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Physics, Chapter 22: Electrostatics

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Part Four

ELECTRICITY AND MAGNETISM

22

Electrostatics

22-1 Introduction

One simple phenomenon of electricity was known to the ancients: that when a piece of amber was rubbed, it acquired the property of attracting small pieces of paper and other light particles. Records show that Thales of Miletus (circa sixth century B.C.) knew of this property of amber; the Greek word for amber is *elektron*, hence the name *electricity*. There was practically no further development of this subject until about the seventeenth century. Otto von Guericke (1602–1686) of Magdeburg built a large sulphur sphere, which, when rotated about an axis and rubbed with his hand, gave off electric sparks. In the eighteenth century it was found that there were two kinds of electricity, one similar to the kind acquired by amber when rubbed with wool, and the other similar to that acquired by glass when rubbed with silk, called *resinous* and *vitreous* electricity, respectively. They are now known as *negative* and *positive* electricity, names first introduced by Benjamin Franklin (1706–1790). Franklin made many important contributions to the subject, experimentally and philosophically. He showed the electrical character of lightning and designed lightning rods for the protection of buildings. The subject of electricity was put on a firm mathematical foundation as a result of the experiments of Coulomb (1785) on the law of force between electrically charged bodies.

22-2 Electrified or Charged Bodies

Any substance, when rubbed, becomes *electrified* or *charged electrically.* If a glass rod is rubbed with a piece of silk, both the rod and the silk become electrified or charged. It is now known that the purpose of rubbing two substances together is to bring parts of their surfaces into sufficiently close contact so that electrically charged particles, called *electrons*, can be transferred from one body to the other. The process of charging bodies by rubbing them together used to be called frictional electrification; it is now more properly called *electrification by contact* or *charging by contact.*

In order to study the behavior of electrically charged bodies, let us suspend a glass rod A by means of a string and then charge the rod electrically by rubbing it with a piece of silk. Suppose we take another glass rod B, which has been rubbed with a piece of silk, and bring it near the first rod A, as shown in Figure 22-1. We shall find that there is a force of *repulsion* between them. If we perform a similar experiment with two rubber rods, each of which has been rubbed with a piece of fur or wool, we shall find that there is a force of repulsion between the two charged rubber rods. If we now take one of the charged rubber rods and bring it near the charged glass rod A, we shall find that there is a force of *attraction* between them. From this set of simple experiments we can conclude, first, that *there are two kinds of electric charges;* and second, that *like charges repel each other and unlike charges attract each other.*

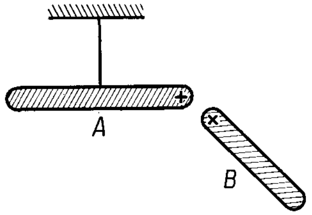

Fig. 22-1 Two glass rods, charged in the same manner, repel each other.

We can use the charged glass rod and the charged rubber rod as our standards and compare all other charged bodies with them. We shall find that there are only two kinds of charges: those which are similar to the charge on the glass rod and are repelled by it, and those which are similar to the charge on the rubber rod and are repelled by it. To distinguish between these two kinds of charges, the charge carried by the glass rod which has been rubbed with silk is arbitrarily called a *positive charge,* and the charge on the rubber rod which has been rubbed with wool or fur, a *negative charge.* The ultimate test of the nature of the charge on any body is to bring this charge near a properly charged glass rod or rubber rod. If the charged body is repelled by the positively charged glass rod, its charge is positive. If it is not repelled by the charged glass rod, it may be brought near the negatively charged rubber rod; if it is repelled by the charged rubber rod, the body carries a negative charge.

22-3 Insulators and Conductors

We have seen that all bodies can be charged electrically by being rubbed. But if a metal sphere is held in the hand and rubbed with a piece of fur and then tested, it will be found to be uncharged. However, if the metal sphere is mounted on a hard-rubber stand or a glass stand and then rubbed with a piece of fur, it will be found to be charged. The reason for this behavior is that metals are *good conductors* of electricity while rubber and glass are *nonconductors* or *insulators*. The human body is also a conductor of electricity, and so is the ground. When the metal sphere was held in the hand, the charge produced on it by rubbing was conducted away by the hand through the body to the ground. But when the sphere was placed on an insulator, the charge produced by rubbing remained on the sphere.

Materials can generally be classified as either conductors or insulators with some degree of accuracy, although there are many borderline cases, including a class of materials called semiconductors which have recently assumed considerable importance in electronics. The quantitative measurement of the conductivity of substances will be considered in a later chapter. For the present, it will be sufficient to give a qualitative discussion of conductivity. All metals in the solid state have been found to be good conductors of electricity and are also fairly good conductors in the liquid state. Some nonmetals, such as carbon and selenium, are also good conductors. Solids such as glass, rubber, amber, and most plastic materials are good insulators. All gases, under ordinary conditions, are poor conductors of electricity. Certain liquid solutions of materials, which in chemistry are known as acids, bases, and salts, are conductors of electricity, while oils, liquid solutions of organic substances such as sugar, and pure water are nonconductors.

22-4 The Electrical Theory of Matter

During the past half century a great deal of information has been obtained concerning the structure of matter and its behavior under the action of various external forces. It will be worth while at this point to outline briefly the electrical theory of matter to aid us in understanding the phenomena to be discussed later. The fact that all bodies can be charged electrically suggests that electric charges are common to all matter. Since, under normal conditions, a body is electrically *neutral*, that is, uncharged, such a body must contain equal amounts of positive and negative electricity so arranged that no electrical effects can be observed outside the body. From investigations of the structure of the atom, it is now known that each atom consists of a *small positively charged massive nucleus made up of positively charged protons and uncharged neutrons*, surrounded by groups of small negatively charged particles called *electrons*. The mass of an electron is very small in comparison with that of the nucleus, it is about 1/1,840th mass of a single proton. The force of attraction between the nucleus and any electron in the atom depends upon the position of the electron with respect to the nucleus; the force on an outer electron is much smaller than that on an inner electron. The charges which can be removed from the atom most readily are the outermost negative electrons. We may think of the process of rubbing two bodies together as bringing electrical forces into play during the time of contact between the two surfaces. These forces produce a displacement of the negative electrons from one body to the other. The body from which electrons have been removed becomes positively charged, while the one to which electrons have been added becomes negatively charged.

When atoms are brought close together, as they are in liquids and solids, the electric charges exert forces on each other, causing some rearrangement among the charges, particularly among the outermost electrons. In the case of metals this rearrangement is such that some of the outermost electrons in each atom can move freely from atom to atom. These are sometimes referred to as the *free electrons* of the metal. In the case of insulators the electrons are tightly bound to the atoms and can be moved from atom to atom only by the application of a comparatively large force.

22-5 Charging by Induction

If a negatively charged rubber rod is brought near an insulated metal sphere A, as shown in Figure 22-2, the electrons in the metal will be repelled; the farther side of the sphere will become negatively charged, and the side near the rubber rod will become positively charged. If the experimenter

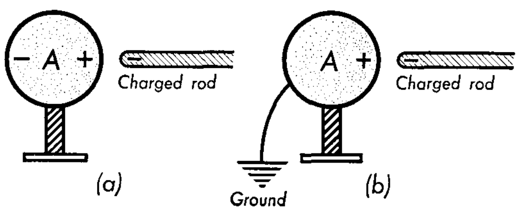

Fig. 22-2 Method of charging a metal sphere by induction.

touches the sphere with his hand for a short time, some electrons will be repelled through him into the ground, thus leaving the sphere with a net positive charge when the hand and rod are removed successively. This method of charging a body by bringing it close to a charged body, without making actual contact, is called *charging by induction.* The fact that the sphere is positively charged can be verified by bringing it near a positively charged glass rod and noting the force of repulsion between them. Grounding of the sphere can also be done by connecting a wire directly from the ground to the sphere. If the insulated positively charged sphere is removed from the neighborhood of other charged bodies and then again connected to ground, electrons will be attracted up from the ground until the sphere is neutral. In this case we say that the sphere has been *discharged* by connecting it to ground.

To charge a metal sphere negatively by induction, we can bring a positively charged body near the sphere. Some of the electrons will be attracted toward the positively charged body and will go to the surface nearest it, leaving the rest of the surface positively charged. If the sphere is momentarily connected to ground, electrons will be attracted up from the ground during the time of contact; the sphere is now negatively charged. If the original positively charged body is now removed, the excess electrons will redistribute themselves uniformly over the surface of the sphere because of the forces of repulsion between them.

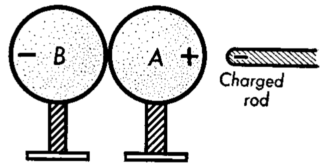

Fig. 22-3 Charging two metal spheres by induction.

An interesting variation of this experiment which does not involve the use of a ground connection is illustrated in Figure 22-3. Two metal spheres A and B, each on an insulated stand, are brought together so that their surfaces touch each other. If a negatively charged body is now brought near A, electrons will be repelled from it to B. If the spheres are now separated, A will be positively charged and B will be negatively charged. Since the total charge on the two bodies was originally zero, the negative charge on B must be numerically equal to the positive charge on A.

22-6 The Electroscope

The electroscope is a simple and sensitive electrical instrument which can be put to a variety of uses. It consists essentially of a metal rod to which two pieces of very thin, light, gold or aluminum foil are attached. The rod is passed into the case through an insulating bushing. The two pieces of foil, or *leaves*, as they are usually called, are enclosed in a container made either of glass or of metal, provided with glass windows, as shown in Figure 22-4. Part of the metal rod projects outside the case so that contact can be made with it. The rod may be provided with any kind of top such as a metal sphere or a flat metal plate. The electroscope can be charged by contact between the rod AB and a charged body. The charge distributes itself over the top, the rod, and the leaves. Since the two leaves acquire like charges, they repel each other and diverge until equilibrium is established. The angle of divergence can be used as a measure of the force between the leaves and also as a measure of the charge on them.

The electroscope can also be charged by induction. Let us suppose that a positively charged body is brought close to the rod of the electroscope. Some of free electrons of the rod-leaf assembly will be attracted to the rod by the positively charged body, with the result that the leaves will be positively charged. The leaves will diverge. By touching the hand, or a ground connection, to the electroscope rod, additional negative charge is attracted to the leaves, and the leaves will collapse. If the hand is removed from the rod, and then the positively charged body is removed, the leaves of the electroscope will diverge again. The entire leaf-rod assembly has now been negatively charged by induction.

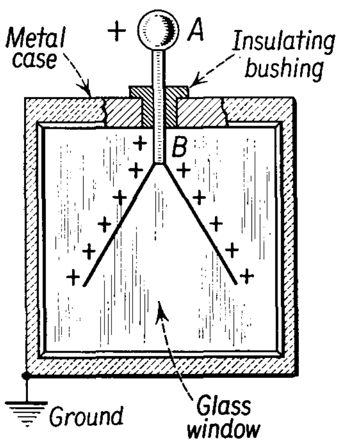

Fig. 22-4 A gold-leaf electroscope.

A charged electroscope is very convenient for determining the sign of the charge on any body brought near it. Suppose that the electroscope is positively charged and that the leaves diverge through a definite angle. If a positively charged body is brought near the electroscope, the leaves will diverge still more. The reason for this is that negative electrons will be attracted up from the leaves by the positively charged body, thus increasing the positive charge on the leaves. Of course if a negatively charged body is brought near the positively charged electroscope, the angle of divergence of its leaves will be decreased.

22-7 The Electrophorus

The process of charging by induction is the basis for the design and construction of many electrostatic machines used for supplying electricity for certain special needs. The *electrophorus* is a simple type of electrostatic machine. It consists of a flat disk made of some insulating material such as hard rubber, sealing wax, or resin, and a metal disk with an insulating handle. The disk A, made of insulating material, is first charged by rubbing it with wool or fur; this gives the disk a negative charge. The metal disk B is now brought very close to the charged disk A. (In actual practice, B is placed on A, but because of the roughness of its surface, contact is made at very few points; the charge transferred at these points is negligibly small.) The electrons in the metal disk are repelled to its upper surface, leaving the lower surface positively charged (see Figure 22-5). The plate B is now grounded momentarily by touching it with the hand or with a grounded wire; the electrons will go to ground, leaving the plate B positively charged. As long as this plate is close to the charged disk A, the positive charge will remain on the lower surface of B, but if this metal disk is removed far enough from the plate A, the charges in the metal disk will redistribute themselves so that both upper and lower surfaces are positively charged. This positive charge is now available for use in any manner we please. For example, the plate can be discharged through a tube containing neon so that it causes the emission of light characteristic of neon.

If we examine the operation of the electrophorus once again, we note first that practically no charge has been removed from the original charged disk A; hence this plate can be used over and over again. Second, in removing the disk B from the vicinity of disk A, we had to work against the forces of attraction between the charges on the two disks. It was because of this work that there was energy available to operate the neon lamp or any other appropriate device. This process can be repeated indefinitely and as often as desired, the work being supplied by the person who separates the charges.

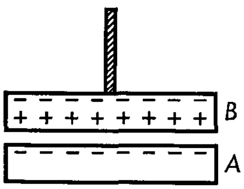

Fig. 22-5 The electrophorus.

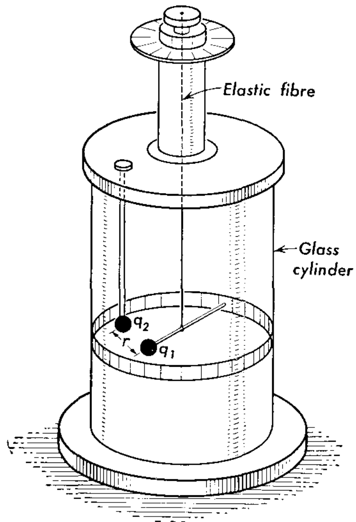

Fig. 22-6 Coulomb type torsion balance for determining the force between two charged bodies.

22-8 Coulomb's Law of Force between Charges

The first quantitative measurement of the force between charged bodies was made in 1785 by Charles Augustin de Coulomb (1736–1806) using an apparatus similar to that used by Cavendish for the determination of the gravitational constant G_0 (Section 6-15). A rod with a small charged sphere at one end was suspended in a horizontal position from an elastic fiber, and another small charged sphere was brought near it, as shown in Figure 22-6. The force between the two charged bodies was measured by noting the amount of twist in the fiber supporting the rod.

Coulomb found that *the force between two small spheres charged with* *electricity was inversely proportional to the square of the distance between them;* that is,

$$F \propto \frac{1}{r^2}, \tag{22-1}$$

where F was the magnitude of the force exerted by each charged body upon the other when the two were separated by a distance r. Although Coulomb's determinations were made with equipment which could be considered crude by modern standards, more recent attempts to determine the law of force between charged particles have shown that the exponent of r in Equation (22-1) does not differ from 2 by more than 1 part in 1 billion. Experiments on the scattering of alpha particles from nuclei have shown that the inverse square law of force between charged particles is correct at distances of the order of 10^{-12} cm. This is of the order of the diameter of the nucleus.

Further research on the law of force between small charged bodies in *vacuum* showed that the force was proportional to the magnitudes of the charges as well. This can be put in the form of an equation

$$\boxed{F = k\frac{q_1 q_2}{r^2},} \tag{22-2}$$

where q_1 is the magnitude of the charge on the first body, q_2 is the magnitude of the charge on the second body, and k is some constant of proportionality which depends upon the system of units used. The force is directed along the line joining the two bodies and is repulsive if the bodies are of like charge but is attractive if the bodies are of unlike charge. While Equation (22-2) is rigorously true only for vacuum, it is very nearly correct for air at atmospheric conditions.

Equation (22-2), which is known as *Coulomb's law,* is in exactly the same form as the law of universal gravitation, as stated in Equation (6-17). There is one important difference between the law of gravitation and Coulomb's law. The constant G_0 of Equation (6-17) relates quantities which had been previously related and defined through Newton's second law. In Equation (22-2) both the constant k and the unit of charge remain to be defined and evaluated. We must decide whether to assign arbitrarily some value to k and thus let Equation (22-2) provide the basis for a definition of a unit of charge, or whether to define arbitrarily a unit of charge and use Equation (22-2) to provide the value of k. In practice, both methods are used.

22-9 The CGS Electrostatic Unit of Charge

In the cgs electrostatic system of units, the constant k of Coulomb's law is arbitrarily set equal to 1 for a vacuum; it is a dimensionless quantity. *The unit of electric charge is defined as one which, when placed one centimeter from a like equal charge in vacuum, will repel it with a force of one dyne.* The unit of charge is called the *statcoulomb* (stcoul), and is sometimes also referred to as the *esu of charge*, as an abbreviation of "electrostatic unit." The esu of charge, or the statcoulomb, is widely used in the older literature of physics and engineering and is currently used in atomic and nuclear physics, but it is too small in magnitude for practical purposes. In the cgs electrostatic system of units, Coulomb's law is written as

$$F = \frac{q_1 q_2}{r^2}, \qquad \textbf{(22-2a)}$$

where F is expressed in dynes, q is expressed in statcoulombs, and r is expressed in centimeters.

The unit of charge which has been universally adopted for practical application is called the *coulomb* (coul) which may be defined for our present purposes as approximately 3×10^9 stcoul. More exactly,

$$1 \text{ coul} = 2.998 \times 10^9 \text{ stcoul.}$$

The number of statcoulombs in a coulomb has been chosen as numerically equal to one tenth the velocity of light in free space, expressed in centimeters per second.

Illustrative Example. A small body carrying a charge of +20 stcoul is placed 6 cm from another small body carrying a charge of +30 stcoul. Both bodies are in vacuum. Determine the force acting on each body.

Since the physical quantities are stated in cgs units, we apply Coulomb's law in the form of Equation (22-2a)

$$F = \frac{20 \text{ stcoul} \times 30 \text{ stcoul}}{36 \text{ cm}^2}$$

$$= 16.7 \text{ dynes.}$$

The force on the 20-stcoul charge is 16.7 dynes and is directed away from the 30-stcoul charge along the line joining the two charges. The force acting on the 30-stcoul charge is equal in magnitude but is opposite in direction.

22-10 Rationalized MKS System of Units

We have already seen that the mks system of units incorporates units of practical size to a far greater extent than does the cgs system of units.

The mks system of units has been extended to include electrical quantities by incorporation of the coulomb as a unit of charge of practical size. The arbitrary choice of a unit of charge implies that the choice of the constant k in Equation (22-2) can no longer be arbitrary but must be determined experimentally. Both the magnitude of k and its dimensions are involved. When the charge is given in coulombs, the value of k is no longer unity but is found to be approximately $9 \times 10^9 \dfrac{\text{nt m}^2}{\text{coul}^2}$. More exactly,

$$k = 8.987 \times 10^9 \frac{\text{nt m}^2}{\text{coul}^2}.$$

In practical engineering computations, equations which are developed from Coulomb's law are used much more frequently than is Coulomb's law itself. In order to simplify these equations, it is convenient to incorporate a factor of 4π into Coulomb's law in order that this factor may be eliminated from many other equations used in routine computation. For this reason the proportionality factor k in Coulomb's law has been redefined in terms of a new constant ϵ_0 (epsilon zero) as

$$k = \frac{1}{4\pi \epsilon_0}, \tag{22-3}$$

so that

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \frac{\text{coul}^2}{\text{nt m}^2}.$$

Coulomb's law in mks units is therefore written as

$$\boxed{F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2},} \tag{22-2b}$$

where F is expressed in newtons, q is expressed in coulombs, and r is expressed in meters. The constant ϵ_0 is called the *permittivity* of vacuum. A system of units in which the factor $\dfrac{1}{4\pi}$ appears in Coulomb's law is called a *rationalized system of units*. The system of units introduced in this section is generally known as the *rationalized mks system of units*. The units of ϵ_0 are sometimes stated in terms of other electrical units than those given above, but these all reduce to $\dfrac{\text{coulomb}^2}{\text{nt m}^2}$, as, for example, $\dfrac{\text{coulomb}^2}{\text{joule m}}$.

Illustrative Example. Two equally charged spheres, each having a mass of 1 gm, are suspended from a common point by silk threads 1 m long, as shown in Figure 22-7(a). Find the charge on each sphere if the angle between the threads is 10°.

The forces acting on each sphere are its weight mg, the tension in the string T, and the electrical repulsive force F_e, as shown in Figure 22-7(b). Since each sphere is in equilibrium under the action of these forces, their vector sum is zero, thus the vector diagram is a triangle. From Figure 22-7(c) we see that

$$F_e = mg \tan \frac{\theta}{2} \cdot$$

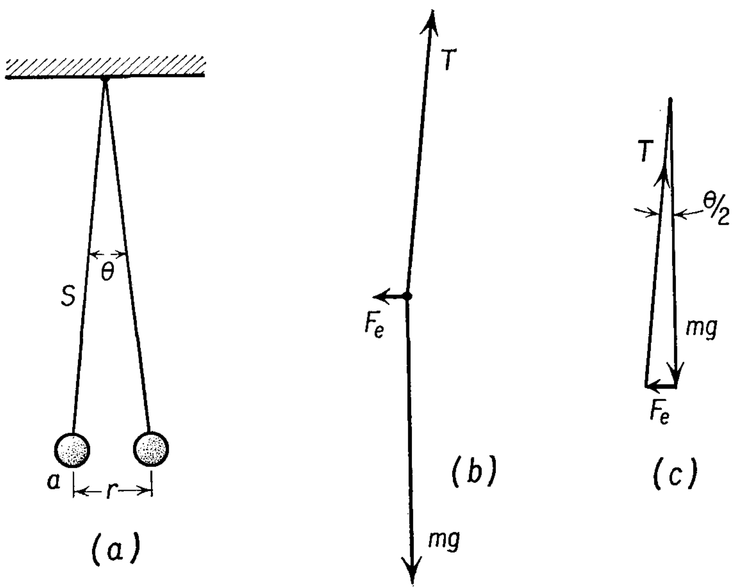

Fig. 22-7 (a) Electrified spheres hung from a common point. (b) Forces on the sphere at a. (c) The vector sum of the forces is zero so the vectors form a triangle.

Putting in numerical values, we get

$$F_e = 0.001 \text{ kg} \times 9.8 \frac{\text{m}}{\text{sec}^2} \times 0.087 = 85 \times 10^{-5} \text{ nt.}$$

Now
$$r = 2s \sin 5^\circ$$
$$= 2 \times 1 \text{ m} \times 0.087$$
$$= 0.174 \text{ m.}$$

From Coulomb's law

$$q^2 = 4\pi\epsilon_0 F_e r^2,$$

so that
$$q^2 = 1.11 \times 10^{-10} \frac{\text{coul}^2}{\text{nt m}^2} \times 85 \times 10^{-5} \text{ nt} \times (0.174)^2 \text{ m}^2.$$

Therefore
$$q = 5.3 \times 10^{-8} \text{ coul.}$$

22-11 Vector Form of Coulomb's Law

The formulas displayed thus far as statements of Coulomb's law have indicated only the magnitude of the force between two charged bodies *in vacuo.* For this reason numerical values without sign have been substituted for q in the illustrative examples. The direction of the force was given by statements not included in the formula itself.

To develop a self-contained vector statement of Coulomb's equation, we must first define a *unit vector.* If $\mathbf{r}$ is the vector drawn from an origin located at the charge q_1 to the charge q_2, as shown in Figure 22-8, the unit vector $\mathbf{1}_r$ may be defined by the relationship

$$\mathbf{1}_r = \frac{\mathbf{r}}{r}. \qquad (22\text{-}4)$$

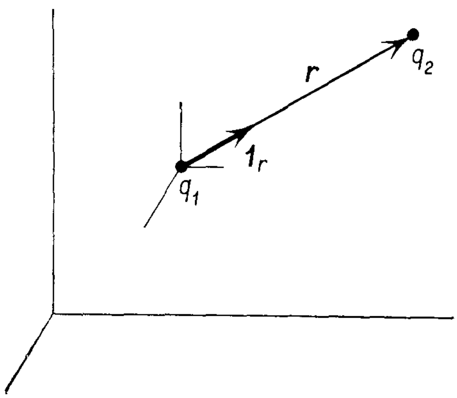

Fig. 22-8 The unit vector $\mathbf{1}_r$ is a vector of unit length directed from q_1 to q_2.

The unit vector $\mathbf{1}_r$ is a vector of unit length which is directed from q_1 to q_2.

Coulomb's law may be expressed in vector form as

$$\mathbf{F}_2 = k\,\frac{q_1q_2}{r^2}\,\mathbf{1}_r, \qquad (22\text{-}5)$$

where $\mathbf{F}_2$ is the force acting on the charge q_2 as a result of the presence of charge q_1. Since Equation (22-5) is a vector equation, it is necessary to write both the magnitude and the sign of the charge when substituting values into it. Thus, if q_1 and q_2 are of the same sign, the force $\mathbf{F}_2$ is parallel to the unit vector $\mathbf{1}_r$, while if the charges are of opposite sign, the force is in the direction of $-\mathbf{1}_r$, or oppositely directed to $\mathbf{1}_r$.

22-12 Atomicity of Charge; Conservation of Charge

Measurements made on a macroscopic scale seem to indicate that the electric charge on a body may have any arbitrary value. However, careful measurements of the electric charges on small droplets show that every electric charge consists of an integral multiple of a certain quantity of charge called the *charge of an electron* and designated by the letter e. The numerical value of this charge is

$$\begin{aligned} e &= 4.802 \times 10^{-10} \text{ stcoul} \\ &= 1.602 \times 10^{-19} \text{ coul.} \end{aligned}$$

No charge smaller than this has ever been found. All charged particles, both positive and negative, no matter how large or small, have always been found to have electric charges which are integral multiples of e.

According to present theory, all matter is composed of one or more elements; a few more than 100 elements are now known (see Table of Elements in the Appendix). Each element is composed of atoms which have certain common properties. The atoms of any one element have the same number of protons; this number is called the *atomic number* of the element and is designated by the letter Z. These Z protons are in a very small volume of the entire atom known as the *nucleus*. The charge of a proton is positive and equal to e; thus the nucleus has a charge Ze. In an electrically neutral atom there are Z electrons outside the nucleus. These electrons extend to distances from the nucleus equal to about 10,000 or 100,000 times the nuclear radius. The nucleus, in spite of its small size, also contains neutral particles, called *neutrons;* the only exception to this is the nucleus of the ordinary hydrogen atom, which is simply the proton. Atoms of any one element may differ in the number of neutrons in the nucleus. The neutron is slightly more massive than the proton.

Nuclei heavier than hydrogen are assembled out of the building blocks of protons and neutrons. The mass of the final nucleus is less than the mass of the appropriate number of unassembled neutrons and protons, but the charge of the nucleus is simply the summed charge of all the constituent protons. Although mass is converted into other forms of energy in the process of assembling the nucleus, the electric charge is conserved.

It is a fundamental concept of electricity that the total charge of the universe is constant; that charge is neither created nor destroyed. This may be called the principle of *conservation of charge*. No violation of this principle has ever been observed.

The principle of conservation of charge can be applied to any closed system. If some object inside this system acquires an excess of one kind of charge, another object or objects in this system must acquire an equal and opposite charge.

When we speak of a substance as being electrically neutral, we mean that the total number of positive charges is equal to the total number of negative charges. Since electrons are comparatively far from the nucleus, it is easier to remove electrons from an atom than it is to remove protons. When a body is charged positively, for example, it generally is done by removing electrons from it; if it is charged negatively, electrons are usually added to it. In *annihilation* reactions studied in nuclear physics (see Chapter 46), an electron and positron are observed to disappear simultaneously. Their mass is converted to radiant energy, called annihilation radiation. But the total charge of the universe remains constant, for the total charge of the electron-positron pair is zero before and after the event.

The process inverse to annihilation is known as *pair production*, in which electromagnetic radiation may be converted into mass by the conversion of the radiant energy into an electron-positron pair. Once again, charge is conserved. No event has been observed in which a single charged particle has either been annihilated or has materialized, and we may assert with some confidence that no such event will be observed in violation of the principle of conservation of charge.

Problems

[NOTE: 1 microcoulomb (abbreviated μcoul) = 10^{-6} coulomb.]

22-1. Two small spheres have charges of +600 stcoul and +300 stcoul and are 4 cm apart. Determine the magnitude and direction of the force exerted by the 300-stcoul charge on the 600-stcoul charge.

22-2. Two small spheres have charges of +200 μcoulombs and −50 μcoul and are 50 cm apart. Determine the magnitude and direction of the force exerted by the −50-μcoul charge on the 200-μcoul charge.

22-3. Determine the force, in newtons, exerted by the proton on the electron of the hydrogen atom if each has a charge of 1.60×10^{-19} coul and their separation is 0.53×10^{-8} cm.

22-4. In the Bohr model of the hydrogen atom, the electron is supposed to rotate about the proton in a circular orbit. Assuming that the centripetal force required for the circular motion is supplied by the force of electrostatic attraction, find (a) the frequency of rotation and (b) the linear speed of the electron in its orbit. Use the constants given in Problem 22-9.

22-5. Three point charges of 3, 4, and 5 stcoul, respectively, are located at the corners of an equilateral triangle of sides 10 cm long. Find the magnitude and direction of the force on the 5-stcoul charge. Place the 5-stcoul charge at the origin, and locate the 4-stcoul charge at coordinates (10, 0).

22-6. Two small spheres of mass 0.1 gm are hung from a common point on silk threads 50 cm long. When the spheres are equally charged, the angle between the threads is 15°. Find the charge on each sphere.

22-7. Two small spheres of equal mass are hung from threads 1 m long. The points of support of the threads are separated by a distance of 10 cm. When the charge on each sphere is 1 μcoul, the angle between the threads is 30°. Determine the mass of each sphere.

22-8. A point charge of 10 stcoul is located at the origin and a second charge of −40 stcoul is located at a point along the x axis 2 cm to the right of the first charge. (a) What is the force on a third charge of magnitude 1 stcoul which is located at coordinates (0, 4 cm)? (b) Where along the x axis may this third charge be placed so that the force acting on it is zero?

22-9. In the Bohr model of the hydrogen atom, an electron of mass 9.107×10^{-28} gm and of charge -4.803×10^{-10} stcoul is located at a distance of 0.529×10^{-8} cm from a proton of mass 1.672×10^{-24} gm and charge $+4.803 \times 10^{-10}$ stcoul. Determine the ratio of the gravitational force of attraction to the electrical force of attraction between the two particles.