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***Physics*, Chapter 41: Polarized Light**

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41

Polarized Light

41-1 Polarization. Transverse Waves

The phenomena of interference and diffraction show that light is propagated as a wave motion, but they do not show whether light is a *longitudinal* wave or a *transverse* wave. The fact that the velocity of light is the same as the velocity of radio waves and the radiation of visible light from accelerated electrons, as in a betatron, indicates that light is an *electromagnetic* wave. We recall from Section 20-9 that a wave can be shown to be transverse if a device can be found which will prevent passage of the wave in one orientation and will allow the wave to be transmitted when in a second orientation at right angles to the first, as in the case of a slit and a transverse wave on a string. Since a longitudinal wave will pass through a slit, however that slit is oriented, longitudinal waves may be distinguished from transverse waves by our inability to demonstrate the property of *polarization*. Such materials as *Polaroid* enable us to demonstrate that light waves are transverse waves. A beam of light in which all the vibrations are in one direction is said to be *linearly polarized*, or *plane polarized*.

On the basis of the electromagnetic theory of light, a linearly polarized monochromatic beam consists of a varying electric field accompanied by a similarly varying magnetic field traveling with the velocity of light, as shown in Figure 41-1. If we take the direction of motion of the beam as the x direction, the vectors representing the electric and magnetic fields at any point along the beam will be in a plane perpendicular to the x axis. The vectors \mathbf{E} and \mathbf{H} are perpendicular to each other and to the direction of propagation in air or vacuum. The direction of propagation of the electromagnetic wave is given by the direction of the vector $\mathbf{E} \times \mathbf{H}$. We shall arbitrarily take the direction of the electric field intensity \mathbf{E} as the direction of vibration of linearly polarized light.

Unless special conditions prevail, the light emitted by a source is unpolarized. The light is emitted in short polarized bursts by independent atoms and molecules in random phase and random polarization, and the net effect is that of an unpolarized beam. We may represent the vibrations

in an unpolarized beam as taking place in all directions at right angles to the direction of propagation of the beam, as shown in Figure 41-2. In a radio or television signal the electromagnetic wave is generally polarized,

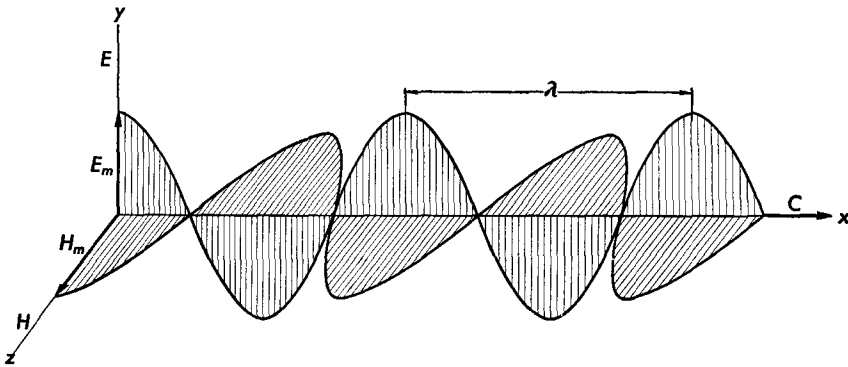
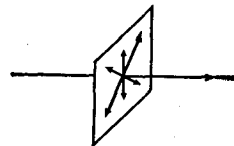


Fig. 41-1 The electric and magnetic field intensities of a linearly polarized beam of light moving in the x direction. The direction of vibration of the beam is taken as the direction of the electric field, in the above case, the y direction.

with the electric vector directed in the plane formed by the direction of propagation and the wire or rod from which the antenna is made. The polarization of a television signal may be demonstrated by rotating an antenna about an axis parallel to the direction of propagation. The signal

Fig. 41-2 The vibrations in a transverse wave are in a plane at right angles to the direction of propagation.



received is a maximum when the receiving antenna is horizontal, that is, parallel to the transmitting antenna.

The human eye is insensitive to the state of polarization of a light beam, so that experiments on the polarization of light must be conducted with the aid of some polarizing substance or device. It has been suggested that the eyes of insects may be sensitive to the polarization of light, and that this may be the mechanism by which bees find their way, as we shall see in the next section.

41-2 Polarization by Scattering

Light can be polarized by scattering from small particles or by molecules of a substance. For example, the blue light of the sky, produced by the scattering of sunlight by air molecules, is partially polarized. When we

look at the sky through a sheet of Polaroid, the intensity of the transmitted light is a minimum when the axis of the Polaroid is at right angles to the direction of vibration of the light.

If unpolarized light is directed vertically down a tube of water containing some fine particles in suspension, the light that is scattered in a horizontal direction will be found to be polarized, as shown in Figure 41-3.

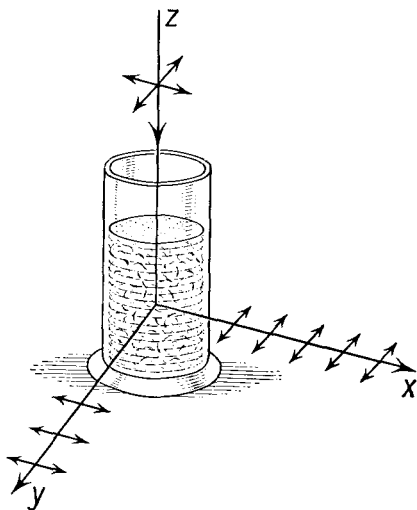


Fig. 41-3 Light traveling downward, z -direction, is scattered by small particles in the liquid. Light scattered in the x -direction has its vibrations in the y -direction. Light scattered in any horizontal direction is linearly polarized.

The direction of vibration of the incident light is in a horizontal plane. The electrons of the substance are caused to vibrate in the horizontal plane by the varying electric field of the incident light. These electrons reradiate the energy absorbed from the incident light, so that the direction of polarization of the scattered light must be horizontal. Since light is a transverse wave motion, the light scattered in the x direction, for example, can have no vibrations in the x direction; hence the only vibrations present when the scattered light is examined along the x axis will be those in the y direction. Light which is scattered in the forward direction or in the backward direction with respect to the incident beam need not be polarized. Only that light scattered at right angles to the original beam is completely polarized, as shown in the figure. Note that the direction of the incident beam can be determined as lying in a plane formed by the direction of the scattered radiation and the normal to the direction of vibration. Thus, by means of a sheet of Polaroid, the direction of the sun can be determined even when the sky is overcast. First one must find the direction in which the Polaroid has the greatest effect on the skylight. The normal to the direction of vibration of the scattered light is found by rotating the Polaroid to minimum intensity. The axis of the Polaroid is then directed along the direction

of the sun. In photography, polarizing filters are sometimes used to yield unusual effects, for by rotating the filter the intensity of the skylight reaching the film can be varied, and one may photograph white clouds against a dark sky to provide good cloud contrast.

The polarization of x -rays by scattering was used by Barkla in 1911 to show that x -rays are transverse waves. As we have seen, a beam of electromagnetic waves scattered at 90° has its direction of vibration normal

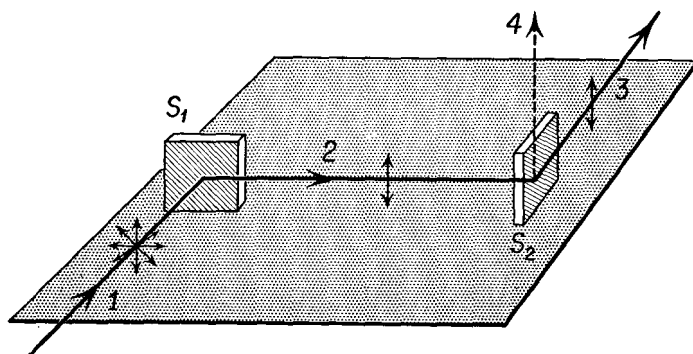


Fig. 41-4 Scattering of x -rays.

to the plane formed by the incident and the scattered beams. If this scattered beam is incident upon another block of scattering material, as in Figure 41-4, the waves scattered by the second block must have their direction of vibration in the same direction as the first scattered beam. Hence if a beam of x -rays is scattered first from S_1 and then from S_2 , the x -rays scattered from the S_2 must have maximum intensity in direction 3 and zero intensity in direction 4. Barkla's experiment showed that this was indeed the case and thus demonstrated the transverse character of x -rays.

41-3 Polarization by Reflection

Light can be polarized by reflection from a plate of glass by the proper choice of the angle of incidence. For example, if the index of refraction of the glass is 1.54, the angle of incidence should be 57° , as shown in Figure 41-5(a). In general, the angle between the reflected ray and the refracted ray is 90° when the reflected light is completely polarized. At this angle of incidence, the reflected light has its direction of vibration parallel to the glass surface. If the reflected ray strikes another glass plate parallel to the first, the light will be reflected from the second plate in the usual manner. Now if the second glass plate is rotated through 90° about the ray incident on it as an axis, it will be found that no light is reflected from it; instead,

all the light is transmitted through it, as shown in Figure 41-5(b). When the second plate is turned through another 90° about the same axis, the light will again be reflected, as in Figure 41-5(c).

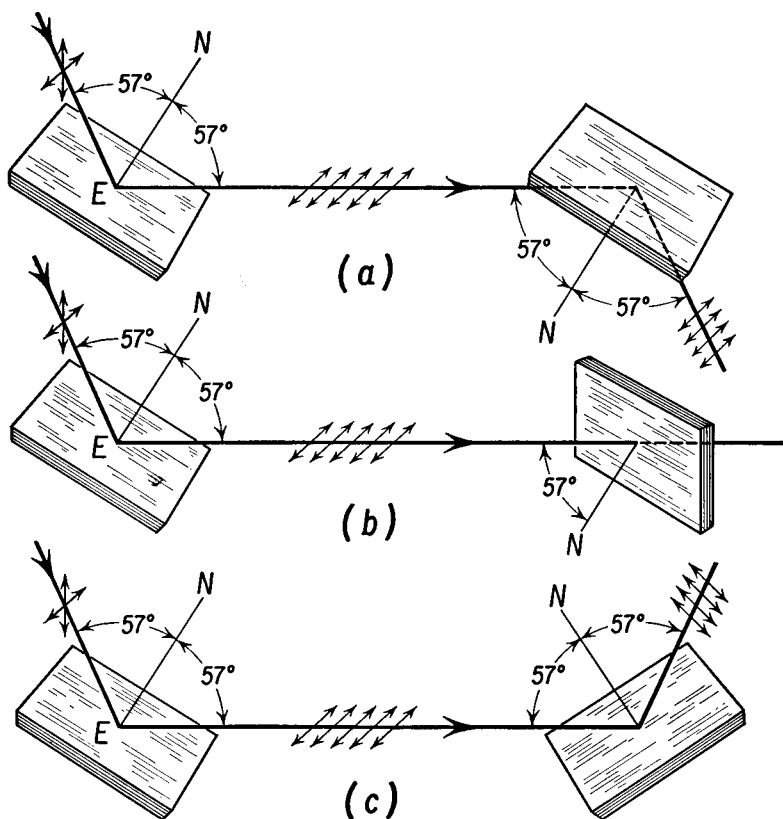
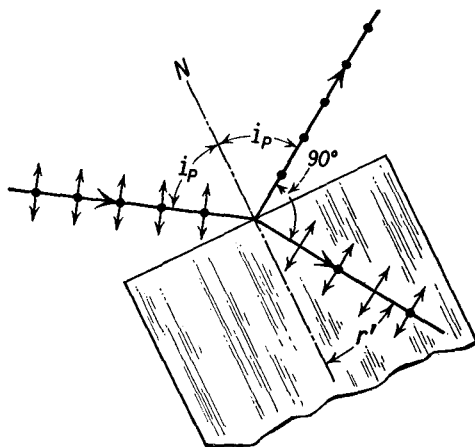


Fig. 41-5 Polarization of light by reflection from a glass plate.

This peculiar behavior of the beam reflected from the first plate can be explained by assuming that only those vibrations which are parallel to the surface of the first plate are reflected from it, when the angle of incidence is such that the angle between the reflected and refracted rays is 90° . Let us call this particular angle of incidence i_p , the *polarizing angle*. The unpolarized incident beam has light of all possible directions of vibration in a plane at right angles to the direction of motion. We may imagine these vibrations to be resolved into two components—one vibrating parallel to the surface of the glass plate, and the other at right angles to this direction. When the angle of incidence is at the polarizing angle, the reflected beam contains only vibrations which are parallel to the glass surface.

When this reflected beam strikes the second glass plate at an angle of incidence equal to the polarizing angle, it will be reflected at maximum intensity when the surface of the plate is parallel to the direction of the vibrations. As the second plate is turned about the ray incident upon it as an axis, the intensity of the light reflected from it decreases and becomes zero after a rotation of 90° , then starts increasing again as the glass plate is turned beyond this position, and becomes of maximum intensity after a rotation of 180° . Thus, as the second glass plate rotates about the ray incident upon it at the polarizing angle, there will be two positions of maximum intensity of the reflected beam and two positions of zero intensity.

Fig. 41-6 The incident ray is unpolarized. The vibrations are shown resolved into two components; the component parallel to the glass surface is represented by dots. The reflected ray is completely polarized; the refracted ray is only partially polarized.



This combination of two glass surfaces arranged so that the light strikes the first surface at the polarizing angle, and the second glass surface mounted so that it can rotate about the reflected beam as an axis, is one form of *polariscope*. The first glass plate, which reflects polarized light, is called the *polarizer*, and the second glass plate, which is used to analyze the light, is called the *analyzer*.

The correct polarizing angle i_p for a particular glass can be found by applying Snell's law to the beam which strikes it. Assuming that the glass is in air, we have

$$\frac{\sin i_p}{\sin r'} = n,$$

where r' is the angle of refraction, as in Figure 41-6. Since the angle of reflection is equal to the angle of incidence, and the reflected ray is perpendicular to the refracted ray, $i_p + r' = 90^\circ$; that is, the angles i_p and r' are complementary. In this case

$$\sin r' = \cos i_p;$$

hence Snell's law becomes

$$\frac{\sin i_p}{\cos i_p} = n = \tan i_p. \quad (41-1)$$

Thus the tangent of the polarizing angle is equal to the index of refraction of the glass. If $n = 1.54$, the polarizing angle is 57° .

Only about 8 per cent of the incident light is reflected from a glass surface at the polarizing angle. By using a bundle of thin glass plates, say seven or eight, the intensity of the reflected beam may be increased to about 40 per cent of the incident beam. This light is all linearly polarized, with the direction of vibration parallel to the glass surfaces. A small percentage of the light will be absorbed by the glass plates, and the remainder will be transmitted. If the number of reflecting surfaces is large, the transmitted beam will consist principally of light whose direction of vibration is perpendicular to the reflecting surface.

41-4 Polarization by Crystal Absorption

Some crystals, such as tourmaline, possess the property of absorbing those vibrations which are perpendicular to the axis of the crystal and of transmitting the vibrations which are parallel to this axis. If a beam of light which is unpolarized is sent through a thin plate of tourmaline, the trans-

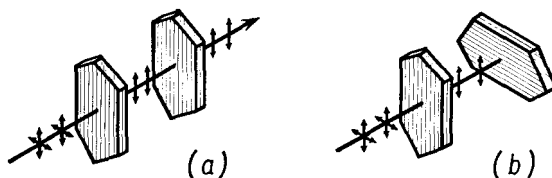


Fig. 41-7 Polarization by absorption in a crystal. (a) Linearly polarized light from the polarizing crystal is transmitted by the analyzer. (b) The linearly polarized light is absorbed by the analyzer when it is turned through 90° .

mitted beam will be linearly polarized parallel to the crystal axis. If this linearly polarized beam is now allowed to fall on a second tourmaline plate set so that its axis is at right angles to the axis of the first crystal, no light will be transmitted by the second crystal, as shown in Figure 41-7. But, if the second tourmaline crystal is rotated through 90° about the incident beam as an axis, thus making the axes of the two crystals parallel, the linearly polarized beam will be transmitted through the second crystal. At intermediate positions the intensity of the transmitted beam will be smaller than that transmitted when the axes are parallel. This phenomenon is known as *dichroism*.

One variety of Polaroid contains crystals of a synthetic material, known as herapathite, oriented in a nitrocellulose film. More recently, specially treated polyvinyl films have been shown to be dichroic and are marketed as varieties of Polaroid.

We have already seen that the energy per unit volume in the electric field in vacuum is $\epsilon_0 E^2/2$, while the energy density of the magnetic field in vacuum is $\mu_0 H^2/2$. In an electromagnetic wave in vacuum, the electric and magnetic fields vary with time, and we may compute the average energy in the field by taking the rms values of these quantities, rather than the instantaneous values of the electric and magnetic field intensities, as in the case of the rms values of alternating current. The flow of energy in the wave depends upon the square of the amplitude of the vibration rather than upon the amplitude itself. We call the energy reaching a unit area of surface per unit time the *intensity* of the wave. Thus the intensity of a light wave depends upon the square of the amplitude of vibration.

When plane polarized light passes through a polarizer whose axis makes an angle θ with the direction of vibration, as shown in Figure 41-8, we may imagine the vibrations of the light wave to have components parallel and perpendicular to the axis of the polarizer. If the amplitude of vibration of the polarized light is E , only the component parallel to the axis of the polarizer passes through it. This component has amplitude $E \cos \theta$. The intensity I of a beam of light is related to its amplitude of vibration through the equation

$$I = \alpha E^2,$$

where α is some constant of proportionality. If the incident beam is of intensity I_0 , the transmitted beam will be of intensity I given by

$$I = I_0 \cos^2 \theta. \quad (41-2)$$

This relationship is known as the *law of Malus*. If unpolarized light is incident upon a perfect polarizer, only half the incident light intensity is transmitted as a plane polarized beam, for the light intensity may be resolved into two equal components one of which is vibrating parallel to the axis of the polarizer and the other is vibrating perpendicular to the axis of the polarizer. When linearly polarized light is transmitted by a perfect analyzer, the intensity of the transmitted light varies from zero to

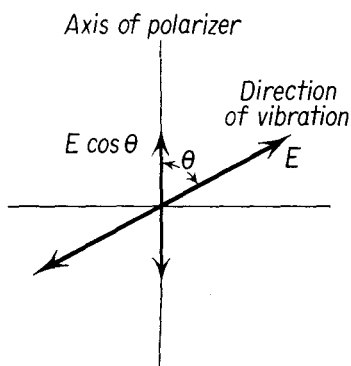


Fig. 41-8

the incident intensity, depending upon the angle θ between the direction of vibration of the light and the axis of the analyzer, in accordance with Equation (41-2).

41-5 Polarization by Double Refraction

When a narrow beam of unpolarized light is sent through certain transparent crystals, such as calcite and quartz, the refracted beam is split into two parts which travel through the crystal and emerge as two separate beams, as shown in Figure 41-9. When an object is viewed through such a crystal, two separate images are seen. If the emergent beams are analyzed

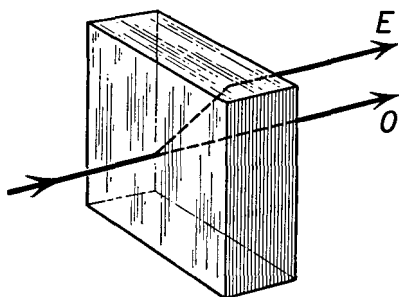


Fig. 41-9 Double refraction by a crystal.

with a Polaroid film, it will be found that each beam is polarized, but that the directions of their vibrations are at right angles to each other. One of these beams obeys the ordinary laws of refraction and is called the *ordinary ray*, designated as *O* in Figure 41-9. The other ray *E* is known as the *extraordinary ray*, for it does not always lie in the plane of incidence; its speed, and hence its index of refraction, depends upon its direction of propagation through the crystal.

In general, if an unpolarized beam of light is sent through a crystal at an arbitrary angle, two linearly polarized beams emerge. However, it is possible to find one direction in the crystal such that both the ordinary ray and the extraordinary ray will travel with the same speed. This particular direction in the crystal is known as the *optic axis*. If the arrangement of atoms in the calcite crystal is examined in a plane perpendicular to the optic axis, the atoms are observed to be symmetrically distributed. In no other direction is this true. The electrical properties of the crystal, such as its permittivity, and the optical properties are found to vary in different directions in the crystal. In all crystals except those displaying cubic symmetry, double refraction is observed. In some crystals more complicated than calcite, there is not one direction in which the ordinary ray and the extraordinary ray travel with the same speed but, rather, there are two such directions. Crystals with one optic axis are called *uniaxial*

crystals, while the latter types are called *biaxial crystals*. We will limit our discussion to the case of uniaxial crystals, which may be understood by a special type of wave surface suggested by Huygens.

In isotropic media the device of spherical wave fronts was used to describe the propagation of light. Let us assume that the ordinary ray is propagated by such spherical wave fronts, but that the extraordinary ray is propagated by ellipsoidal wave fronts. A point source of light within such a crystal is thought to emit two wave fronts, corresponding to different directions of vibration, as shown in Figure 41-10. In the direction of the optic axis, the two wave fronts are tangent.

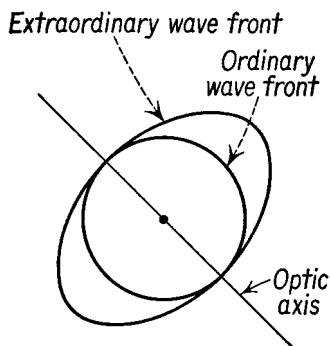


Fig. 41-10

Following the Huygens construction, let us take the case of a plane wave AB incident upon the plane surface S of a uniaxial crystal, in which the optic axis lies in the plane of incidence, as shown in Figure 41-11. By the time the portion of the incident wave front at B reaches the surface at C , the O and E wavelets at A have reached the indicated size. Drawing the tangent to the O wavelet from C , we find the refracted ordinary wave front to be CD , and the refracted ray to be AD . In a similar way the E wave front is CF , and the refracted E ray is AF . The vibrations of the O ray are normal to the plane of the figure, while the vibrations of the E ray lie in the plane of the figure. Both vibrations are tangent to their respective wavelets. In general, it is found that the O vibrations are normal to a plane containing the refracted O ray and the optic axis, called the *principal plane of the O ray*; the E vibrations lie in a plane containing the E ray and the optic axis, called the *principal plane of the E ray*. Note that the vibrations of the E ray are not perpendicular to the direction of propagation of the E ray. The incident light is doubly refracted.

In the above case the refracted E ray lies in the plane of incidence. When the optic axis does not lie in the plane of incidence, the tangent plane to the E wave front may be skewed so that the refracted E ray also does not lie in the plane of incidence. The E ray is truly extraordinary. The behavior of a crystal under the influence of the varying electric field of the light ray is related to the behavior of a long bar of rectangular cross section subjected to a periodic driving force. When the force is normal to one face of the rod, waves are generated in the rod which are propagated with one velocity, appropriate to the stiffness of the rod in the direction of the applied force. Thus there are two velocities of wave propagation down the rod, for the stiffness of the rod is different in the two normal

directions. If the applied force is not normal to one face of the rod, the rod responds as though it were subjected to two independent normal forces—the components of the applied force normal to the faces of the rod. Two separate wave motions are generated which travel down the rod with

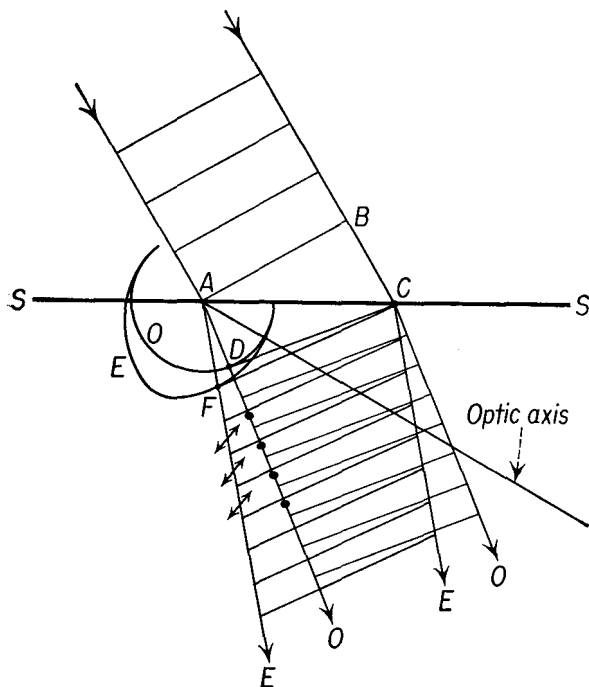


Fig. 41-11

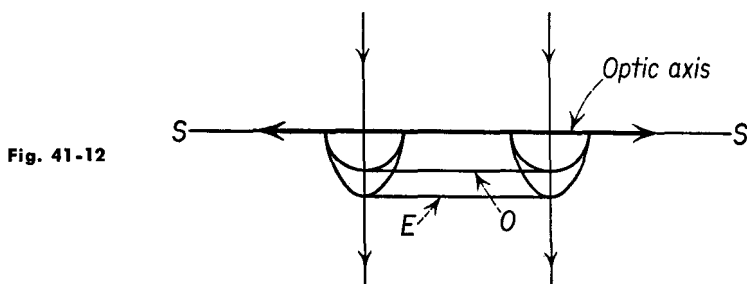


Fig. 41-12

different speeds; each speed is characteristic of a particular direction of vibration. The displacement of the rod at any point is the vector sum of the two displacements due to the two wave motions, at right angles to each other.

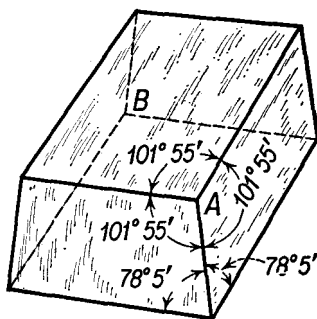
The Huygens ellipsoid which describes the propagation of the extraor-

dinary wave requires that we have knowledge of the velocity of propagation of the E ray in every direction. Once this is known, the behavior of the E ray and the O ray, under arbitrary conditions, can be determined. Thus, if a plane wave of unpolarized light is incident normally upon a section cut from a uniaxial crystal so that the optic axis is parallel to the crystal face, the O ray and the E ray are propagated along the normal to the crystal face with different speeds but in the same direction, as shown in Figure 41-12.

41-6 The Nicol Prism

An ingenious method of producing linearly polarized light was devised by W. Nicol in 1828 and is known as a *Nicol prism*. A natural crystal of calcite is shown in Figure 41-13. In such a crystal the direction of the optic

Fig. 41-13 Sketch of calcite crystal.



axis is parallel to a line making equal angles with the three intersecting angles at A or B . Using a fairly long, clear crystal, the end faces are cut so as to obtain a piece whose cross section is a parallelogram with an

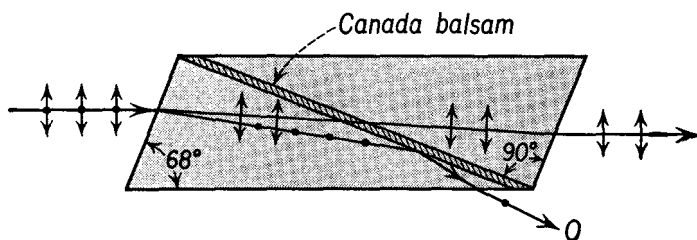


Fig. 41-14 Nicol prism.

included angle of 68° , with the sides of the section parallel to the principal axis of the crystal. This piece is cut in two along the diagonal, as shown in Figure 41-14, in cross section. The two parts of the piece are then

cemented together with a cement widely used in optical work, known as Canada balsam. As shown in Figure 41-14, a plane wave incident upon the crystal is doubly refracted. The index of refraction of the ordinary ray in calcite is 1.66, and in Canada balsam it is 1.55. The angle of incidence of the ordinary ray when it strikes the Canada balsam is greater than the critical angle, so that this ray is totally reflected. The index of refraction of the extraordinary ray depends upon its direction. For the possible directions in the Nicol prism, the index of refraction of the E ray is about 1.49; hence it will be transmitted through the Canada balsam and emerge from the prism.

Nicol prisms are used in optical devices as analyzers, polarizers, or both. The Nicol prism suffers the disadvantage, compared to sheets of Polaroid, that it is limited in aperture, but it has the advantage that it produces a completely polarized beam of light with all the vibrations in one direction.

41-7 Elliptically Polarized Light

In our study of simple harmonic motion in Chapter 12, we have seen that the projection of the motion of a particle in uniform circular motion onto two mutually perpendicular axes yields two simple harmonic motions of equal amplitude and frequency which are 90° out of phase with each other. One of these projected motions was taken along the x axis and the other along the y axis in Chapter 12. Thus we may think of these two simple harmonic motions as the x and y components of a uniform circular motion.

The general case of motion of a particle subjected to two perpendicular harmonic motions is quite complex. The path followed by the particle is known as a *Lissajous* figure. In studying the behavior of polarized light, we shall be interested in the special case where the two motions are of the same frequency f , and therefore of the same angular frequency ω , although there may be an arbitrary phase difference ϕ between the two simple harmonic motions. Let us examine the path of a particle subjected to two simple harmonic motions, of the same frequency but of different amplitudes, when these are directed along two perpendicular directions, say the x and y axes. We have

$$x = A \sin \omega t,$$

and

$$y = B \sin (\omega t + \phi).$$

The simplest case is one in which $\phi = 0$, that is, the two motions are *in phase*. Here we find that

$$x = \frac{A}{B} y;$$

that is, the path of the vibrating particle is a straight line.

The second case of interest to us is that in which ϕ is equal to 90° , so that $y = B \cos \omega t$; for this case

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1.$$

This is the equation of an ellipse with semiaxes A and B ; hence a particle subjected to two simple harmonic motions of different amplitudes which are 90° out of phase experiences an elliptical motion. If the two motions are of the same amplitude, the path is circular.

From Chapter 35 we know that light can be emitted by an accelerated charged particle. When the charged particle oscillates with a frequency f , the emitted light is of the same frequency and is linearly polarized, with its direction of polarization perpendicular to the direction of propagation and lying in the plane formed by the acceleration vector and the direction of propagation. If the charged particle moves in an ellipse, the light emitted is elliptically polarized; that is, it may be resolved into two linearly polarized components which are of different amplitudes but of the same frequency, and which are 90° out of phase with each other. The two components are along the axes of the ellipse.

We can generate circularly polarized light and elliptically polarized light from plane polarized light with the aid of a thin section of a doubly refracting crystal known as a *quarter-wave plate*. Such a plate is also useful in identifying elliptically polarized light. We shall find it convenient to discuss these plates by means of the concept of the *optical path length*, which is defined as the distance the light would travel in vacuum in the time it takes to traverse a given path in a medium. The time to traverse a distance s in a medium is s/v , where v is the velocity of light in the medium. If n is the index of refraction of the medium, the optical path length s' is given by the equation

$$s' = \frac{s}{v} c = ns. \quad (41-3)$$

If a plane wave is incident normally upon a slab of medium of thickness s , the phase difference between light just incident upon the front face of the slab and the light just leaving the rear face of the slab is given by $\Delta\phi$, where

$$\Delta\phi = \frac{2\pi}{\lambda} ns, \quad (41-4)$$

and λ is the wavelength of the light in vacuum. Note that there is no phase change on transmission, and that with this consideration Equation (41-4) is in agreement with Equation (40-5).

When light is incident normally upon a slab of a uniaxial crystal with parallel faces, cut so that the optic axis lies in the plane of the crystal face,

the ordinary and extraordinary rays are refracted normally, as shown in Figure 41-12, so that there is no angular separation between the two rays. Since the refractive indices n_O and n_E are different in this direction, we find that the phase changes of the two rays are different. Plates which are cut of such thickness that the difference in phase between the emergent O and E rays is π are called *half-wave plates*, while if the difference in phase between the two emergent rays is $\pi/2$, the plate is called a *quarter-wave plate*.

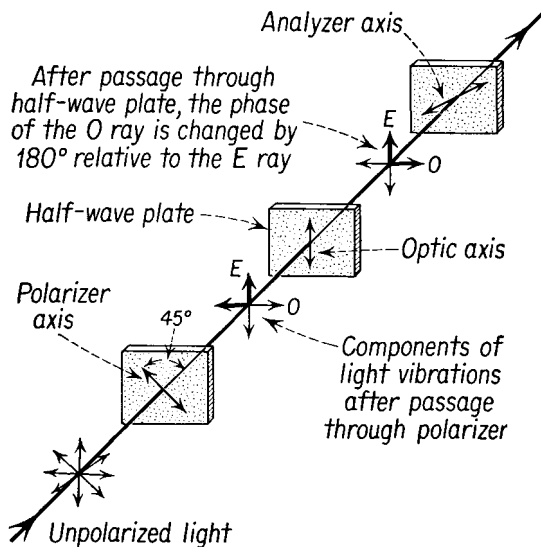


Fig. 41-15 A half-wave plate rotates the plane of polarization of linearly polarized light.

We have seen that linearly polarized light may be resolved into two components whose directions of vibration are mutually perpendicular. Referring to Figure 41-15, suppose that a beam of monochromatic light first goes through a polarizer and then strikes a half-wave plate with the direction of its optic axis at an angle of 45° with the direction of vibration of the incident beam. We can consider the light on its passage through the half-wave plate as resolved into two components, the E ray with its direction of vibration parallel to the optic axis, and the O ray with its direction of vibration perpendicular to the optic axis. Although these two vibrations are in phase when the light enters the crystal, the effect of traversing the half-wave plate is to introduce a phase difference of π between them, as shown in the figure. The sum of these two vibrations is again a linear vibration but at right angles to the direction of the vibrations of the incident beam. This can be verified by rotating an analyzer

until extinction of the beam is produced. It must be remarked that a half-wave plate is such only for a particular wavelength.

If a quarter-wave plate is substituted for the half-wave plate of Figure 41-15, then the phase difference between the E and the O vibrations of the emergent beam will be $\pi/2$; that is, when the E beam has maximum amplitude, the O beam has zero amplitude, and when the E beam has zero amplitude, the O beam has maximum amplitude. The emergent beam is *circularly polarized*. If this beam is now examined with the analyzer, it will be transmitted at all angles with equal intensity, similar to the behavior of an unpolarized beam. To detect the fact that the emergent beam is circularly polarized, it is necessary to introduce another quarter-wave plate in the path; the effect will be to convert the circularly polarized beam into a linearly polarized beam.

If the thickness of the crystal is such as to produce a phase difference of $\pi/4$, for any particular color, it will convert a linearly polarized beam into an *elliptically polarized* beam. When examined with an analyzer, the intensity of one direction of vibration will be different from that in a direction at right angles to it.

41-8 Interference with Polarized Light

We have already considered the phenomenon of interference using ordinary or unpolarized light. From the preceding discussion and from the vector superposition principle, it must be clear that destructive interference can only take place with waves which are linearly polarized in the same di-

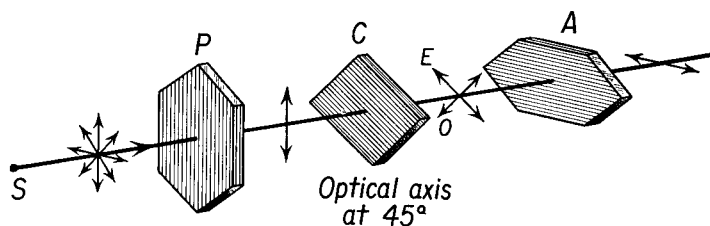


Fig. 41-16 A doubly refracting crystal C placed between the polarizer P and the analyzer A when P and A are in crossed positions.

rection. It was unnecessary to make this restriction in the case of ordinary interference phenomena, for the requirement of coherent radiation led us to consider the interference as composed of the sum of the effects produced by the light emitted from individual atoms. In the present discussion we wish to consider special interference effects which are produced with linearly polarized light.

Consider, for example, a polariscope arranged in the *crossed* position, as shown in Figure 41-7(b), so that no light passes through the analyzer. If a piece of doubly refracting material, such as a quartz crystal C , with its optic axis in the plane of the crystal face and oriented at 45° with the axis of the polarizer P , is placed between the polarizer and the analyzer A , colored light passes through the analyzer (see Figure 41-16). When white light is incident on the polarizer, linearly polarized white light is

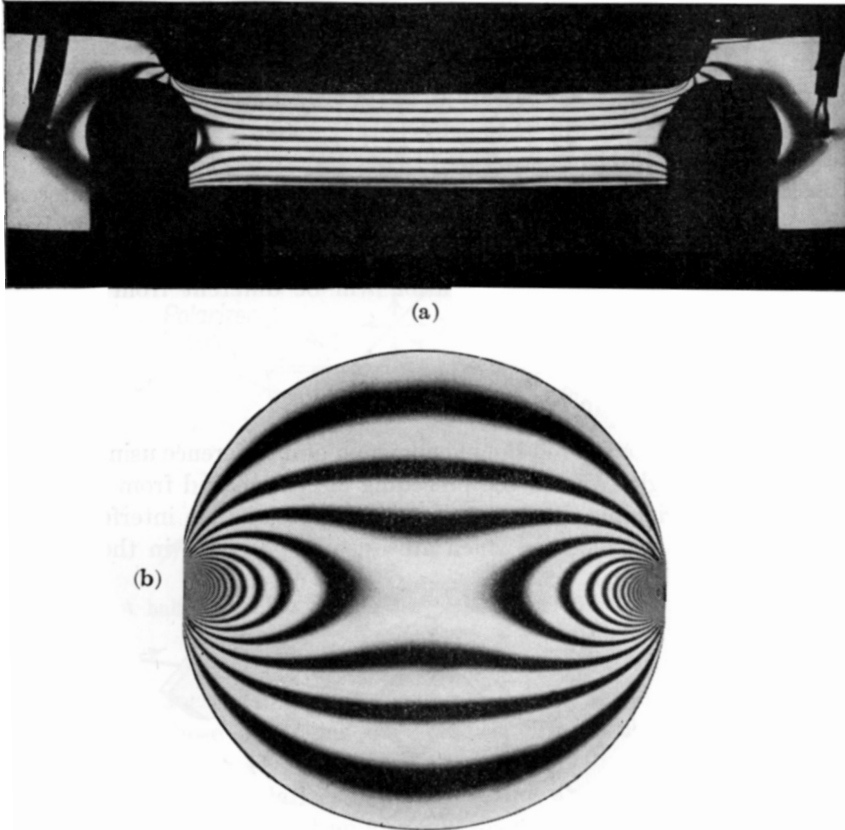


Fig. 41-17 Interference pattern produced by passing polarized light through a piece of transparent plastic in a crossed polariscope. (a) Strain pattern when a bar is subjected to a bending moment. (b) Strain pattern in a cylindrical disk subject to a diametral compression. (Photographs by George Gerard, New York University.)

incident upon the crystal. The beam is divided into two components, the extraordinary ray vibrating parallel to the optic axis, and the ordinary ray vibrating perpendicular to it. Since they travel with different velocities, there will be a difference in phase when they emerge. But when the two vibrations pass through the analyzer, only the horizontal components

of these vibrations will be transmitted, and interference will occur between these two transmitted rays. If the path difference due to passage through the crystal is a half wavelength, or an odd number of half wavelengths, these two rays will annul each other. When this is the case for the yellow light, the light transmitted through the analyzer will be "minus yellow," or purple. Hence specimens of clear, colorless, doubly refracting material will appear colored when viewed through a polariscope. These effects may be observed with pieces of mica or Scotch tape, and colored decorative patterns may be obtained from an overlaid mosaic of transparent substances.

Many transparent materials, such as glass and clear Bakelite, which are not noticeably doubly refracting under ordinary conditions, become so when subjected to stresses. When such a substance is placed between the polarizer and analyzer of a polariscope, a colored pattern may be observed. The pattern of colors can be related to the strains in the substance produced by the stresses. In modern engineering practice, models of structures are built of transparent Bakelite, and the stresses induced in elements of the structure due to applied loads may be determined with the aid of polarized light. This type of study of the stress in a structure is called *photoelasticity*. Strain patterns in loaded specimens illuminated with monochromatic light are shown in Figure 41-17. When glassware is cooled too rapidly, internal stresses often develop owing to nonuniform contraction. These internal stresses may be studied by placing the object in a polariscope.

41-9 Further Effects and Applications

(a) *Optical Activity*. Many solids and liquids *rotate* the direction of polarization of linearly polarized light. If a tube of sugar solution is placed between a polarizer and an analyzer, it is found that the sugar solution rotates the direction of polarization by an amount which depends upon the length of the tube, the sugar concentration, and the type of sugar used. In some cases the rotation is clockwise, or right-handed, and is called *dextrorotatory*, while in other cases the rotation is left-handed, or *levorotatory*. Such a polariscope may be used to measure the concentration of the solution of a known sugar; it is called a *saccharimeter*. This type of optical activity is associated with a corkscrewlike structure of the molecules of the substance.

(b) *Kerr Effect*. Many substances acquire the property of *birefringence* and become doubly refracting when placed in a sufficiently strong electric field. The effect is due to anisotropic alignment of the molecules of the substance when placed in the electric field. This effect is especially marked in carbon disulphide and nitrobenzene. A cell of nitrobenzene of proper thickness can be made into a temporary half-wave plate, under

the influence of an electric field, and can act as a very fast optical shutter when placed between crossed sheets of Polaroid. The Kerr effect is used as the optical shutter in the *geodimeter*, mentioned in Section 36-8, and in other devices to measure the velocity of light.

(c) *Cotton-Mouton Effect*. Some substances placed in a strong transverse magnetic field become doubly refractive, in this magnetic analogue of the Kerr effect. The Cotton-Mouton effect is associated with the alignment of the molecules in a magnetic field. Both the Kerr effect and the Cotton-Mouton effect vary with temperature, as we might expect, for the tendency of the molecules to become disaligned depends on their random motion, which increases with temperature.

(d) *Glare Reduction*. Light incident upon a diffusely reflecting surface at large angles is sometimes reflected at an angle of reflection approximately equal to the angle of incidence. This results in *glare*. When the reflected light is analyzed with a Polaroid, it is found that the component with its vibrations parallel to the surface is much more intense than the component whose vibrations are perpendicular to the surface. Glare may be reduced considerably by using polarizing spectacles oriented to absorb the horizontal vibrations.

Problems

41-1. The index of refraction of one type of flint glass is 1.65. Determine the polarizing angle when the glass is (a) in air and (b) immersed in water of index 1.33.

41-2. The indices of refraction of yellow light of 6,000 Å in a doubly refracting medium are 1.710 for the ordinary ray and 1.740 for the extraordinary ray. (a) Determine the velocity of each of these waves in this medium transverse to the optic axis. (b) Determine the minimum thickness of this material necessary to produce a path difference of half a wavelength for these two rays. (c) If white light is incident on a polariscope in the crossed position, what will be observed when this doubly refracting material is placed between the polarizer and analyzer? What will be the intensity of the light passing through the analyzer compared to the light incident upon the doubly refracting plate?

41-3. Derive a formula relating the critical angle θ_c of a transparent substance to its polarizing angle θ_p .

41-4. A beam of unpolarized light is incident upon a polarizer. (a) If the incident beam has a luminous flux of 5 lumens, what will be the luminous flux passing through the polarizer? (b) The light is then incident upon an analyzer whose axis is at an angle of 53° with the axis of the polarizer. What will be the luminous flux which passes through the analyzer?

41-5. A beam of unpolarized light traveling in the $+x$ direction is scattered in all directions by a cloud of fine particles. (a) Is the light scattered at an angle of 37° with the $+x$ direction completely polarized? (b) If this scattered light is

examined by a Polaroid, what are the relative intensities of the maximum and minimum light transmitted by the Polaroid?

41-6. The index of refraction for the ordinary ray n_o of quartz is 1.544, while the index for the extraordinary ray n_e is 1.553 for sodium light. (a) Is the ordinary Huygens wavelet inside or outside the extraordinary wavelet? (b) A ray of polarized light is incident normally upon a slab of quartz in which the optic axis is in the plane of incidence and makes an angle of 30° with the normal. What is the angle of refraction of the ordinary ray? (c) What is the angle of refraction of the extraordinary ray? Use a Huygens construction.

41-7. A slab of quartz has its optic axis in the plane of incidence. (a) What is the thickness of a quarter-wave plate for sodium light? (b) What is the thickness of a half-wave plate? (c) Will the phase of the ordinary ray be advanced or retarded with respect to the extraordinary ray? Use the data of Problem 41-6.

41-8. Circularly polarized light is incident upon a polariscope. Compared to the incident beam, (a) what is the intensity of the light passing through the polarizer? (b) When the analyzer is in the crossed position, what is the intensity of the light passing through the analyzer? (c) When the axis of the analyzer makes an angle of 53° with that of the polarizer, what is the intensity of the beam passing through the analyzer?

41-9. Elliptically polarized light in which the amplitude of the x vibration is three times the amplitude of the y vibration is incident upon a polarizer. Compared to the incident beam, (a) what is the maximum intensity transmitted through the polarizer? (b) What is the minimum intensity transmitted through the polarizer? (c) What is the intensity transmitted through the polarizer when it is rotated to a position midway between the maximum and minimum position?

41-10. Elliptically polarized light, in which the amplitude of the x vibration is twice the amplitude of the y vibration, is incident upon a quarter-wave plate, and the light passing through the plate is then examined by an analyzer. Compared to the incident beam, (a) what is the maximum intensity of the light transmitted by the analyzer and (b) what is the minimum intensity of the light transmitted by the analyzer?

41-11. Three perfect polarizers are placed in the path of a beam of unpolarized light so that the axis of the second polarizer makes an angle of 30° with the first, while the axis of the third polarizer makes an angle of 90° with the first and an angle of 60° with the second. Find the percentage of the incident light transmitted by the combination.

41-12. A television antenna is found to receive a maximum signal when its axis lies along the direction of propagation and it is oriented horizontally. What fraction of the maximum signal is received when the plane of the antenna is rotated about the direction of propagation so that it lies at an angle of 37° with the horizontal?