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A Partial Taxonomy of Substitutability & Interchangeability

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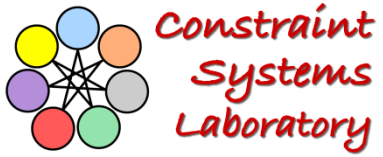

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Outline

- Introduction
  - Basic form & extensions
  - Features & use
  - Further developments
- Taxonomy using a partial order
  - One example
- Relation to
  - General forms of symmetry
  - Symmetry breaking during search
- Future research & conclusions

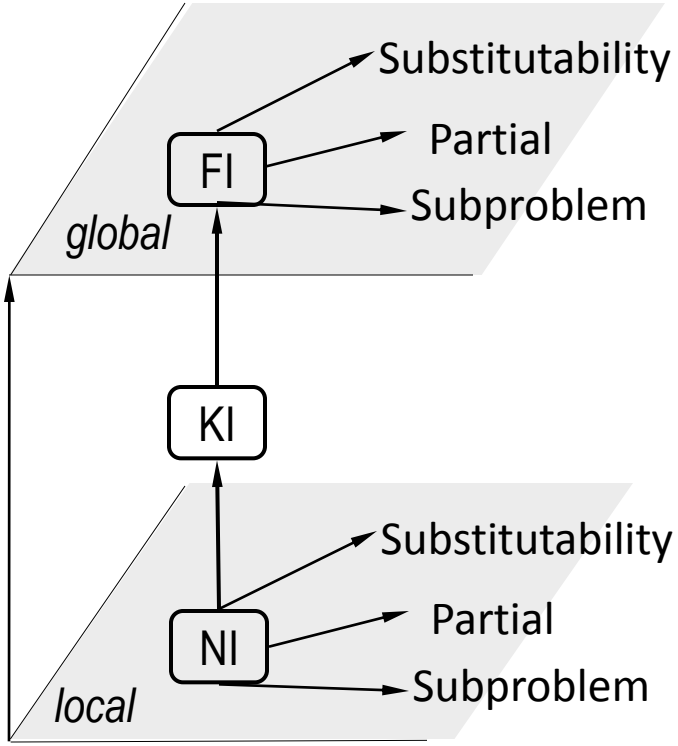
Interchangeability & Symmetry

- Eliminating Interchangeable Values in Constraint Satisfaction Problems [Freuder, AAAI 91]
- "The detection of symmetries is a research avenue pioneered by Freuder [AAAI 1991] and subsequently investigated by many others." [Van Hentenryck, SARA 2006]
- Interchangeability is a form of 'solution symmetry'
  - Symmetry is not specified, but is detected
- We survey work on interchangeability & substitutability
  - Identifying & proving relationships among different forms of interchangeability/substitutability
  - We welcome your input

Basics

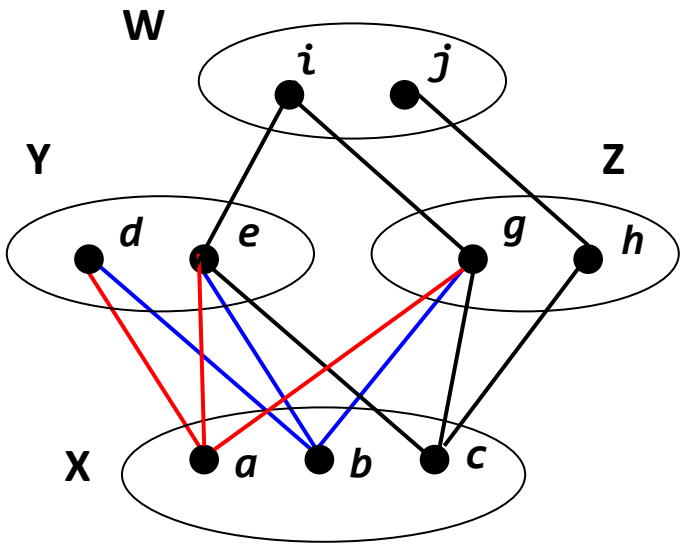
[Freuder 91]

- Local vs Global
  - Neighborhood Interchangeability (NI)
  - K-Interchangeability (KI)
  - Full Interchangeability (FI)
- Weakening
  - Substitutability (ref. dominance)
  - Partial interchangeability
  - Subproblem interchangeability
- Generalization
  - Dynamic interchangeability (ref. SBDS & SBDD)
  - Meta interchangeability
  - Functional/isomorphic interchangeability: mapping values between different variables (ref. symmetry)



NI and FI

- FI: Global, semantic level, likely intractable
- NI: Local, syntactic level, efficiently determined
- NI $\Rightarrow$ FI



a,b,c are FI
a,b are NI

Interchangeability Researchers

Further Developments

- Exploration
  - Interchangeability types
  - Their detection cost
  - Their benefits for problem solving
- Context
  - Finding all solutions
  - Problem decomposition
- CSP Extensions
  - Distributed CSPs
  - Quantified CSPs
  - Soft CSPs

Features & Use

- May be viewed as an extension of the fundamental CP concept of inconsistency filtering & propagation
  - Can remove values without removing *all* solutions
  - Trade amount of filtering against difficulty of recovering removed solutions
- Automatic symmetry detection
- Bundling interchangeable values for the same variable
  - Yields a compact representation of a CSP
  - Yields ‘robust/flexible’ solutions
  - Nogood bundling dramatically reduces search cost
- Shown to be beneficial in
  - Backtrack search & local search, interaction w/ users
  - Random CSPs, benchmarks, resource allocation problems

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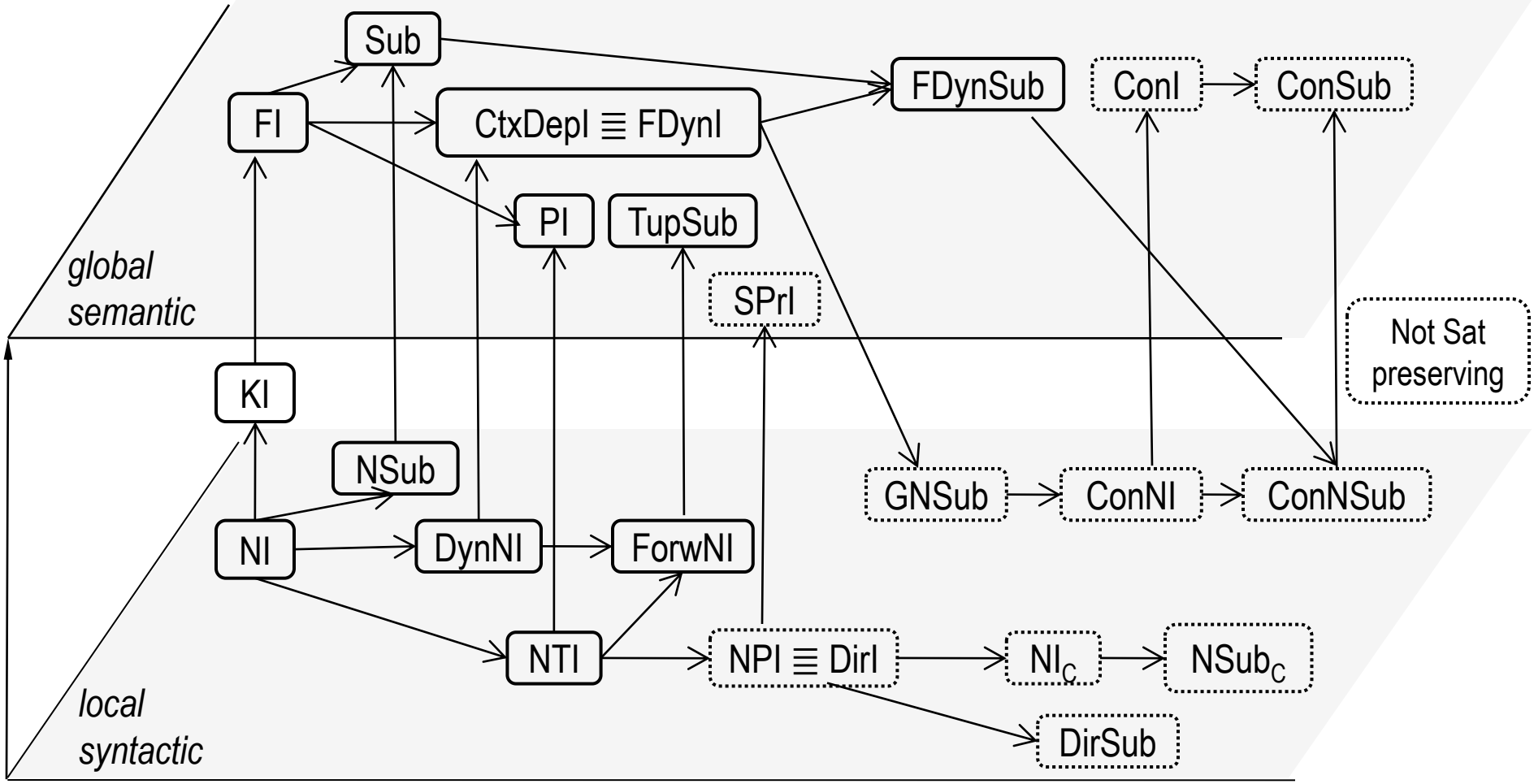
Taxonomy

- Surveyed & analyzed interchangeability concepts
- Identified those that are satisfiability preserving
- Classified them in terms of implication

$$X \Rightarrow Y \quad \textit{iff} \quad \forall a, b \; X(a,b) \Rightarrow Y(a,b)$$

- Identified 22 interchangeability concepts
  - 231 relations between concepts
  - 94 relations are covered in paper
- In extended paper, we will justify the remaining 137 incomparability results

The Interchangeability Landscape



Substitutability

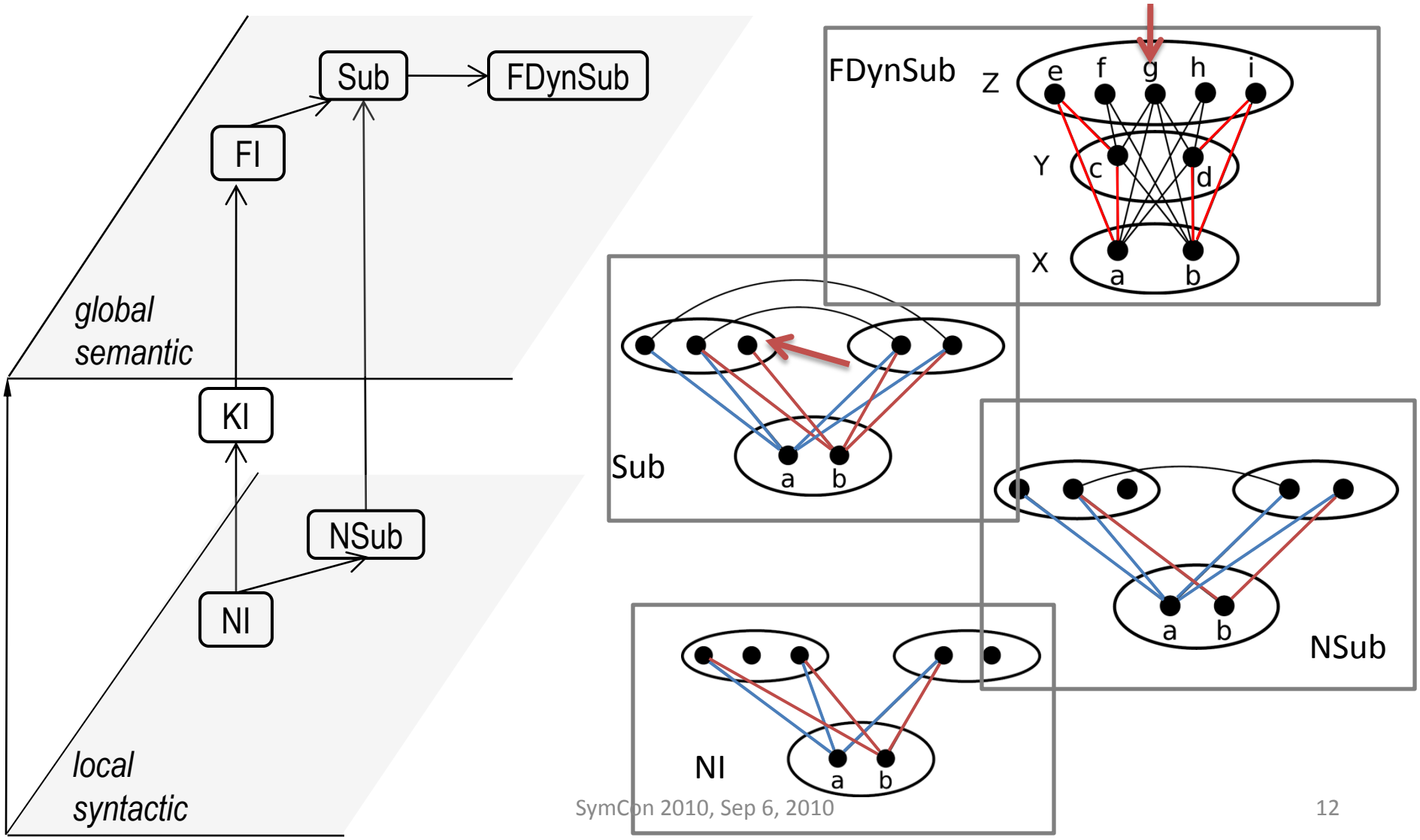


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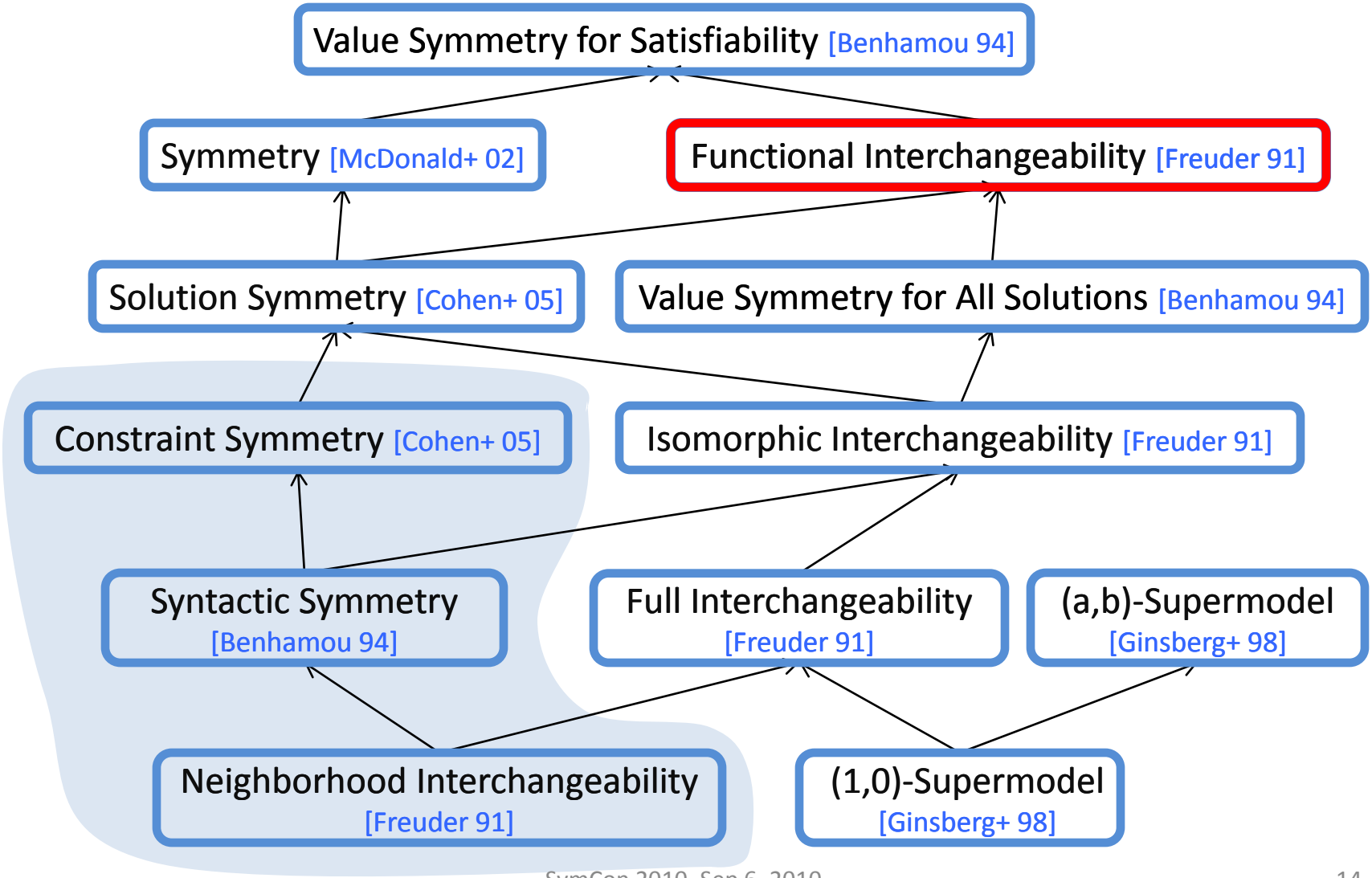
Diagram of Symmetry Concepts



Relation to SBDS & SBDD

- Dynamic interchangeability
  - New opportunities for interchangeability appear during search
  - Forms proposed: DynNI, FDynI, DynSub & ForwNI
- SBDS & SBDD are related to dynamic interchangeability
  - Break symmetries during search
  - Can implement dynamic interchangeability

	Dynamic Interchangeability	SBDS/SBDD
Discovers symmetry	Yes	No
Overhead	Polynomial	Exponential
Space complexity	Polynomial	Exponential/Polynomial
Broken symmetries	Expressed by the concept	All specified symmetries
Advantages	Time & space complexity	Breaks more symmetries

High-Level Observations

	Interchangeability	Symmetry
Research focus	Efficient detection techniques	Efficient breaking techniques
Detected by…	Examining supports & nogoods	• Given by user • Using graph automorphism tools, e.g. NAUTY
Defined over	• Individual variable-value pairs, tuples • Partial assignments • Solutions	
Variations	Substitutability ≈ Dominance	
	Meta interchangeability ≈ Indistinguishable variables	
	Partial interchangeability ≈ Super-solutions	
	Dynamic variations ≈ Symmetry breaking during search	
State of affairs	Many concepts proposed yet to be exploited	Has received intensive attention in recent years

Future Research

- Analysis of symmetry definition was started by [Cohen+ 2005], and is still an ongoing effort
- In interchangeability, many concepts are yet to be investigated
  - Detection algorithms
  - Exploitation in problem solving
- New opportunities: building hybrids of
  - Concepts
  - Algorithms

… where the whole is more powerful than the sum of its parts

Thank you