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A Partial Taxonomy of Substitutability & Interchangeability

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Outline

• Introduction
  – Basic form & extensions
  – Features & use
  – Further developments
• Taxonomy using a partial order
  – One example
• Relation to
  – General forms of symmetry
  – Symmetry breaking during search
• Future research & conclusions
Interchangeability & Symmetry

- Eliminating Interchangeable Values in Constraint Satisfaction Problems [Freuder, AAAI 91]
- “The detection of symmetries is a research avenue pioneered by Freuder [AAAI 1991] and subsequently investigated by many others.” [Van Hentenryck, SARA 2006]
- Interchangeability is a form of ‘solution symmetry’
  - Symmetry is not specified, but is detected
- We survey work on interchangeability & substitutability
  - Identifying & proving relationships among different forms of interchangeability/substitutability
  - We welcome your input
Basics

• Local vs Global
  – Neighborhood Interchangeability (NI)
  – K-Interchangeability (KI)
  – Full Interchangeability (FI)

• Weakening
  – Substitutability *ref. dominance*
  – Partial interchangeability
  – Subproblem interchangeability

• Generalization
  – Dynamic interchangeability *ref. SBDS & SBDD*
  – Meta interchangeability
  – Functional/isomorphic interchangeability: mapping values between different variables *ref. symmetry*
NI and FI

• FI: Global, semantic level, likely intractable
• NI: Local, syntactic level, efficiently determined
• \( NI \Rightarrow FI \)

\[ a, b, c \text{ are } FI \]
\[ a, b \text{ are } NI \]
Interchangeability Researchers

Audemard
Chmeiss
Agren
Prestwich
Naanaa
Mancini
Vilare
Cadoli

Benson
Bistarelli
Kokeny
Choueiry
Bowen
Wilson
Brown
Heus

Benhamou
Neagu
Weil
Haselboeck
Freuder
Sais
Bisdatting
Van Hentenryck

Mazure
Bleik
Bellicha
Capelle
Davis
Flener
Meisels
Bordeaux

Razgon
Pearson
Bisson

Etc.
Further Developments

• Exploration
  – Interchangeability types
  – Their detection cost
  – Their benefits for problem solving

• Context
  – Finding all solutions
  – Problem decomposition

• CSP Extensions
  – Distributed CSPs
  – Quantified CSPs
  – Soft CSPs
Features & Use

- May be viewed as an extension of the fundamental CP concept of inconsistency filtering & propagation
  - Can remove values without removing all solutions
  - Trade amount of filtering against difficulty of recovering removed solutions
- Automatic symmetry detection
- Bundling interchangeable values for the same variable
  - Yields a compact representation of a CSP
  - Yields ‘robust/flexible’ solutions
  - Nogood bundling dramatically reduces search cost
- Shown to be beneficial in
  - Backtrack search & local search, interaction w/ users
  - Random CSPs, benchmarks, resource allocation problems
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Taxonomy

• Surveyed & analyzed interchangeability concepts
• Identified those that are satisfiability preserving
• Classified them in terms of implication
  \[ X \Rightarrow Y \iff \forall a, b \ X(a,b) \Rightarrow Y(a,b) \]
• Identified 22 interchangeability concepts
  – 231 relations between concepts
  – 94 relations are covered in paper
• In extended paper, we will justify the remaining 137 incomparability results
The Interchangeability Landscape
Substitutability

FDynSub

FI

KI

NSub

NI

Sub

global

semantic

local

syntactic

SymCon 2010, Sep 6, 2010
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Diagram of Symmetry Concepts

- **Symmetry** [McDonald+ 02]
- **Solution Symmetry** [Cohen+ 05]
- **Constraint Symmetry** [Cohen+ 05]
- **Syntactic Symmetry** [Benhamou 94]
- **Neighborhood Interchangeability** [Freuder 91]
- **Value Symmetry for Satisfiability** [Benhamou 94]
- **Value Symmetry for All Solutions** [Benhamou 94]
- **Functional Interchangeability** [Freuder 91]
- **Isomorphic Interchangeability** [Freuder 91]
- **Full Interchangeability** [Freuder 91]
- **(a,b)-Supermodel** [Ginsberg+ 98]
- **(1,0)-Supermodel** [Ginsberg+ 98]
Relation to SBDS & SBDD

- Dynamic interchangeability
  - New opportunities for interchangeability appear during search
  - Forms proposed: DynNI, FDynI, DynSub & ForwNI

- SBDS & SBDD are related to dynamic interchangeability
  - Break symmetries during search
  - Can implement dynamic interchangeability

<table>
<thead>
<tr>
<th></th>
<th>Dynamic Interchangeability</th>
<th>SBDS/SBDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discovers symmetry</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Overhead</td>
<td>Polynomial</td>
<td>Exponential</td>
</tr>
<tr>
<td>Space complexity</td>
<td>Polynomial</td>
<td>Exponential/Polynomial</td>
</tr>
<tr>
<td>Broken symmetries</td>
<td>Expressed by the concept</td>
<td>All specified symmetries</td>
</tr>
<tr>
<td>Advantages</td>
<td>Time &amp; space complexity</td>
<td>Breaks more symmetries</td>
</tr>
</tbody>
</table>
### High-Level Observations

<table>
<thead>
<tr>
<th></th>
<th><strong>Interchangeability</strong></th>
<th><strong>Symmetry</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Research focus</strong></td>
<td>Efficient detection techniques</td>
<td>Efficient breaking techniques</td>
</tr>
<tr>
<td><strong>Detected by...</strong></td>
<td>Examining supports &amp; nogoods</td>
<td>• Given by user</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Using graph automorphism tools, e.g. <strong>Nauty</strong></td>
</tr>
<tr>
<td><strong>Defined over</strong></td>
<td>• Individual variable-value pairs, tuples</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Partial assignments</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Solutions</td>
<td></td>
</tr>
<tr>
<td><strong>Variations</strong></td>
<td><strong>Substitutability</strong> ≈ <strong>Dominance</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Meta interchangeability</strong> ≈ <strong>Indistinguishable variables</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Partial interchangeability</strong> ≈ <strong>Super-solutions</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Dynamic variations</strong> ≈ <strong>Symmetry breaking during search</strong></td>
<td></td>
</tr>
<tr>
<td><strong>State of affairs</strong></td>
<td>Many concepts proposed yet to be exploited</td>
<td>Has received intensive attention in recent years</td>
</tr>
</tbody>
</table>

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Future Research

• Analysis of symmetry definition was started by [Cohen+ 2005], and is still an ongoing effort

• In interchangeability, many concepts are yet to be investigated
  – Detection algorithms
  – Exploitation in problem solving

• New opportunities: building hybrids of
  – Concepts
  – Algorithms

... where the whole is more powerful than the sum of its parts
Thank you