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Part Three

WAVE MOTION AND SOUND

20

Wave Motion

20-1 Vibrating Bodies and Wave Motion

Wave motion is an important method of transferring energy from one place to another without involving the actual transfer of matter. When a pebble is dropped into a still pool, some of the kinetic energy of the pebble is used to generate the ripples which spread out in all directions over the surface of the pool. When the ripples pass by a floating object, such as a bit of cork, the cork bobs up and down, having acquired its kinetic energy of vibration from the ripple system. The ripples thus serve to deliver some of the energy of the pebble to the distant cork. Although the ripple system is seen to move, there is no net flow of the water, for the cork simply bobs up and down and does not move in the direction of the ripples. Since the cork merely provides visible evidence of the behavior of the water, we may infer that the motion of the water is one of vertical oscillation, while the motion of the wave is horizontal; that is, *the oscillations of the water are transverse to the direction of propagation of the wave*. A wave motion in which the vibrations of the medium are perpendicular to the direction of propagation is called a *transverse wave*. The waves set up in a taut string when one end of the string is vibrated in simple harmonic motion are transverse waves.

A second type of wave motion may be demonstrated by the use of a long helical spring. If such a spring is suspended with its axis vertical, and one end of the spring is caused to oscillate in the vertical direction, these oscillations are transmitted down the spring as a succession of compressions and rarefactions of the spring. The direction of oscillation of any part of the spring is parallel to its axis and therefore parallel to the direction of propagation of the disturbance. A wave motion in which the oscillations of the medium are parallel to the direction of propagation of the wave is called a *longitudinal wave*.

Both transverse and longitudinal waves may be propagated within a continuous medium, provided that the medium has appropriate elastic properties. For the transmission of a longitudinal wave, a displacement of one element of the medium in the direction of propagation must be capable of exerting a force on an adjacent element. In a solid, this type of stress-strain relationship is described by Young's modulus of elasticity, while in a fluid medium the bulk modulus is the means through which the displacement of one volume element generates a force on an adjacent element. Thus both solids and liquids may transmit longitudinal waves.

The transmission of a transverse wave in a continuous medium requires the existence of a shear modulus, for the displacement of one part of the medium in a direction perpendicular to the direction of propagation must generate a force on an adjacent element which is transverse to the direction of propagation. Gases are not capable of exerting shearing forces, hence they cannot transmit transverse waves. Since sound is a wave motion generated by a vibrating body and transmitted through air, we must infer that sound waves are longitudinal waves.

20-2 Equation of Wave Propagation

When a wave is propagated in an elastic medium, each particle of the medium vibrates in simple harmonic motion. The *frequency* f of each vibrating particle is the same as the frequency of the source of vibration. Just as in the case of simple harmonic motion, it is sometimes convenient to describe this oscillation by its *period* T , or its *angular frequency* ω . If we examine the appearance of the wave in space at any one instant of time, the displacements of the particles of the medium from their equilibrium positions follow a sine or cosine function of the space coordinates. The adjacent particles of the medium are out of phase with each other. The wavelength λ is the distance between two successive crests, or two successive troughs, or between any two successive particles whose simple harmonic motion is at the same phase angle. We may relate the velocity v at which the wave is propagated to the wavelength λ and the period T by observing that in the time of one period the wave has advanced by a distance of one wavelength. Thus

$$v = \frac{\lambda}{T} = \lambda f, \quad (20-1)$$

for the frequency is the reciprocal of the period: $f = 1/T$.

When a wave moves to the right a small distance Δx in a time Δt , as shown in Figure 20-1, a point P may be displaced upward to a new position

P' , while a second point Q is displaced downward to a new position Q' . The particles at P and Q are therefore clearly out of phase with each other.

The general equation of simple harmonic motion was given in Chapter 12 as

$$y = A \sin(\omega t + \phi),$$

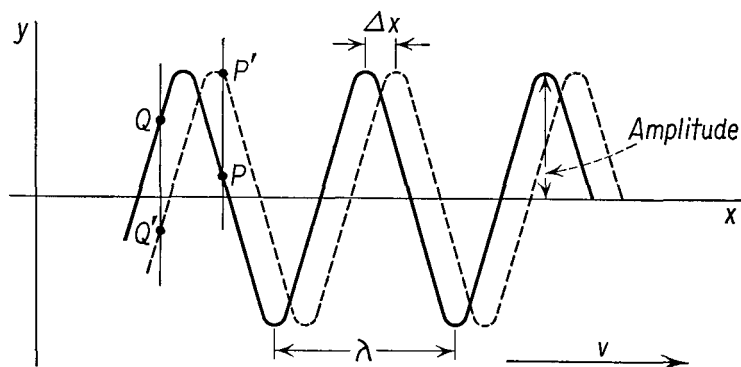


Fig. 20-1 When a wave moves to the right a distance Δx , the point Q of the medium is displaced downward to Q' , while the point P is displaced upward to P' .

where A is the amplitude of the motion, and the angular frequency ω is given by

$$\omega = 2\pi f = \frac{2\pi}{T}.$$

Substituting for ω , we find

$$y = A \sin\left(\frac{2\pi t}{T} + \phi\right) \quad (20-2)$$

as another form of the general equation of simple harmonic motion; that is, this equation describes the simple harmonic motion of any one particle when suitable values are inserted for the parameters A , T , and ϕ . As we have seen, the phase of a particle along a wave depends upon its position. The phase changes at a regular rate with displacement along the wave, and in a distance of one wavelength the phase changes in amount by 2π . For a wave which is moving to the right, we may set

$$\phi = -\frac{2\pi x}{\lambda}$$

in Equation (20-2) to obtain

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right). \quad (20-3)$$

For a particular instant of time t , fixed in value, we observe that the displacement y of a particle from its equilibrium position is given by a sine curve of amplitude A . The displacement of a point at any position along the wave is repeated a distance λ farther along. If the position x is held fixed, the value of the displacement y at this point in space varies periodically, repeating its motion after a time interval T ; this is the period of its motion.

Let us consider the direction in which the wave described by Equation (20-3) is propagated. When we watch the motion of a system of ripples we can tell which way the wave is going by noting the translational motion of a particular high light of one of the ripples of the system. At the position of the high light, the displacement of the water from its equilibrium position is always the same. Thus the high light is a position of constant phase angle. In Equation (20-3) a constant phase angle is represented by a constant value of the quantity within the parentheses. Thus Equation (20-3) must describe a wave of amplitude A which moves in the direction of increasing x ; that is, the wave must move to the right. If, instead of the minus sign in the parentheses, we had written a plus sign, it would have been necessary for x to decrease as t increased in order to keep the value of the parentheses constant. The wave would then have moved to the left.

The speed of the wave represented by Equation (20-3) may be obtained by noting that, if t is increased by one period T , the wave need have moved a distance of one wavelength λ in that time for the parentheses to remain at constant value. Thus the speed of the wave is given by Equation (20-1). The speed v represented by the equation $v = \lambda/T$ is called the *phase velocity* of the wave, for it represents the speed of points of constant phase.

Equation (20-3) is a general equation of wave motion and is used much as one uses the general equation for a straight line in analytic geometry. Specific cases of wave motion are described by substituting appropriate numbers for A , λ , and T into the general equation.

Illustrative Example. A sine wave of amplitude 1 m and of wavelength 4 m is moving to the right with a speed of 2 m/sec. Find the displacement of a point 25 cm to the left of the origin when $t = 1.5$ sec.

A sine wave moving to the right is described by the equation

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right).$$

As indicated in the problem, $A = 1$ m, $\lambda = 4$ m; T may be obtained from the equation

$$v = \frac{\lambda}{T},$$

so that

$$T = \frac{\lambda}{v} = \frac{4 \text{ m}}{2 \text{ m/sec}} = 2 \text{ sec.}$$

We wish to find y when $x = -0.25$ m and $t = 1.5$ sec. Substituting into the equation, we have

$$\begin{aligned}
 y &= 1 \text{ m} \times \sin 2\pi \left(\frac{1.5 \text{ sec}}{2 \text{ sec}} - \frac{-0.25 \text{ m}}{4 \text{ m}} \right) \\
 &= 1 \text{ m} \times \sin 2\pi \left(\frac{13}{16} \right) \\
 &= 1 \text{ m} \times \sin 292^\circ \\
 &= -0.93 \text{ m}.
 \end{aligned}$$

20-3 Huygens' Principle

When a source of waves is placed in a uniform continuous medium, waves spread out from the source in all directions. The locus of points of constant phase is a spherical surface, just as the locus of points of constant phase in a two-dimensional ripple is a circle. Such a wave in a continuous medium

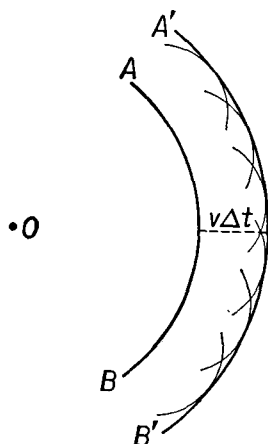


Fig. 20-2 Huygens' construction for determining the position of wave front AB after a time interval Δt . $A'B'$ is the new position of the wave front.

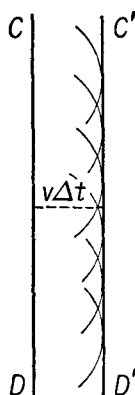


Fig. 20-3 Huygens' construction for determining $C'D'$, the new position of the plane wave front which started at CD at a time Δt earlier.

is called a *spherical wave*. We may restrict the spherical wave from a distant source by means of appropriate apertures so that only a very small portion of the wave passes through the aperture. In this case we may approximate the wave front by a plane surface. Such a wave is referred to as a *plane wave*.

The progress of a wave under a variety of conditions can be predicted

with the aid of a principle first enunciated by the Dutch physicist Christian Huygens (1629–1695). According to *Huygens' principle*, each point in a wave front can be considered as a source of waves, and the new position that the wave front will occupy after the lapse of a small time interval Δt can be found by drawing the envelope to all of the small waves from all of the individual points on the initial wave front at the beginning of this time interval. To illustrate the use of Huygens' principle, let us consider the progress of a spherical wave front from a point source O , as shown in Figure 20-2. If the arc of circle AB represents the position of a section of the wave front at a certain time t , each point on the wave front is imagined to emit spherical waves which are of radius $v \Delta t$ at time $t + \Delta t$. The new wave front $A'B'$ is the envelope of all these small spheres. The same type of construction can be used to find the subsequent position of any type of wave front. Figure 20-3 shows a plane wave front CD progressing with speed v . The new position of the wave front after a short time interval Δt is $C'D'$, the envelope of all the small waves, each of radius $v \Delta t$ emitted by each of the points on the wave front CD .

20-4 Reflection of a Wave

When a wave which is traveling in one medium reaches the surface of a second medium, part of the wave is *reflected* back into the first medium, and the rest penetrates into the second medium and is said to be *refracted* into

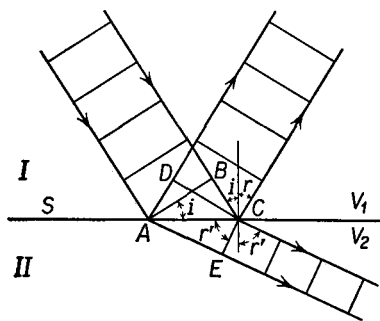


Fig. 20-4 Reflection and refraction of a plane wave at a surface separating two media.

the second medium. Let us suppose that a plane surface S separates two media I and II in which the velocities of propagation of the wave are v_1 and v_2 respectively, as shown in Figure 20-4. The position of the reflected wave and the direction of its motion relative to that of the incident wave can be found with the aid of Huygens' principle. The angle that each incident wave front makes with the surface is called the *angle of incidence* i , while the angle each reflected wave front makes with the surface is the *angle of reflection* r . A line drawn in the direction of motion of the wave is called a *ray*; a ray is perpendicular to a wave front. A line drawn perpendicular to the interface between the two media is called a *normal*. Thus the angle of incidence is equal to the angle between the incident ray and the normal, while the angle of reflection is the angle between the reflected ray and the normal. Figure 20-5 shows

the plane wave advancing toward the surface S at an angle of incidence i . Let us consider the plane wave front represented by the line AB such that the point A has just reached the surface S , while B is still moving toward it; The angle between AB and the surface S is the angle i . If the surface had not been there, the wave front AB would have advanced to the position $A'C$ in a time Δt . But as the different parts of the wave front reach the surface, each point on the surface, such as $A, e, f, g, h, \dots C$, becomes a source of waves. By the time the point B on the incident wave front reaches the position C on the surface S , the Huygens wavelet which started

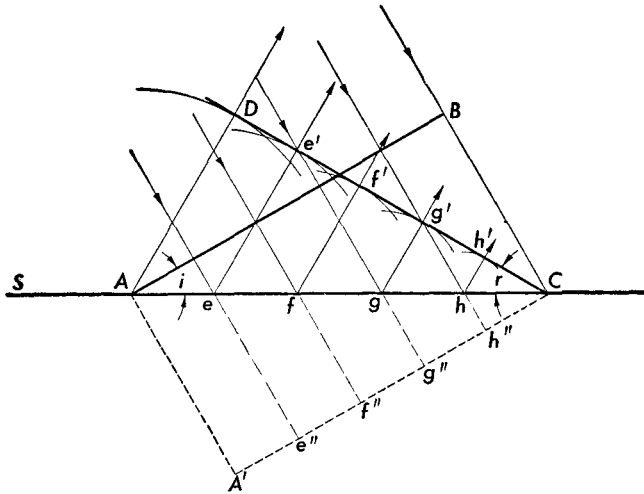


Fig. 20-5 Huygens' construction for determining the position and direction of motion of the wave front reflected by surface S . AB is the incident wave front and CD is the reflected wave front.

from A has grown to radius AD . Similarly, the wavelet initiated at point e when the incident wave front struck the surface has grown to radius ee' , and so on. The envelope $De'f'g'h'C$ tangent to these spherical wave fronts is the plane wave front reflected by the surface. The angle of reflection r is the angle between CD and the surface S . Since the velocity of propagation is a property of the medium and is the same for the incident and reflected waves, we see from the construction that the *angle of incidence is equal to the angle of reflection*. The direction of the reflected wave is indicated by the arrows on the rays AD, ee' , and so on. The reflected waves appear to originate somewhere behind the reflecting surface.

If a wave from some point source P strikes a flat surface and is reflected from it, the reflected wave will appear to come from a point P' behind the surface. This point P' is the *image* of P and is located directly

behind the surface so that the surface is the perpendicular bisector of the line PP' , as shown in Figure 20-6. An observer in the region QQ' who wishes to determine the source of the waves does so by extending the direction of the rays reaching him in the backward direction until they intersect in a point.

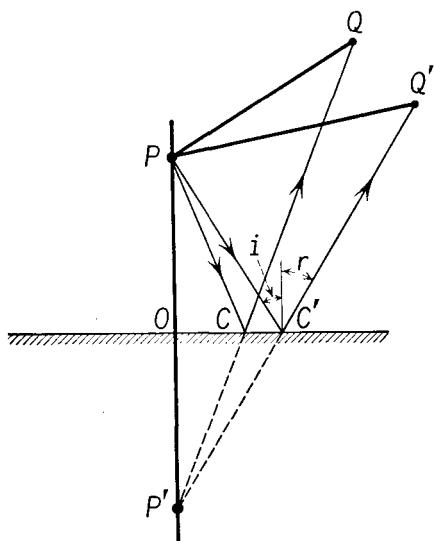


Fig. 20-6 Image formation by a plane mirror; P' is the image of P .

The direct rays PQ and PQ' thus appear to originate at point P , while the reflected rays CQ and $C'Q'$ appear to originate at the point P' , the image point. It may be readily shown from the construction that the distance OP is equal to the distance OP' by making use of the fact that the angle of incidence is equal to the angle of reflection.

The walls of a room usually act as mirrors for the sounds produced in the room. In rooms of average size these reflections add to the intensity of the sounds. In a large auditorium the reflected sound may reach the hearer a considerable time after he has received the directly transmitted sound. In the case of

speech this effect may be very objectionable; in the case of music the overlapping of different sounds may be pleasing to the ear.

If a taut string is struck a blow near one end, a single *pulse* is formed which travels down the string toward the other end, as shown in Figure 20-7(a). When the pulse strikes the wall at the end of the string, it is reflected so that both the direction of propagation and the direction of the deflection are opposite to that of the original pulse as shown in Figure 20-7(b). If we regard the upward deflection of the string as positive, the downward deflection of the reflected pulse is negative, and we may describe the alteration in the sense of the pulse by saying that a *phase change of 180° occurs on reflection*.

Another way of viewing the phenomenon of reflection is to imagine that the surface from which a wave is reflected is truly a mirror, as in Figure 20-7(c), and that all events which occur in fact on the left-hand side are reproduced in the opposite sense on the right-hand, or image, side. Thus if an upward pulse is initiated at the point P of a real string, a downward pulse is initiated at the image point P' of the image string. Both pulses may be imagined to move toward and through the wall at the same speed. When the two pulses reach the wall, the two opposite deflections

cancel, and there is no deflection of the string, in agreement with the physical restriction that the string is tied to the wall. We may imagine that the image pulse then passes through the wall and becomes visible as a real reflected pulse, as in Figure 20-7(d).

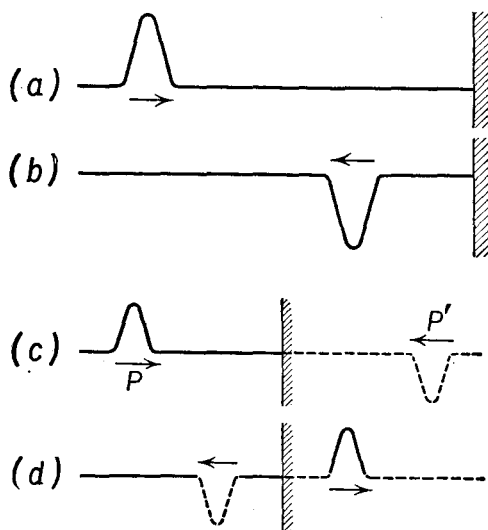


Fig. 20-7 Change of phase of a pulse on reflection from a wall.

20-5 Standing Waves

When a particle is simultaneously subjected to two vector displacements, its resultant displacement is the vector sum of the two individual displacements. At a point in a medium where the paths of two waves intersect, the medium is simultaneously displaced by the two waves, so that its resultant displacement is the vector sum of the individual displacements. This is known as the *principle of superposition*. We have already utilized the concept of superposition in the last paragraph of the preceding section.

When a wave strikes a reflecting surface normally, that is, at zero angle of incidence, the wave is reflected back at the same angle, and consequently along the same line. If a continuous wave is propagated in this medium, the incident and reflected waves will *interfere* with each other. If the two waves traveling in opposite directions through the medium have the same wavelength and the same amplitude, their effect is to set up steady vibrations, called *standing waves*.

The existence of standing waves may be demonstrated analytically by use of the general equation of wave motion. Two waves of the same wavelength, velocity, and amplitude, moving in opposite directions, may be

represented by the equations

$$y_1 = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$$

for the wave moving to the left, and

$$y_2 = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

for the wave moving to the right. The resultant displacement y of the medium by the simultaneous application of these two waves is the sum of the deflections due to the individual waves. Thus

$$y = y_1 + y_2.$$

From trigonometry we recall that

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y.$$

Applying this formula to the equations for y_1 and y_2 , and adding to find y , we obtain

$$y = 2A \cos 2\pi \frac{x}{\lambda} \sin 2\pi \frac{t}{T}. \quad (20-4)$$

Unlike Equation (20-3), the above equation no longer displays a wave in motion, for there is no longer a moving point of constant phase. Instead we see that the amplitude of vibration of a point in the medium of coordinate x is given by the factor multiplying $\sin 2\pi \frac{t}{T}$ and is given by $2A \cos 2\pi \frac{x}{\lambda}$. The amplitude of vibration varies from place to place on the string, and, at positions separated by half a wavelength, the amplitude of the vibration is zero. Points of zero vibration are called *nodes*. Midway between two nodes the vibration is a maximum, at points called *loops* or *antinodes*. From the above equation we note that two successive nodes are separated by a distance of half a wavelength.

If two wave motions are simultaneously imposed on a string by attaching identical tuning forks to its opposite ends, or, more simply, by attaching a vibrating tuning fork to one end of the string while the other end is attached to a rigid reflecting wall, the conditions for the generation of standing waves are fulfilled. If the string is properly adjusted, it appears to vibrate in segments. Since the eye cannot follow the vibrations with sufficient rapidity, an observer sees the string clearly where it is moving slowly and sees a blur where it is moving rapidly. Consequently, the observer sees only the envelope within which all vibrations take place, described by the function

$$y = \pm 2A \cos 2\pi \frac{x}{\lambda}.$$

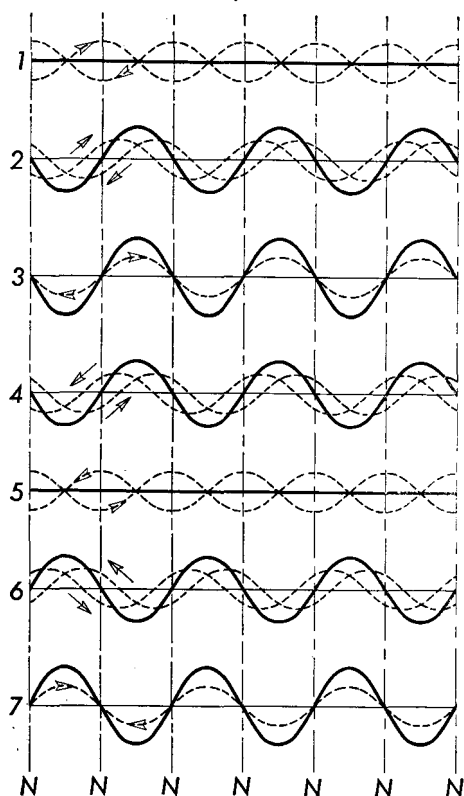


Fig. 20-8 Method of adding two waves of equal wavelengths and equal amplitudes traveling in opposite directions to produce a stationary wave.

The addition of two waves of equal amplitude and wavelength traveling in opposite directions is illustrated graphically in Figure 20-8. The dashed lines indicate the individual waves moving in opposite directions, while the heavy lines show the sum of the two waves.

If a source of sound of constant frequency and intensity is operated in a room, the walls, floor, and ceiling of the room are reflectors of sound waves, and standing waves are set up in the room. At some positions in the room, the

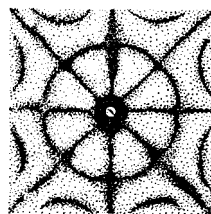


Fig. 20-9 Standing wave pattern of a vibrating plate.

sound intensity is a minimum; these are the nodes of the standing wave pattern. At other positions in the room, the intensity is a maximum; these are the positions of the antinodes.

The vibrations of tuning forks, of strings in musical instruments, of air columns such as those in organ pipes, and of bars and plates can all be analyzed into sets of standing waves in the substance. To detect the standing waves on a vibrating plate, sand or fine powder is sprinkled onto the plate. The standing wave pattern obtained on a horizontal plate under one specific set of conditions is shown in Figure 20-9. Each particular standing wave pattern is called a *mode of vibration* and is excited at a particular frequency. If a structure, or part of a structure, is subjected to vibration at its own resonant frequency, a standing wave pattern is generated. At the position of an antinode, the structure is subjected to alter-

nating strains; it experiences large alternating stresses, and is likely to crack. This process is known as *fatigue failure*. Cracks which develop in strange places in old automobile fenders can often be explained by this mechanism.

20-6 Diffraction of Waves

Many common phenomena, such as the bending of sound waves around obstacles and the spreading of sound waves after passing through a small aperture, are examples of the *diffraction of waves*. We use the term “diffraction” to describe wave phenomena in which the wave front is limited, as by a barrier or an aperture. Diffraction phenomena occur with all types of wave motion and may be most readily understood through the applica-

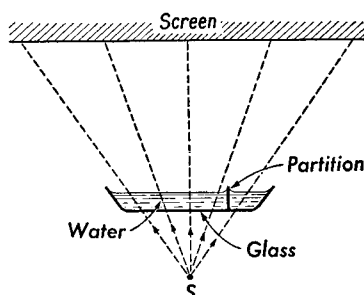


Fig. 20-10 Shallow rectangular tank for demonstrating wave phenomena. Side view showing source of light S under the glass bottom of the tank, for projection of wave phenomena.

tion of Huygens' principle. When the size of a barrier is small compared to the incident wavelength, the wave bends around the barrier. If the barrier is large compared to the wavelength, the barrier seems to cast a sharp shadow. Thus sound waves, whose length ranges from a few centimeters to many meters, may be heard in all parts of a room when sound comes in through an open window. Light coming through the same window directly illuminates only a small part of a room, for the wavelength of light is in the neighborhood of 5×10^{-5} cm. We may infer that man's physical size determines his use of sound as a communication medium and light as a means of localizing objects in space, for sound waves bend easily around man-sized objects, while light waves cast sharp shadows of these objects. The ability to produce diffraction effects is one of the criteria used to determine whether we are dealing with a wave phenomenon.

A simple method for demonstrating the diffraction of waves is illustrated in Figure 20-10. A shallow glass tank containing water to a depth of about 1 in. is illuminated from a light source S below the tank. Waves, in the form of ripples, may be generated in the tank by an oscillating source, which forms circular ripples when the source is a small rod, while plane waves are generated by a flat stick. The progress of a wave can be followed

by viewing its image on a screen. Photographs obtained with such a ripple tank are shown in Figure 20-11. The photograph is brightest where the layer of water is thinnest, as in the trough of a ripple, while the darkest regions correspond to the crest of a ripple. The distance between two successive bright bands is the wavelength. In Figure 20-11(a) a plane wave is shown incident upon an aperture of nearly the same width as the wave-

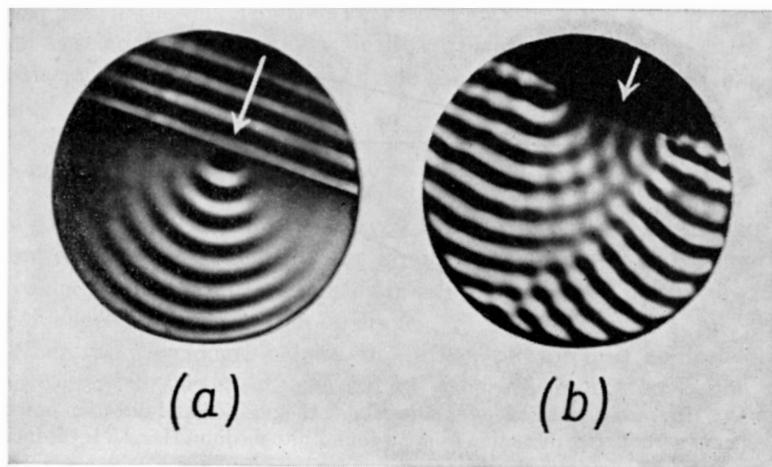


Fig. 20-11 (a) Diffraction of plane waves through an aperture (8–10 mm) in a metal barrier. (b) Diffraction of plane waves round an obstacle about 20 mm wide and production of diffraction fringes. (Courtesy the Ealing Corporation.)

length of the ripples. The waves are diffracted into the shadow area beyond the aperture. In Figure 20-11(b) the waves are diffracted into the shadow area behind a barrier. If we think of each point on the wave front of the incident plane wave as a source of circular wavelets, whose effects must be added in accordance with their phase relationships by the principle of superposition, we may account for these diffraction phenomena. We will consider diffraction effects in a quantitative way in Chapter 40, Light as a Wave Motion.

20-7 Refraction of a Wave

When a wave traveling in a medium I, in which its velocity is v_1 , reaches an interface S between the first medium and a second in which the wave velocity is v_2 , part of the wave enters the second medium at an angle r' , called the *angle of refraction*. We may determine the relationship between the angle of incidence i and the angle of refraction r' with the aid of Huygens' principle, as illustrated in Figure 20-12. Suppose that the incident

wave is a plane wave, and that a portion of a wave front is represented by the line AB . The point A on the wave front has just reached the surface S . The position of the wave front in medium II, after a time interval Δt when the part of the wave front at B has reached the interface at C , may be found from Huygens' construction. From A as center we draw a spherical wave front of radius $v_2 \Delta t$. Carrying out this construction for all points on the

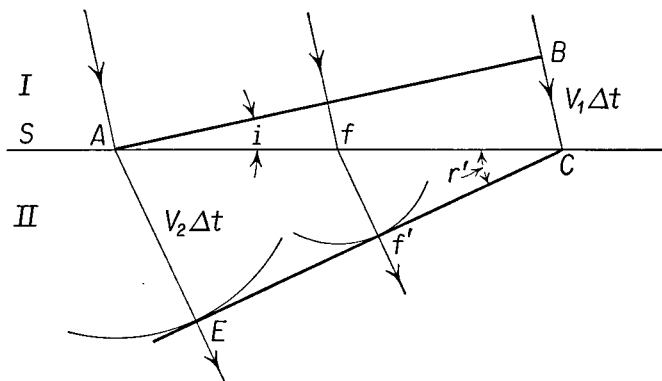


Fig. 20-12 Huygens' construction for determining the position and direction of motion of a plane wave front after refraction from medium I into medium II. AB is the incident wave front; EC is the refracted wave front.

incident wave front, we find the refracted wave front to be along the line EC , and the direction of propagation of the refracted plane wave front is normal to the wave front, or along the direction ff' . If the speed of the wave v_2 in medium II is greater than the speed of the wave v_1 in medium I, then the transmitted part of the wave will be bent away from the normal. From the figure we find

$$BC = v_1 \Delta t,$$

$$AE = v_2 \Delta t,$$

$$\sin i = BC/AC,$$

$$\sin r' = AE/AC,$$

from which

$$n_r = \frac{\sin i}{\sin r'} = \frac{v_1}{v_2}; \quad (20-5)$$

that is, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for all plane waves transmitted from medium I to medium II. This constant is represented by the symbol n_r , called the *relative index of refraction* of the two media.

If a wave is refracted into a medium in which its speed v_2 is less than

the speed v_1 in the first medium, the wave will be bent toward the normal: that is, the angle r' will be smaller than the angle of incidence i , according to Equation (20-5).

20-8 Surface Waves on Water

The speed of surface waves on water depends upon the depth and the wavelength. Where the wavelength is very short and the water is deep compared to the wavelength, the waves are called *ripples* and are propagated by a combination of surface tension and the force of gravity with a velocity v given by

$$v = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho} \right)^{1/2}, \quad (20-6)$$

where g is the acceleration of gravity, λ is the wavelength, T is the surface tension, and ρ is the density of the liquid. Equation (20-6) is sometimes used to determine the surface tension of a liquid of known density by measuring the speed of propagation of ripples.

When the wavelength is long, the surface tension may be neglected in comparison with the effect of gravity. The speed of propagation then depends on the wavelength and the depth and is given by

$$v = \left(\frac{g\lambda}{2\pi} \tanh \frac{2\pi D}{\lambda} \right)^{1/2}, \quad (20-7)$$

where D is the depth of the water and the symbol \tanh means hyperbolic tangent and is a symbol for the expression

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

Equation (20-7) may be approximated for short waves in deep water, when the depth is greater than half a wavelength, by the expression

$$v = \left(\frac{g\lambda}{2\pi} \right)^{1/2}, \quad (20-8)$$

so that the speed depends directly on the square root of the wavelength. The appropriate approximation to Equation (20-7) for long waves in shallow water, where the wavelength is greater than about 25 times the depth, is

$$v = (gD)^{1/2}. \quad (20-9)$$

Equation (20-9) is of great interest in determining the depth and the character of beaches, and it accounts for the riffled nature of the water above a reef. As a wave passes from one medium to a second in which it travels with a lower speed, the wave is refracted, and, in addition, its wave-

length is changed in accordance with Equation (20-1), the frequency remaining constant. Since the water is shallow over a reef, the wave velocity, and therefore the wavelength, is reduced. In the invasion of North Africa in World War II, the nature of the shore line and the location of invasion beaches was determined by measurement of the wave velocity of water waves along the coast line by observation from the air.

The motion of water waves is complex and cannot be adequately covered here. The motion of small volume elements of the water is circular or elliptical rather than purely longitudinal or transverse, and extends in depth beneath the surface, but at a depth of one wavelength the amplitude is less than $1/500$ of the amplitude at the surface. Generally speaking, there is no translational motion of the water in a water wave. When the depth of water is too shallow for wave propagation, the energy of the waves is converted into translational energy of water, giving rise to the familiar breakers and surf along the shore line.

Ocean waves are generated by winds and storms at sea. Since the waves travel at much greater speed than the storm, wave observation stations have been set up along the coasts of Britain to determine the direction from which the waves come and the amplitude with which waves reach the coast. By comparing data from several stations, the progress of storms at sea can be followed, and useful weather information can be transmitted to ships at sea.

20-9 Transverse Waves

The waves transmitted down a string are transverse waves. If a slotted board is passed over a vibrating string, the slot will permit passage of the waves when it is parallel to the direction of vibration of an element of the string, but it will prevent passage of the waves when it is perpendicular to the direction of vibration. A transverse wave motion in which all of the vibrations of the medium are in the same direction is said to be *polarized*. From an experimental point of view, a wave is said to be polarized if a slit or slitlike device can be found which will permit passage of the wave in one orientation, but which will forbid passage of the wave in a second orientation at right angles to the first. Since a longitudinal wave will pass through a slit, however oriented, longitudinal waves are not polarized. The wave propagation in air, which we call sound, must be a longitudinal wave motion, by virtue of the fact that air does not possess a shear modulus. Sound waves in air are not polarized and are not polarizable.

If a wire is given an impulsive blow, the deflection of the wire is transmitted as a single pulse down the wire, with a speed v , and the pulse may be assumed to retain its shape as it moves. Any short section of the deflected part of the wire may be approximated by the arc of a circle. To

analyze the motion we imagine that the deflected portion of the wire is stationary, and that the wire moves past a template of the form of the pulse in a direction opposite to the motion of the pulse with a speed v , as shown in Figure 20-13(a). If the mass per unit length of the wire is m , the mass of a short section of the wire, whose arc is of radius r and which sub-

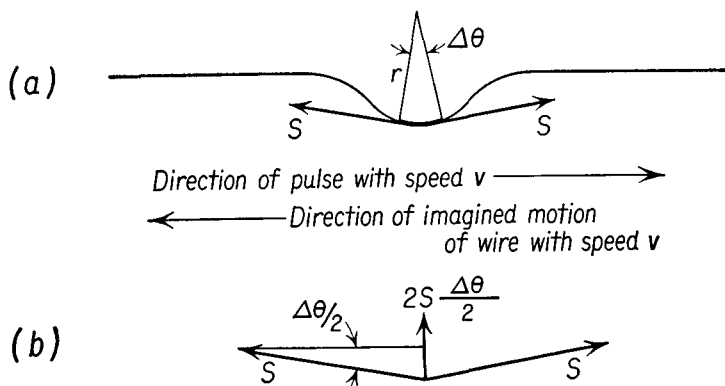


Fig. 20-13

tends an angle $\Delta\theta$ at the center of its circle of curvature, is $mr \Delta\theta$. The centripetal force F required to keep this element of wire moving in a circular arc is

$$F = \frac{mr \Delta\theta v^2}{r}.$$

From Figure 20-13(b) the required centripetal force is supplied by the radial components of the tension S in the wire at both ends of the circular arc. For small angles these are given by

$$F = 2S \frac{\Delta\theta}{2} = S \Delta\theta.$$

Thus we have $m \Delta\theta v^2 = S \Delta\theta$,

or $v = (S/m)^{1/2}$. (20-10)

Illustrative Example. A long steel wire under a tension of 10^5 dynes has one end attached to a prong of a tuning fork which vibrates with a frequency of 128 vib/sec. The linear density of the wire is 0.03 gm/cm. Determine (a) the speed of the transverse wave in the wire and (b) the wavelength of this wave.

The speed of the wave in the wire can be obtained from Equation (20-10), which yields

$$v = \sqrt{\frac{10^5 \text{ dynes}}{0.03 \text{ gm/cm}}} = 1,830 \text{ cm/sec.}$$

Solving Equation (20-1) for λ , we get

$$\lambda = \frac{v}{f},$$

from which

$$\lambda = \frac{1,830}{128} \text{ cm} = 14.3 \text{ cm}.$$

20-10 Longitudinal Waves

Waves can be set up in any elastic body by producing some disturbance at a point in the body. For example, consider a helical spring stretched between two fixed points A and B , as shown in Figure 20-14, and suppose that we compress a small section of the spring near the bottom by taking one of its coils and pushing it down. When this coil is released, it will be

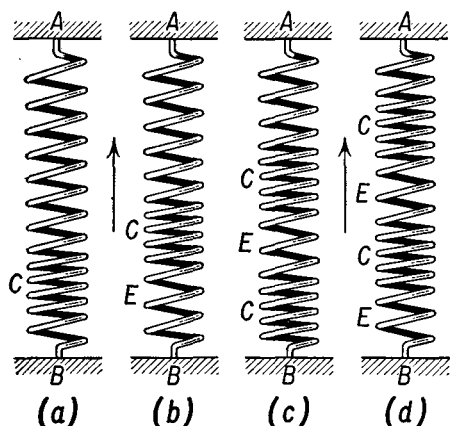


Fig. 20-14 Setting up a longitudinal wave in a coiled spring stretched between two points A and B . (a) Portion of spring C near the bottom is compressed. (b) Compression C has moved up the spring and is followed by an extension E below it. (c) and (d) Compressions C and extensions E moving up the spring.

pulled up by the coil above it and pushed up by the coil below. The displaced coil will move up to its equilibrium position and will continue on upward because of its inertia. Thus the compressed region of the spring will be displaced upward, and an *extension* will appear in the part of the spring which formerly was in *compression*. While the compressions and extensions of a coil spring move along the spring, the individual coils vibrate up and down with simple harmonic motion; the vibration is parallel to the direction of propagation of the wave, so that the wave is longitudinal.

The wave motion in air is also a longitudinal wave. Let us consider the wave in air produced by the vibratory motion of a stick, as shown in Figure 20-15. When the end of the stick moves to the right, the air next to it is compressed; this, in turn, produces a *compression* in the next layer of air, and so on, so that a compression travels out from the vibrating stick. While the compression is moving out to the right, the end of the stick starts

moving back, leaving a *rarefaction*, or region of reduced pressure, on its right. The adjacent layer of air starts moving back, so that the rarefaction travels out from the vibrating stick. This succession of compressions and rarefactions traveling out from the stick constitutes a *longitudinal wave* in air. The layers of air vibrate with the same frequency as the stick.

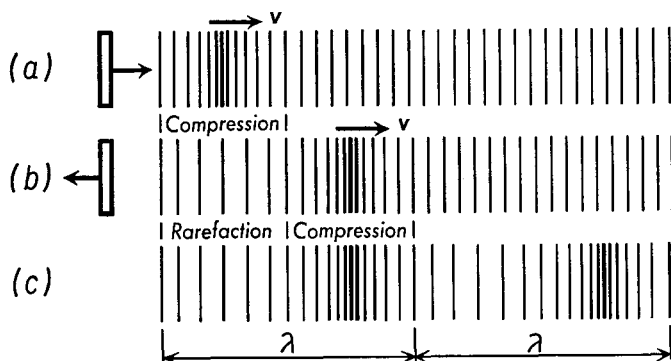


Fig. 20-15 (a) Compression in the air produced by the motion of the end of the rod to the right, followed by (b) a rarefaction produced by the motion of the end of the rod to the left. (c) Longitudinal wave in air consists of a series of compressions and rarefactions moving outward with velocity v .

The changes in pressure which occur during the compressions and rarefactions are generally very small in comparison with the atmospheric pressure. A graphical method for representing the wave is shown in

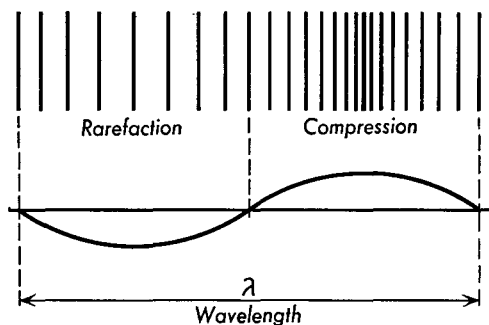


Fig. 20-16 Representation of a longitudinal wave.

Figure 20-16, in which the change in pressure from normal atmospheric pressure is plotted along the vertical axis, and the position where this pressure difference exists is plotted along the horizontal axis. The section of the curve below the axis represents the rarefaction, while the section above the axis represents the compression.

It may be shown that the speed of propagation of a longitudinal wave within a fluid medium is given by the equation

$$v = (B/\rho)^{1/2}, \quad (20-11)$$

where B is the bulk modulus of the fluid and ρ is its density. Although there is a temperature gradient between a compression and a rarefaction, the conductivity of a gas is so low, and the distance between them, which is a half wavelength, is so long, that very little heat can be transmitted in the available time of half a period. The processes of compression and rarefaction in a sound wave are practically adiabatic at audio frequencies. Hence we must use the value for the *adiabatic bulk modulus* in Equation (20-11). This is given by

$$B_{(\text{adiabatic})} = \gamma P, \quad (20-12)$$

where γ is the ratio of the specific heats of the gas and P is its pressure. Thus the speed of propagation of longitudinal waves in gases is given by

$$v = \left(\frac{\gamma P}{\rho} \right)^{1/2}. \quad (20-13)$$

From the general gas law

$$PV = nRT,$$

where n is the number of moles of the gas, and R is the gas constant per mole. If a mass m of a gas of molecular weight M is enclosed in a container, the number of moles of gas within the container is given by

$$n = \frac{m}{M}.$$

If we substitute this expression into the gas law and rearrange terms, we find that

$$\frac{P}{m/V} = \frac{RT}{M},$$

and, since the term on the left-hand side of the above equation is equal to P/ρ , we may substitute this result into Equation (20-13) to obtain

$$v = \left(\frac{\gamma RT}{M} \right)^{1/2}. \quad (20-14)$$

Thus the velocity of sound in a gas depends upon the absolute temperature T , but is independent of the pressure of the gas. The velocities of propagation of longitudinal waves in several substances are given in Table 20-1.

A longitudinal wave is transmitted in a wire or rod with a speed given by

$$v = (Y/\rho)^{1/2}, \quad (20-15)$$

TABLE 20-1 SPEED OF SOUND (Longitudinal Waves)

Substance	Temperature in °C	Speed in meters per sec
Air	0	331.46
Hydrogen	0	1,262
Carbon dioxide	0	258.0
Water	15	1,447
Sea water	13	1,492
Glass	0	5,500
Steel		4,700-5,200

where Y is Young's modulus. In many problems in engineering design, it is important to know Young's modulus for a particular substance at elevated and reduced temperatures. One of the simplest methods for making this determination is to measure the velocity of propagation of a longitudinal wave in the substance at the desired temperature, and to apply Equation (20-15) to this measurement.

Problems

20-1. A steel wire is stretched between two pegs 80 cm apart under a tension of 10^6 dynes. The linear density of the wire is 0.25 gm/cm. Determine the speed of a transverse wave in this wire.

20-2. One end of a horizontal string is attached to a prong of an electrically driven tuning fork which is vibrating with a frequency of 256 vib/sec, while the other end passes over a pulley and has a weight of 6 lb attached to it. The weight of 1 ft of string is 0.02 lb. (a) Determine the speed of transverse waves in the string. (b) Determine the wavelength of the waves set up in the string.

20-3. A copper wire 2 m long whose mass is 8 gm has one end attached to a fixed post and the other end attached to a prong of a tuning fork which vibrates with a frequency of 1,000 cycles/sec. A standing wave is set up in the wire, and the distance between the nodes is 8.0 cm. Determine (a) the wavelength of the transverse wave in the wire, (b) the speed of the wave, and (c) the tension in the wire.

20-4. A stone is dropped in a well, and the sound of the stone's splash is heard 4.0 sec later. How deep is the well?

20-5. A steel pipe 200 ft long is struck at one end. A person at the other end hears the sound that traveled through the pipe and also the sound that traveled through the air. Determine the time interval between the two sounds.

20-6. (a) Determine the index of refraction of hydrogen with respect to air for a sound wave. (b) Determine the index of refraction of glass with respect to air for a sound wave. A glass partition 0.5 cm thick separates a volume of air from a volume of hydrogen, each at atmospheric pressure. A sound wave from the air strikes the glass surface at an angle of 3° with respect to the normal. Determine the angle at which the sound wave is refracted (c) into the glass and (d) into the hydrogen.

20-7. Derive an equation similar to Equation (20-4) which displays the existence of standing waves by beginning with the cosine representation of a wave

$$y = A \cos 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right).$$

20-8. Show that Equation (20-3) satisfies an equation of the form

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2},$$

which is called the *wave equation*.

20-9. A standing wave is set up in a string. The distance between the nodes is 50 cm, and the measured width of the vibration at the antinodes is 10 cm. The standing wave pattern is set up by a tuning fork which vibrates at a frequency of 100 cycles/sec. (a) Rewrite Equation (20-4), substituting appropriate numerical values. (b) Find the width of the vibration pattern in the string at a distance of 20 cm from a node.

20-10. A transverse wave in a string is represented by the equation

$$x = 5 \cos (30t - 15y),$$

where all dimensions are in cgs units. Find (a) the direction of propagation of the wave, (b) the wavelength, (c) the frequency, and (d) the velocity.

20-11. A wind blows from the north at a speed of 50 ft/sec. In what direction should a beamed source of sound be pointed for the sound to travel due east?

20-12. Assume a ship to have vertical sides near the water line. (a) Find the natural frequency of vibration of a ship of cross-sectional area A and mass M when floating in water of density ρ . (b) When the ship is at anchor in deep water, what is the wavelength of ocean waves which will excite the ship to resonance?

20-13. A plane wave of length 25 cm, velocity 100 cm/sec, and having an amplitude of 5 cm, is propagated in the $-x$ direction. At time $t = 0$, a point at the origin of coordinates experiences its maximum positive displacement. (a) Find the displacement of a point whose coordinates are (0, 5 cm) when $t = 5$ sec. (b) Find the displacement of a point whose coordinates are (+5 cm, -3 cm) when $t = 8$ sec.

20-14. A wave motion is described by the equation

$$y = 10 \sin \left(2t - 5x + \frac{5\pi}{6} \right).$$

(a) In which direction and along which coordinate axis is the wave moving? (b) Is the wave longitudinal or transverse? (c) What is the frequency? (d) The wavelength? (e) the velocity? (f) At $t = 0$ what is the displacement and the direction of motion of a point located at $x = 0$?

20-15. The motion of a vibrating string is given by

$$y = 3 \cos \frac{2\pi}{5} x \sin \pi t.$$

Find the velocity of the segment of the string located at $x = 2.5$ when $t = 0.5$ sec.