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A Partial Taxonomy of Substitutability & Interchangeability

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Outline

• Introduction
  – Basic form & extensions
  – Features & use
  – Further developments
• Taxonomy using a partial order
  – One example
• Relation to
  – General forms of symmetry
  – Symmetry breaking during search
• Future research & conclusions
Interchangeability & Symmetry

• Eliminating Interchangeable Values in Constraint Satisfaction Problems [Freuder, AAAI 91]

• “The detection of symmetries is a research avenue pioneered by Freuder [AAAI 1991] and subsequently investigated by many others.” [Van Hentenryck, SARA 2006]

• Interchangeability is a form of ‘solution symmetry’
  – Symmetry is not specified, but is detected

• We survey work on interchangeability & substitutability
  – Identifying & proving relationships among different forms of interchangeability/substitutability
  – We welcome your input
Basics

• Local vs Global
  – Neighborhood Interchangeability (NI)
  – K-Interchangeability (KI)
  – Full Interchangeability (FI)

• Weakening
  – Substitutability (ref. dominance)
  – Partial interchangeability
  – Subproblem interchangeability

• Generalization
  – Dynamic interchangeability (ref. SBDS & SBDD)
  – Meta interchangeability
  – Functional/isomorphic interchangeability: mapping values between different variables (ref. symmetry)

[Freuder 91]
NI and FI

• **FI**: Global, semantic level, likely intractable
• **NI**: Local, syntactic level, efficiently determined
• **NI \implies FI**

![Diagram](image-url)
Interchangeability Researchers

Audemard
Benhamou
Bistarelli
Benson
Bellicha
Bowen
Borodov
Brown
Budish
Budish
Budish
Budish
Budish
Budish
Budish
Budish
Budish
Budish
Budish
Further Developments

• Exploration
  – Interchangeability types
  – Their detection cost
  – Their benefits for problem solving

• Context
  – Finding all solutions
  – Problem decomposition

• CSP Extensions
  – Distributed CSPs
  – Quantified CSPs
  – Soft CSPs
Features & Use

• May be viewed as an extension of the fundamental CP concept of inconsistency filtering & propagation
  – Can remove values without removing all solutions
  – Trade amount of filtering against difficulty of recovering removed solutions

• Automatic symmetry detection

• Bundling interchangeable values for the same variable
  – Yields a compact representation of a CSP
  – Yields ‘robust/flexible’ solutions
  – Nogood bundling dramatically reduces search cost

• Shown to be beneficial in
  – Backtrack search & local search, interaction w/ users
  – Random CSPs, benchmarks, resource allocation problems
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Taxonomy

• Surveyed & analyzed interchangeability concepts
• Identified those that are satisfiability preserving
• Classified them in terms of implication
  \[ X \implies Y \quad \text{iff} \quad \forall a, b \quad X(a,b) \implies Y(a,b) \]
• Identified 22 interchangeability concepts
  – 231 relations between concepts
  – 94 relations are covered in paper
• In extended paper, we will justify the remaining 137 incomparability results
The Interchangeability Landscape

SymCon 2010, Sep 6, 2010
Substitutability

- Global semantic
- Local syntactic
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• **Future research & conclusions**
Diagram of Symmetry Concepts

- **Value Symmetry for Satisfiability** [Benhamou 94]
- **Functional Interchangeability** [Freuder 91]
- **Symmetry** [McDonald+ 02]
- **Isomorphic Interchangeability** [Freuder 91]
- **Constraint Symmetry** [Cohen+ 05]
- **Syntactic Symmetry** [Benhamou 94]
- **Neighborhood Interchangeability** [Freuder 91]
- **Solution Symmetry** [Cohen+ 05]
- **Value Symmetry for All Solutions** [Benhamou 94]
- **Isomorphism Symmetry** [McDonald+ 02]
- **Full Interchangeability** [Freuder 91]
- **(a,b)-Supermodel** [Ginsberg+ 98]
- **(1,0)-Supermodel** [Ginsberg+ 98]
Relation to SBDS & SBDD

• Dynamic interchangeability
  – New opportunities for interchangeability appear during search
  – Forms proposed: DynNI, FDynI, DynSub & ForwNI

• SBDS & SBDD are related to dynamic interchangeability
  – Break symmetries during search
  – Can implement dynamic interchangeability

<table>
<thead>
<tr>
<th></th>
<th>Dynamic Interchangeability</th>
<th>SBDS/SBDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discovers symmetry</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Overhead</td>
<td>Polynomial</td>
<td>Exponential</td>
</tr>
<tr>
<td>Space complexity</td>
<td>Polynomial</td>
<td>Exponential/Polynomial</td>
</tr>
<tr>
<td>Broken symmetries</td>
<td>Expressed by the concept</td>
<td>All specified symmetries</td>
</tr>
<tr>
<td>Advantages</td>
<td>Time &amp; space complexity</td>
<td>Breaks more symmetries</td>
</tr>
</tbody>
</table>
## High-Level Observations

<table>
<thead>
<tr>
<th></th>
<th>Interchangeability</th>
<th>Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Research focus</td>
<td>Efficient detection techniques</td>
<td>Efficient breaking techniques</td>
</tr>
<tr>
<td>Detected by...</td>
<td>Examining supports &amp; nogoods</td>
<td>• Given by user</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Using graph automorphism tools, e.g. Nauty</td>
</tr>
<tr>
<td>Defined over</td>
<td>• Individual variable-value pairs, tuples</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Partial assignments</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Solutions</td>
<td></td>
</tr>
<tr>
<td>Variations</td>
<td>Substitutability ≈ Dominance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Meta interchangeability ≈ Indistinguishable variables</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Partial interchangeability ≈ Super-solutions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dynamic variations ≈ Symmetry breaking during search</td>
<td></td>
</tr>
<tr>
<td>State of affairs</td>
<td>Many concepts proposed yet to be exploited</td>
<td>Has received intensive attention in recent years</td>
</tr>
</tbody>
</table>

SymCon 2010, Sep 6, 2010
Future Research

• Analysis of symmetry definition was started by [Cohen+ 2005], and is still an ongoing effort
• In interchangeability, many concepts are yet to be investigated
  – Detection algorithms
  – Exploitation in problem solving
• New opportunities: building hybrids of
  – Concepts
  – Algorithms
  ... where the whole is more powerful than the sum of its parts
Thank you