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Identifying Controlling Variables for Math Computation Fluency Through Experimental Analysis: The Interaction of Stimulus Control and Reinforcing Consequences

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Abstract
This study investigated a method for conducting experimental analyses of academic responding. In the experimental analyses, academic responding (math computation), rather than problem behavior, was reinforced across conditions. Two separate experimental analyses (one with fluent math computation problems and one with non-fluent math computation problems) were conducted with three elementary school children using identical contingencies while math computation rate was measured. Results indicate that the experimental analysis with non-fluent problems produced undifferentiated responding across participants; however, differentiated responding was achieved for all participants in the experimental analysis with fluent problems. A subsequent comparison of the single-most effective condition from the experimental analyses replicated the findings with novel computation problems. Results are discussed in terms
of the critical role of stimulus control in identifying controlling consequences for academic deficits, and recommendations for future research refining and extending experimental analysis to academic responding are made.

**Keywords:** academic performance, experimental analysis, math computation fluency, stimulus control

As an assessment technology, functional analysis has had remarkable success in guiding treatment selection (Beavers, Iwata, & Lerman, 2013) because it is grounded in the experimental analysis of empirically derived controlling variables that have a great deal of explanatory power. Its clear utility in areas such as developmental disabilities led researchers to seek to extend functional analysis to schools where educators also encounter problem behavior (Asmus, Vollmer, & Borrello, 2002; Doggett, Edwards, Moore, Tingstrom, & Wilczynski, 2001; O’Neill et al., 1997). More research, however, is needed to adapt functional analysis to the unique features of the school setting if its use is to expand. The almost exclusive focus on behavioral excesses may be a significant limitation to their adoption in schools where the primary objective is to improve academic performance. Expanding the scope of variables that can be examined in a school-based functional analysis would significantly increase its utility to educators. The problem, however, with targeting academic performance through a functional analysis is that there are no existing functional relations to detect in the first place. A lack of responding on the part of the student—the very problem that needs to be resolved—reflects a deficit in stimulus control: The instructional exercise (e.g., a reading passage, a math worksheet) fails to evoke a correct response because responding is not under the control of the appropriate stimuli (e.g., the configuration of letters or numbers and other symbols on a page). Therefore, although existing forms of reinforcement may be readily available to the student in the instructional setting for a variety of behaviors—some appropriate (e.g., hand raising) and some inappropriate (e.g., aggressive behavior)—a functional analysis could not directly determine whether these potential reinforcers would be suitable for academic responding due to a lack of stimulus control.

Investigators have been successful in extending the basic principles of functional analysis to academic performance problems through the assessment method that has come to be known as brief experimental analysis (BEA; Daly, Martens, Hamler, Dool, & Eckert, 1999; Daly, Witt, Martens, & Dool,
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BEA is characterized by single-case experimental design elements and rapid alternation of conditions. Although the vast majority of BEA research has focused on reading fluency (e.g., Daly, Andersen, Gortmaker, & Turner, 2006; Daly, Bonfiglio, Mattson, Persampieri, & Foreman-Yates, 2005; Daly et al., 1999; Daly, Murdoch, Lillenstein, Webber, & Lentz, 2002; Eckert, Ardoin, Daisey, & Scarola, 2000; Eckert, Ardoin, Daly, & Martens, 2002; Jones & Wickstrom, 2002), this assessment method has also been applied to a variety of academic areas, including reading comprehension, math, and writing (Carson & Eckert, 2003; Duhon et al., 2004; McComas et al., 1996; Mong & Mong, 2012; see also the 2009 mini-series in the Journal of Behavioral Education). The critical difference with the traditional method of functional analysis is that BEA tests instructional strategies directly instead of testing for behavioral function. The instructional strategies invoke various forms of prompting, error correction, and increased opportunities to respond in an effort to increase responding. Although programmed consequences are sometimes included to test for possible reinforcer effects (e.g., Daly, Bonfiglio, et al., 2005; Jones & Wickstrom, 2002), the primary emphasis has been on antecedent control (e.g., modeling and prompting) and error correction strategies. To date, BEAs have not been designed to capitalize on the types of naturalistic forms of reinforcement that traditional functional analyses use to detect behavioral function (e.g., social attention and escape), which virtually excludes these forms of reinforcement from any treatment recommendations derived from a BEA. However, the lack of stimulus control exerted by the instructional exercise would render a traditional form of functional analysis useless because the results will surely be undifferentiated between conditions.

Establishing a broader use of functional analysis technologies in the schools will be possible if robust and replicable test conditions appropriate to instructional settings can be developed through further experimentation. Experimental analyses that account for a lack of stimulus control—the functional basis for poor academic responding—and yet tap into currently existing forms of reinforcement may lead to more comprehensive and ecologically valid forms of BEA in the schools. The challenge is to determine whether natural consequences such as those used in traditional functional analyses (e.g., teacher or peer attention and escape) have the potential to serve as effective reinforcers for behavior deficits such as math or reading problems. Experimental analyses for skills such as math and reading that can directly identify naturally existing reinforcers for further strengthening those response classes will provide a stronger functional basis for treatment selection while capitalizing on naturally occurring sources of reinforcement in the current instructional context.
Treatment selection based on a traditional functional analysis for problem behavior assumes that the existing stimulus function detected by a functional analysis is generalizable to alternative behaviors. For example, if the functional analysis reveals a social attention function, a treatment in which contingent social attention is applied to an alternative behavior (e.g., appropriate social or adaptive behavior) as a part of a differential reinforcement of alternative behavior treatment is recommended without knowing whether that reinforcer will be effective for the alternative behavior. Directly testing reinforcement contingencies for academic response classes, however, would eschew the need to make inferences about the generalizability of stimulus functions from one behavior (e.g., problem behavior) to another behavior (e.g., academic responding) because the reinforcer has been identified for the desired response class (academic responding in this case). To accomplish this objective, it would be necessary to demonstrate reliable differentiation of function across conditions for an otherwise weak (or inexistent) but desirable response class.

To examine whether this is possible, a sequence of experimental analyses would be necessary. First, an experimental analysis would need to establish that there is a behavior deficit. This could be accomplished by producing results that suggest (a) low levels of responding and (b) undifferentiated function across multiple stimulus conditions. If the target skill was then brought under stimulus control and a second experimental analysis subsequently produced differentiated function across conditions, the results would suggest that one (or more) consequences might serve as effective reinforcers for that target response class. In the current study, stimulus conditions representative of traditional functional analyses (teacher social attention, peer social attention, and escape) were used in two sets of analysis to determine whether stimulus function could be determined for writing answers to math computation problems. In both experimental analyses, the same response class—math computation—was targeted. The first experimental analysis was conducted with stimulus items with which participants were not fluent to see whether a behavior deficit existed. The second experimental analysis replicated the first experimental analysis in terms of stimulus conditions. However, stimulus items with which participants were fluent were used to see whether differentiated function could be obtained within and across participants. To assure equivalence of stimulus items, participants received instruction with the non-fluent items following the first experimental analysis and prior to the second experimental analysis. A third and final comparison of the condition producing the highest rate of academic responding across fluent and non-fluent stimulus items for a novel skill was conducted to rule out possible sequence effects and examine the generality of results across math computation skills.
Method

Participants and Setting

The participants included three female students enrolled in a private elementary school in the Midwest. They were referred by their classroom teachers when the experimenter explained that students experiencing difficulties in math were being sought for participation in the study. Patty was an 8-year-old, White female with no previous diagnoses or special education services. Jennifer was a 9-year-old, White female, who was previously diagnosed with Attention-Deficit Hyperactivity Disorder (AD/HD). Jennifer was prescribed Adderall, which she took during school hours throughout the study. Erin was an 11-year-old, White female with no previous diagnoses or special education services. Screening of math computation skills (described below) revealed that all three students displayed poor fluency rates. Medication status was held constant throughout the study for all participants. Approval was obtained by the authors from their institution’s Human Subjects Institutional Review Board (IRB).

All assessment and intervention activities were conducted in empty office spaces and classrooms during class-wide mathematics activities. The first author carried out all experimental analysis procedures in addition to all screening and instructional sessions. Two trained advanced school psychology doctoral students conducted observations of student and experimenter behavior during experimental analysis conditions via video recording.

Materials

Math fact index cards. Blank, white 3 × 5-inch index cards with skill-specific (i.e., addition, subtraction, multiplication, division) math computation problems printed on one side were used for screening and instructional sessions. Addition, subtraction, multiplication, and division cards contained all possible problems between 1 and 12 using all possible digit combinations for each respective skill. Laminated, colored 3 × 5-inch index cards were used for experimental analysis sessions using items selected for each participant based on the screening (described below). Four different color note cards were used, with each color assigned to an experimental analysis condition to facilitate discrimination between conditions (i.e., pink cards for experimenter attention, green cards for peer attention, yellow cards for escape, and orange cards for control).

Video camera. All screening, instruction, and experimental analysis sessions were videotaped using a camera that was built into a laptop computer. The laptop
was placed on the table throughout each session to capture experimenter and participant behavior for treatment integrity purposes.

Rewards

Rewards offered for appropriate behavior during sessions included free time, game play, drawing time, or time to play with modeling clay. Cards with pictures of each reward option were created to facilitate selection of activities at the end of the instructional session.

Measurement of Dependent Variables, Observer Training, and Interobserver Agreement

In each experimental session, participants wrote answers to math problems on math computation cards for 5 min. Responding was measured as rate of completed problems (CP per min) and rate of digits correct (DC per min). For CP per min, those responses with an identifiable number under the equals line on the math computation card were counted. For DC per min, responses were scored as correct if the correct number was written in the appropriate place column. For example, for “3 × 4 =, ” a student answering “22” would receive a score of 0 CP and 1 DC (the 2 placed in the ones column is correct whereas the 2 in the tens column is incorrect). Using a stopwatch, the exact number of seconds was recorded for each session, and the time provided on each video recording was used to confirm the exact number of seconds elapsed for each session. To calculate rate of CP, the number of completed problems per session was divided by the time (in seconds) and multiplied by 60 to determine the number of CP per min. The same formula was applied to DC. For the escape condition, the calculations had to be adjusted to account for the 15-s breaks that were provided contingent on the target behavior. As such, 15 s were subtracted from the time for every break provided before results were calculated. The CP per min and the DC per min were calculated across each condition as measures of mathematics computation rate.

Interrater agreement was calculated for CP and DC per min. An impartial, trained school psychology doctoral student independently scored 34% of sessions (randomly selected) across participants in each condition. Agreements and disagreements were compiled either on a problem-by-problem basis (CP) or on a digit-by-digit basis (DC). An agreement included identical scoring across observers for the same math computation problem. Any discrepancy between observers regarding the same math computation problem was a disagreement. For both variables, the total number of agreements was divided either by the total number of problems (CP) or digits (DC) and multiplying by 100. The mean interrater agreement was 100% for CP and 99.9% (range, 99% to 100%) for DC.
Experimental Conditions

Four conditions were used in both experimental analyses. The fifth condition— instruction—was delivered between the first and second experimental analyses. Each condition is described in turn. The experimental conditions were conducted individually with each student.

Experimenter attention. During the experimenter attention condition, the participants were instructed to remain seated and work quietly on individually presented mathematics computation cards. The experimenter sat at the table with the participant, presenting pink math computation cards on completion of each problem. The experimenter provided attention (e.g., praise, smile, high five) contingent on problem completion, using a continuous schedule of reinforcement. All other responses were actively ignored by averting eye contact and physically turning away from the student, while continuing to present math problems at regular intervals. If no response was given, another problem was presented after 10 s.

Peer attention. A peer confederate was identified by each teacher prior to conducting the experimental analysis. Teachers nominated children who they thought would be cooperative and able to learn how to respond to the participants. Each confederate was asked to work with the experimenter and the participant to assist with problem completion. After peers were selected by classroom teachers, the experimenter met individually with each peer. In this session, the experimenter asked the peer for his or her assent to participate and, if given, asked the peer to praise the participant for writing answers, but to not tell the participant the answer or pay attention to anything other than math work. The peer was provided with sample praise statements (e.g., “You are working very hard,” “Wow, you got another one done!”) to be used during sessions, and the experimenter modeled contingent ignoring. Participants were told that a peer would help them to remain seated and work quietly on individually presented math problems by giving them attention for completing math problems. The peer was then seated at the table beside or across from the participant. The peer was prompted by the examiner to present green math computation cards on completion of each problem or after 10 s if there was no response. Throughout each peer attention session, the examiner was seated approximately 5 ft from the peer, but ignored all student behavior.

Escape. During the escape condition, the participants were instructed to remain seated and work quietly. They were told that they would be given a 15-s break from working for each problem completed. The experimenter was seated
at the table beside the student, and yellow math computation cards were presented at fixed 10-s intervals if no response was given or after each 15-s escape period when a correct response was given. During the 15-s escape period, the completed math card was physically removed and the experimenter turned away from the student. After each break, the experimenter continued to present yellow math computation fact cards at fixed, 10-s intervals or after each escape period.

Control. During the control condition, participants were instructed to stay seated and work quietly at their own pace, completing as many or as few problems as they desired. The participants were then provided with a stack of orange, non-fluent math computation cards, and all behavioral responses were ignored. The participant controlled the rate of stimulus presentation, and the examiner remained approximately 4 to 6 ft away from the participant with averted eye gaze throughout the control condition.

Instruction. After the initial experimental analysis, but prior to the second analysis, instruction was delivered to create a set of “fluent” math problems for the second experimental analysis. This condition allowed the experimental analysis to examine stimulus function with instructional items that had stimulus control over academic responding. Non-fluent math problems, which were identified during screening, were taught using a modified version of the Strategic Incremental Rehearsal (SIR) intervention procedures (Kupzyk, Daly, & Andersen, 2011) in combination with programmed incentives. Prior to implementing the intervention procedures, each participant was presented with all non-fluent math facts on white note cards and prompted to provide the answer. Problems answered correctly within 3 s were excluded from the subsequent experimental analysis with fluent problems.

During the first instructional session, the first non-fluent problem (NF1) was presented and the experimenter modeled the correct response. The participant was then prompted to repeat the answer. For incorrect responses, the experimenter provided corrective feedback by repeating the problem and answer and prompting the student to say the correct answer. The same procedure was used to introduce the second non-fluent problem (NF2), and the same steps were repeated for NF1 and NF2 before a prompt delay was implemented. With subsequent presentations of NF1 and NF2, the examiner modeled the correct answer if the student answered incorrectly or failed to respond within 3 s. If the student provided the correct answer within 3 s, the next stimulus item was presented. After students provided correct answers to both problems (NF1 and NF2) without a prompt, a new non-fluent problem
was presented (NF3). Modeling, prompting, and corrective feedback were used when NF3 was presented for the first time, then NF1 and NF2 were presented in random order, using the prompt delay and corrective feedback procedures. All three problems (NF1, NF2, NF3) were then shuffled and presented using prompt delay and corrective feedback. This process continued until all three problems were answered correctly within 3 s without prompting. Once all instructional items were answered correctly, a new non-fluent problem (NF4) was introduced. These procedures were repeated until the conclusion of each instructional session.

During subsequent instructional sessions, all previously instructed items were shuffled and presented with a prompt delay and corrective feedback. Math fact index cards were shuffled between each presentation and prior to adding a new non-fluent problem. Modeling was used to introduce each new non-fluent problem, and a prompt delay was used with all subsequent problem presentations. These instructional procedures continued until 80% of all non-fluent problems were answered correctly within 3 s without prompting (i.e., fluent).

**Experimental Design and Procedures**

Two successive experimental analyses were conducted using a multielement design (Sidman, 1960). Experimental analysis conditions were rapidly alternated in a pre-determined, semi-random order (i.e., sampling without replacement until all conditions were administered and then re-randomizing the order of conditions and repeating). The same design was also used for the extended analysis. The experimental analyses followed an initial screening to identify computation problems.

**Screening.** Each student’s math computational skills were assessed using math fact cards until reaching a criterion of 25% non-fluent facts within one skill area, either addition or multiplication. During the screening session, each participant was presented with all possible digit combinations of addition and multiplication facts 1 to 12. Correct responses provided within 3 s were counted as correct. Errors were scored for omissions, incorrect answers, and responses occurring after 3 s. The percentage of non-fluent problems was calculated by dividing the number of errors by the total number of problems and multiplying by 100. The mathematics operation (i.e., addition or multiplication) resulting in the highest percentage of non-fluent problems was selected as the target skill.

The screening process also included the structured use of incentives to maintain acceptable levels of task engagement. Specifically, during the screening session, both stacks of 144-math-fact flashcards were shuffled and divided
into four stacks of 36 cards each. The participants were told that they would earn a sticker for each completed stack, and that a reward would be offered when eight stickers were earned. Reward options included 5 min of free time, 5 min of game play, a positive note home, or 5 min of drawing time. Cards with pictures of each reward option were presented to the participant, and the selected reward was immediately provided. Participants selected game play, drawing time, and time to play with modeling clay (a free-time option).

**Experimental analysis with non-fluent facts.** Using 100% non-fluent facts derived from screening in one skill area (addition for Jennifer or multiplication for Patty and Erin), all four experimental analysis conditions (teacher attention, peer attention, escape, and control) were randomly presented to each participant 2 to 3 times a day with breaks between sessions. All conditions were presented once in random order and the order was then randomized again to determine the sequence of administration for the next set of conditions. This step was followed 1 more time, so that all conditions were presented 3 times in all. Session length for each condition was 5 min. All conditions were videotaped.

**Instruction.** After completion of the experimental analysis with non-fluent facts, instruction using SIR was conducted with each participant until 80% of math facts were answered correctly. The remaining 20% of non-fluent problems were retained for the control condition in the subsequent experimental analysis. Instructional sessions lasted approximately 20 min and were conducted 1 to 2 times per week for each participant.

**Experimental analysis with fluent facts.** This experimental analysis was conducted for each participant after all instructional sessions were completed. The same procedures and steps were followed for selecting and administering conditions as were used for the experimental analysis with non-fluent facts. The only difference in this experimental analysis was that fluent instructional items (of the same problem types as in the first experimental analysis) were used for the teacher attention, peer attention, and escape conditions. To assure consistency in the control condition across experimental analyses, only non-fluent math computation problems were presented.

**Extended analysis.** The most effective condition for each participant was further examined in the extended analysis. Specifically, both versions of the single-most effective condition from the previous experimental analyses (with and without prior instruction) were compared to determine whether results from separate experimental analyses would be replicated within a single phase. Replication
within the same phase permitted direct comparisons of like conditions across fluent and non-fluent stimulus materials. Different problem types were used in this phase (subtraction for Jennifer and division for Patty and Erin), which allowed for examining the degree to which results could be replicated with a different computation skill. Prior to the experimental sessions, instruction (SIR) was delivered exclusively using the math facts assigned to the condition with fluent facts. Then, each condition was administered repeatedly in the manner described above.

**Procedural Integrity**

Procedural integrity was calculated via video recording. After completion of the study, 33% of videotaped sessions for each condition were randomly selected to determine procedural adherence across participants. For the experimental analyses, the correct use of consequences was examined. A trained observer recorded student responses to problem presentation as well as experimenter and peer responses to calculate the percentage of responses correctly consequated for each condition. Condition-specific procedural adherence was calculated to ensure that contamination across conditions did not occur. Academic and behavioral responses exhibited by the participant along with experimenter and peer responses (i.e., experimenter attention, peer attention, task removal) were recorded continuously. If a condition-appropriate response (i.e., experimenter attention during the experimenter attention condition, escape during the escape condition, peer attention during the peer attention condition, or no action/response during the control condition) occurred within 5 s of student academic responding, it was coded as a correct consequence. When a student response was not followed by a condition-appropriate consequence, no consequence was coded. Finally, when condition-inappropriate responses (e.g., experimenter attention during the escape condition, experimenter attention in response to disruptive behavior) occurred, they were coded as incorrect consequences. To calculate procedural integrity, the number of appropriate consequences (i.e., correct responses) was divided by the total number of consequences provided (i.e., correct consequences plus no consequences plus incorrect consequences) and multiplied by 100. Average procedural integrity was 97.8% (range, 77% to 100%) across participants.

For instruction, correct adherence to treatment steps was examined. All instructional sessions were videotaped, and a procedural checklist was completed by an independent observer for 50% of sessions (randomly selected). The percentage of procedural adherence was calculated for each session by dividing the number of steps completed by the total number of intervention steps and multiplying by 100. The percentage of procedural integrity was 100 for all instructional sessions.
Results

Experimental Analysis With Non-Fluent Facts

Figures 1 through 3 display the results for the experimental analysis with non-fluent items (left panel of each figure). Undifferentiated patterns of responding across participants were obtained for all participants—Patty (Figure 1, left panel), Jennifer (Figure 2, left panel), and Erin (Figure 3, left panel). The consistently undifferentiated patterns across all three participants suggest that controlling variables could not be identified when non-fluent computation problems were used. It is noteworthy that responding was above zero in all sessions and that there were even some performance increases for some conditions (e.g., experimenter attention and control in CP per min for Patty, all conditions for Jennifer, and all conditions for Erin for CP per min), indicating a weakened form of stimulus control rather than a total absence of stimulus control during the initial experimental analysis.

Experimental Analysis With Fluent Facts

The experimental analysis with fluent problems was intended to examine whether stimulus function could be identified when stimulus items (i.e., math computation problems) were under appropriate stimulus control, which was lacking in the first experimental analysis. The results reveal that controlling variables differed across all participants. In Patty's case (Figure 1, middle panel), academic performance (both CP per min and DC per min) was highest in the experimenter attention condition. The peer attention and escape conditions produced variable results for both of the dependent variables, and the control condition produced the lowest levels of responding. For Jennifer (Figure 2, middle panel), clear differentiation between conditions is visible for both CP per min and DC per min. Experimenter attention and peer attention produced the highest and most accurate rates of responding. The results were comparable for both conditions. The escape condition produced the lowest levels of responding, lower even than the control condition. Finally, for Erin (Figure 3, middle panel), differentiation between conditions occurred for DC per min only. The escape condition was associated with higher rates of accurate performance (i.e., DC per min) than the other conditions. Overall, experimental control was established for all participants when fluent stimulus items were used. The clearest differentiation was present for DC per min across all participants, suggesting that it may be the most sensitive variable for detecting changes in responding for math computation.
Identifying Controlling Variables for Math Computation Fluency

Extended Analysis

The extended analysis was designed to replicate two like conditions that differed only in the types of stimulus items that were used—fluent versus non-fluent. The condition producing the highest level of responding was selected for

Figure 1. Functional analysis results for Patty. Note. CP = completed problems; DC = digits correct.
each participant. The extended analysis also was designed to examine whether conclusions were generalizable to a different computation skill (i.e., division or subtraction). The conditions chosen for the extended analysis were selected on
the following basis. Patty produced clearly differentiated responding for both CP per min and DC per min after antecedent instruction. Experimenter attention produced higher CP per min and DC per min relative to other conditions and was therefore selected for the extended analysis. Clear differentiation was
also visible in Jennifer’s results for both CP per min and DC per min. Two conditions—experimenter attention and peer attention—were considered effective. However, because the experimenter attention condition produced a more consistent level of responding, the experimenter attention condition was selected for the extended analysis. Erin produced a clearly differentiated pattern of responding for DC per min only. The escape condition produced higher rates of responding than all other conditions. Therefore, escape was selected for the extended analysis.

Overall, visual inspection of the extended analysis results (right panel of each of the figures) reveals that conditions in which antecedent instruction was delivered produced higher rates of responding compared with the same contingencies without antecedent instruction for all three of the participants. For Patty (Figure 1, right panel), the experimenter attention with fluent problems condition produced a visibly higher rate of CP per min, DC per min, and percentage of academic engagement relative to the experimenter attention with non-fluent problems condition. Interestingly, increasing trends are evident in both conditions, a finding that is not unusual for a skilled behavior, which is progressively coming under appropriate sources of stimulus control. Jennifer’s extended analysis results (Figure 2, right panel) were very similar to Patty’s. The experimenter attention with fluent problems condition produced a visibly higher rate of CP per min and DC per min relative to the experimenter attention with non-fluent problems condition. Increasing trends are also evident in both conditions. Finally, the pattern in Erin’s data was the same (Figure 3, right panel), except that the condition differed (escape) and there was no increasing trend for academic responding (CP per min and DC per min). In the same manner, academic responding was higher in the escape condition containing fluent problems than in the condition containing non-fluent problems.

Discussion

Very few experimental analysis studies have been conducted targeting academic performance as an outcome (e.g., Broussard & Northup, 1995; Filter & Horner, 2009), let alone examining possible interaction effects between stimulus control and reinforcement. The current study examined whether differentiated function could be obtained using typical functional analysis conditions when non-fluent and fluent stimulus items were used. Unsurprisingly, no differentiation was observed when non-fluent stimulus items were used in the experimental analysis. The weak stimulus control makes it virtually impossible to identify potentially reinforcing consequences. However, when stimulus
control was strengthened through instruction, clear differences emerged between conditions. It is almost as if the reinforcing properties of the various consequences (e.g., social attention, escape) were latent and became functional once a threshold of stimulus control was achieved for the targeted response class. The experimental analyses conducted in this study directly targeted a skilled behavior (math computation fluency), which may be advantageous for several reasons. The topography of reinforcement (e.g., type of attention) has been shown to differentially affect behavior (Kodak, Northup, & Kelley, 2007), raising questions about the effectiveness of a putative reinforcer when the topography changes from the functional analysis (e.g., reprimand for inappropriate behavior) to treatment (e.g., praise for appropriate behavior). Furthermore, under some conditions, the variables maintaining problem behavior may not maintain replacement behavior (Holden, 2002).

Further development of the type of experimental analysis reported in this study as a school-based functional analysis would allow behavior analysts to bypass inferences about the generalizability of reinforcer effectiveness from behavioral excesses to behavioral deficits. Indeed, the method used in this study might permit a more direct and more efficient analysis of the potential effectiveness of differential reinforcement of alternative behavior when students referred for behavior problems have concurrent skill deficits. The current study may encourage future research along these lines. Success in this endeavor would probably expand and solidify the importance of functional analysis in the schools.

In the current study, the critical difference between both methods of experimental analysis was the type of instructional items that were used. Although the consequences remained the same (in terms of the types of reinforcement and the schedule of reinforcement) in both experimental analyses, an antecedent variable—the presentation of fluent versus non-fluent stimulus items—differed across analyses. It would appear, therefore, that experimental analysis targeting academic responding using fluent stimulus items is a more productive method for application and future investigation. The first method used in this study provided differential consequences for a behavioral deficit that was not under appropriate stimulus control (participants were not fluent with stimulus items), which is quite different from an experimental analysis conducted with a behavioral excess, which capitalizes on already existing forms of stimulus control and provides high doses of various types of putative reinforcers to identify behavioral function. The weak stimulus control characteristic of the first experimental analysis reduced the likelihood that the target behavior would occur in the first place and ultimately affected the frequency of reinforcement, making it virtually
impossible to detect behavioral function. The math problems in the current study had not yet become associated with the reinforcers present across conditions, resulting in low rates of academic performance. Although a continuous schedule of reinforcement was used across conditions, the low frequency of behavior resulted in a low frequency of reinforcement, probably due to the absence of antecedent instructional strategies (modeling and prompting), which would evoke correct responses that could then be reinforced. For academic performance and other behavioral deficits, it appears that establishing some level of stimulus control for the instructional task that brings the participant into contact with the contingencies for instructional items may be necessary to obtain accurate information regarding potentially effective consequences.

One salient characteristic of the second experimental analysis is that the overall level of responding is higher for CP and DC than for the first experimental analysis, which is not at all surprising in light of the fact that the sessions now included items with which participants were fluent. More importantly, the experimental analysis with fluent problems produced consistent within-subject variability across (but not within) conditions, so that discernible and consistent patterns of responding across conditions were identified for each participant. In addition, idiosyncratic responding was observed across participants: Responding varied across participants, both in level and in patterns of responding across conditions. The positive impact of incorporating relevant stimulus items for which discriminative control had been established through instruction was further demonstrated in the extended analysis. Indeed, the rates of behavior were consistent with observed patterns in the prior experimental analyses. This finding rules out the possibility of a mere sequence effect accounting for prior differences across experimental analyses and indicates that behavioral function was generalizable across math computation problem type. Thus, the data provide individualized information regarding reinforcers that may promote academic responding. As such, the current findings suggest that applying experimental analysis methodology to alternative response classes is a viable method for producing consistent results regarding function when instructional items exert antecedent control over responding. The results of the experimental analysis with fluent problems produced results consistent with those observed for functional analyses of problem behavior, in which sources of influence are identified for up to 96% of cases (Iwata, Pace, et al., 1994).

The results of the experimental analysis with fluent problems extend both functional analysis (e.g., Iwata, Dorsey, et al., 1994) and treatment-based analysis research (e.g., Harding, Wacker, Cooper, Millard, & Jensen-Kovalan, 1994; Millard et al., 1993) by demonstrating that behavioral function can be identified for at least some replacement behaviors (i.e., math computation). This
type of analysis may be more acceptable and feasible for educators who do not have to evoke problem behaviors (as is the case in traditional functional analyses) and who can design functional analysis conditions that more accurately reflect the types of instructional events that are going on in the classroom using variables that can potentially affect both behavioral excesses and academic deficits. These findings may also be relevant to classrooms in which similar types of reinforcers may be available to children (e.g., contingent teacher attention for correct answers, contingent breaks for completing assignments). For a child exhibiting behavioral excesses and deficits, teachers may be more likely to design interventions to address the behavioral excesses, as these are the most salient behaviors (Sterling-Turner, Robinson, & Wilczynski, 2001). This approach may reduce the behavioral excesses observed, but those interventions are likely to be missing critical variables that are needed to establish appropriate forms of academic responding. The current findings suggest that effective instruction brings academic responding into contact with sources of reinforcement that are common in classrooms, and that those reinforcers may be idiosyncratic across children. Nonetheless, those forms of reinforcement will not be effective if instruction is not appropriately configured to make correct responding possible in the first place (Heward, 1994). For this reason, for children exhibiting both behavioral excesses and academic deficits, exclusive emphasis on behavioral interventions may be neglecting the development of academic skills and may be less efficient than analyses explicitly targeting skills that need to be strengthened.

The method used in the experimental analysis with fluent problems appears to be promising for identifying the conditions under which optimal academic responding can be achieved. However, the analyses were not carried out in the natural setting in the current study. Additional research is necessary to determine whether the results of experimental analyses of academic performance lead to effective classroom-based treatments. For example, interventions derived from experimental analyses for academic performance could be compared with interventions derived from functional analyses of disruptive behavior to determine which interventions are more efficient and efficacious. It would also be beneficial to conduct a comparison of conditions in the natural setting to examine outcomes across settings and examine the impact of setting on the relative effectiveness of conditions. Finally, future research could examine just how accurate and fluent responding needs to be during the functional analysis to produce accurate identification of controlling variables. In the current study, instructional items were either non-fluent or fluent. In the classroom, the teacher is generally adding new instructional items over time. It may be productive in the future to conduct parametric evaluations of ratios of fluent
to non-fluent stimulus items to determine optimal ratios that achieve the most valid and generalizable identification of controlling variables. Future research is also necessary to determine the extent to which these results generalize to other academic skills (e.g., reading). For instance, researchers could investigate the emergence of functional relations for new instructional tasks as response strength grows (accuracy, fluency, generalization). It would be particularly interesting to determine whether experimental analyses using fluent stimulus items predict which reinforcers would be more effective for instructional items that have yet to be learned while teachers use antecedent prompting strategies to evoke high levels of responding during acquisition training.

The current study emphasized the importance of discriminative control in identifying existing reinforcement contingencies for a skilled behavior—math computation fluency in this case. It did not, however, explicitly target motivating operations as a part of the analysis. Traditional functional analyses are generally designed to contrive motivating operations that will enhance the likelihood of identifying stimulus function during the functional analysis. In the current study, reliable trends within data series and clear differences between conditions suggest that motivating operations were stable for the participants and, therefore, further manipulations were not necessary. Future experimental analyses of the type conducted here might not have as much success. Direct manipulation of antecedent conditions through procedures such as restricting prior access may be necessary if undifferentiated results are obtained under appropriate conditions of discriminative control. Future research examining potential manipulating operations in experimental analyses of skilled behaviors would greatly increase the field’s understanding of how both types of antecedent conditions—discriminative control and motivational control—interact in ways that are productive or counterproductive to the development of those skilled behaviors during classroom instruction. Research of this type takes the field a step closer to offering educators a form of analysis that may have great appeal to them as they try to resolve both the academic and behavioral problems of children in schools. As researchers refine this methodology in the future, they are advised to choose their academic stimuli very carefully to assure that the appropriate antecedent conditions are present to make the accurate detection of behavioral function possible.

Authors’ Note
This study was completed in partial fulfillment of the requirements for the first author’s doctoral dissertation.
Identifying Controlling Variables for Math Computation Fluency

References


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