

Groundwater flow to a horizontal or slanted well in an unconfined aquifer

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[1] New semianalytical solutions for evaluation of the drawdown near horizontal and slanted wells with finite length screens in unconfined aquifers are presented. These fully three-dimensional solutions consider instantaneous drainage or delayed yield and aquifer anisotropy. As a basis, solution for the drawdown created by a point source in a uniform anisotropic unconfined aquifer is derived in Laplace domain. Using superposition, the point source solution is extended to the cases of the horizontal and slanted wells. The previous solutions for vertical wells can be described as a special case of the new solutions. Numerical Laplace inversion allows effective evaluation of the drawdown in real time. Examples illustrate the effects of well geometry and the aquifer parameters on drawdown. Results can be used to generate type curves from observations in piezometers and partially or fully penetrating observation wells. The proposed solutions and software are useful for the parameter identification, design of remediation systems, drainage, and mine dewatering. *INDEX TERMS*: 1829 Hydrology: Groundwater hydrology; 1831 Hydrology: Groundwater quality; 1832 Hydrology: Groundwater transport; *KEYWORDS*: horizontal well, slanted well, groundwater flow, unconfined aquifer, anisotropy, type curves

1. Introduction

[2] Horizontal and slanted wells have screened sections that can be positioned parallel to or under some angle of inclination to the static water table. These wells have several important traits: (1) groundwater flow has a significant vertical velocity component in large areas of the aquifer including the zone near the water table; (2) more uniform proximity to the water table provides more control over the dynamics of the water table; (3) well screens have better contact with the horizontal aquifer units; (4) drilling operations are feasible near the ground surfaces that are obstructed by permanent structures (airport runways, roads, buildings, etc.); and, finally, (5) long screen sections can be installed even in aquifers of small thickness.

[3] These traits offer some practical advantages over the use of the vertical wells in many applications. Generally, a much larger contact zone between well and contaminated groundwater, vapor, or oil improves the effective recovery of fluids. In a case study in a thin oil reservoir, *Seines et al.* [1994] showed that a horizontal well was equivalent to about four vertical wells. *Parmentier and Klemovich* [1996] have pointed out that a single horizontal well may have the contact area equal to that of 10 vertical wells. In the petroleum industry, horizontal wells improve oil recovery from petroleum reservoirs [*Joshi*, 1988; *Seines et al.*, 1994; *Maurer*, 1995; *Penmatcha et al.*, 1997]. In aquifer remediation, horizontal or slanted wells are used to recover contaminated groundwater, nonaqueous phase liquids, and soil vapor from

the subsurface [*Morgan*, 1992; *Environmental Protection Agency (EPA)*, 1994; *Murdoch*, 1994; *Falta*, 1995; *Parmentier and Klemovich*, 1996; *Sawyer and Lieuallen-Dulam*, 1998; *Zhan*, 1999; *Zhan and Cao*, 2000]. These wells are commonly used for groundwater supply, drainage, and mine dewatering [*Hantush and Papadopoulos*, 1962].

[4] An early study of fluid flow to a horizontal well by *Hantush and Papadopoulos* [1962] investigated a collector well as a series of jointed horizontal well distributed in a horizontal plane. However, further studies of horizontal and slanted wells have seen limited progresses because of high drilling costs. With the advancement of drilling technology over the last 15 years, this methodology stimulated significant interest among petroleum engineers and groundwater hydrologists. Horizontal wells have been increasingly utilized for oil and gas exploration and production. Problems of oil and gas flow to a horizontal well in a confined reservoir were studied by *Goode and Thambynayagam* [1987], *Joshi* [1987], *Daviau et al.* [1988], *Ozkan et al.* [1989], and *Kuchuk et al.* [1991]. In recent years, the use of horizontal and slanted wells was investigated for hydrogeological applications. That includes wells in confined aquifers [*Cleveland*, 1994; *EPA*, 1994; *Murdoch*, 1994; *Falta*, 1995; *Zhan*, 1999; *Zhan and Cao*, 2000; *Kawecki*, 2000] and underneath the reservoirs [*Sawyer and Lieuallen-Dulam*, 1998; *Zhan and Cao*, 2000]. However, the transient three-dimensional flow to the horizontal or slanted wells in unconfined aquifers has not been studied yet.

[5] The objective of this paper is to present solutions for drawdown near horizontal and slanted wells in a three-dimensionally anisotropic unconfined aquifer. As a basis, a fundamental solution for drawdown created by a point

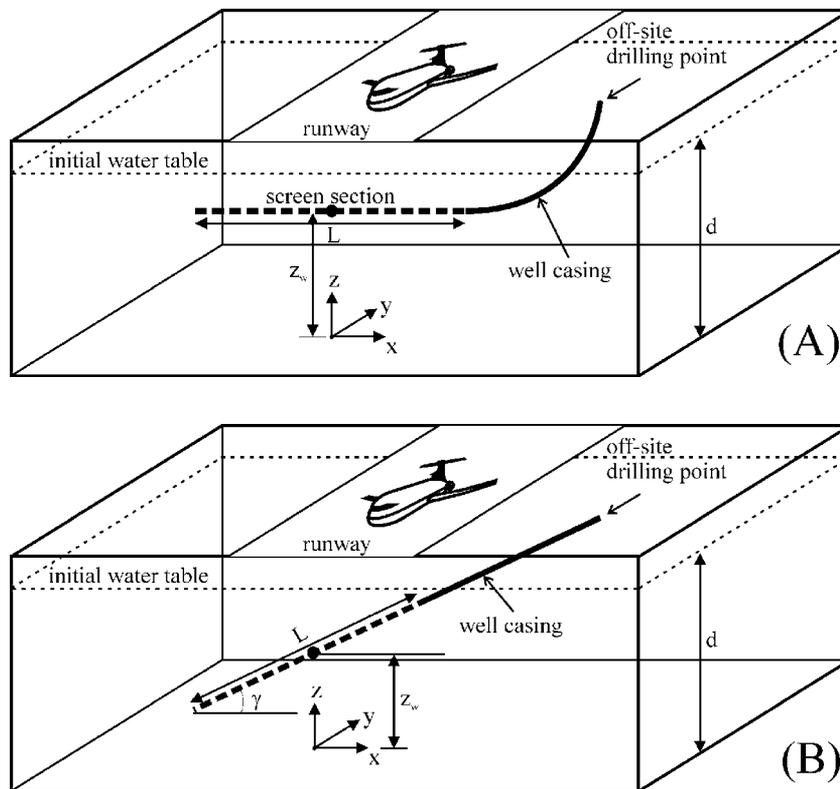


Figure 1. The schematic diagrams of (a) a horizontal well and (b) a slanted well in water table aquifers.

source in a uniform anisotropic unconfined aquifer is derived in Laplace domain. Using superposition, the point source solution is extended to linear sinks that represent the horizontal and slanted wells. It is shown that previous solutions for vertical wells can be described as a special case of the new solutions. Numerical Laplace inversion is used for evaluation of the drawdown in real time. Examples will illustrate the effects of the aquifer parameters on the drawdown. Results can be used to generate type curves from observations in piezometers or partially or fully penetrating observation wells.

2. Flow to a Point Sink

[6] The origin of the Cartesian coordinate system is set at the aquifer base with the x and y axes along the horizontal directions and the z axis along the vertical direction (Figure 1). To simplify the problem statement, we position the screen sections of length L of the horizontal or slanted wells in the xz plane with the z axis through the center of the screen. A horizontal well is located in an unconfined aquifer with a distance z_w to the aquifer base (Figure 1a). A screened section of the slanted well has an angle γ with the xy plane, and its center is located at a distance z_w to the bottom boundary (Figure 1b). The aquifer is assumed to be infinite laterally.

[7] The water table is a free moving boundary that makes the flow problem nonlinear [Polubarinova-Kochina, 1962]. However, in many cases it is assumed that the drawdown of the water table due to the pumping is much smaller than the initial saturated thickness of the aquifer, and the problem can be linearized [e.g., Dagan, 1967a; Neuman, 1972, 1974; Moench, 1995, 1997]. This assumption is not very restric-

tive in our case because most horizontal and slanted wells used in environmental applications have quite low pumping rates. Discussion of nonlinear effects is beyond the scope of this paper and can be found elsewhere [e.g., Polubarinova-Kochina, 1962; Dagan, 1967a, 1967b, 1968; Papatzakos, 1992; Gjerde and Tyvand, 1992; MacDonald and Kitandis, 1993].

[8] In mathematical treatment of the physical processes near the moving water table, two approaches are commonly used [Boulton, 1954, 1955; Neuman, 1972, 1974; Moench, 1995, 1997; Zlotnik, 1998]. One approach assumes the instantaneous drainage of the unsaturated zone above the falling water table [Boulton, 1954; Neuman, 1972, 1974]; such treatment is mathematically straightforward but may be an oversimplification of the physical reality. An alternative approach assumes the delayed response of drainage of the unsaturated zone; such a model seems physically sound, but the difficulty of explaining the empirical “delay index” still remains [Boulton, 1955; Moench, 1995, 1997]. Use of instantaneous drainage model for early and intermediate time may introduce some errors. Further discussion on the delay index is presented in section 5.1.4. We develop solutions for both instantaneous drainage and delayed yield models.

2.1. Problem Statement

[9] Transient groundwater flow in a three-dimensionally anisotropic medium near a point source at (x_0, y_0, z_0) obeys the following governing equation:

$$S_s \frac{\partial h}{\partial t} = K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} - Q \delta(x - x_0) \delta(y - y_0) \delta(z - z_0), \quad (1)$$

subject to the initial condition

$$h(x, y, z, 0) = h_0, \quad (2)$$

no-flow condition at the base of the aquifer,

$$\partial h(x, y, 0, t) / \partial z = 0, \quad (3)$$

boundary condition at the water table (instantaneous drainage):

$$K_z \partial h(x, y, d, t) / \partial z + S_y \partial h(x, y, d, t) / \partial t = 0, \quad (4)$$

and lateral boundaries at the infinite horizontal distance from the well,

$$h(\pm\infty, y, z, t) = h(x, \pm\infty, z, t) = h_0, \quad (5)$$

where h is hydraulic head; t is time; h_0 is initial hydraulic head; K_x , K_y , and K_z are values of hydraulic conductivity in x , y , and z directions, respectively; Q is pumping rate (positive for pumping); $\delta(u)$ is the Dirac delta function; d is saturated aquifer thickness in static condition; S_s is specific storage; and S_y is specific yield.

[10] The water table change is assumed to be much smaller than d , and an instantaneous change of water table is assumed when using the boundary condition equation (4).

[11] If a delayed yield model is used, the boundary condition equation (4) is replaced by

$$K_z \frac{\partial h(x, y, d, t)}{\partial z} = -\alpha_1 S_y \int_0^t \frac{\partial h(x, y, d, t')}{\partial t'} \exp[-\alpha_1(t - t')] dt', \quad (6)$$

where α_1 is the empirical constant for drainage from the unsaturated zone; $1/\alpha_1$ is the so-called ‘‘delay index’’ [Boulton, 1954; Bear, 1979; Moench, 1997]. If $\alpha_1 \rightarrow \infty$, equation (6) reduces to the instantaneous drainage condition equation (4).

2.2. Point Sink Solution in Laplace Domain

[12] Solution for a point sink in a vertically anisotropic aquifer can be obtained from the Neuman [1974] or Moench [1995] solutions by considering the infinitesimal screen length well. However, a solution of the initial boundary value problem equations (1)–(5) is required in the case of more general anisotropy. Using drawdown $s = h_0 - h$ and dimensionless parameters labeled with subscript ‘‘D,’’

$$\begin{aligned} s_D &= \frac{4\pi K d}{Q} s, & t_D &= \frac{K}{S_s d^2} t, & x_D &= \alpha_x \frac{x}{d}, & y_D &= \alpha_y \frac{y}{d}, \\ z_D &= \alpha_z \frac{z}{d}, & \sigma &= \frac{S_s d}{S_y \alpha_z}, & \alpha_x &= (K/K_x)^{1/2}, & \alpha_y &= (K/K_y)^{1/2}, \\ \alpha_z &= (K/K_z)^{1/2}, & K &= (K_x K_y K_z)^{1/3}, \end{aligned} \quad (7)$$

one arrives to the following initial boundary value problem:

$$\frac{\partial s_D}{\partial t_D} = \frac{\partial^2 s_D}{\partial x_D^2} + \frac{\partial^2 s_D}{\partial y_D^2} + \frac{\partial^2 s_D}{\partial z_D^2} + 4\pi\delta(x_D - x_{0D})\delta(y_D - y_{0D})\delta(z_D - z_{0D}), \quad (8)$$

$$s_D(x_D, y_D, z_D, 0) = 0, \quad (9)$$

$$\partial s_D(x_D, y_D, 0, t_D) / \partial z_D = 0, \quad (10)$$

$$\sigma \partial s_D(x_D, y_D, \alpha_z, t_D) / \partial z_D + \partial s_D(x_D, y_D, \alpha_z, t_D) / \partial t_D = 0, \quad (11)$$

$$s_D(\pm\infty, y_D, z_D, t_D) = s_D(x_D, \pm\infty, z_D, t_D) = 0, \quad (12)$$

where $x_{0D} = \alpha_x x_0/d$, $y_{0D} = \alpha_y y_0/d$, and $z_{0D} = \alpha_z z_0/d$. This problem statement permits more general treatment of anisotropy than commonly used [e.g., Bear, 1979; Zlotnik, 1994]. Application of the Laplace transforms to equations (8)–(12) yields the following boundary value problem:

$$p \bar{s}_D = \nabla^2 \bar{s}_D + \frac{\partial^2 \bar{s}_D}{\partial z_D^2} + \frac{4\pi\delta(\mathbf{r}_D - \mathbf{r}_{0D})\delta(z_D - z_{0D})}{p}, \quad (13)$$

$$\partial \bar{s}_D(\mathbf{r}_D, 0, p) / \partial z_D = 0, \quad (14)$$

$$\sigma \partial \bar{s}_D(\mathbf{r}_D, \alpha_z, p) / \partial z_D + p \bar{s}_D(\mathbf{r}_D, \alpha_z, p) = 0, \quad (15)$$

$$\bar{s}_D(\infty, z_D, p) = 0, \quad (16)$$

where ∇^2 is a two-dimensional Laplace operator in cylindrical coordinates, p is the Laplace transform parameter that corresponds to the dimensionless time t_D , \bar{s}_D is the Laplace transform for dimensionless drawdown s_D , and $\mathbf{r}_D = (x_D, y_D)$ and $\mathbf{r}_{0D} = (x_{0D}, y_{0D})$ are dimensionless radial vectors of the observation point and sink point, respectively, in two-dimensional space.

[13] The boundary condition with delayed drainage in Laplace domain is

$$\begin{aligned} \sigma \partial \bar{s}_D(\mathbf{r}_D, \alpha_z, p) / \partial z_D + \frac{p \alpha_{1D}}{p + \alpha_{1D}} \bar{s}_D(\mathbf{r}_D, \alpha_z, p) &= 0, \\ \alpha_{1D} &= S_s d^2 \alpha_1 / K. \end{aligned} \quad (17)$$

[14] The details of solving equations (13)–(16) are shown in Appendix A. The drawdown for the instantaneous drainage case is as follows:

$$\begin{aligned} \bar{s}_D(p) &= \frac{8}{p} \sum_{n=0}^{\infty} \frac{\cos(\omega_n z_{0D}) \cos(\omega_n z_D)}{b(\omega_n)} K_0(\Omega_n |\mathbf{r}_D - \mathbf{r}_{0D}|), \\ b(\omega_n) &= 2\alpha_z + \sin(2\omega_n \alpha_z) / \omega_n, & \Omega_n &= [\omega_n^2 + p]^{1/2}, \end{aligned} \quad (18)$$

where $K_0(x)$ is the second kind, zero-order modified Bessel function and parameters ω_n satisfy the following equation:

$$\omega_n \tan(\omega_n \alpha_z) = p / \sigma, \quad n = 0, 1, \dots \quad (19)$$

[15] For a delayed drainage model, parameters ω_n are computed from another equation:

$$\omega_n \tan(\omega_n \alpha_z) = \frac{p \alpha_{1D}}{\sigma(p + \alpha_{1D})}, \quad n = 0, 1, \dots \quad (20)$$

[16] Fundamental solution (18) for drawdown distribution invoked by a point sink of unit strength in an unconfined aquifer will be used for treatment of wells with various screen configurations.

3. Drawdown Near Wells With Different Screen Configurations

[17] Solution for a horizontal or slanted well of length L with pumping rate Q can be obtained by superposition of sinks of strength $q_3(x, y, z)$, defined as a pumping rate per unit volume of the aquifer. In the case of a horizontal well (Figure 1a) this function can be represented as follows:

$$q_3(x, y, z) = q(x)\delta(y)\delta(z - z_w), \quad Q = \int_{-L/2}^{L/2} q(x)dx, \quad (21)$$

where $q(x)$ is defined as pumping rate per unit screen length.

[18] In the case of a slanted well (Figure 1b), screen is located between points $(-(L/2)\cos\gamma, 0, z_w - (L/2)\sin\gamma)$ and $((L/2)\cos\gamma, 0, z_w + (L/2)\sin\gamma)$. Here $z_{lw} = z_w - (L/2)\sin\gamma$ and $z_{uw} = z_w + (L/2)\sin\gamma$ are elevations of the upper and the lower screen ends of the slanted well. In this case,

$$q_3(x, y, z) = q(l)\delta(y)\delta(z - z_w - l\sin\gamma), \quad Q = \int_{-L/2}^{L/2} q(l)dl, \quad (22)$$

where l is the coordinate along the slanted well axis with respect to the well center ($l > 0$ if $x > 0$; $l < 0$ if $x < 0$). Generally, the distributions of sink strength $q(x)$ or $q(l)$ are unknown, and an additional condition is needed for evaluation of this function.

3.1. Flux Distribution Along the Well Screen

[19] Fluid dynamics inside the well, in the screen, and in the aquifer creates spatially variable head due to pumping [Tarshish, 1992]. Head variations along the screen inside the well are much smaller than the head gradients in the aquifer, and a constant head condition on the screen is an appropriate approximation [Bear, 1979]. Still, evaluation of sink strength $q(x)$ or $q(l)$ is not straightforward.

[20] Originally, Muskat [1937] formulated a mixed-type boundary value problem for partially penetrating vertical wells; however, only a few accurate solutions of such problems have been found since then. Kirkham [1959] derived a fully analytical solution for steady state flow toward a partially penetrating well, but it saw little practical use due to computational complexity. Dagan [1978], Haitjema and Kraemer [1988], Cole and Zlotnik [1994], and Tartakovsky et al. [2000] presented semianalytical solutions for various steady state mixed-type boundary value problems. Hayashi et al. [1987] and Cassiani and Kabala [1998] found several transient solutions. In general, mixed-type boundary value problems can be reduced to a system of integral equations that require intensive computations, and these solutions can yield the distribution of the sink strength.

[21] Muskat [1937] proposed an alternative approximate approach for flow analysis near the vertical wells in homo-

geneous aquifers. This approach represents wells as a system of sinks that are uniformly distributed along the borehole axis with the uniform strength, while the sink strength at the edges of a screen is higher than the average over the screen length [Polubarinova-Kochina, 1962]. However, the error produced by the uniformity assumption is small (less than a few percent) if the screen length-to-radius ratio is large enough: $L/r_w > 20-40$ [Hayashi et al., 1987; Haitjema and Kraemer, 1988; Cole and Zlotnik, 1994; Ruud and Kabala, 1997]. More importantly, this error rapidly decays with the distance from the well screen [Luther and Haitjema, 2000]. Because of this, the uniformity approximation is commonly used for vertical wells in homogeneous aquifers [e.g., Hantush, 1961; Dagan, 1967a; Neuman, 1972, 1974; Moench, 1995, 1997].

[22] We tested a hypothetical case of a 34 m long horizontal well in a 20 m thick unconfined aquifer under both uniform flux and uniform head well bore conditions using a three-dimensional finite difference method (see the Visual Modflow website at <http://www.flowpath.com>). The uniform head was simulated by assigning very large hydraulic conductivity (10^5 m/d) to each cell that represented the horizontal well. The hydraulic conductivity of the aquifer was 10 m/d. Numerical simulations show that the discrepancy of the uniform flux and uniform head results is $<10\%$ when the distance between a measured point to a well end is 5 times the well diameter. When that distance increases, the discrepancy of the uniform flux and uniform head solutions decreases quickly to insignificant level ($<1\%$). Similar conclusions hold for slanted wells.

[23] In heterogeneous aquifers, flux distribution over the screen length is dependent on the orientation of the screen with respect to the aquifer heterogeneities. Analytical methods for evaluation of sink distribution have limited capabilities, and the assumption of uniformity has to be assessed by numerical techniques in each specific case [e.g., Braester and Thunvik, 1984; Ruud and Kabala, 1997; Hemker, 1999]. For the most practical applications of horizontal or slanted wells in homogeneous aquifers, one can safely employ the approximation of the uniform sink strength, i.e., $q(x) = q(l) = Q/L = \text{const}$.

3.2. Drawdown Near a Horizontal Well

3.2.1. Case 1: Drawdown in a piezometer

[24] Using the uniform sink distribution and fundamental solution (18), one can integrate the effect of sinks along the well axis and obtain the point drawdown in a piezometer:

$$\bar{s}_{HD}(p) = \frac{8}{pL_D} \sum_{n=0}^{\infty} \frac{\cos[\omega_n z_D] \cos[\omega_n z_{wD}]}{b(\omega_n)} \int_{-L_D/2}^{L_D/2} K_0[\Omega_n F(x'_D)] dx'_D, \quad (23)$$

where $\bar{s}_{HD}(p)$ is the dimensionless drawdown in the Laplace domain for a horizontal well, and it is defined in the same way as s_D is in equation (7), $L_D = \alpha_x L/d$, and $F(x'_D) = [(x_D - x'_D)^2 + y_D^2]^{1/2}$.

3.2.2. Case 2: Drawdown in an observation well

[25] The drawdown in an observation well that is screened from z_l to z_u ($z_u > z_l$) can be estimated as the

average of the point drawdown (23) along the screen [Hantush, 1961]:

$$\bar{s}_{HD}(p) = \frac{8}{pL_D(z_{uD} - z_{lD})} \sum_{n=0}^{\infty} \frac{[\sin(\omega_n z_{uD}) - \sin(\omega_n z_{lD})] \cos(\omega_n z_{wD})}{\omega_n b(\omega_n)} \cdot \int_{-L_D/2}^{L_D/2} K_0[\Omega_n F(x'_D)] dx'_D, \quad (24)$$

where $z_{uD} = \alpha_z z_u/d$ and $z_{lD} = \alpha_z z_l/d$.

3.2.3. Case 3: Drawdown created by a long well in a piezometer

[26] If the horizontal well is much longer than the aquifer thickness ($L_D \rightarrow \infty$), a simplification of the equation (23) is possible:

$$\lim_{L_D \rightarrow \infty} [\bar{s}_{HD}(p)L_D] = \frac{8\pi}{p} \sum_{n=0}^{\infty} \frac{\cos[\omega_n z_D] \cos[\omega_n z_{wD}]}{b(\omega_n) \Omega_n \alpha_x} \exp[-\Omega_n |y_D|], \quad (25)$$

using the identity $\int_{-\infty}^{\infty} K_0[\Omega_n \sqrt{x^2 + u^2}] dx = (\pi/2\Omega_n) \exp[-\Omega_n u]$ from Gradshĭyn and Ryzhik [1994, p. 727, 6.596.3]. Note that $\bar{s}_{HD}(p) \rightarrow 0$ when $L_D \rightarrow \infty$, but their product is finite if pumping rate per unit screen length Q/L is a finite value. That implies that a large total pumping rate is needed to create a finite drawdown near a very long horizontal well.

[27] Generalization of equation (25) for drawdown created by a long well in an observation well is straightforward by taking the average of the drawdown along the screen of the observation well from z_l to z_u ($z_u > z_l$).

3.3. Drawdown Near a Slanted Well

[28] For a slanted well with the uniform sink distribution the Laplace transform of dimensionless drawdown in the piezometer $\bar{s}_{lD}(p)$ is obtained through the integration of fundamental solution (18) along the well axis:

$$\bar{s}_{lD}(p) = \frac{8}{pL_D} \sum_{n=0}^{\infty} \frac{\cos(\omega_n z_D)}{b(\omega_n)} \int_{-L_D/2}^{L_D/2} \cos \left[\omega_n \left(z_{wD} + l \frac{\alpha_z}{\alpha_x} \sin \gamma \right) \right] \cdot K_0[\Omega_n F(l \cos \gamma)] dl, \quad (26)$$

where $\bar{s}_{lD}(p)$ is defined in the same way as s_D is in equation (7).

[29] If $\gamma = 0$, the slanted well becomes a horizontal well, and equation (26) reduces to equation (23). The case of $\gamma = \pi/2$ corresponds to a vertical well. In this case, function $F(0) = \sqrt{x_D^2 + y_D^2}$ is independent of l , and the integration yields the following equation:

$$\bar{s}_{lD}(p) = \frac{8}{p(z_{uD} - z_{lD})} \sum_{n=0}^{\infty} \frac{\cos(\omega_n z_D) [\sin(\omega_n z_{uwD}) - \sin(\omega_n z_{lwD})]}{\omega_n b(\omega_n)} \cdot K_0 \left(\Omega_n \sqrt{x_D^2 + y_D^2} \right), \quad (27)$$

where $z_{uwD} = \alpha_z z_{uw}/d$ and $z_{lwD} = \alpha_z z_{lw}/d$. Equation (27) is identical to Moench's [1997] equation (19) if one assumes an infinitesimal pumping well radius and the absence of the

storage effect. This can be easily verified as follows. Using notation and definition of dimensionless terms by Moench [1997, Table 1], one obtains $r_w \rightarrow 0$, $W_D \rightarrow 0$, $\beta_w \rightarrow 0$, $t_D \rightarrow \infty$, $p \rightarrow 0$, and $q_n \rightarrow 0$. From the identity $\lim_{q_n \rightarrow 0} q_n K_1(q_n) = 1$ [Abramowitz and Stegun, 1972, p. 379, equation (9.8.7)] it follows that Moench's [1997] equation (19) is equivalent to our equation (27).

4. Numerical Evaluation of the Drawdown

[30] Inversions of the Laplace transforms of equations (23)–(27) by obtaining the closed-form analytical solutions require special techniques [e.g., Neuman, 1972; Zlotnik and Ledder, 1992]. The efficiency of inversion can be improved by the numerical Laplace transforms [e.g., Stehfest, 1970; Talbot, 1979; Davies and Martin, 1979; de Hoog et al., 1982]. Moench [1995] showed that the Stehfest [1970] algorithm is quite adequate for applications in unconfined aquifers. Our numerical experiments confirmed that this algorithm is devoid of numerical oscillations or other problems at times of interest.

[31] The computer program uses equations (23), (24), and (26) and several routines of the WTAQ code [Barlow and Moench, 1999]. The integrations in these equations are based on the Gaussian quadrature method [Abramowitz and Stegun, 1972, p. 916; Press et al., 1989]. Equation (A2), for an instantaneous drainage case, or equation (A3), for a delayed yield case, is solved for ω_n using the Newton-Raphson method [Press et al., 1989]. Function $K_0(x)$ is approximated after Abramowitz and Stegun [1972, p. 379].

[32] Note that this solution is valid in the case of small drawdown values, i.e.,

$$s \ll d, \quad s_D \ll \frac{Q}{4\pi K d^2}. \quad (28)$$

[33] In practice, criterion $s_{\max}/d < 0.5$ is commonly used, where s_{\max} denotes the maximal drawdown in the pumping well [Boulton, 1954; Hantush, 1964, p. 364; Bear, 1979, p. 332].

[34] The computer program employs the coordinate system used in Figure 1 and calculates the drawdowns in real time at any location of the aquifer. It can also generate dimensionless type curves that present the drawdown in log-log coordinates.

5. Discussion

[35] In the following discussion our analysis of solutions is based on the drawdown in piezometers that are located in different positions near the pumping well. Unlike the case of vertical well, new solutions lack axial symmetry. In addition to the aquifer properties, screen length, pumping rate, and piezometer location, the drawdown in the piezometer is dependent on well location with respect to the aquifer base and the static water table for horizontal wells. In the case of the slanted well the angle of inclination will be an additional parameter. To simplify this analysis, the type curves for the dimensionless drawdown are plotted versus t_D/r_D^2 in log-log coordinates. Here t_D is the dimensionless time defined in equation (7) and $r_D = (x_D^2 + y_D^2)^{1/2}$ is the projection of dimensionless radial distance from the center of the well to the piezometer on the horizontal plane. Each plot is

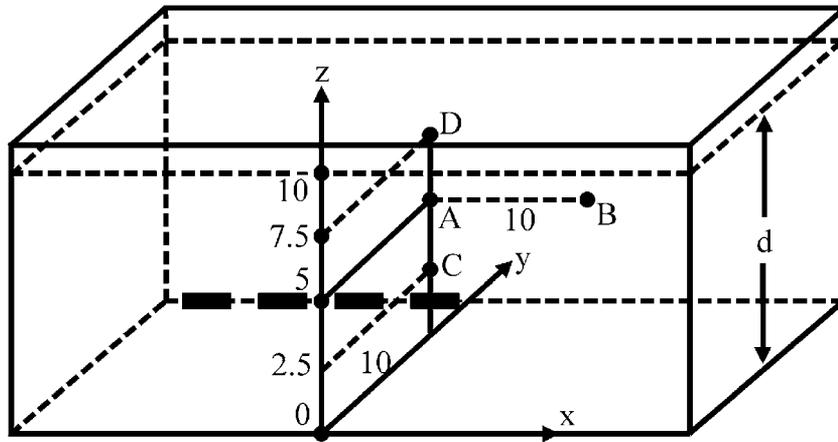


Figure 2. Locations of piezometers at points A (0, 10 m, 5 m), B (10 m, 10 m, 5 m), C (0, 10 m, 2.5 m), and D (0, 10 m, 7.5 m).

accompanied with two Theis type curves for a vertical well using dimensionless times t_D and $t_{Dy} = t_D(S_s d/S_y)$ that provide the direct comparison between vertical and nonvertical wells for small and large times. Figures 3–6 show that the type curves for horizontal wells at early times generally do not agree with the Theis curve for a fully penetrating vertical well due to the absence of the axial symmetry. The degree of discrepancy depends on the piezometer location and well screen length and orientation. At large time this discrepancy vanishes.

5.1. Drawdown Near a Horizontal Well

[36] Figure 2 shows four different piezometer locations in points A (0, 10 m, 5 m), B (10 m, 10 m, 5 m), C (0, 10 m, 2.5 m), and D (0, 10 m, 7.5 m). The following set of typical parameters was used to generate Figures 3: initial saturated thickness of the unconfined aquifer is $d = 10$ m. The specific storage and specific yield are $S_s = 2.0 \times 10^{-5} \text{ m}^{-1}$ and $S_y = 0.2$, respectively. The hydraulic conductivity is isotropic: $K_x = K_y = K_z = 10^{-4} \text{ m/s}$, and the pumping rate is $Q = 2.0 \times 10^{-2} \text{ m}^3/\text{s}$. Sections 5.1.1–5.1.3 present the instantaneous drainage model, and the delayed yield model is used in section 5.1.4.

5.1.1. Effect of piezometer location

[37] The horizontal well is 20 m long and is located at the center of the aquifer ($z_w = 5$ m). Points A and B are located at the same elevation as the horizontal well, but point A is closer to the well center ($x = 0$), and point B is closer to the end of the well screen ($x = 10$ m). Points C and D are located in the yz plane but have different elevations. Thus comparison of the drawdown between points A and B and points C and D can show the dependency of drawdown on x and z coordinates, respectively.

[38] Analysis of the dimensionless drawdown on Figure 3 in these four points indicates the following features. The types curves in log-log coordinates show three stages of the drawdown that are typical of unconfined aquifers [e.g., Neuman, 1972, 1974; Moench, 1995, 1997]: a rapid increase stage at the early time, a “flat” stage at the intermediate time, and another rapid increase stage at the late time. These stages are also characteristic of horizontal wells in confined aquifer

or reservoir [Goode and Thambynayagam, 1987; Daviau et al., 1988].

[39] When time is sufficiently large, all drawdown curves converge to the Theis curve with respect to t_{Dy}/r_D^2 . At that time the groundwater flow is essentially horizontal, and water is released from the aquifer by gravity drainage [Neuman, 1972, 1974; Moench, 1997; Barlow and Moench, 1999]. Our results show that the drawdown near the horizontal well at the late stage of pumping is similar to the drawdown of a large diameter vertical well.

[40] At the intermediate pumping stage, drawdown at point A is generally higher than that at point B, as expected for the finite screen length of the pumping well. The drawdown at point C (near the aquifer bottom) is higher than that at point D (near the water table) at the intermediate pumping stage. This indicates the presence of the downward vertical flow component (drainage). Comparison of the

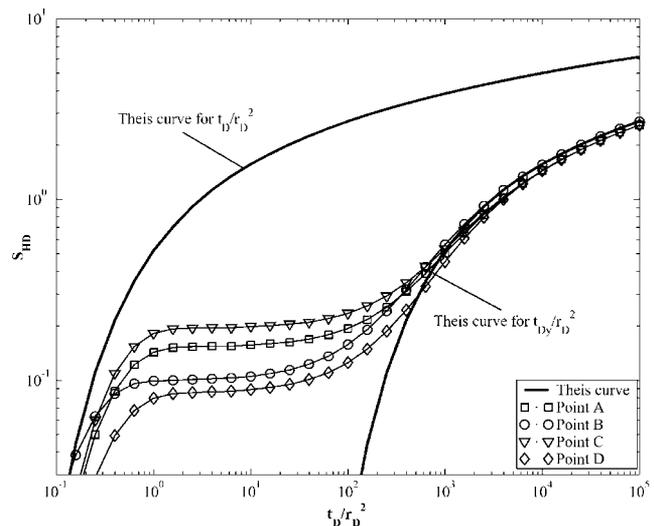


Figure 3. Type curves at points A, B, C, and D of Figure 2 near the horizontal well. Their curves for vertical well are plotted in coordinates t_D/r_D^2 and t_{Dy}/r_D^2 .

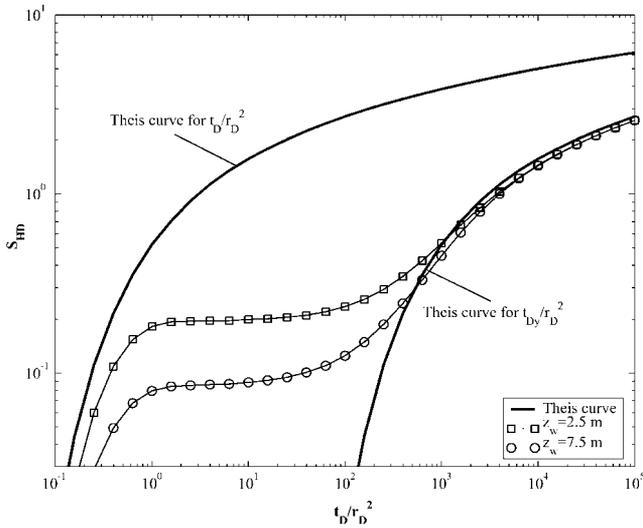


Figure 4. Type curves in piezometer location (0, 10 m, 5 m) for different depths of horizontal well ($z_w = 2.5$ and 7.5 m above the bottom of the aquifer).

drawdown between points A and B and between points C and D indicates that the drawdown is more sensitive to changes of the z coordinate than changes of the x coordinate at this stage.

5.1.2. Effect of the horizontal well elevation

[41] Figure 4 compares the drawdown that is created by the horizontal wells located near the aquifer bottom ($z_w = 2.5$ m) and closer to the water table ($z_w = 7.5$ m). The piezometer is located at point A (0, 10 m, 5 m) for both horizontal well elevations. The results show that a higher well location creates a smaller drawdown at point A than a lower well location at the intermediate pumping stage. A higher well collects more water by draining the pore space near the water table and less water from the aquifer storage than the lower

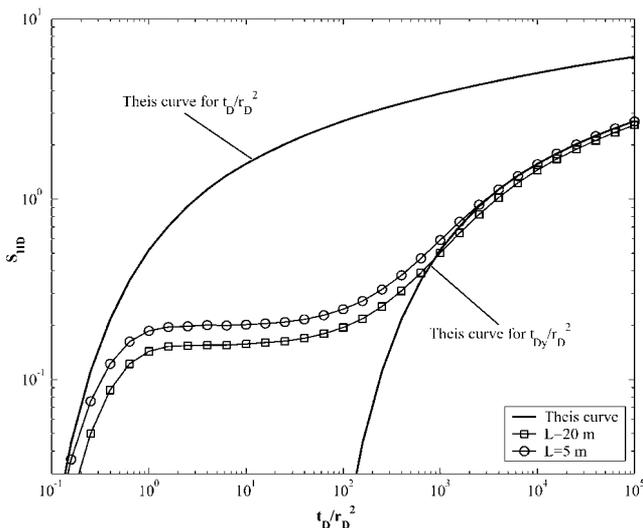


Figure 5. Type curves in piezometer location (0, 10 m, 5 m) for two different lengths of horizontal well ($L = 20$ and 5 m).

well. At larger times t_{Dy}/r_D^2 both curves converge to the Theis curve; that is, the horizontal well behaves like a large diameter vertical well, and the location of the horizontal well does not influence the drawdown at the later stages.

5.1.3. Effect of the screen length

[42] Figure 5 compares the drawdown that is created by the horizontal wells with different screen lengths: $L = 20$ and 5 m. The horizontal well is located at the middle elevation of the aquifer, and the piezometer is located at point A (0, 10 m, 5 m) in both cases. The drawdown created by the shorter well is higher than that invoked by the longer well at the intermediate stage. This can be explained by the higher strength of sinks that represent the well screen and by a closer proximity of these sinks to point A in the case of a shorter screen. Later time responses for both configurations are similar.

5.1.4. Effect of the delay index

[43] Figure 6 compares models of instantaneous drainage ($\alpha_1 = \infty$) and delayed yield with $\alpha_1 = 10^{-2}$, 10^{-4} , and 10^{-6} s^{-1} for a piezometer at point A (0, 10 m, 5 m). The horizontal pumping well is 20 m long and is located at the middle elevation $z_w = 5$ m. The difference between the instantaneous drainage model and a delayed yield model with $\alpha_1 = 10^{-2} \text{ s}^{-1}$ is negligible. For this piezometer the instantaneous drainage model is suitable, even when the delayed drainage ($\alpha_1 > 10^{-2} \text{ s}^{-1}$) exists.

[44] With the decrease of parameter α_1 the drawdown curve becomes closer to that of a confined aquifer. The mathematical explanation is apparent from equation (6). The physical rationale is as follows: with the decrease of α_1 the “delay index” ($1/\alpha_1$) increases, and the drainage response time increases. In the extreme case when $\alpha_1 \rightarrow 0$ an infinite time is needed for drainage; that is, the water table boundary becomes impermeable, and the aquifer becomes a confined aquifer.

[45] First efforts to elucidate relationship between the delay index and soil type were made by Prickett [1965]. Later, Moench [1995, 1997] addressed the physical aspects of the

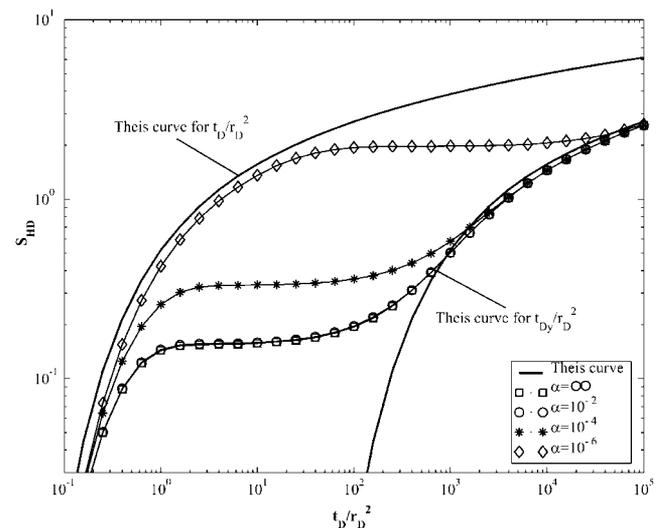


Figure 6. Type curves for horizontal well in piezometer location (0, 10 m, 5 m) for different drainage parameters α_1 in isotropic aquifers.

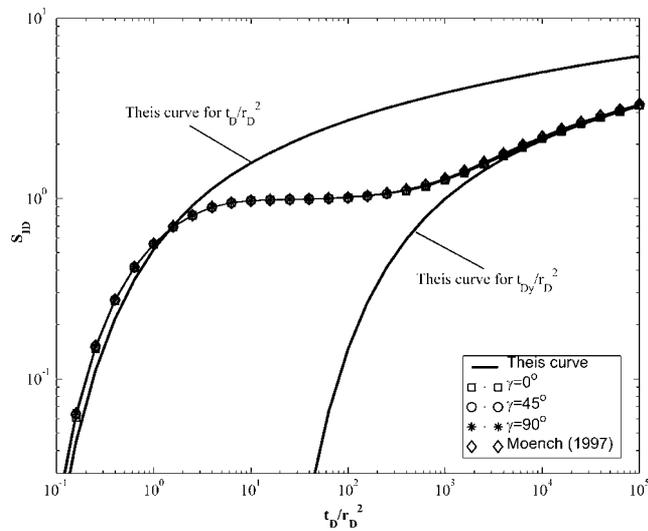


Figure 7. Type curves for slanted well with different angles of inclination (0° , 45° , and 90°) and comparison with the *Neuman* [1974] and *Moench* [1997] solutions for piezometer location (0, 10 m, 15 m).

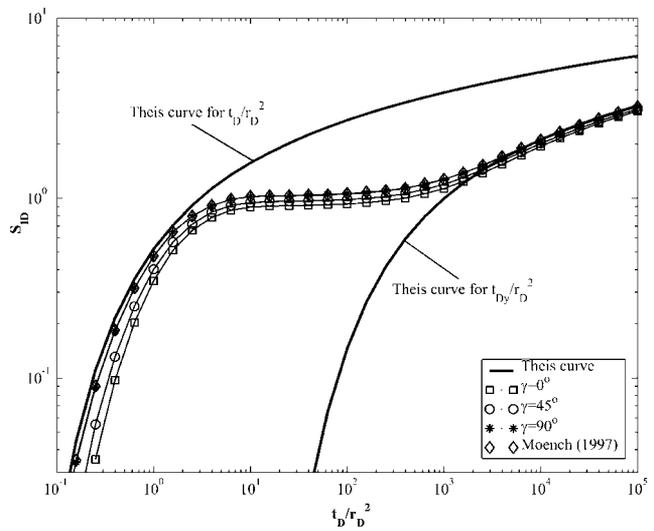


Figure 8. Type curves for slanted well with different angles of inclination (0° , 45° , and 90°) and comparison with the *Neuman* [1974] and *Moench* [1997] solutions for piezometer location (0, 10 m, 5 m).

delayed index. Recently, *Moench et al.* [2001] presented data related to the delay index for the Borden and Cape Code sites.

5.2. Drawdown Near the Slanted Well

[46] We will discuss the influence of the angle of inclination on the drawdown. The initial aquifer thickness is $d = 30$ m. The center of the slanted well is located at the middle elevation of the aquifer ($z_w = 15$ m). The screen is $L = 20$ m long. The aquifer hydraulic properties are the same as those used in the horizontal well analysis. For the sake of brevity, only the instantaneous drainage model is considered.

[47] We start with the analysis of the drawdown at point A (0, 10 m, 15 m). This point is located on the rotational axis of the slanted well, where the rotational axis is defined as the line perpendicular to the xz plane and across the center of the well. Figure 7 shows the drawdown at that point for different values of the angle of inclination: 0° (horizontal well), 45° , and 90° (vertical well). The differences between the drawdowns that are created by wells with different angles of inclination are small in this particular observation point. The rotational symmetry is only weakly affected by the presence of the horizontal aquifer boundaries, and the drawdown at the rotational axis of this slanted well is insensitive to this angle. The drawdown for a 90° angle is identical to the *Neuman* [1974] or *Moench* [1997] solutions when the well storage of the pumping and observation wells and delayed response are excluded.

[48] Figure 8 shows the results of pumping from the slanted wells with different angles of inclination at point B (0, 10 m, 5 m), which is below point A. The drawdown at point B exhibits some sensitivity to the screen orientation. The case of the 90° angle is identical to the *Neuman* [1974] or *Moench* [1997] results. Further departure of the piezometer from the rotational axis will enhance this sensitivity. In general, the traits of drawdown near the slanted wells are

similar to those for horizontal wells and are determined by the well orientation and location of piezometers.

5.3. Water Table Elevation

[49] An important application of horizontal or slanted wells is drainage of mines, waterlogged terrain, etc., where the effects of pumping on water table elevations are of interest. The water table elevation is obtained by setting the vertical coordinate of the observation point to the initial aquifer thickness ($z = d$) in equation (23) or equation (26).

[50] Figure 9a is the three-dimensional view of the water table at time $t = 10^5$ s = 1.16 days of pumping from a horizontal well. The following parameters were used in this analysis: $d = 20$ m, $L = 40$ m, the well is located at the middle elevation of the aquifer ($z_w = 10$ m), the hydraulic

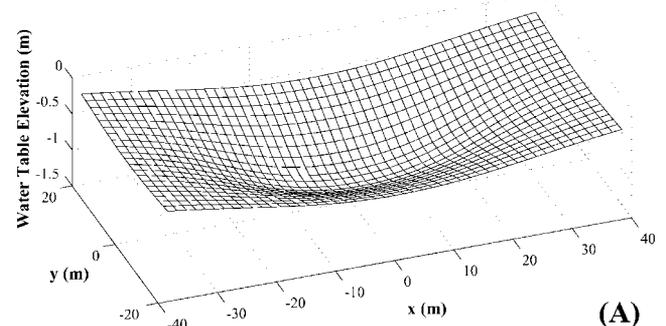


Figure 9. (a) Water table elevation at time $t = 10^5$ s near the pumping horizontal well; (b) cross section of water table near horizontal well for a vertical plane parallel to the well axis at $y = 2$ m.

conductivity is isotropic ($K_x = K_y = K_z = 10^{-4}$ m/s), the specific storage is $S_s = 2.0 \times 10^{-5} \text{ m}^{-1}$, the specific yield is $S_y = 0.2$, and the pumping rate is $Q = 0.01 \text{ m}^3/\text{s}$.

[51] Figure 9a shows that the largest drawdown s_{\max} is reached at point $(x, y) = (0, 0)$. The drawdown distribution does not have axial symmetry; however, there is mirror symmetry with respect to vertical planes $x = 0$ and $y = 0$. The bottom of the three-dimensional water table profile is relatively flat. We also notice that a significant area has been dewatered by the horizontal well. Addition of several parallel horizontal wells at the same or similar elevations can yield a more uniform horizontal water table beneath the drained area.

[52] Figure 9b shows a vertical cross section of the water table along the horizontal well axis (x axis) at three different times: $t = 10^5, 10^6$, and 10^7 s. Figure 9b indicates that the change of the water table is approximately linear function of $\ln t$. This is consistent with the results of Figures 3–8, which show that the type curves converge to the Theis curve (for t_{Dy}/r_D^2), and the Theis function is linear function of $\ln t$ at the large time.

[53] One should note that after $t = 10^6 \text{ s} \approx 11.6$ days, the largest drawdown is $s_{\max} \approx 2 \text{ m}$ or ~ 0.1 of the aquifer saturated thickness d , and the model is very accurate for all practical purposes. With increasing time the linearized model becomes less accurate. However, the relative error increases very slowly with time as a linear function of $\ln t$. Bear [1979, p. 332] indicated that the linearized boundary conditions can be valid if $s_{\max} < 0.5 d$. Some practical recommendations on improving the accuracy of the linearized model were presented by Neuman [1975, p. 334].

6. Summary and Conclusion

[54] To our knowledge, this paper presents the first three-dimensional solutions of the transient groundwater flow near horizontal and slanted wells in unconfined aquifers. These solutions are presented for a broad range of hydrogeological conditions, aquifer properties, and well geometry. The solutions can be used for analysis of the instantaneous drainage or delayed yield processes near the water table in the aquifer. In addition, these solutions permit treatment of the general and practically important case of aquifer anisotropy in hydraulic conductivity, when principal directions of anisotropy coincide with the axes of Cartesian coordinates ($K_x \neq K_y \neq K_z$). Solution for slanted wells include arbitrary angle of inclination of well axis.

[55] These fully three-dimensional solutions were obtained after the linearization of the boundary condition on the water table. The linearized boundary value problem was solved in two stages. First, flow to a point sink was studied in the Laplace transform domain. Then, the solution for flow to a horizontal well or a slanted well in the Laplace transform domain was obtained by the superposition of sinks of uniform strength. These solutions are valid for small water table inflection.

[56] The computer program for numerical Laplace transform inversion was developed to calculate the drawdown in the real-time domain or to generate type curves for horizontal or slanted wells. This program considers both full and partial penetration of wells and piezometers.

[57] The type curves for horizontal or slanted wells in log-log coordinates exhibit a three-stage response typical of

an unconfined aquifer: an early rapid increase, an intermediate “flat” stage, and late rapid increase stage. At the late stage, the type curves converge to the Theis type curves when the matching coordinate t_{Dy}/r_D^2 is used. This convergence is an indicator of essentially horizontal flow at the late stage of pumping, when water is released from the aquifer only by gravity drainage.

[58] Analysis of the drawdown that is created by the horizontal well was based on comparison of the differences in the piezometer responses. We investigated the effects of the well and piezometer locations, well screen length, and drainage parameters. This analysis indicates the important role of three-dimensional geometry (position, orientation, and size of the well and piezometer) and aquifer parameters in the piezometer responses in the zone near the well. In this zone, which has a characteristic size about that of the screen length, the following effects were observed: (1) drawdown is more sensitive to the change of the z coordinate than the change of the x coordinate at the intermediate pumping stage; (2) a higher position of the pumping well creates a lower drawdown at the intermediate aquifer depths at the intermediate pumping stage; (3) at the stabilized stage, drawdown generated by a shorter pumping well is higher than that generated by a longer well in the vertical plane of symmetry that is normal to the well axis; (4) the drawdown curve becomes closer to that of a confined aquifer when the delayed drainage parameter $\alpha_1 \rightarrow 0$. In this case it takes infinite time to drain the water table, and the water table becomes an aquiclude.

[59] The investigation of the drawdown near the slanted wells with different angles of inclination indicates that the drawdown at the rotational axis of the slanted well is insensitive to this angle, while the drawdown at a point off the rotational axis weakly depends on the angle of inclination. The degree of dependence is determined by the location of the piezometer with respect to the well, aquifer boundaries, and well geometry. For an angle of inclination of 90° , the solution is identical to the Neuman [1974] and Moench [1997] solutions when the well storage and the delayed response of the observation well are not considered. The proposed solutions can be used for parameter identification, design of remediation systems, drainage, and mine dewatering.

Appendix A

[60] The solution satisfying boundary condition equation (14) can be written as [Dougherty and Babu, 1984; Moench, 1997, 1998]

$$\bar{s}_D(p) = \sum_{n=0}^{\infty} H_n(\mathbf{r}_D, \mathbf{r}_{0D}, p) \cos(\omega_n z_D). \quad (\text{A1})$$

For an instantaneous drainage model, substituting equation (A1) into equation (15) results in

$$\omega_n \tan(\omega_n \alpha_z) = p/\sigma. \quad (\text{A2})$$

For a delayed drainage model, substituting equation (A1) into equation (17) results in

$$\omega_n \tan(\omega_n \alpha_z) = \frac{p \alpha_{1D}}{\sigma(p + \alpha_{1D})}. \quad (\text{A3})$$

For a given p , the value of ω_n is determined by solving equation (A2) or (A3).

[61] After substituting (A1) into equation (13), multiplying by $\cos(\omega_n z_D)$, integrating from 0 to α_z in the z_D direction, one obtains an equation for H_n :

$$\nabla^2 H_n - \Omega_n^2 H_n = -4\pi f_n \delta(\mathbf{r}_D - \mathbf{r}_{0D}) \quad (\text{A4})$$

$$\Omega_n = [\omega_n^2 + p]^{1/2}, \quad f_n = 4 \frac{\cos(\omega_n z_{0D})}{p b(\omega_n)}, \quad (\text{A5})$$

$$b(\omega_n) = 2a_z + \sin(2\omega_n \alpha_z) / \omega_n.$$

[62] Here we used formula $\int_0^{\alpha_z} \cos(\omega_n z_D) \cos(\omega_m z_D) dz_D = \delta_{m,n} b(\omega_n) / (4\omega_n)$, where $\delta_{m,n} = 1$ if $n = m$, and $\delta_{m,n} = 0$ if $n \neq m$. Note that equation (A4) is the equation for Green's function for the two-dimensional modified Helmholtz equation in an infinite two-dimensional domain [Arfken and Weber, 1995, Table 8.5]. Considering factor $4\pi f_n$, solution of this problem is as follows:

$$H_n = \frac{8 \cos(\omega_n z_{0D})}{p b(\omega_n)} K_0(\Omega_n |\mathbf{r}_D - \mathbf{r}_{0D}|), \quad (\text{A6})$$

where $K_0(u)$ is the second kind, zero-order modified Bessel function. Substituting equation (A6) into equation (A1) results in the solution for \bar{s}_D .

Notation

$b(\omega_n)$	$= 2\alpha_z + \sin(2\omega_n \alpha_z) / \omega_n$.
d	saturated thickness of the unconfined aquifer before pumping [L].
f_n	factor in Green's function.
$F(x'_D)$	$= [(x_D - x'_D)^2 + y_D^2]^{1/2}$.
h	hydraulic head [L].
h_0	static hydraulic head [L].
K	effective hydraulic conductivity [L/T], equal to $(K_x K_y K_z)^{1/3}$.
K_x, K_y, K_z	hydraulic conductivity in the x , y , and z directions, respectively [L/T].
$K_0(x)$	the second kind, zero-order modified Bessel function.
L	screen length of the horizontal or slanted well [L].
L_D	dimensionless screen length of the horizontal or slanted well, equal to $\alpha_x L / D$.
p	Laplace transform parameter referring to dimensionless time $[T^{-1}]$.
$q_3(x, y, z)$	three-dimensional sink strength $[T^{-1}]$.
$q(x)$	pumping rate per unit screen length for horizontal well $[L^2/T]$.
$q(l)$	pumping rate per unit screen length for slanted well $[L^2/T]$.
Q	pumping rate, positive for pumping $[L^3/T]$.
r_D	dimensionless radial distance, equal to $(x_D^2 + y_D^2)^{1/2}$.
\mathbf{r}_D	vector of point (x_D, y_D) , equal to (x_D, y_D) .
r_{0D}	dimensionless radial distance between the point source and origin.
\mathbf{r}_{0D}	vector of point (x_{0D}, y_{0D}) in horizontal plane, equal to (x_{0D}, y_{0D}) .

s	drawdown near point source [L].
s_D	dimensionless drawdown near point source.
\bar{s}_D	dimensionless drawdown in Laplace domain near the point source.
\bar{s}_{HD}	dimensionless drawdown in Laplace domain near the horizontal well.
\bar{s}_{ID}	dimensionless drawdown in Laplace domain near slanted well.
s_{\max}	maximal drawdown [L].
S_s	specific storage $[L^{-1}]$.
S_y	specific yield.
t	time [T].
t_D	dimensionless time.
x, y, z	Cartesian coordinates [L].
x_0, y_0, z_0	coordinates of the point sink [L].
x_D, y_D, z_D	dimensionless Cartesian coordinates.
x_{0D}, y_{0D}, z_{0D}	dimensionless coordinates of the point sink.
z_w	elevation of the horizontal well or center of the slanted well [L].
z_{wD}	dimensionless elevation of the horizontal well or center of the slanted well.
z_{us}, z_l	elevations of the upper and the lower points of the piezometer screen [L].
z_{uD}, z_{lD}	dimensionless elevations of the piezometer screen.
z_{uw}, z_{lw}	elevations of the upper and the lower point of the slanted well screen [L].
z_{uwD}, z_{lwD}	dimensionless elevations of the slanted well screen.
α_1	empirical constant for drainage from the unsaturated zone $[T^{-1}]$.
α_{1D}	$= S_s d^2 \alpha_1 / K$ (dimensionless).
$\alpha_x, \alpha_y, \alpha_z$	anisotropy coefficients $(K/K_x)^{1/2}, (K/K_y)^{1/2}, (K/K_z)^{1/2}$, respectively.
γ	angle of inclination between the slanted well and the horizontal plane.
$\delta(u)$	Dirac delta function $[L^{-1}]$.
σ	$= (S_s d^2) / (S_y \alpha_z)$.
Ω_n	$= [n^2 \pi^2 + p]^{1/2}$.

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