

2011

Reformulating the Dual Graphs of CSPs to Improve the Performance of RNIC

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Woodward, Robert J.; Karakashian, Shant; Choueiry, Berthe Y.; and Bessiere, Christian, "Reformulating the Dual Graphs of CSPs to Improve the Performance of RNIC" (2011). *CSE Conference and Workshop Papers*. 185.

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Reformulating the Dual Graphs of CSPs to Improve the Performance of RNIC

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Acknowledgements

- Elizabeth Claassen and David B. Marx of the Department of Statistics @ UNL
- Experiments conducted at UNL's Holland Computing Center
- Robert Woodward supported by a B.M. Goldwater Scholarship and NSF Graduate Research Fellowship
- NSF Grant No. RI-111795

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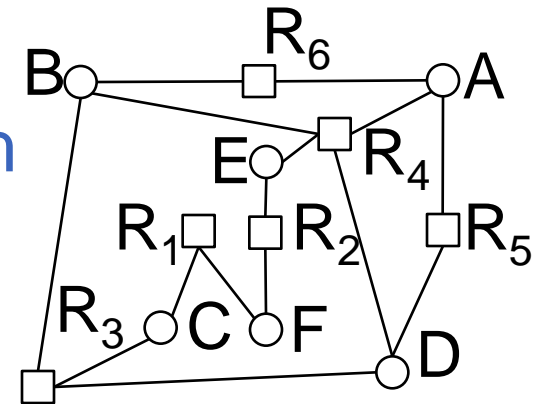
Outline

- Introduction
- Relational Neighborhood Inverse Consistency
 - Property & algorithm
- Reformulating the Dual Graph by
 1. Removing redundant edges, yields property wRNIC
 2. Triangulation, yields property triRNIC
- Selection strategy: four alternative dual graphs
- Experimental Results
- Conclusion

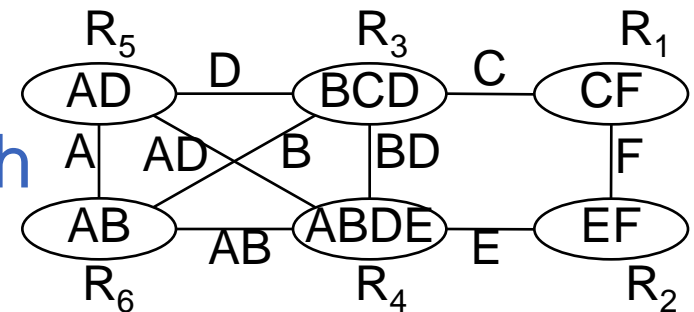
Constraint Satisfaction Problem

- CSP
 - Variables, Domains
 - Constraints: binary / non-binary
- Representation
 - Hypergraph
 - Dual graph
- Solved with
 - Search
 - Enforcing consistency
- Warning
 - Consistency properties vs. algorithms

Hypergraph

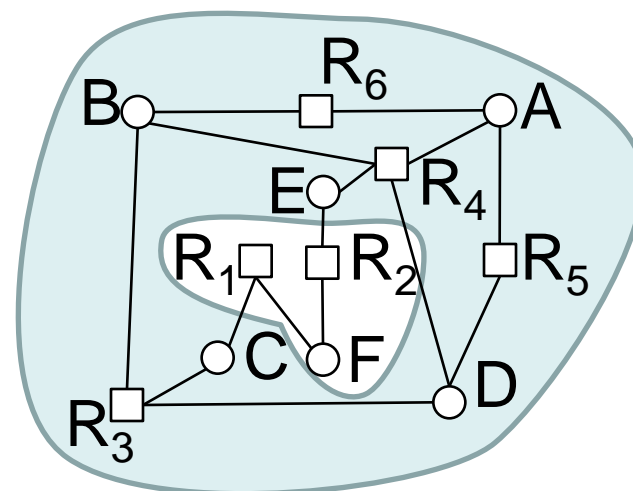
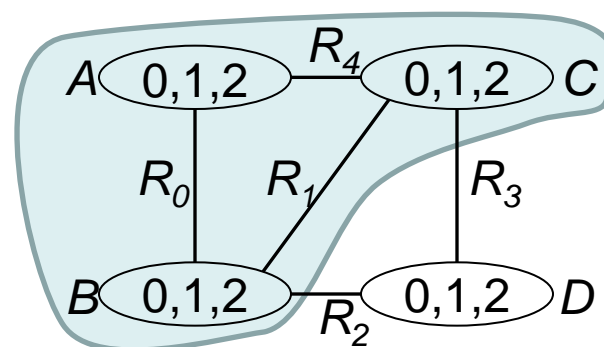


Dual graph



Neighborhood Inverse Consistency [Freuder+ 96]

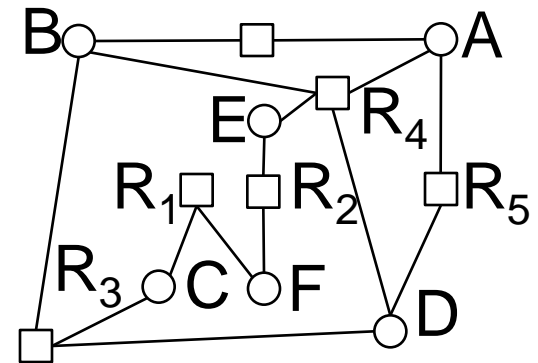
- Property
 - Defined for binary CSPs
 - Every **value** can be extended to a solution in its variable's neighborhood
- Algorithm
 - + No space overhead
 - + Adapts to the connectivity
 - Not effective on sparse problems
 - Too costly on dense problems
- Non-binary CSPs?
 - Neighborhoods likely too large



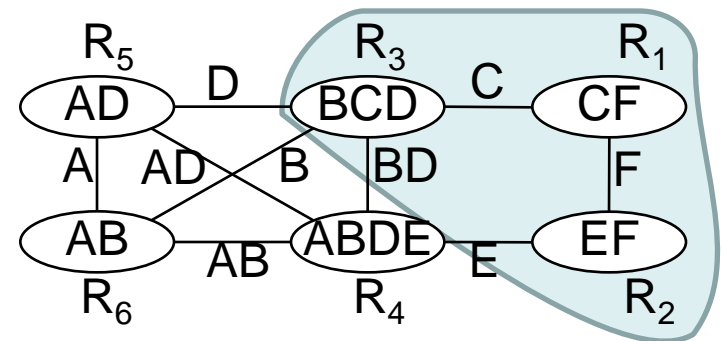
Relational NIC

[Woodward+ AAAI11]

- Property
 - Defined for dual graph
 - Every tuple can be extended to a solution in its relation's neighborhood
- Algorithm
 - Operates on dual graph
 - ... filter relations (not domains!)
- Domain filtering
 - Property: RNIC+DF
 - Algorithm: Projection



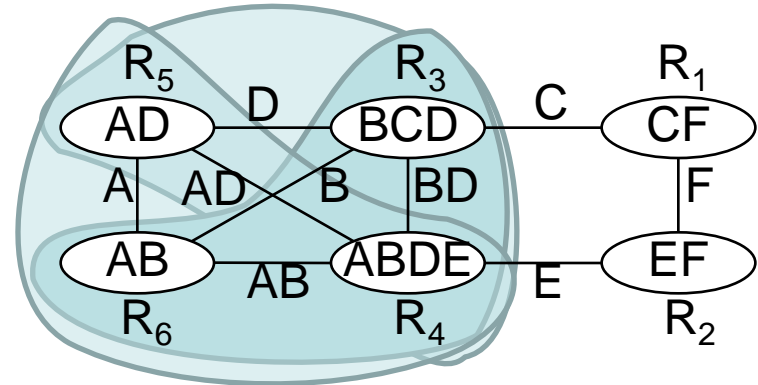
Hypergraph



Dual graph

Reformulation: Removing Redundant Edges

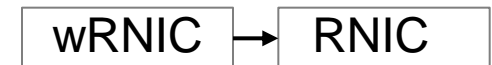
- High density
 - Large neighborhoods
 - Higher cost of RNIC
- Minimal dual graph
 - Equivalent CSP
 - Computed efficiently [Janssen+ 89]



$$d^{Go} = 60\%$$

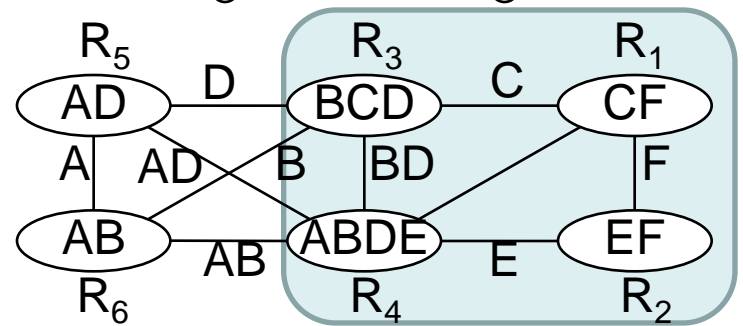
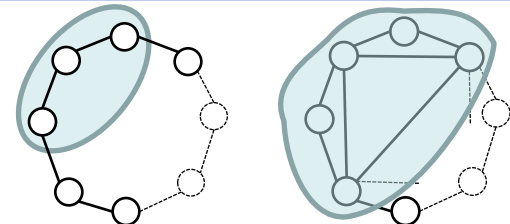
$$d^{Gw} = 40\%$$

- Run algorithm on a minimal dual graph
 - + Smaller neighborhoods, solution set not affected
 - wRNIC: a strictly weaker property



Reformulation: Triangulation

- Cycles of length ≥ 4
 - Hampers propagation
- Triangulating dual graph
 - Equivalent CSP
 - Min-fill heuristic
- Run algorithm on a triangulated dual graph
 - + Created loops enhance propagation
 - triRNIC: a strictly stronger property



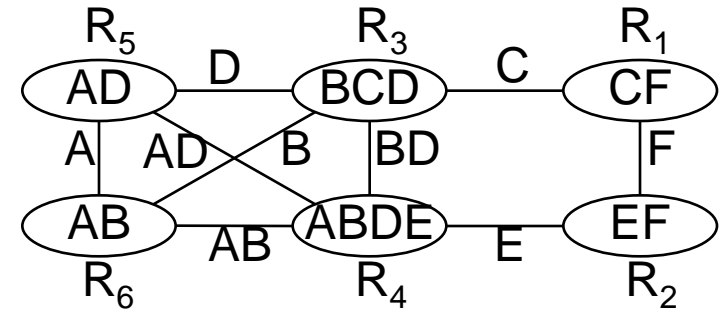
$d^{Go} = 60\%$

$d^{Gtri} = 67\%$

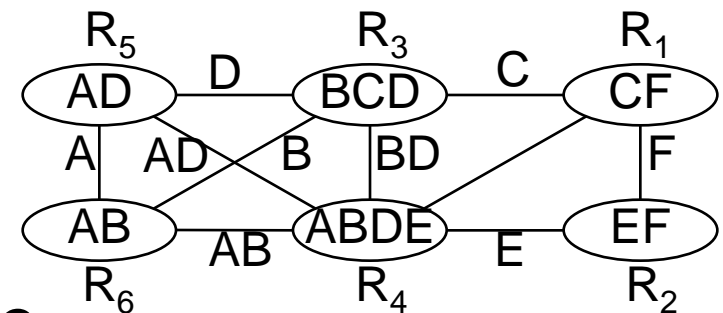


Reformulation: RR & Triangulation

- Fixing the dual graph
 - RR copes with high density
 - Triangulation boosts propagation
- RR+Tri
 - Both operate locally
 - Are complementary, do not ‘clash’
- Run algorithm on a RR+tri dual graph
 - CSP solution set is not affected
 - wtriRNIC is not comparable with RNIC



$d^{Go} = 60\%$

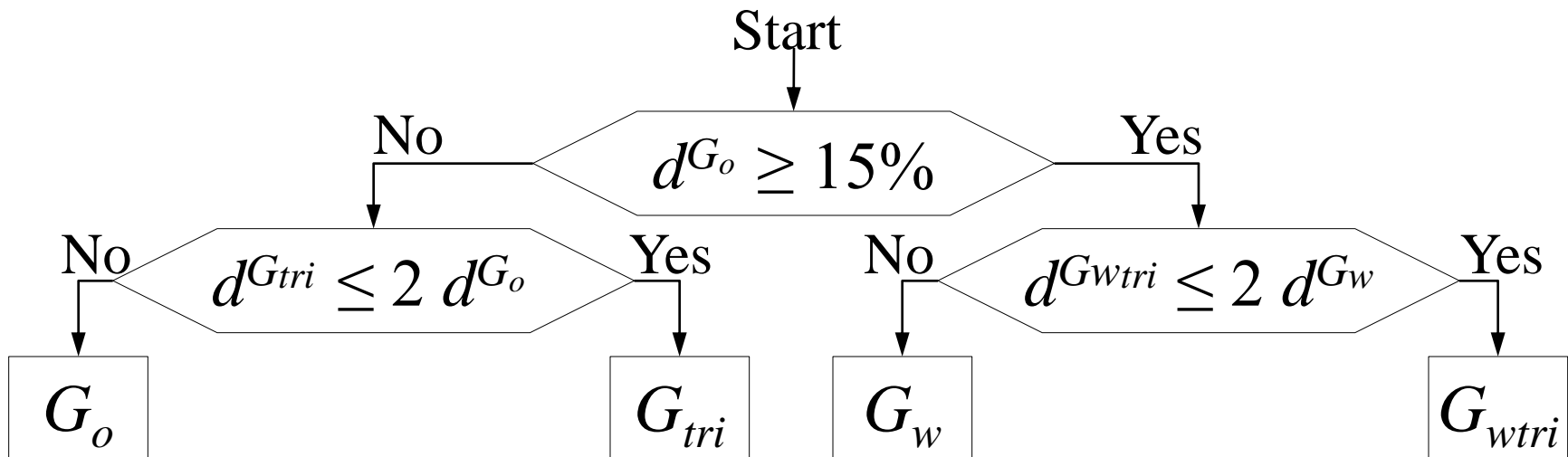


$d^{Gwtri} = 47\%$



Selection Strategy: Which? When?

- Density $\geq 15\%$ is too dense
 - Remove redundant edges
- Triangulation increases density no more than two fold
 - Reformulate by triangulation
- Each reformulation executed at most once



Experimental Results

- Statistical analysis on CP benchmarks
- Time: Censored data calculated mean
- R: Censored data rank based on probability of survival data analysis
- S: Equivalence classes based on CPU
- S_B : Equivalence classes based on completion
- #C: Number of instances completed
- #F: Number of instances fastest
- #BF: # instances solved backtrack free

Algorithm	Time	#F	R	S	#C	S_B	#BF
169 instances: aim-100,aim-200,lexVg,modifiedRenault,ssa							
wR(*,2)C	944924	52	3	A	138	B	79
wR(*,3)C	925004	8	4	B	134	B	92
wR(*,4)C	1161261	2	5	B	132	B	108
GAC	1711511	83	7	C	119	C	33
RNIC	6161391	19	8	C	100	C	66
triRNIC	3017169	9	9	C	84	C	80
wRNIC	1184844	8	6	B	131	B	84
wtriRNIC	937904	3	2	B	144	B	129
seIRNIC	751586	17	1	A	159	A	142

Conclusions

- Contributions
 - Algorithm
 - Polynomial in degree of dual graph
 - BT-free search: hints to problem tractability
 - Various reformulations of the dual graph
 - Adaptive, unifying, self-regulatory, automatic strategy
 - Empirical evidence, supported by statistics
- Future work
 - Extend to constraints given as conflicts, in intension
 - Extend to singleton type consistencies