Physics, Chapter 46: Nuclear Reactions

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Nuclear Reactions

46-1 Special Relativity

One of the most important developments of twentieth-century physics was the formulation of the special theory of relativity. This theory was an outgrowth of the failure of all attempts to show that the motion of the source of light relative to the observer had any effect on the speed of light. It is impossible to account for these experimental findings of Michelson and Morley, and others, on the basis of classical mechanics and electromagnetic theory. In 1905, Albert Einstein put forth the suggestion that all experimental findings would be clarified if it were assumed that the speed of light is a constant and is independent of the relative motion of the source and the observer. This statement forms the first postulate of the special, or restricted, theory of relativity. The second postulate of the theory is that all systems which are in uniform motion relative to one another are equally valid frames of reference, and all fundamental physical laws must have the same mathematical forms in each of these reference frames. Einstein expressed the viewpoint that all motion was relative motion, that there was no absolute coordinate frame, and that it was impossible to distinguish between a state of rest and a state of uniform translational motion by any physical experiment whatever. Thus, if the statement that the velocity of light was $3 \times 10^{10}$ cm/sec was a fundamental physical law, every observer in uniform translational motion who measures the velocity of light must obtain this value, regardless of the motion of the source of light.

Let us consider some of the immediate implications of the first postulate. Suppose that we have two observers on two coordinate frames which are in relative motion with respect to each other. For simplicity we will orient the axes parallel to each other and will call one of these the unprimed frame and the other the primed frame. Let us suppose that the primed frame moves in the $x$ direction with velocity $v$ as seen in the unprimed frame. At the instant the origin of the two frames overlap, we cause a pulse of light to be emitted. From the first postulate both observers must
see the light spreading out as a spherical Huygens wave. The equation of this sphere as seen by the observer in the unprimed frame is

\[ x^2 + y^2 + z^2 = c^2 t^2, \]  

(46-1)

which is the equation of a sphere of radius \(ct\). At the moment we can assume nothing whatever about the primed frame except that the velocity of light is \(c\). Thus we write that the Huygens wave front, as seen in the primed frame, may be represented by the equation

\[ x'^2 + y'^2 + z'^2 = c^2 t'^2. \]  

(46-2)

We seek to find a set of equations representing the coordinate transformations between the primed and the unprimed frames. Such a transformation must involve only the first powers of the coordinates, for otherwise a single point in one system might become two or more points in the second system. Furthermore, we have no right to assume that the time \(t\) measured in the unprimed frame is the same as the time \(t'\) measured in the primed frame. When these conditions are applied, it is found that the appropriate transformation equations are

\[ x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma \left(t - \frac{vx}{c^2}\right), \]  

(46-3)

where

\[ \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}. \]

This result may be verified by substituting into Equation (46-2) to obtain Equation (46-1). Equations (46-3) are called the Lorentz transformation equations. These equations imply that an event which takes place in the unprimed frame at coordinates \(x, y, z, t\) will be reported by an observer in the primed frame as taking place in his frame at coordinates \(x', y', z', t'\). We assume that both observers have meter sticks and clocks with which to make measurements, and that these sticks and clocks are in agreement with each other when set side by side. The transformation equations work in both directions. To find the coordinates in the unprimed frame of an event taking place in the primed frame, we may solve Equations (46-3), or, more simply, we note that the unprimed frame is moving in the \(-x\) direction, with velocity \(-v\), with respect to the primed frame, so that we obtain

\[ x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma \left(t' + \frac{vx'}{c^2}\right). \]  

(46-4)

In the Lorentz transformation equations we note that time is reduced to the status of a coordinate, that it has no special properties.

While we cannot derive all the important results of relativity theory, two important consequences may be simply obtained. These are the *Lorenz-Fitzgerald contraction* of length, and *time dilatation*. 


Let us suppose that a rod of length \( l \) is at rest in the unprimed system, and that the coordinates of its end points are given by \( x_2 \) and \( x_1 \) such that 
\[
 l = x_2 - x_1.
\]
An observer in the primed system who wishes to determine the length of the rod measures the coordinates of the ends of the rod at the same time \( t' \), as read on his clocks, and finds that the rod is of length 
\[
l' = x_2' - x_1'.
\]
We may determine the relationship between \( l \) and \( l' \) by applying Equations (46-4). Thus we note that 
\[
x_2 - x_1 = \gamma (x_2' + vt') - \gamma (x_1' + vt'),
\]
or 
\[
l = \gamma l'.
\]
Since the speed of the coordinate frames with respect to each other is less than the velocity of light, \( \gamma \) is greater than 1, and we note that the length \( l' \) is less than the length \( l \). The observer in the moving coordinate frame sees the rod as contracted, as compared to the length seen by the observer in the frame in which the rod was at rest. Note that we have used the transformation equations in the form of Equations (46-4) rather than Equations (46-3) for we knew that the measurements were made at the same time \( t' \) in the primed frame. We have no right to assume that the observations were made at the same time \( t \) in the unprimed frame, for the measurements were made at two different points in the primed frame. From Equations (46-4) this implies that the observer in the unprimed frame will infer that the measurements were made at two different times \( t_1 \) and \( t_2 \).

Suppose a clock is located at a fixed point \( x_1 \) in the unprimed system, and that this clock is used to measure the time interval \( \Delta t \) between two events which occur at times \( t_1 \) and \( t_2 \), such that 
\[
\Delta t = t_2 - t_1.
\]
Applying transformation Equations (46-3), we find that an observer in the primed system would observe that the time interval between the two events was 
\[
\Delta t' = t_2' - t_1'
\]
such that 
\[
t_2' - t_1' = \gamma \left( t_2 - \frac{v x_1}{c^2} \right) - \gamma \left( t_1 - \frac{v x_1}{c^2} \right),
\]
or 
\[
\Delta t' = \gamma \Delta t.
\]
Again \( \gamma \) is greater than 1, so that the observer in the moving frame will claim that the time interval between the two events is greater than the interval recorded by the observer who has no relative motion with respect to the clock. This is called time dilatation. Since the Lorentz transformation equations are based upon the finite velocity of light, the same for all observers, these strange effects of length contraction and time dilatation are associated with the fact that, in actual measurement, information is obtained by means of light signals. When an object is moving with speeds approaching the velocity of light, we must examine with great care what we mean by length and time.
Among the results of relativity theory, we find that the simple rules for the vector addition of velocity must be modified. Thus we may no longer write

\[ \mathbf{v} = \mathbf{w} + \mathbf{u} \]

for the vector addition of two velocities, but rather we find that, if the two velocities are directed along the same line, the resultant velocity is given by

\[ v = \frac{w + u}{1 + \frac{uw}{c^2}}. \]  \hfill (46-7)

The momentum \( p \) of a particle of mass \( m \) moving with velocity \( v \) is given by

\[ p = \frac{mv}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} = \gamma mv. \]  \hfill (46-8a)

The de Broglie wavelength to be associated with a particle of mass \( m \) moving with velocity \( v \) and momentum \( p \) is given by

\[ \lambda = \frac{h}{p} = \frac{h}{\gamma mv}. \]  \hfill (46-8b)

The energy of a moving particle is given by

\[ \mathcal{E} = \gamma mc^2 = \frac{mc^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}, \]  \hfill (46-9)

and the kinetic energy \( \mathcal{E}_k \) of a moving particle is the difference between its energy in motion and its mass energy, or

\[ \mathcal{E}_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1), \]  \hfill (46-10)

which reduces to \( \mathcal{E}_k = \frac{1}{2}mv^2 \) in the case of slowly moving objects.

Equation (46-9) implies that the limiting speed for the motion of a particle is \( c \); that is, any particle moving with speed \( c \) would have infinite energy.

If we solve the equation defining \( \gamma \) for \( v^2 \) we find

\[ v^2 = c^2 \left(1 - \frac{1}{\gamma^2}\right); \]

squaring Equation (46-8a) and substituting this result, we obtain

\[ p^2 = \gamma^2 m^2 v^2 = m^2 c^2(\gamma^2 - 1). \]
We may solve this equation for $\gamma^2$ to find

$$\gamma^2 = \left(1 + \frac{p^2}{m^2c^2}\right).$$

Substituting this result into the square of Equation (46-9), we find

$$E^2 = \gamma^2 m^2 c^4 = p^2 c^2 + m^2 c^4. \quad (46-11)$$

Equation (46-11) is extremely interesting, for it illustrates the unity between mass particles and quanta. The relation between energy and momentum for particles of zero rest mass reduces to the familiar relation for quanta $E = pc$, while a particle whose momentum is zero has energy given by the mass-energy equation.

One further result is of interest to us at this time. We have seen that a charged particle whose velocity is transverse to a field of flux density $B$ moves in a circular path whose radius $r$ is given by $Be = (mv)/r$, where $e$ is the charge of the particle, and $m$ is its mass. At high velocities we must replace $mv$ by the relativistic expression for momentum to find

$$Be = \frac{\gamma mv}{r}. \quad (46-12)$$

The relationships and concepts of the theory of special relativity are presented in the preceding paragraphs to indicate some of the fundamental changes in our concept structure and in the equations of mechanics which must be introduced in the study of rapidly moving particles. These equations reduce to more familiar forms at the velocities of ordinary experience. These equations must be used in the analysis of the data obtained with a beta-ray spectrometer and in the analysis of nuclear radiations. Here the equations find ample experimental verification. Perhaps the most detailed verification of relativity theory lies in its application to the design, construction, and successful operation of modern high-energy accelerators.

### 46-2 Particle Accelerators

There are many different types of devices designed and built to accelerate particles to high energies. We have already described one of these, the betatron, which accelerates electrons to energies up to 300 Mev. Using other methods, electrons can now be accelerated up to energies of about 2 Bev (billion electron volts), and protons can be accelerated up to about 6 Bev. Designs and experiments are now in progress for a device to accelerate protons to 100 Bev.

The forerunner of this development of particle accelerators is the cyclotron, which was originally designed and built by E. O. Lawrence and M. S. Livingston in 1931. It consists essentially of a short, hollow, metal
cylinder, divided into two sections $A$ and $B$, as shown in Figures 46-1 and 46-2. The cylinder is placed between the poles of a large electromagnet.

**Fig. 46-1** The paths of particles in a cyclotron.

Some of the magnets built have pole pieces from 30 to 60 in. in diameter, and a modified form of cyclotron in use in California has a diameter of
The 184-in. Berkeley cyclotron at the Radiation Laboratory of the University of California at Berkeley. The vacuum chamber is in place between the pole pieces of the electromagnet. The tube extending into the chamber at the left carries the target, which is bombarded by the high-energy ions in the cyclotron chamber. (Photograph supplied by Prof. R. I. Thornton, Radiation Laboratory, University of California, Berkeley, Calif.)

circle of radius \( r \) given by Equation (46-12). Each time the particle nears the gap, the potential of the half cylinders, or dees, as they are called, is reversed, so that the particle is accelerated through the gap. If the timing is just right, the potential change is always synchronized with the passage of the particle through the gap, and the particle may be accelerated to high velocities. The potential difference between the sections \( A \) and \( B \) may have any value from about 10,000 to 200,000 volts. The particle emerging from the cyclotron may have an energy of several million volts due to the successive accelerations it experiences. The cyclotron is thus a low-voltage source of high-energy particles.
From Equation (46-12) we see that the radius of the orbit of the particle is given by
\[ r = \frac{\gamma mv}{Be}. \]

The maximum magnetic induction \( B \) obtainable with the use of iron is about 20,000 gausses, so that the radius of the pole face of the magnet presents a limit on the maximum velocity which can be achieved with a cyclotron. The time \( t \) required for a particle to traverse a semicircular path is
\[ t = \frac{\pi r}{v} = \frac{\pi \gamma m}{Be}. \]

As long as the velocity of the ion is considerably less than \( c \), \( \gamma \) is close to 1, and the time required to traverse the semicircle is independent of the speed of the particle. Thus the cyclotron may be driven by a fixed-frequency oscillator. When the velocity is such that \( \gamma \) is appreciably different from 1, it is necessary to vary the frequency of the oscillator to maintain the proper timing. The frequency is varied in synchronism with the changes in \( t \), and the accelerator is then called a synchro-cyclotron. The synchro-cyclotron of the University of California is shown in Figure 46-3.

There are many other important types of accelerators which cannot be discussed here, for the principles of their operation are beyond the scope of this book. The particles accelerated are always charged particles and include electrons, protons, deuterons, helium ions, and ions of more massive atoms.

46-3 Nuclear Reactions with Protons, Deuterons, and Neutrons

With the high-energy particles now available, many different types of nuclear reactions have been produced. For example, when lithium of mass 7 is bombarded with protons, the compound nucleus thus formed breaks up into two alpha particles having an energy of about 8.63 Mev each. The reaction is
\[ ^3\text{Li}^7 + p \rightarrow (^4\text{Be}^8) \rightarrow ^2\text{He}^4 + ^2\text{He}^4 + Q; \]
the reaction energy \( Q \) is 17.28 Mev, which checks very well with the value obtained from the difference in the masses of the initial and final nuclei. Sometimes the proton captured by the lithium nucleus remains with the compound nucleus, beryllium, and the excess energy is emitted in the form of a gamma ray of 17 Mev energy. The \(^4\text{Be}^8\) nucleus subsequently decays into two alpha particles with a half life which is probably less than \( 10^{-14} \) sec. A reaction in which a proton is absorbed and a gamma ray is emitted is known as a \((p, \gamma)\) reaction.
Another interesting reaction occurs when high-energy deuterons bombard deuterium. Two different reactions have been observed:

\[ _1H^2 + _2H^2 \rightarrow ({}^4He) \rightarrow _1H^3 + _1H^1 + Q, \]

and

\[ _1H^2 + _2H^2 \rightarrow ({}^4He) \rightarrow _2He^3 + _0n^1 + Q. \]

The first of these reactions results in the production of tritium, an isotope of hydrogen of mass number 3, identified with the aid of the Wilson cloud chamber. The value of \( Q \) for this reaction was determined from the range of the particles in air, yielding a value of \( Q = 4.03 \text{ Mev} \). With the value of \( Q \), and the known masses of hydrogen and deuterium, the mass of tritium can be determined very accurately. This is at present the most accurate method for determining the mass of \( H^3 \). This method is now widely used for determining the atomic masses of isotopes formed in nuclear reactions for which accurate \( Q \)-values are known. The second of the above reactions results in the production of neutrons and is one of the simplest methods for obtaining neutrons of known energy. Neutrons have been observed for comparatively low values of incident deuteron energies, as low as 6 kev. The neutron yield increases with reaction energy. The value of \( Q \) for this reaction has been found to be \( Q = 3.18 \text{ Mev} \).

When nitrogen is bombarded with neutrons, a radioactive isotope of carbon of mass number 14 is produced, according to the following reaction:

\[ _7N^{14} + _0n^1 \rightarrow (_7N^{15}) \rightarrow _6C^{14} + _1p^1. \]

The carbon isotope produced in this reaction has a half life of 5,580 years. This same reaction takes place in the atmosphere, induced by neutrons liberated by cosmic rays, and is responsible for the existence of radiocarbon dating.

### 46-4 Nuclear Cross Section

The concept of a nuclear cross section is a very useful one in nuclear physics. In general, we do not refer to the geometrical cross section, as though the nucleus were a sphere; rather, we use the term “cross section” as a measure of the probability of the occurrence of a given process. Thus, if there are \( n \) nuclei per unit volume of a given substance, a foil of thickness \( t \) will contain \( nt \) nuclei per unit of surface area of the foil. The probability that a single particle will strike one of these nuclei if it is directed at a unit area is simply the fraction of the surface area of the foil occupied by nuclei. Thus if \( \sigma \) is the nuclear cross section, the probability that an incident particle will strike a nucleus is \( \sigma nt \). If an incident beam whose flux is \( N \) particles per unit area per second strikes the foil, the number of interactions with nuclei, per second, will be \( N_i \), according to the equation

\[ N_i = N\sigma nt. \]
Equation (46-13) may be taken as the defining equation for the cross section for a particular process. Thus we have scattering cross sections, capture cross sections, and so on. A large number of nuclear cross sections are of the order of $10^{-24}$ cm$^2$, so that it has been convenient to use a separate name for this area. The name adopted is a *barn*; that is, 1 barn = $10^{-24}$ cm$^2$. Nuclear forces are short-range forces, so that the scattering cross section for fast neutrons might be expected to reflect the physical size of the nucleus, and, indeed, this cross section is one of the means for determining the nuclear radius as given in Equation (45-6).

46-5 Nuclear Fission

All the nuclear disintegrations described thus far have concerned the emission of comparatively light particles, such as electrons, positrons, protons, and alpha particles. A new type of process known as *nuclear fission* was discovered by Hahn and Strassman in 1939 in a series of experiments in which uranium was bombarded by neutrons. Chemical analysis of the products of disintegration showed the presence of barium, $Z = 56$, and lanthanum, $Z = 57$, and other elements of medium atomic weights. The interpretation of these results is that when a neutron is captured by a uranium nucleus, the compound nucleus formed becomes unstable and disintegrates into two particles of intermediate masses; for example, if one of the particles is barium, $Z = 56$, the other particle is krypton, $Z = 36$. Cloud-chamber photographs, as in Figure 46-4, have verified this hypothesis.

The particles which are produced in the fission of uranium have energies of the order of 200 Mev. The source of this energy is the difference in mass between the reacting particles, the neutron and the uranium nucleus, and the product or fission particles. There is a decrease in mass of about 0.1 per cent in this process; thus, in the nuclear fission of 1 kg of uranium, there is a decrease in mass of about 1 gm, and this corresponds to about $25 \times 10^6$ kw hr of energy.

The masses of the fission products are found to be those of unstable isotopes; that is, they have many more neutrons than the stable isotopes of the corresponding elements. One of the first questions investigated was the manner in which these unstable fission products disintegrated, particularly whether any of the excess neutrons were emitted in this process. Early experiments showed that between 2 and 3 neutrons were emitted per nuclear fission. The process can now be represented schematically, as shown in Figure 46-5: when a neutron is captured by a uranium nucleus of mass number 238, a new isotope of mass number 239 is formed; in the process of nuclear fission the latter splits into two isotopes of intermediate masses, say barium and krypton, with the prompt emission of 2 neutrons. A variety of other pairs of nuclei may be produced in the fission process,
Fig. 46-4  Cloud-chamber photograph showing the fission of uranium. The foil in the center of the cloud chamber is coated with uranium and bombarded by neutrons. The tracks of the two heavy fission particles can be seen coming from the foil where a uranium atom has undergone fission as a result of the capture of a neutron. (From a photograph by J. K. Boggild, K. K. Brostom, and T. Lauritsen.)

Fig. 46-5  Nuclear fission of uranium. A fast neutron is captured by a nucleus of uranium of mass number 238 forming uranium 239; the latter splits into two comparatively massive particles, in the above case krypton and barium, with the simultaneous emission of two fast neutrons.
all of them radioactive, most of them decaying to a stable isotope by the
emission of beta rays; gamma rays are also emitted by many of these
isotopes.

In addition to uranium, thorium, $Z = 90$, and protoactinium, $Z = 91$,
have been found to be fissionable by the capture of neutrons, and a new
element, plutonium, $Z = 94$, is also fissionable by the capture of neutrons.
Fission may also occur spontaneously, by excitation of a nucleus with
high-energy gamma rays, and by the bombardment of heavy nuclei with
protons, deuterons, or alpha particles. The fission cross section varies
with the energy and type of incident particles. In the following sections,
we shall discuss only neutron-induced nuclear fission.

46-6  A Nuclear Chain Reaction

The concept of a nuclear chain reaction is very simple: if a single nuclear
fission process involving the capture of one neutron results in the release
of energy and simultaneously the release of more than one neutron, it
should be possible to so arrange the mass of fissionable material to ensure
the capture of the newly released neutrons. Or, stated another way, the
mass of fissionable material should be so arranged that at any one place the
number of new neutrons produced should be equal to the number of free
neutrons originally present at that place. The ratio of these two numbers
of neutrons is called the multiplication factor $K$. If $K = 1$, the chain reac-
tion will be self-sustaining; if $K$ is less than 1, the process will ultimately
come to a halt; if $K$ is greater than 1, the neutron density will increase
and may lead to an explosive reaction. A mass of fissionable material so
arranged that the multiplication factor is equal to or greater than 1 con-
stitutes a nuclear reactor.

In order to be able to design a nuclear reactor, it is essential to know
the conditions under which neutrons are captured by nuclei and the con-
ditions under which such capture of neutrons results in the fission of the
product nuclei. We shall restrict this discussion to the fission of uranium.
Ordinary uranium consists of 3 isotopes: one of mass number 238, another
of mass number 235, and a third of mass number 234. The most abundant
of these is U238—about 99.3 per cent abundance. The amount of U234
in ordinary uranium is negligible. U235 constitutes about 0.7 per cent of
ordinary uranium. Experiments show that U238 is fissionable only if it
captures fast neutrons, that is, neutrons having energies of 1 Mev or greater.
On the other hand, U235 is fissionable with neutrons of any speed, and
the fission cross section is particularly high with slow neutrons, that is,
neutrons having energies corresponding to the thermal energies at ordinary
temperatures. These energies are much less than 1 ev.

The neutrons released in nuclear fission have a wide range of energies.
In the case of the fission of U235, these energies extend up to about 17 Mev, with a maximum number having energies of about 0.75 Mev. If such neutrons are captured by other uranium nuclei, they produce nuclear fission. However, not every collision between a fast neutron and a uranium nucleus results in capture of the neutron; the collision may simply produce a decrease in the energy of the neutron. Thereafter the probability of its capture will be very small; additional collisions will produce further reductions in the energy of the neutrons. At some particular values of energy, the neutron will be readily captured by U238, but such a capture does not result in nuclear fission. Instead, the newly formed isotope of uranium, U239, emits a gamma-ray photon and then becomes radioactive, emitting a beta ray with a half life of 23 min. The nuclear reaction equations are

\[ ^{92}\text{U}^{238} + _{0}\text{n}^{1} \rightarrow (^{92}\text{U}^{239}) \rightarrow ^{92}\text{U}^{239} + \text{gamma ray}, \]

then

\[ ^{92}\text{U}^{239} \rightarrow ^{93}\text{Np}^{239} + \beta^{-}. \quad T = 23 \text{ min.} \quad (46-14) \]

The new element thus formed, called neptunium, Np, is itself radioactive, emitting a beta particle with a half life of 2.3 days. The product nucleus formed in this reaction is plutonium, Pu, of atomic number 94. The reaction in which this is formed is

\[ ^{93}\text{Np}^{239} \rightarrow ^{94}\text{Pu}^{239} + \text{beta}. \quad T = 2.3 \text{ days.} \]

It is followed by

\[ ^{94}\text{Pu}^{239} \rightarrow ^{92}\text{U}^{235} + _{2}\text{He}^{4} \quad T = 24,000 \text{ years.} \quad (46-15) \]

The isotope of plutonium is radioactive, emitting an alpha particle, but it has a very long half life—24,000 years. In this sense, it is a comparatively stable element. It will be noted that neptunium and plutonium are transuranic elements, that is, elements with atomic numbers greater than that of uranium. When plutonium disintegrates with the emission of an alpha particle, the resulting nucleus is U235. The plutonium isotope formed in the above process is fissionable by the capture of neutrons of any energy and is thus similar to U235 as far as the fission process is concerned. Since it is chemically different from uranium, it can be separated more readily from the uranium metal than the uranium isotope of mass number 235.

If ordinary uranium is to be used in a nuclear reactor, it is essential to avoid loss of neutrons by nonfission capture. Since slow neutrons can produce fission in U235, and since the probability of capture varies inversely with the speed of the neutron, one method of ensuring its fissionable capture is to slow down the neutrons very rapidly to thermal energies. This is done with the aid of a moderator, that is, a light element in which the probability of nuclear capture of a neutron is negligible, but in which collisions between neutrons and nuclei will cause a rapid decrease of the energy of the
neutron. Deuterium and carbon are two elements suitable for use as moderators.

The first nuclear reactor, or *uranium pile* as it is sometimes called, was operated successfully in Chicago on December 2, 1942; it was built under the direction of E. Fermi and operated by groups headed by W. H. Zinn and H. L. Anderson. A schematic diagram of the construction of a graphite-moderated uranium pile is shown in Figure 46-6. Rods of uranium metal are embedded in blocks of graphite; rods of boron metal are inserted at various places in the pile to control the flux of neutrons; boron nuclei capture neutrons very readily. No special source of neutrons is needed to start this pile; there are always neutrons present from cosmic rays, or from spontaneous fission, to start the nuclear reactor. The mode of its operation can be understood by referring to Figure 46-7. Suppose that a neutron is captured by a uranium nucleus, so that fission results and that two new neutrons are released with energies of about 1 Mev each. These neutrons then make several collisions with nuclei of the moderator, graphite, until their energies are reduced to thermal energies. Whenever one of these slow neutrons is captured by U235, fission will again occur with the release of, say, 2 neutrons. Some neutrons may be lost through the surface of the reactor; one way to reduce this loss is to make the reactor very large; the increase in the surface area is proportional to the square
of its linear dimension, while the volume is proportional to the cube of the linear dimension. Other neutrons may be lost through capture by impurities or through nonfissionable capture by U238. But if \( K = 1 \), the reaction will be self-sustaining. To prevent the multiplication factor from becoming excessive, boron rods are inserted to various depths in the pile to absorb the excess neutrons. One other control factor may be mentioned here; that is, not all of the neutrons are emitted promptly in nuclear fission; a small percentage of the neutrons are delayed, some by 0.01 sec, others by as much as 1 min.

![Diagram of neutron action in uranium pile](image)

**Fig. 46-7** Schematic diagram of the action of a neutron in a uranium pile based on the assumption that each fission process yields two neutrons. The shaded circles represent rods of uranium; the small circular dots represent neutrons. Sudden changes in direction of the neutron path are due to collisions with nuclei of the moderator graphite.

A whole new field of nuclear science and engineering has been opened as the result of the discovery of nuclear fission and following the successful construction of the first nuclear reactor. Nuclear reactors designed for many different purposes are now in operation throughout the world. Some are used as sources of energy for power plants; others are used for experimental purposes. A nuclear reactor is one of the best sources of neutrons for use in physical, chemical, and biological experiments. It is also a source of radioactive isotopes for medical and industrial uses. A nuclear reactor may also be designed as a military weapon known as an atomic bomb or A-bomb. The latter is a type of nuclear reactor in which the multiplication factor \( K \) is greater than 1. It may consist of uranium containing a large percentage of U235 or of plutonium 239. If the mass of fissionable material is less than a certain critical amount, \( K \) will be less than 1, and there will be no chain reaction. If the mass is built up rapidly so that the total
exceeds the critical mass, a very fast chain reaction will be produced. One of the problems in exploding an atomic bomb is to hold the material together for a sufficient time, probably several millionths of a second, so that a large quantity of the material will take part in the fission process. It has been estimated that the energy released in an atomic bomb is sufficient to raise the temperature of this material to several million degrees and produce pressures upon explosion of perhaps a few million atmospheres. In addition, great quantities of radioactive materials and gamma rays are produced.

46-7 The New Particles of Physics

There were only two fundamental particles known in physics before 1932, the proton and the electron. It was then believed that all matter in the universe was composed of these two particles. The picture changed when the neutron was discovered in 1932, followed shortly by the discovery of the positron in 1934. Among the theories proposed to explain nuclear forces and nuclear phenomena was one put forward by H. Yukawa in 1935 in which it was assumed that a nuclear field of force, called a meson field, exists between nucleons. Furthermore, this field has particles, called mesons, associated with it in a manner analogous to the association of photons with an electromagnetic field. The Yukawa theory predicted that the mass of the mesons should be intermediate between the mass of an electron and that of a proton. This Yukawa particle or meson could have a positive charge, a negative charge, or zero charge, the charge being equivalent to that of one electron.

Many such particles have since been discovered; the first one, known as a mu meson (μ meson) or muon, was discovered in 1937 by S. H. Neddermyer and C. D. Anderson and independently by J. C. Street and E. C. Stevenson. All the others were discovered after 1947. Most of these new particles were discovered in the study of cosmic rays, a type of very energetic radiation which reaches the earth’s atmosphere from outside, penetrates the atmosphere, and, in so doing, reacts with atmospheric particles giving rise to many nuclear disintegrations. The cosmic rays as observed at various places in the atmosphere will usually consist of a combination of primary cosmic rays, which consist almost entirely of protons and other nuclei, plus the secondary radiations or particles produced by the interactions of the primaries with matter. With the development of high-energy particle accelerators, many of the new particles can be produced and studied under controlled conditions.

The particle which reacts most strongly with nuclei and is assumed to be the Yukawa particle is the pi meson (π meson) or pion. This was first discovered by Lattes, Occhialini, Powell, and D. H. Perkins in 1947 in
high-altitude cosmic-ray investigations using special photographic plates. In 1948, E. Gardner and M. Lattes, using the 184-in. Berkeley cyclotron, bombarded a carbon target with 380-Mev alpha particles and showed that pi mesons are emitted by the carbon nuclei as a result of this bombardment.

![Diagram showing the masses and charges of the so-called elementary particles; the mass of the electron is taken as unity.](image)

The mesons were bent in circular paths by the magnetic field of the cyclotron, the positively charged pions traveling in one direction, say clockwise, the negative ones counterclockwise. They were detected by special photographic plates placed at suitable positions along these paths. The existence of a neutral pi meson ($\pi^0$ meson) was verified in other experiments.
The pi and mu mesons are only two of the types of particles discovered. Figure 46-8 shows the other new particles known at present (1958). The mass of each particle is shown on the vertical scale for charge \(+e\), 0, and \(-e\). The unit of mass is the mass of one electron. The particle most recently discovered (1955) is the negative proton, that is, a particle with a negative charge and a mass equal to that of the proton. All of these particles, with the exception of the electron, proton, and deuteron, are unstable; that is, they either disintegrate when in the free state or combine with other particles. For example, the neutron is radioactive with a half life of about 12 min, decaying into a proton, electron, and neutrino. The positron combines with an electron and the energy of the two, which is essentially
the rest mass energy $2m_0c^2$, is converted into the energy of one or more, usually two, gamma-ray photons. The charged pi meson disintegrates into a charged mu meson and a neutrino in about $10^{-8}$ sec, while the mu meson disintegrates into an electron and two neutrinos with an average lifetime of about 2 µsec.

One of the newest devices for studying high-energy reactions is the bubble chamber first developed by Donald A. Glaser in 1953. (See Section 17-7.) Figure 46-9 shows two photographs of tracks made by some of the elementary charged particles in their passage through the liquid of the chamber. Figure 46-9(a) is reproduced to show that each of these tracks actually consists of a series of successive bubbles. The heavy tracks on the right are those made by protons whose paths end in the chamber. An interesting event, the disintegration of a heavy meson, a $K^+$-meson into a $\mu^+$-meson, is shown in this photograph, and two such events are shown in Figure 46-9(b); the latter are more suitable for measurements. In order to conserve linear momentum in this process, another particle must have been emitted simultaneously with the $\mu$-meson. This second particle must be a neutral particle since it leaves no track in the chamber; it may be a neutrino. Because it disintegrates into two particles, a $\mu$-meson and a neutral particle, this $K$-meson is designated as a $K_{\mu2}$ particle.

The discovery of these new particles, sometimes called elementary particles, has opened up a whole new field of physics, coming to be known as particle physics. It is hoped that a study of particle physics will shed new light on nuclear forces and nuclear process.

**Problems**

46-1. A thin rod of length $L_0$, when measured by an observer at rest with respect to it, has a velocity of $v = 3c/4$ with respect to a second observer. The direction of its velocity is parallel to its length. Determine the length $L$ measured by the second observer.

46-2. A small particle is in the form of a sphere of radius $R_0$ when at rest. Determine its shape as seen by an observer if this particle is moving in the $x$ direction with a velocity $c/2$ with respect to this observer.

46-3. Two charged particles are emitted by a substance in opposite directions, each moving with a velocity $v = 0.9c$ with respect to the emitting substance. Determine the velocity of one particle relative to the other.

46-4. The $K$ conversion electron from $\text{Cs}^{137}$ produces a sharp line in its beta-ray spectrum for which $Br = 3.381$ gausses cm. The binding energy of the $K$ electron is 37.44 kev. (a) Determine the velocity of the electron. (b) Determine the kinetic energy of the electron. (c) Determine the energy of this gamma ray emitted in a transition between the same two levels.

46-5. A gamma-ray photon from $\text{Cs}^{137}$, when incident upon a piece of uranium, ejects photoelectrons from its $K$ shell. These photoelectrons follow
a circular path in a beta-ray spectrometer for which $Br = 3,083$ gausses cm. The binding energy of a $K$ electron in uranium is $115.59$ kev. Determine (a) the velocity of the photoelectrons, (b) the kinetic energy of the photoelectrons, and (c) the energy of the gamma-ray photons.

46-6. Verify the validity of the Lorentz transformation equations.

46-7. By expanding $\gamma$ according to the binomial expansion, show that the relativistic expression for the kinetic energy of a moving particle reduces to the classical expression in the limit of low velocities.

46-8. Let us assume that a cyclotron will operate with a fixed-frequency oscillator as long as $\gamma$ is no more than 1.01. (a) What is the maximum energy to which electrons can be accelerated in a cyclotron? (b) Protons? (c) Alpha particles?

46-9. The cyclotron of the Nobel Institute of Physics has a pole face of 88.5 in. diameter, a maximum flux density of 18,000 gausses, and an oscillator frequency of $8.7 \times 10^6$ cycles/sec. Find the maximum energy to which (a) protons, (b) deuterons, and (c) helium nuclei can be accelerated with this cyclotron. Neglect relativistic effects.

46-10. The Oak Ridge reactor has a flux of thermal neutrons of $10^{12}$ neutrons/cm$^2$ sec. The activation cross section of Te$^{126}$ is 90 millibarns, in a process in which this isotope absorbs a neutron to become radioactive Te$^{127}$. If 1 gm of Te$^{126}$ is placed in the Oak Ridge reactor, how many grams of Te$^{127}$ are manufactured in each second?

46-11. (a) Write the nuclear reaction equation for the fission process illustrated in Figure 46-5. (b) Determine the difference in mass between the initial components of the reaction and its final products. (c) Express the result of part (b) in million electron volts. (d) How much energy is released in this reaction? Assume the following atomic masses: Ba = 142.955; Kr = 93.935.