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Application of generalized wall function for complex turbulent flows*

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Abstract. This paper describes a generalized wall function for three-dimensional turbulent boundary layer flows. Since the formulation is valid for various pressure gradients including those associated with zero skin friction, it can be applied to wall bounded complex flows with acceleration, deceleration and recirculation. This generalized wall function is extended to the whole surface layer (or inner layer), covering the viscous sublayer, buffer layer and inertial sublayer; therefore, it is a unified wall function. This 'unified' feature is particularly useful for computational fluid dynamics (CFD) to deal with flows with complex geometries, because it allows a flexible grid resolution near the wall to provide accurate wall boundary conditions. This paper also describes a systematic procedure for implementing the wall function in a general CFD code. Finally, a few examples of complex turbulent flows are presented to show the performance of the generalized wall function.

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Contents

1	Introduction	2
2	Generalized wall function	3
2.1	Inertial sublayer	3
2.2	Viscous sublayer	4
2.3	Unified wall function	4
3	Comparison with experiments	5
4	Procedure of wall function implementation	6
4.1	Wall stresses	7
4.2	Determination of $(\partial U/\partial \eta, \partial V/\partial \eta, \partial W/\partial \eta)_w$	8
4.3	Tangential mean velocity and tangential wall pressure gradient	9
4.4	Calculation of $(\partial \eta/\partial x, \partial \eta/\partial y, \partial \eta/\partial z)_w$	9
5	Performance in CFD applications	9
5.1	Flat-plate turbulent boundary layer	10
5.2	Backward-facing step flow	11
5.3	Shedding cylinder flow	12
5.4	Three-dimensional reacting flow	14
6	Conclusion	15

1. Introduction

A standard wall function for the inertial sublayer can be written as

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{u_\tau y}{\nu} \right) + C \quad (1)$$

where U is the wall-tangential mean velocity, u_τ is the skin friction velocity defined by the wall stress τ_w as $u_\tau = \sqrt{|\tau_w|/\rho}$, y is the normal distance from the wall and ν and ρ are the kinematic viscosity and density of the fluid. $\kappa \approx 0.41$ and $C \approx 5.0$. For boundary layer flows, equation (1) is valid only for a zero-pressure-gradient boundary layer, but it has been applied to other wall bounded flows with some success despite its formal validity. For a boundary layer with a large adverse pressure gradient and zero wall stress, Tennekes and Lumley [1] derived another asymptotic solution:

$$\frac{U}{u_p} = \alpha \ln \left(\frac{u_p y}{\nu} \right) + \beta \quad (2)$$

where u_p is defined by the adverse wall pressure gradient as $u_p = [(\nu/\rho) dP_w/dx]^{1/3}$, and the constants $\alpha \approx 5$ and $\beta \approx 8$ were determined according to the experimental data of Stratford [2]. Equation (2) has not received much attention in CFD. Apparently, equation (1) will become erroneous for flows near separation or re-attachment points, since the wall stress, and hence the skin friction velocity, will be nearly zero. On the other hand, equation (2) will not be valid for boundary layer flows with a small or zero pressure gradient because u_p will be nearly zero. Shih *et al* [3] followed and extended the method used by Tennekes and Lumley in analysing the effect of pressure gradients on the mean flow near the wall and obtained a generalized wall function for

Figure 1. Effects of favourable and adverse pressure gradients on the flow in the inertial sublayer.

a three-dimensional turbulent boundary layer. This new wall function is valid for both zero-wall-stress and zero-wall-pressure-gradient turbulent boundary layers. In fact, the new wall function will exactly become equation (1) for flows with zero pressure gradient and equation (2) for flows with zero wall stress.

The basic idea in the analysis of a three-dimensional turbulent boundary layer is to assume that at large Reynolds numbers there exists a so-called surface layer (or inner layer) distinct from the outer layer. The existence of the law of the wall in the surface layer and the existence of the velocity-defect law in the outer layer will lead to an asymptotic solution in an overlapping region where

$$\frac{y}{\delta} \ll 1, \quad \frac{y}{\ell_\nu} \gg 1. \quad (3)$$

In equation (3), δ is the thickness of the boundary layer; ℓ_ν is the length scale related to the viscous effect and is defined as ν/u_c . u_c is the velocity scale of the turbulent boundary layer, which will be defined later. The region in which equation (3) holds is called the inertial sublayer since the viscous stress in this region is negligibly small compared to the turbulent stresses. In the vicinity of the wall where y/ℓ_ν is of order one, the turbulent stress is significantly suppressed by the viscosity. This region is referred to as the viscous sublayer. An asymptotic solution can be obtained for both inertial and viscous sublayers since there is no turbulence closure problem in these two sublayers. The region between these two sublayers is named the buffer layer, where the turbulent and viscous stresses are of the same order. Therefore, no analytical solution can be obtained in the buffer layer due to the turbulence closure problem. However, following Spalding [4] by introducing a turbulent stress model in the buffer layer, an analytical expression which matches both asymptotic solutions in the viscous and inertial sublayers can be obtained, which makes the wall function a unified expression throughout the whole surface layer.

2. Generalized wall function

2.1. Inertial sublayer

Let u_τ and u_p be the velocity scales defined by the magnitude of wall stress and the magnitude of tangential wall pressure gradient, respectively

$$u_\tau = \sqrt{|\tau_{wi}|/\rho}, \quad u_p = \left(\frac{\nu}{\rho} \left| \frac{dP_w}{dx_i} \right| \right)^{1/3}, \quad (4)$$

and u_c be a hybrid velocity scale:

$$u_c = u_\tau + u_p. \quad (5)$$

Note that u_c is always a non-zero and positive quantity. In the inertial sublayer, the generalized wall function gives the following relationship between the tangential velocity U_{it} , wall shear stress τ_{wi} and tangential wall pressure gradient $\partial P_w/\partial x_i$:

$$\frac{U_{it}}{u_c} = \frac{\tau_{wi}}{\rho u_\tau^2} \left[\frac{1}{\kappa} \frac{u_\tau}{u_c} \ln \left(\frac{u_c y}{\nu} \right) + C_1 \right] + \frac{\nu}{\rho} \frac{\partial P_w / \partial x_i}{u_c^3} \left[\alpha \ln \left(\frac{u_c y}{\nu} \right) + \beta \right], \quad (6)$$

where $\alpha = 5.0$, $\beta = 8.0$ and the coefficient

$$C_1 = \frac{u_\tau}{u_c} \left[\frac{1}{\kappa} \ln \left(\frac{u_\tau}{u_c} \right) + C \right], \quad (7)$$

where $\kappa = 0.41$ and $C = 5.0$. Figure 1 shows the effects of favourable and adverse pressure gradients on the inertial sublayer. Apparently, the effects become significant as the pressure gradients increase.

2.2. Viscous sublayer

In the viscous sublayer, the turbulent stresses are negligible and the generalized wall function becomes

$$\frac{U_{it}}{u_c} = \frac{\tau_{wi}}{\rho u_c^2} \left(\frac{u_c y}{\nu} \right) + \frac{1}{2} \frac{\nu}{\rho} \frac{\partial P_w / \partial x_i}{u_c^3} \left(\frac{u_c y}{\nu} \right)^2. \quad (8)$$

Figure 2 shows that the effects of pressure gradients on the flow in the viscous sublayer are also significant.

2.3. Unified wall function

The inertial sublayer and the viscous sublayer are blended by the buffer layer. Following the approach originated by Spalding [4], a new turbulent stress model is introduced for the buffer layer. Together with equations (6) and (8), this buffer layer model leads to a unified expression of the generalized wall function [3], which covers three regions of the surface layer: viscous sublayer, buffer layer and inertial sublayer. However, the original analytical formula is not very convenient for CFD applications, hence a curve-fitting formula is introduced

$$\frac{U_{it}}{u_c} = \frac{\tau_{wi}}{\rho u_\tau^2} \frac{u_\tau}{u_c} f_1(Y_\tau^+) + \frac{\nu}{\rho} \frac{\partial p_w / \partial x_i}{u_c^3} f_2(Y_c^+) \quad (9)$$

where $Y_\tau^+ = u_\tau y / \nu$, $Y_c^+ = u_c y / \nu$. f_1 and f_2 are the piecemeal fitting functions defined as

$$f_1(Y_\tau^+) = \begin{cases} a_1 Y_\tau^+ + a_2 (Y_\tau^+)^2 + a_3 (Y_\tau^+)^3 & \text{if } Y_\tau^+ \leq 5; \\ b_0 + b_1 Y_\tau^+ + b_2 (Y_\tau^+)^2 + b_3 (Y_\tau^+)^3 + b_4 (Y_\tau^+)^4 & \text{if } 5 \leq Y_\tau^+ \leq 30; \\ c_0 + c_1 Y_\tau^+ + c_2 (Y_\tau^+)^2 + c_3 (Y_\tau^+)^3 + c_4 (Y_\tau^+)^4 & \text{if } 30 \leq Y_\tau^+ \leq 140; \\ \frac{1}{\kappa} \log(Y_\tau^+) + C & \text{if } 140 < Y_\tau^+. \end{cases} \quad (10)$$

$$f_2(Y_c^+) = \begin{cases} a_2 (Y_c^+)^2 + a_3 (Y_c^+)^3 & \text{if } Y_c^+ \leq 4; \\ b_0 + b_1 Y_c^+ + b_2 (Y_c^+)^2 + b_3 (Y_c^+)^3 + b_4 (Y_c^+)^4 & \text{if } 4 \leq Y_c^+ \leq 15; \\ c_0 + c_1 Y_c^+ + c_2 (Y_c^+)^2 + c_3 (Y_c^+)^3 + c_4 (Y_c^+)^4 & \text{if } 15 \leq Y_c^+ \leq 30; \\ \alpha \log(Y_c^+) + \beta & \text{if } 30 < Y_c^+. \end{cases} \quad (11)$$

Figure 2. Effects of favourable and adverse pressure gradients on the flow in the viscous sublayer.

Figure 3. Effects of favourable and adverse pressure gradients on the whole surface layer.

Table 1. The coefficients in f_1 .

a_1	a_2	a_3		
1.0	1.0E-02	-2.9E-03		
b_0	b_1	b_2	b_3	b_4
-0.872	1.465	-7.02E-02	1.66E-03	-1.495E-05
c_0	c_1	c_2	c_3	c_4
8.6	0.1864	-2.006E-03	1.144E-05	-2.551E-08

The coefficients in f_1 and f_2 are listed in tables 1 and 2.

The unified wall function equation (9) is illustrated in figure 3 for various favourable and adverse pressure gradients.

3. Comparison with experiments

As a validation, we directly compare the generalized wall function with the experimental data for turbulent boundary layers under various adverse and favourable pressure gradients. A set of classical experimental data of turbulent boundary layers with various pressure gradients were collected in the proceedings of the *AFOSS-IFP-Stanford Conference* [5]. We choose some of them for direct comparison with the generalized wall function. In order to make a direct

Table 2. The coefficients in f_2 .

a_2	a_3			
0.5	$-7.31\text{E}-03$			
b_0	b_1	b_2	b_3	b_4
-15.138	8.4688	$-0.819\,76$	$3.7292\text{E}-02$	$-6.3866\text{E}-04$
c_0	c_1	c_2	c_3	c_4
11.925	$0.934\,00$	$-2.7805\text{E}-02$	$4.6262\text{E}-04$	$-3.1442\text{E}-06$

Figure 4. Comparison with experimental data of Ludwig and Tillmann [5].

comparison, all the experimental data are rescaled using u_c instead of u_τ . Two sets of mild and strong adverse pressure gradient data by Ludwig and Tillmann are shown in figure 4. The generalized wall function agrees very well with the experimental mean velocity profiles in the surface layer. The data deviate from the generalized wall function only in the core region.

Two sets of mild and strong favourable pressure gradient data of Herring and Norbury are shown in figure 5. The generalized wall function matches the data reasonably well, but the data with the strong favourable pressure gradient deviate somewhat from the generalized wall function in the lower part of the inertial sublayer. We note that a comment made by the editor of the proceedings for this particular flow indicates that this flow was not in equilibrium. Finally, two sets of severe adverse pressure gradient (zero-skin-friction data of Stratford) are shown in figure 6. It can be seen that the generalized wall function fits the experimental data very well in the whole surface layer including the buffer layer.

4. Procedure of wall function implementation

All CFD codes need the values of wall stresses or their equivalents, for example, the mean velocity gradient at the wall. Numerical calculations of these values typically need fine grids near the

Figure 5. Comparison with experimental data of Herring and Norbury [5].

wall to capture the rapid variation of the mean flow near the wall. The wall function allows us to provide the wall stresses or their equivalents without numerical calculations of mean velocity derivatives; consequently, we do not need very fine grids near the wall. In this section, the details of a systematic procedure for obtaining the wall stresses via the generalized wall function will be illustrated. A finite volume approach is assumed and all variables are in Cartesian coordinates. The momentum equation can be written as

$$\frac{\partial \rho U_i}{\partial t} + \frac{\partial \rho U_i U_j}{\partial x_j} = \frac{\partial}{\partial x_j} \tau_{ij} - \frac{\partial P}{\partial x_i}. \quad (12)$$

The integration of $\frac{\partial}{\partial x_j} \tau_{ij}$ in a volume element gives

$$\sum_{k=1}^n \tau_{ix,k} \frac{A_{x,k}}{\Delta v} + \sum_{k=1}^n \tau_{iy,k} \frac{A_{y,k}}{\Delta v} + \sum_{k=1}^n \tau_{iz,k} \frac{A_{z,k}}{\Delta v} \quad (13)$$

where $A_{x,k}$, $A_{y,k}$ and $A_{z,k}$ are the three components of the k th face of the element. Δv is the volume of the element. For the element sitting on a solid wall, the values of nine stress components τ_{ij} at the wall (only six of them are independent) are required by CFD. These wall stress components can be obtained using the wall function.

4.1. Wall stresses

Let us denote $x_i = (x, y, z)$ as the Cartesian coordinates, and $\xi_i = (\xi, \eta, \zeta)$ as the curvilinear coordinates. By the definition of τ_{ij} and the chain rule, we may write

$$\tau_{ij} = (\mu + \mu_T) \left(\frac{\partial \xi_n}{\partial x_j} \frac{\partial U_i}{\partial \xi_n} + \frac{\partial \xi_n}{\partial x_i} \frac{\partial U_j}{\partial \xi_n} \right) + \dots \quad (14)$$

Figure 6. Comparison with experimental data of Stratford [5].

where $U_i = (U, V, W)$ are the velocity components in the Cartesian coordinates. Now, consider a wall surface defined as the curvilinear coordinate surface $\eta = \text{constant}$, and let its normal direction lie in the η -direction. At the wall, the derivatives of the mean velocity with respect to the surface coordinates ξ and ζ are all zero due to the no-slip condition. Therefore, the six independent τ_{ij} at the wall can be written as

$$\begin{aligned} \tau_{xx} &= 2\mu \left(\frac{\partial \eta}{\partial x} \frac{\partial U}{\partial \eta} \right)_w, & \tau_{xy} &= \mu \left(\frac{\partial \eta}{\partial y} \frac{\partial U}{\partial \eta} + \frac{\partial \eta}{\partial x} \frac{\partial V}{\partial \eta} \right)_w, & \tau_{xz} &= \mu \left(\frac{\partial \eta}{\partial z} \frac{\partial U}{\partial \eta} + \frac{\partial \eta}{\partial x} \frac{\partial W}{\partial \eta} \right)_w, \\ \tau_{yy} &= 2\mu \left(\frac{\partial \eta}{\partial y} \frac{\partial V}{\partial \eta} \right)_w, & \tau_{yz} &= \mu \left(\frac{\partial \eta}{\partial z} \frac{\partial V}{\partial \eta} + \frac{\partial \eta}{\partial y} \frac{\partial W}{\partial \eta} \right)_w, & \tau_{zz} &= 2\mu \left(\frac{\partial \eta}{\partial z} \frac{\partial W}{\partial \eta} \right)_w. \end{aligned} \quad (15)$$

Equation (15) indicates that τ_{ij} at the wall are determined by three derivatives, $\partial U/\partial \eta$, $\partial V/\partial \eta$ and $\partial W/\partial \eta$, at the wall. We will show that these three derivatives can be provided by the wall function.

4.2. Determination of $(\partial U/\partial \eta, \partial V/\partial \eta, \partial W/\partial \eta)_w$

The generalized wall function, equation (9), gives the relationship between the wall stress τ_{wi} and the tangential velocity U_{it} in the surface layer, and it can be rewritten as

$$\tau_{wi} = \left\{ \frac{U_{it}}{u_c} - \frac{\nu}{\rho} \frac{(\partial P_w / \partial x_i)}{u_c^3} f_2 \right\} \frac{\rho u_\tau^2}{u_c} f_1. \quad (16)$$

On the other hand, we may write the wall shear stress as

$$\tau_{wi} = \mu \left(\frac{\partial U}{\partial \eta}, \frac{\partial V}{\partial \eta}, \frac{\partial W}{\partial \eta} \right)_w. \quad (17)$$

Therefore, equations (16) and (17) will provide us with the normal derivative of the mean velocity at the wall:

$$\left(\frac{\partial U}{\partial \eta}, \frac{\partial V}{\partial \eta}, \frac{\partial W}{\partial \eta}\right)_w = \left\{ \frac{U_t}{u_c} - \frac{\nu}{\rho} \frac{(\frac{\partial P_w}{\partial x})}{u_c^3} f_2, \frac{V_t}{u_c} - \frac{\nu}{\rho} \frac{(\frac{\partial P_w}{\partial y})}{u_c^3} f_2, \frac{W_t}{u_c} - \frac{\nu}{\rho} \frac{(\frac{\partial P_w}{\partial z})}{u_c^3} f_2 \right\} \frac{\rho u_\tau^2}{\mu \frac{u_\tau}{u_c} f_1} \quad (18)$$

where U_t , V_t and W_t are the three Cartesian components of the tangential velocity in the surface layer, and $(\partial P_w/\partial x)$, $(\partial P_w/\partial y)$ and $(\partial P_w/\partial z)$ are the three Cartesian components of the tangential wall pressure gradient. For CFD calculations with standard wall function, the first grid point from the wall is usually required to be located in the inertial sublayer. However, if the first grid point happens to be close to the wall such that $u_c y/\nu$ is of order one, the above relation is still valid since the generalized wall function is valid for the whole surface layer. Therefore, the generalized wall function offers a great convenience regarding the grid generation since it allows a much more flexible grid resolution near the wall to produce accurate boundary conditions for CFD.

4.3. Tangential mean velocity and tangential wall pressure gradient

Let n be the normal distance of the first grid point from the wall and δx_i be the associated normal vector. Furthermore, let (U, V, W) be the velocity at the first grid point. Then, we may calculate the three components of the tangential mean velocity as

$$U_t = U - (U_j \delta x_j) \frac{\delta x}{n^2}, \quad V_t = V - (U_j \delta x_j) \frac{\delta y}{n^2}, \quad W_t = W - (U_j \delta x_j) \frac{\delta z}{n^2}. \quad (19)$$

The tangential wall pressure gradient can be approximated by the values at the first grid point from the wall, say, the point A:

$$\left(\frac{\partial P_w}{\partial x}\right) = \left(\frac{\partial P}{\partial x}\right)_A, \quad \left(\frac{\partial P_w}{\partial y}\right) = \left(\frac{\partial P}{\partial y}\right)_A, \quad \left(\frac{\partial P_w}{\partial z}\right) = \left(\frac{\partial P}{\partial z}\right)_A. \quad (20)$$

This approximation is reasonable since the normal variation of the pressure near the wall is negligible.

4.4. Calculation of $(\partial \eta/\partial x, \partial \eta/\partial y, \partial \eta/\partial z)_w$

Let (x, y, z) be the location of the first grid point from the wall, (xf, yf, zf) be the location of the wall surface element and (S_x, S_y, S_z) be the three components of the surface element vector with the area of S ($S_x^2 + S_y^2 + S_z^2 = S^2$). Then the normal direction vector of the surface element will be $(S_x/S, S_y/S, S_z/S)$. The normal distance n (the first grid point away from the wall) can be calculated as

$$n = (x - xf) \frac{S_x}{S} + (y - yf) \frac{S_y}{S} + (z - zf) \frac{S_z}{S}. \quad (21)$$

From equation (21), we obtain

$$(\partial \eta/\partial x)_w = S_x/S, \quad (\partial \eta/\partial y)_w = S_y/S, \quad (\partial \eta/\partial z)_w = S_z/S. \quad (22)$$

Now, with all the above relations, we will be able to calculate the six τ_{ij} at the wall using equation (15).

5. Performance in CFD applications

In this section, we will show the effectiveness of using the wall function in CFD and present the numerical results of a few complex turbulent flows. The results presented in section 5.1 were obtained with a parabolic boundary-layer code. The calculations of complex flows presented in sections 5.2–5.4 were performed using the NCC code which is an unstructured CFD code for calculations of steady or time-accurate three-dimensional reacting flows.

Figure 7. Skin friction of boundary layer with zero pressure gradient.

Figure 8. Mean velocity of boundary layer with zero pressure gradient.

5.1. Flat-plate turbulent boundary layer

By using a flat-plate turbulent boundary layer with a $k-\varepsilon$ model [6], we demonstrate how effective a numerical calculation can be when the wall function is applied. It shows that the skin friction and mean velocity profile can be captured with a coarse grid resolution. In one extreme case, it uses only five grid points across the whole turbulent boundary layer. All calculations start with uniform profiles of mean velocity, turbulent kinetic energy and its dissipation rate at the inlet of the computation domain. The calculated boundary layer development depends on the number of grid points used in the calculations. Numerically, faster development usually associates with more grid points. Our calculations with various grid resolutions across the boundary layer, from five grid points to 100 grid points, all approach the fully developed turbulent boundary layer. Figure 7 shows the skin friction versus the Reynolds number based on the momentum thickness. The reference Reynolds number is 10 000. Calculations with five different grid resolutions (uniformly distributed across the boundary layer except for the case with five grid points, which was stretched toward the wall) are shown in this figure and compared with the experimental data.

The left-hand plot in figure 8 shows the computational results of the mean velocity profile compared with the experimental data at $Re_\theta = 7700$. All calculations agree well with the experimental data when the calculated turbulent boundary is fully developed from their initial uniform profiles. The right-hand plot in figure 8 shows the results with different reference

Figure 9. Comparison of re-attachment length in KKK flow: standard wall function, generalized wall function and low- Re model.

Reynolds numbers, ranging from 10^3 to 10^7 . The mean velocity profiles are plotted at $Re_\theta = 7700$ and compared with experimental data. All cases show good agreement between the calculated results and available experimental data. We conclude that the present wall function has produced a grid-independent as well as reference-Reynolds-number-independent solution for the flat-plate turbulent boundary layer.

5.2. Backward-facing step flow

A two-dimensional backward-facing step flow is one of the simplest recirculation flows. We use the KKK experimental data [7] as one of the test flows to benchmark the performance of the unified wall function versus the standard wall function. Figure 9 is a global comparison of numerical results between the unified wall function and the standard wall function. The numerical results from a low-Reynolds-number $k-\varepsilon$ model are also included as a reference. The same grid distribution is used for these three cases. The size of the recirculation zone from the standard wall function is much smaller than the size calculated from the unified wall function. The reattachment length normalized by the step height, L/h , measured by KKK is about 7.0 ± 0.5 . The CFD prediction using the unified wall function is about 6.56. But the prediction using the standard wall function is only about 5.05. The predicted reattachment length from the low-Reynolds-number turbulence model is about 6.54.

Other calculated quantities, such as the pressure distribution along the bottom wall (see the left-hand plot in figure 10) and the mean velocity profiles (see the right-hand plots in figure 10)

Figure 10. Pressure coefficients and mean velocity profiles compared with experimental data.

at several downstream locations from the step, also show a better comparison when the unified wall function is used. The KKK flow indicates that the effect of the wall pressure gradient on the wall bounded recirculation flow is important. The standard wall function misses this effect.

5.3. Shedding cylinder flow

A more complex 2D turbulent flow is the shedding cylinder flow. It is an unsteady flow with vortex shedding. Many measurements have been made from laminar vortex shedding to turbulent vortex shedding. The experimental data become scattered as the Reynolds number increases, especially in the regime of turbulent vortex shedding. The left-hand plot in figure 11 shows the Strouhal number of experimental data versus the Reynolds number. We chose a turbulent shedding cylinder flow with Reynolds number of 2×10^6 to check the performance of the generalized wall function versus the standard wall function. A calculation using the low-*Re* model was also carried out for comparison. Calculated Strouhal numbers are all around 0.22, which is well within the scattered experimental data. However, the detailed flow fields of numerical results are quite different for different wall functions used in the calculations. For example, the right-hand plots in figure 11 show the contours of the *U* velocity component from three numerical results: generalized wall function, standard wall function and low-*Re* model. Apparently, there are significant differences in the results between the generalized wall function and standard wall function, while the results between the generalized wall function and low-*Re* model are very close. We conclude that the effect of pressure gradient on flow separation and recirculation is important, and this is appropriately captured by the generalized wall function and low-*Re* model, but is missed by the standard wall function.

Application of generalized wall function for complex turbulent flows

Figure 11. Experimental Strouhal number and computed velocity contours of shedding cylinder flows.

Figure 12. A schematic and numerical grid of the Anderson burner.

Figure 13. Temperature and pressure contours in the symmetry plane of the Anderson burner.

5.4. Three-dimensional reacting flow

The Anderson burner [8] was used to study a premixed hydrogen/air combustor. The left-hand plot in figure 12 is a schematic diagram of the burner. Hydrogen is injected into a preheated air stream upstream of the flame-holder to form a well mixed hydrogen/air stream in the inlet duct. The water cooled flame-holder consists of 80 small tubes. The flame temperature and the burner pressure were measured for various test conditions. Combustion products were measured along the burner's centreline. A computational study [9] of the Anderson burner was carried out using the NCC code. The inlet and outlet boundary conditions were taken from the Anderson experiments. The flow was assumed to have a symmetry plane. About 1.3 million tetrahedral elements were used to represent one-half of the Anderson burner shown in the right-hand plot in figure 12. Numerical results along the centreline compared well with Anderson's experimental data. The left-hand plot in figure 13 shows the temperature contour in the symmetry plane. The peak value of the temperature is about 1420 K which matches the experimental data of 1380 K within a 3% error. The right-hand plot in figure 13 shows the contour of the pressure distribution in the symmetry plane. The calculated burner pressure is about $373\,935\text{ N m}^{-2}$ which matches the experimental data of $380\,000\text{ N m}^{-2}$ within 2% error. The calculated NO_x emissions along the centreline of the burner, shown in the left-hand plot in figure 14, also agree

Figure 14. Predicted NO_x compared with experimental data: flow structures behind the flame-holder.

well with the experimental data. The right-hand plot in figure 14 shows the flow and flame structures behind the flame-holder.

6. Conclusion

The generalized wall function for a three-dimensional turbulent boundary layer and its implementation procedure have been described. The theory of the generalized wall function has been compared with experimental data of turbulent boundary layers under various pressure gradients. Good performance has also been shown in several complex turbulent flows. We have demonstrated that the effect of pressure gradients on surface flows is significant in many practical flows. The standard wall function should be replaced by the new generalized wall function for flows with large pressure gradients.

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