bar{s}_D(\vec{r}_D, z_D, p)$ for $z_D > 0$, the Laplace transform solution $\bar{s}_{aD}(\vec{r}_D, z_D, p)$ for $z_D < 0$ is obtained from equation (10). The solutions in the real time domain can be obtained after applying the inverse Laplace transform [e.g., *Moench*, 1997, 1998; *Zhan et al.*, 2001; *Zhan and Zlotnik*, 2002].

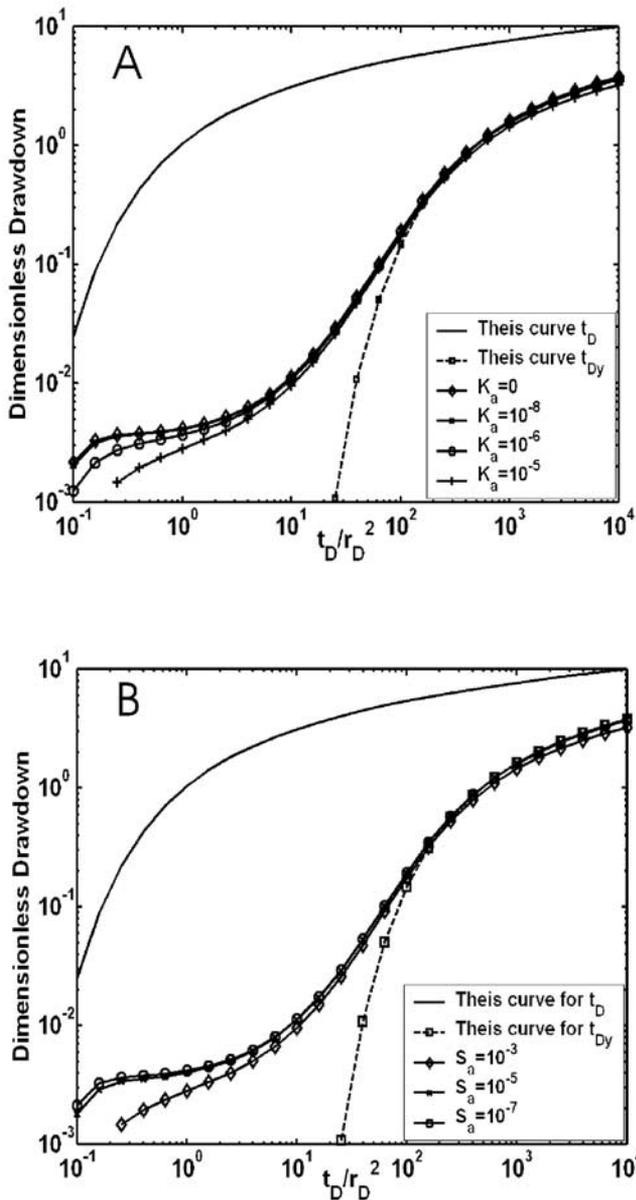


Figure 2. Type curves of dimensionless drawdown in an unconfined aquifer as a function of scaled dimensionless time for different aquitard parameters: (a) for a near field $(x, y, z) = (5 \text{ m}, 5 \text{ m}, 10 \text{ m})$ and (b) for a far field $(x, y, z) = (50 \text{ m}, 50 \text{ m}, 10 \text{ m})$. The early Theis curve (t_D) and later Theis curve (t_{Dy}) are plotted for reference.

3.2. Solution for a Partially Penetrating Well

[21] For a partially penetrating vertical well located at $(x_0, y_0) = (0, 0)$ and screened from z_l to z_u , the solution is obtained by superposing the point sink solution equation (22) along z_l to z_u :

$$\bar{s}_{w,D} = \frac{8}{z_{uD} - z_{lD}} \sum_{n=0}^{\infty} \frac{\cos(\omega_n z_{lD} + \mu_n) K_0(\Omega_n r_{lD})}{\omega_n p b(\omega_n, \mu_n)} \cdot [\sin(\omega_n z_{uD} + \mu_n) - \sin(\omega_n z_{lD} + \mu_n)], \tag{24}$$

where \bar{s}_{wD} represents the dimensionless drawdown in the Laplace domain, and dimensionless z_{lD} and z_{uD} correspond to z_l and z_u (Table 1). Substitution of $z_l = 0$ and $z_u = d$, or $z_{lD} = 0$ and $z_{uD} = \alpha_z$ yields solution for a fully penetrating well.

4. Discussion

[22] We will use several examples to illustrate the major traits of the drawdown in the unconfined aquifer stemming from nonzero aquitard permeability. We are particularly interested in situations where the aquitard has a much smaller value of hydraulic conductivity but much smaller value of specific storage than those of the aquifer. This relates to similar diffusivity values of both units. (Note that the Laplace transforms for the drawdown in the aquitard for a point sink or a well can be obtained from equations (22) and (24) respectively after substitution into equation (10). The solutions in the real time domain can be evaluated by using the inverse Laplace transform as mentioned above.)

[23] The default hydrologic parameters of the unconfined aquifer are as follows: the initial saturated thickness is $b = 20 \text{ m}$, isotropic and homogeneous hydraulic conductivity $K = 10^{-3} \text{ m/s}$, specific storage $S_s = 2.0 \times 10^{-5} \text{ m}^{-1}$, specific yield $S_y = 0.2$ in the first example and varies among 0.002, 0.02, and 0.2 in the second example. The later Theis curves shown in both examples are plotted with specific yield of 0.2. The fully penetrating pumping well has a pumping rate $Q = 0.01 \text{ m}^3/\text{s}$. Other necessary parameters are given in figure captions and legends. Definitions of dimensionless terms are given in Table 1.

[24] In the first two examples, the effect of the unconfined aquifer drawdown on differing aquitard hydraulic conductivity and specific storage is illustrated. The first example (Figure 2a) shows the sensitivity of aquifer drawdown to aquitard hydraulic conductivity $K_a = 0$ (impermeable), 10^{-8} , 10^{-6} , 10^{-5} m/s with the same $S_a = 10^{-3} \text{ m}^{-1}$ at a near field at a point $(x, y, z) = (5 \text{ m}, 5 \text{ m}, 10 \text{ m})$. The aquifer-aquitard contrast in hydraulic conductivity ranges from ∞ to 10^2 . The second example (Figure 2b) shows the sensitivity to different aquitard specific storage values $S_a = 2 \times 10^{-3}$, 2×10^{-5} , and $2 \times 10^{-7} \text{ m}^{-1}$ with the same $K_a = 10^{-5} \text{ m/s}$ at a far field $(x, y, z) = (50 \text{ m}, 50 \text{ m}, 10 \text{ m})$. Several points are notable. There is a weak influence of the aquitard parameters upon the type curves at the early or intermediate stages (Figures 2a and 2b). The drawdown at later time is slightly less than that predicted by the later Theis curve. In general, the difference is negligible for near field points and moderate for far field points.

[25] Specific yield is one of the key parameters of an unconfined aquifer, and two more examples are used to compare the sensitivity of the drawdown to specific yield and aquitard parameters. The first comparison (Figure 3a) involves a series of type curves for specific yields of 0.2, 0.02, and 0.002 in an aquifer with an impermeable base with a case of a leaky aquitard ($K_a = 10^{-5} \text{ m/s}$ and $S_a = 2 \times 10^{-3} \text{ m}^{-1}$) and a permeable base at the near field point (coordinates are given in the first example). All curves are identical at early time and separate to different branches starting at intermediate stage, however, there is a negligible difference between the cases of the leaky and impermeable

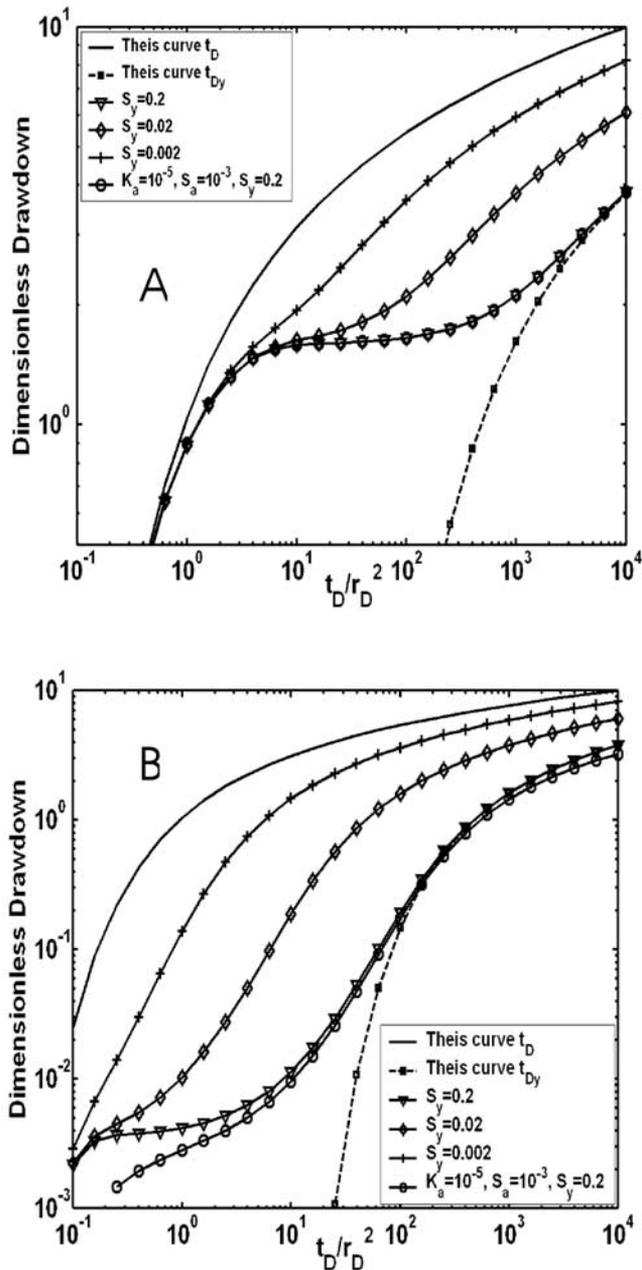


Figure 3. Type curves of dimensionless drawdown in an unconfined aquifer as a function of scaled dimensionless time for different aquifer and aquitard parameters: (a) for a near field $(x, y, z) = (5 \text{ m}, 5 \text{ m}, 10 \text{ m})$ and (b) for a far field $(x, y, z) = (50 \text{ m}, 50 \text{ m}, 10 \text{ m})$. The early Theis curve (t_D) and later Theis curve (t_{Dy}) are plotted for reference.

aquifer base for $S_y = 0.2$ at the near field. There is a slight difference between the cases of leaky and impermeable bases with the same specific yield 0.2 in the far field (Figure 3b). In general, the unconfined aquifer drawdown is more sensitive to the specific yield than to the aquitard parameters. This means that drainage of water from the unsaturated zone starts at later stages, and slow water release from the aquitard becomes unimportant. For a

practical case of a finite diameter well, an aquitard should have even smaller effect.

5. Conclusions

[26] We have investigated the aquitard effect on the results of pumping tests in unconfined aquifers using semi-analytical drawdown solutions for the aquifer and aquitard. The following conclusions can be reached from this study: (1) The boundary condition that describes water release from an aquitard to the aquifer is similar to a drainage-type boundary condition. (2) Water release from aquitard can be described using kernel $1/\sqrt{t}$, instead of the $\exp(-\alpha_1 t)$ kernel, which is used for interpretation of pumping tests in water table aquifers. (3) Aquifer-aquitard water exchange plays a minor role often at short and intermediate times. At large times, the role of the aquitard seems negligible.

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