# Physics, Chatper 24: Potential 

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## 24

## Potential

## 24-1 Potential Difference

A positive charge $q$ situated at some point $A$ in an electric field where the intensity is $\mathbf{E}$ will experience a force $\mathbf{F}$ given by Equation (23-1a) as

$$
\mathbf{F}=\mathrm{E} q .
$$

In general, if this charge $q$ is moved to some other point $B$ in the electric field, an amount of work $\Delta \mathscr{Y}$ will have to be performed. The ratio of the work done $\Delta W$ to charge q transferred from point A to point B is called the difference of potential $\Delta \mathrm{V}$ between these points; thus

$$
\begin{equation*}
\Delta V=V_{B}-V_{A}=\frac{\Delta \mathscr{W}}{q} \tag{24-1}
\end{equation*}
$$

where $V_{A}$ is the potential at $A$, and $V_{B}$ is the potential at $B$.
The quantity of charge $q$ should be so small that it does not disturb the distribution of the charges which produce the electric field $E$. Or we may imagine that successively smaller charges are used, and that the work done in moving each such charge is determined. The limit of the ratio of the work done to the charge transferred as the charge gets progressively smaller is the potential difference between the two points $A$ and $B$. Difference of potential is thus the work per unit charge that would be done in transferring charge from one point to another. A difference of potential can exist between two points even though no charge is actually transferred between them.

Potential difference is a scalar quantity, since both work and charge are scalar quantities. If a positive charge $q$ is transferred from $A$ to $B$, and if the work is done by some outside agency against the forces of the electric field, then point $B$ is said to be at a higher potential than $A$; if the work is
done by the electric field in moving a positive charge from $A$ to $B$, then the potential at $A$ is higher than that at $B$.

The unit of potential difference in the mks system is the volt. From Equation (24-1) we see that one volt is equal to one joule per coulomb.

In the cgs system of units, the unit of potential difference is the statvolt, which is equal to one erg per statcoulomb. We may find the relationship between the volt and the statvolt by following the usual unit conversion procedure. Thus

$$
\begin{aligned}
& 1 \text { volt }=\frac{1 \text { joule }}{1 \text { coul }} \times \frac{10^{7} \mathrm{ergs}}{1 \text { joule }} \times \frac{1 \text { coul }}{3 \times 10^{9} \text { stcoul }} \\
& =\frac{10^{7} \mathrm{ergs}}{3 \times 10^{9} \text { stcoul }}=\frac{1}{300} \text { statvolt. } \\
& \text { More exactly, } \quad 1 \text { statvolt }=299.6 \text { volts. }
\end{aligned}
$$

In the above discussion we have used the term potential at a point, while the definition was in terms of the difference of potential between two points. The term "potential at a point" can have meaning if we decide upon some reference point as a point of zero potential. In practical work this reference point or zero level of electrical potential is usually taken as the earth or the ground, and the potential at any other point is measured with respect to it. Electrical equipment is practically always connected to earth or to ground at some point, and other potentials are spoken of as being so many volts above or below ground potential. In many calculations in physics, in dealing with the properties of finite charge distributions, without reference to their position with respect to the earth, it is convenient to refer potentials to the potential of a point at infinity. In such calculations a point infinitely distant from the charge distribution is considered as the zero of potential.

The assignment of a zero of electrical potential is thus somewhat arbitrary and is analogous to the assignment of the position of zero potential energy in dealing with a particle in the earth's gravitational field. Although we did not develop the concept of gravitational potential in mechanics, the gravitational potential difference may be defined in terms of the work per unit mass in moving a mass between two points in the field. Since there is no work done in moving a mass along a frictionless level surface in the gravitational field, all points on a level surface have the same gravitational potential. The work done in raising a mass $m$ through a height $h$ in a field of gravitational intensity $g$ is $m g h$, and, dividing by $m$, we see that the gravitational potential difference is $g h$. Thus altitude is a measure of gravitational potential, which is customarily referred to an arbitrary zero of altitude at sea level.

## 24-2 Potential Due to a Point Charge in Vacuum

An isolated point charge in vacuum generates an electric field which is given by Equation (23-2) as

$$
\mathbf{E}=\frac{q}{4 \pi \epsilon_{0} r^{2}} \mathbf{1}_{r} .
$$

We recall that in this equation $r$ is the distance from the charge $q$ to the point where the field is being evaluated, and the unit vector $\mathbf{1}_{r}$ is directed from the charge to the field point.


Fig. 24-1 Let us place the charge $q$ at the origin of coordinates and calculate the work which must be done by some outside agency in moving a positive charge $q^{\prime}$ from a point $A$ in the electric field at a distance $r_{a}$ from the origin, to a point $B$ in the field at a distance $r_{b}$ from the origin, as shown in Figure 24-1.

Let us first show that the work done does not depend upon the path by which the charge $q^{\prime}$ is moved from $A$ to $B$. To do this we shall consider two alternate routes. In the first of these we shall move $q^{\prime}$ radially along the line $A C$, and then along the are of a circle $C B$. In this path, work is done only along the radial portion of the path, for here the force which must be exerted is equal and opposite to the force experienced by $q^{\prime}$ due to the electric field of $q$. No work is done by the agency displacing $q^{\prime}$ along the circular portion of the path, for here the force exerted is radial and is perpendicular to the displacement, which is tangential.

We may approximate the second route $A D B$ as closely as we please by a succession of radial and circular displacements. Again we see that work is done only during a radial displacement, for in each of the circular displacements the displacement is perpendicular to the force. Since the magnitude of the electric intensity depends only upon the distance from the origin and not upon the angular position, we see that the force exerted by the outside agency, and therefore the work done in a displacement between radial coordinates $r_{2}$ and $r_{1}$, is the same whether this radial dis-
placement takes place along the path $A C B$ or along the path $A D B$, or along any other path between $A$ and $B$.

The work done in carrying the charge $\mathrm{q}^{\prime}$ between two points in the field of a point charge $q$ is therefore independent of the path. The potential difference between the two points depends only on their position with respect to the charge $q$. To simplify the calculation of the potential difference between the points $A$ and $B$, we choose the path $A C B$ of Figure 24-1, where, as we have already seen, it is only necessary to calculate the work done along the radial portion of the path, $A C$. The force $F$ which must be exerted by an external agency is equal and opposite to the force exerted by the electric field on this charge. The force on a positive charge $q^{\prime}$ is given by

$$
\begin{equation*}
\mathbf{F}=-\mathbf{E} q^{\prime}=-\frac{q q^{\prime}}{4 \pi \epsilon_{0} r^{2}} \mathbf{1}_{r} \tag{24-2}
\end{equation*}
$$

The mechanical work $\Delta \mathscr{Y}$ done by the force $\mathbf{F}$ which is exerted on the charge $q^{\prime}$ in displacing it radially through a distance $\Delta r$ toward the point charge $q$ is given by

$$
\begin{equation*}
\Delta \mathscr{Y}=F \Delta r=-\frac{q q^{\prime}}{4 \pi \epsilon_{0} r^{2}} \Delta r \tag{24-3}
\end{equation*}
$$

To find the potential difference between the points $A$ and $B$, we must sum up the work done in transporting the charge $q^{\prime}$ over all the increments of path. In the limit of small increments of displacement, the work done $\Delta W$ is

$$
\Delta \mathscr{W}=-\int_{r_{a}}^{r_{b}} \frac{q q^{\prime}}{4 \pi \epsilon_{0} r^{2}} d r ;
$$

remembering that
we find

$$
\begin{align*}
\int \frac{d r}{r^{2}} & =-\frac{1}{r}+\text { const } \\
\Delta \mathscr{Y} & =\frac{q q^{\prime}}{4 \pi \epsilon_{0}}\left(\frac{1}{r_{b}}-\frac{1}{r_{a}}\right) \tag{24-4}
\end{align*}
$$

From the definition of the potential difference as the work done in transporting a test charge $q^{\prime}$ divided by the magnitude of that test charge, we have

$$
\begin{equation*}
\Delta V=\frac{\Delta \mathscr{W}}{q^{\prime}}=\frac{q}{4 \pi \epsilon_{0} r_{b}}-\frac{q}{4 \pi \epsilon_{0} r_{a}} . \tag{24-5}
\end{equation*}
$$

From Equation (24-1)

$$
\Delta V=V_{b}-V_{a}
$$

If the initial point $A$ is taken at $r_{a}=\infty$, it will be convenient to assign the
value zero to its potential. The potential at point $B$ will then be given by the equation

$$
V_{b}=\frac{q}{4 \pi \epsilon_{0} r_{b}} .
$$

Dropping the subscript $b$, we find that the potential $V$ at a point located at a distance $r$ from a point charge $q$ in vacuum is given by

$$
\begin{equation*}
V=\frac{q}{4 \pi \epsilon_{0} r} \tag{24-6}
\end{equation*}
$$

Note that in this expression $V$ is an algebraic scalar quantity, which may be either positive or negative. The distance $r$ is always a positive number, while the charge $q$ must be replaced by a positive number for a positive charge and by a negative number if the charge is negative.

## 24-3 Potential Due to a Distribution of Charge in Vacuum

The potential at a point $P$ due to a single point charge is a scalar quantity which represents the work per unit positive charge done in transporting charge from infinity to the field point $P$. Let us suppose that the electric field is generated by several point charges $q_{1}, q_{2}, \ldots$, and so on. In transporting the test charge from infinity, work must be done against the electric field contributed by each of the charges $q_{1}, q_{2}$, and so on. The work done against the field of each of these charges separately is given by Equation (24-6). Since work, and therefore potential, is a scalar quantity, the total work done may be computed by finding the work done against the field due to charge $q_{1}$, the work done against the field of charge $q_{2}$, and so on, and then adding these algebraically. If the potential at $P$ due to $q_{1}$ alone is $V_{1}$, the potential at $P$ due to $q_{2}$ alone is $V_{2}$, and so on, and the potential $V$ at $P$ due to the entire charge distribution is

$$
V=V_{1}+V_{2}+\cdots
$$

Thus we have, for the potential of a collection of point charges,

$$
\begin{equation*}
V=\sum \frac{q_{i}}{4 \pi \epsilon_{0} r_{i}} \tag{24-7}
\end{equation*}
$$

where $r_{i}$ is the distance from the $i^{\prime}$ th charge $q_{i}$ to the field point $P$ at which the potential is being evaluated, and the summation is to be extended to all the charges in the distribution.

Illustrative Example. Two point charges, $q_{1}=5 \mu$ coul and $q_{2}=-5 \mu$ coul, are separated by a distance of 8 cm , as shown in Figure 24-2. Find the potential at points $a$ and $b$ of that figure.

Since only two charges generate the field, the summation of Equation (24-7)
reduces to a sum of two terms. At point $a, r_{1}=0.1 \mathrm{~m}$ and $r_{2}=0.02 \mathrm{~m}$. The potential $V_{a}$ at point $a$ is

$$
\begin{aligned}
V_{a} & =\frac{5 \times 10^{-6} \text { coul }}{4 \pi \times 8.85 \times 10^{-12} \frac{\text { coul }^{2}}{\mathrm{nt} \mathrm{~m}^{2}} \times 0.1 \mathrm{~m}}+\frac{-5 \times 10^{-6} \text { coul }}{4 \pi \times 8.85 \times 10^{-12} \frac{\operatorname{coul}^{2}}{\mathrm{nt} \mathrm{~m}^{2}} \times 0.02 \mathrm{~m}} \\
& =4.5 \times 10^{5} \frac{\mathrm{nt} \mathrm{~m}}{\text { coul }}-22.5 \times 10^{5} \frac{\mathrm{nt} \mathrm{~m}}{\mathrm{coul}} \\
& =-18 \times 10^{5} \text { volts. }
\end{aligned}
$$

Fig. 24-2


At the point $b$ we see from the figure that $r_{1}=0.1 \mathrm{~m}$, while $r_{2}=0.06 \mathrm{~m}$. Substituting into Equation (24-7), we find

$$
\begin{aligned}
V_{b} & =\frac{5 \times 10^{-6} \text { coul }}{4 \pi \times 8.85 \times 10^{-12} \frac{\text { coul }^{2}}{\mathrm{nt} \mathrm{~m}^{2}} \times 0.1 \mathrm{~m}}+\frac{-5 \times 10^{-6} \text { coul }}{4 \pi \times 8.85 \times 10^{-12} \frac{\text { coul }^{2}}{\mathrm{nt} \mathrm{~m}^{2}} \times 0.06 \mathrm{~m}}, \\
V_{b} & =4.5 \times 10^{5} \text { volts }-7.5 \times 10^{5} \mathrm{volts} \\
& =-3 \times 10^{5} \text { volts. }
\end{aligned}
$$

In comparing this example to the corresponding illustrative example of Section 23-3, we see that the calculation of the potential is far simpler than the calculation of the electric intensity. This follows from the scalar nature of the potential and the vector nature of the electric intensity.

In the event that we have a continuous distribution of charge rather than a collection of discrete point charges, we may find the potential by integration rather than by summation. Equation (24-7) becomes

$$
\begin{equation*}
V=\int \frac{d q}{4 \pi \epsilon_{0} r} \tag{24-8}
\end{equation*}
$$

Illustrative Example. Calculate the potential at a point $P$ on the axis of the uniformly charged narrow ring of Section 23-4, illustrated in Figure 23-5.

Every element of charge $d q$ is at the same distance

$$
r=\left(a^{2}+z^{2}\right)^{1 / 2}
$$

from the point $P$ and contributes the same amount

$$
d V=\frac{d q}{4 \pi \epsilon_{0} r}
$$

to the potential at this point. Hence the potential at point $P$ due to the entire ring of charge $q$ is

$$
\begin{aligned}
V & =\frac{q}{4 \pi \epsilon_{0} r} \\
& =\frac{1}{4 \pi \epsilon_{0}} \frac{q}{\left(a^{2}+z^{2}\right)^{1 / 2}} .
\end{aligned}
$$

## 24-4 Equipotential Surfaces

An equipotential surface is a surface along which a charged body may be displaced without any work having been required in the process. The equipotential surface is defined as a locus of points in space at a common potential. In consequence, lines of force must intersect an equipotential surface at right angles to the surface, for if there were any component of the electric intensity parallel to an equipotential surface, work would be required to displace a charged body along that surface, in contradiction with our definition of an equipotential surface.

The surface of a conductor is an equipotential surface. The equipotential surfaces surrounding a point charge are concentric spherical surfaces centered at that charge; the potential of each such surface is given by Equation (24-6). The equipotential surfaces surrounding a uniformly charged cylinder are coaxial cylinders, with their common axis as the axis of the charged cylinder. In general, equipotential surfaces are drawn so that equal increments in potential separate each pair of surfaces. A few of the equipotential surfaces surrounding a point charge are shown in Figure 24-3, and some of the equipotential surfaces surrounding a charged metallic sphere are shown in Figure 24-4.

The knowledge that equipotential surfaces and lines of force intersect perpendicularly everywhere enables us to solve graphically many problems in electrostatics to a good approximation, even when these problems are too difficult for mathematical solution. Furthermore, any equipotential surface may be replaced by a metallic surface which is maintained at the appropriate potential without altering the electric field outside the conductor. Thus, for example, we may replace the appropriate imaginary equipotential surface of Figure $24-3$ by a real metallic sphere which is maintained at the potential of that equipotential surface. If the charge inside the spherical shell is removed, there can be no lines of force inside
the conducting shell, and the inside of the shell is an equipotential volume. The lines of force and the equipotential surfaces outside the shell are unaltered. We may find the complete set of equipotential surfaces and lines


Fig. 24-3 The equipotential surfaces around a point charge are concentric spherical surfaces with the point charge at the center.


Fig. 24-4 The equipotential surfaces outside a charged metallic sphere are spherical surfaces concentric with the charged sphere.
of force of the sphere simply by erasing the lines of force and the equipotential surfaces within the conducting shell, as in Figure 24-4.

Two equal and opposite point charges $+q$ and $-q$ separated by a distance $s$ constitute a dipole. The set of equipotential surfaces of a dipole intersect the plane of the diagram in the dotted lines shown in Figure 24-5, while the lines of force are shown as solid lines. At all points along a plane perpendicular to the line joining the two charges at the mid-point of the line, the potential is zero. At a field point $P$ located in this plane the potential is

$$
V=\frac{-q}{4 \pi \epsilon_{0} r}+\frac{q}{4 \pi \epsilon_{0} r}=0
$$

This imaginary equipotential plane at zero potential may be replaced by a conducting plane at zero potential (obtained by connecting the plane to ground) without altering the field distribution. Thus if we are interested in obtaining the potential distribution of a charge $q$ located a distance $s / 2$ to the right of a grounded conducting plane, we may compute this field by finding the field and potential due to the dipole, and then erasing the lines of force and equipotential surfaces which appear to the left of the conducting plane.


Fig. 24-5 Lines of force (solid) and equipotentials (dotted) about a dipole.


Fig. 24-6

## 24-5 Potential Gradient

When a charge $q$ is displaced an amount $\Delta$ s from point $A$ to an adjacent point $B$, as shown in Figure 24-6, the force which must be exerted on the charge by an external agency is oppositely directed to the electric field and is given by

$$
\mathbf{F}=-q \mathbf{E}
$$

and the work done by this force is

$$
\Delta \mathscr{Y}=\mathbf{F} \cdot \Delta \mathbf{s}
$$

Substituting for $\mathbf{F}$ its value from the above equation, we find

$$
\begin{equation*}
\Delta \mathscr{Y}=-q \mathbf{E} \cdot \Delta \mathbf{s} . \tag{24-9}
\end{equation*}
$$

If we divide Equation (24-9) by the charge $q$, the quantity on the left-hand side of the equation is equal to the potential difference $\Delta V$ between the initial and final points of the displacement, giving

$$
\begin{equation*}
\Delta V=-\mathrm{E} \cdot \Delta \mathrm{~s}=-E \Delta s \cos \phi \tag{24-10}
\end{equation*}
$$

In the limit of small displacements, we may find the potential from the electric field by integration. Symbolically, we write

$$
\begin{equation*}
\Delta V=\int_{a}^{b}-\mathbf{E} \cdot d \mathbf{s}=\int_{a}^{b}-E d s \cos \phi \tag{24-11}
\end{equation*}
$$

where $\Delta V$ represents the potential difference $V_{B}-V_{A}$ between the points $A$ and $B$, and $\phi$ is the angle between $d s$ and $E$.

If we divide Equation (24-10) by the magnitude of the displacement $\Delta s$, we obtain the result that

$$
\begin{equation*}
\frac{\Delta V}{\Delta s}=-E \cdot\left(\frac{\Delta \mathrm{~s}}{\Delta s}\right)=-\mathrm{E} \cdot \mathbf{1}_{s} \tag{24-12}
\end{equation*}
$$

where $1_{s}$ is a unit vector in the direction of the displacement. Thus the rate of change of potential with distance in any direction, as specified by the direction of the unit vector $1_{s}$, is equal to the negative of the component of the electric field intensity in that direction. If the direction of the unit vector is along the line of force, the rate of change of potential is greatest. At a given point the rate of change of the potential in the direction of most rapid change is called the potential gradient at that point. When the unit vector is directed along a line of force the angle $\phi$ between the unit vector $\mathbf{1}_{s}$ and the electric intensity E is zero, and we may write

$$
E=-\frac{\Delta V}{\Delta s}
$$

which may be rewritten as

$$
\begin{equation*}
E=-\frac{d V}{d s} \tag{24-13}
\end{equation*}
$$

in the limit of small displacements.
The units of electric field intensity are therefore the same as the units of potential gradient. In the mks system of units, we may use either newtons per coulomb or volts per meter to represent either electric intensity or potential gradient. As we have seen in Section 23-9, the dielectric strength of air is approximately $3 \times 10^{6} \mathrm{nt} /$ coul, or $3 \times 10^{6}$ volts $/ \mathrm{m}$. After walking across a carpeted room in the wintertime when the air of the room is quite dry, sparks as long as 5 cm may be observed to jump from one's knuckles to a doorknob. This implies that a difference of potential of approximately $1.5 \times 10^{5}$ volts exists between the doorknob and the knuckle.

If the potential is known as a function of the coordinates, we may find the components of the electric field parallel to any of the coordinate axes by imagining the displacement $\Delta s$ to be parallel to that axis. Thus the component of the electric field intensity in the $x$ direction, $E_{x}$, is given by the negative of the derivative of $V$ with respect to $x$. In finding $E_{x}$ by this means, we would examine the variation in $V$ with respect to $x$ holding the other coordinates constant. In the usual notation this is called a partial derivative and is represented by the symbol $\partial$ rather than the symbol $d$.

Following this convention, we write

$$
\begin{array}{ll} 
& E_{x}=-\frac{\partial V}{\partial x} ; \\
\text { similarly, } & E_{y}=-\frac{\partial V}{\partial y}, \\
\text { and } & E_{z}=-\frac{\partial V}{\partial z} . \tag{24-14c}
\end{array}
$$

The electric field intensity E may then be expressed in terms of its components and the unit vectors in the coordinate directions; thus

$$
\begin{equation*}
\mathrm{E}=E_{x} \mathbf{1}_{x}+E_{y} \mathbf{1}_{y}+E_{z} \mathbf{1}_{z} . \tag{24-15}
\end{equation*}
$$

Illustrative Example. From the second example in Section 24-3, we have shown that the potential generated by a uniform ring of charge at a point along its axis is

$$
V=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{\left(a^{2}+z^{2}\right)^{1 / 2}} .
$$

Find the electric intensity at a point on the axis of the ring by application of Equations (24-14).

First we note that the coordinates $x$ and $y$ do not appear in the expression for the potential. Thus the derivative of $V$ with respect to $x$ is zero, and the derivative of $V$ with respect to $y$ is zero. Hence there is no component of the electric field in the $x$ or the $y$ direction. To find the component of the field in the $z$ direction, we apply Equation (24-14c).

$$
E_{z}=-\frac{\partial V}{\partial z}=-\frac{\partial}{\partial z}\left[\frac{1}{4 \pi \epsilon_{0}} \frac{q}{\left(a^{2}+z^{2}\right)^{1 / 2}}\right] .
$$

Carrying out the indicated differentiation, we find

$$
E_{z}=\frac{1}{4 \pi \epsilon_{0}} \frac{q z}{\left(a^{2}+z^{2}\right)^{3 / 2}} .
$$

This result is identical with the formula obtained by direct integration in the illustrative example of Section 23-4.

In general, the electric field intensity may be obtained by the methods of Section 23-4. This requires three separate integrations to be performed, one for each of the components of the electric intensity. It is often much simpler to integrate once to find the potential, since this is a scalar quantity, according to the procedure of Section 24-3, and then to differentiate this result, following Equations (24-14) to find the electric field intensity.

TABLE 24-1 PRINCIPAL EQUATIONS IN MKS AND CGS UNITS

| Equation | MKS | CGS |  |
| :--- | :---: | :---: | :--- |
| $(24-1)$ | $\Delta V=\frac{\Delta \mathscr{W}}{q}$ | Same as mks | Potential difference |
| $(24-6)$ | $V=\frac{q}{4 \pi \epsilon_{0} r}$ | $V=\frac{q}{r}$ | Point charge in vacuum |
| $(24-12)$ | $\frac{\Delta V}{\Delta s}=-\mathbf{E} \cdot 1_{s}$ | Same as mks | Potential gradient |
| $(24-14 a)$ | $E_{x}=-\frac{\partial V}{\partial x}$ | Same as mks | Potential gradient |

TABLE 24-2 CONVERSION FACTORS RELATING MKS AND CGS UNITS

| Quantity | Symbol | MKS Unit | CGS Unit |
| :---: | :---: | :---: | :---: |
| Potential | V | 1 volt | $\begin{equation*} =\frac{1}{300} \text { stat volt } \tag{esu} \end{equation*}$ |
| Electric intensity | E | $1 \frac{\mathrm{nt}}{\text { coul }}=1 \frac{\mathrm{volt}}{\mathrm{~m}}$ | $=\frac{1}{3 \times 10^{4}} \frac{\text { dyne }}{\text { stcoul }}=\frac{1}{3 \times 10^{4}} \frac{\text { statvolt }}{\mathrm{cm}}(\mathrm{esu})$ |
| Charge | $q$ | 1 coul | $=3 \times 10^{9}$ stcoul (esu) |
| Work | w | 1 joule | $=10^{7} \mathrm{ergs}$ |

$$
\text { Permittivity of free space: } \epsilon_{0}=8.85 \times 10^{-12} \frac{\operatorname{coul}^{2}}{\text { joule } \mathrm{m}}
$$

## Problems

24-1. A small charge of +12 stcoul is placed in a uniform electric field whose intensity is 5,000 dynes/stcoul. (a) What is the force acting on this charge? (b) How much work is done by the electric field in moving this charge a distance of 4 cm in the direction of the field? (c) What is the difference in potential between its initial and final positions?

24-2. A small body carrying a charge of $72 \mu$ coul is placed 0.60 m from another small body fixed in position, carrying a charge of $180 \mu$ coul. If the $72-\mu$ coul body moves to a place 0.90 m from the $180-\mu$ coul body, what will be its kinetic energy?

24-3. A small charged body of $1 \mu$ coul is released from rest in a region of space where the electric field intensity is 100 volts $/ \mathrm{m}$. What will its kinetic energy be when it has been displaced a distance of 150 cm ?

24-4. Two plane metallic plates are located a distance of 1 cm apart. If the electric field between them is uniform, what must the potential difference between these plates be if the force on a $5-\mu$ coul charge between the plates is to be $10^{-3} \mathrm{nt}$ ?

24-5. Two equal charges, each of +250 stcoul, are placed 24 cm apart on the $x$ axis. Determine the potential at a point 15 cm from each charge.

24-6. An electric charge of +15 stcoul is located at the origin, and a charge of -40 stcoul is located at a point whose coordinates are ( $0,20 \mathrm{~cm}$ ). Find the potential at the following points: (a) ( $0,-5 \mathrm{~cm}$ ), (b) ( $15 \mathrm{~cm}, 0$ ), (c) ( $-15 \mathrm{~cm}, 0$ ).

24-7. A charge of $-13 \mu$ coul is located at a point whose coordinates are $(-5 \mathrm{~m}, 0)$ and a second charge of $+30 \mu$ coul is located at a point whose coordinates are $(+9 \mathrm{~m}, 0)$. (a) Find the potential at the origin and (b) at a point whose coordinates are ( $0,+12 \mathrm{~m}$ ). (c) How much work must be done by an external agency to move a $5-\mu$ coul charge from the origin to the point $(0,+12 \mathrm{~m})$ ?

24-8. An isolated conducting hollow sphere of radius 50 cm is charged to a potential of 100 statvolts. (a) What is the potential of the center of the sphere? (b) What charge placed at the center of the sphere would give an identical electric field distribution outside the sphere, if the conducting shell were removed? (c) What is the charge on the conducting sphere?

24-9. A uniformly charged sphere of radius $a$ and charge density $\rho$ coul $/ \mathrm{m}^{3}$ gives rise to an electric field outside the sphere of charge which is identical to the field generated by a point charge at the center of the sphere whose charge is $q=\frac{4}{3} \pi a^{3} \rho$. Inside the charged sphere the electric field intensity is given by $E=\left(\rho / 3 \epsilon_{0}\right) r$. Find a formula for the potential at a point inside the charged sphere a distance $r_{0}$ from the center of the sphere.

24-10. The electric field intensity from a uniformly charged rod is directed radially and is given by the formula $E=\lambda / 2 \pi \epsilon_{0} r$, where $\lambda$ is the charge per unit length and $r$ is the distance from the center of the cylinder. Find the potential difference between two points whose radial coordinates are $r_{a}$ and $r_{b}$.

24-11. An electron volt (ev) is a unit of energy used in atomic and nuclear physics. It represents the energy acquired by an electron in falling through a potential difference of 1 volt. How many electron volts are there in 1 erg? The charge of the electron is $1.60 \times 10^{-19}$ coul.

24-12. Two horizontal metallic plates are placed 1.5 cm apart, and a potential difference of 3,000 volts is applied between them so that the electric field is uniform and directed vertically. A small oil drop containing a charge of $32 \times 10^{-19}$ coul and a mass of $10^{-10} \mathrm{gm}$ is between the plates. (a) Determine the electrical force on the oil drop. (b) Determine the net force on the oil drop. (c) What potential difference must be applied to the plates for the oil drop to be in equilibrium under the action of both electrical and gravitational forces?

24-13. The binding energy of a hydrogen atom is 13.6 ev . What energy, in calories, would be required to separate the electron and proton to infinite distance from each other?
$24-14$. The heat of formation of water vapor is 57.8 kilocal/mole. What is the energy, in electron volts, which must be added to a molecule of water to dissociate it into hydrogen and oxygen?
$24-15$. An electron is liberated from the filament of a vacuum tube and is accelerated to the plate which is maintained at a potential of 300 volts above the filament. With what speed is the electron moving when it strikes the plate? The mass of the electron is $9.11 \times 10^{-31} \mathrm{~kg}$.

