Resonant phenomena in laser-assisted radiative attachment or recombination

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An electron colliding with an atom in the presence of an intense laser field can efficiently convert the combined energy of a large number of laser photons into the energy of a spontaneously emitted photon [1]. This process occurs, in particular, in laser-assisted radiative attachment (LARA) or recombination (LARR), in which the emission of a spontaneous photon is accompanied by absorption of $n$ laser photons and the formation of a negative ion or neutral atom. Although experimental studies of these processes have only begun recently [2], theoretical studies began more than a decade ago. The first theoretical investigations employed the strong field approximation (SFA) [3–5], in which the effects of the atomic potential $U(r)$ on the scattering state of the incident electron are neglected. These studies show that (even in lowest order in the potential $U(r)$) the LARR cross sections as a function of $n$ (or the energy of the spontaneous photon) exhibit a plateau structure, whose shape and extent can be described by treating the LARR process classically [6]. The first-order correction in the potential $U(r)$ to the SFA LARR amplitude was introduced in [7] (see also [1, 8]) taking into account $U(r)$ perturbatively, using the Born approximation. Inclusion of higher-order corrections in $U(r)$ (or rescattering effects) into the LARA/LARR amplitudes results in a second, high-energy (or rescattering) plateau in the LARA/LARR spectra [7, 9]. However, as for laser-induced processes, such as high-order harmonic generation (HHG) and above-threshold ionization (cf [1]), the cross sections for the high energy rescattering plateau are orders of magnitude smaller than those for the low-energy plateau. Hence, mechanisms for increasing the high-energy plateau cross sections are of great interest. For laser-assisted collisions, one way to achieve such an increase is to tune the incoming electron energy so that it can be temporarily captured (by stimulated emission of $\mu$ laser photons) to a bound state of the potential $U(r)$. Obviously, such a resonance phenomenon cannot be described in the Born approximation and requires an accurate account of the potential $U(r)$. Significant enhancement of plateau structures in resonant laser-assisted electron–atom scattering (LAES) was predicted recently [10]. However, resonant phenomena in LARA/LARR processes remain unexplored.

In the present communication, we extend the study of laser-induced resonant phenomena in collision problems to the case of LARA/LARR. We present a general parametrization for the resonant LARA/LARR cross sections and show the following at resonant electron energies: (i) the shape of LARA/LARR spectra as a function of $n$ coincides with that for HHG; (ii) the electron energy dependence of the $n$-photon LARA/
LARR cross section exhibits an asymmetric profile similar to the Fano profile in photoionization cross sections [11]; and (iii) LARR cross sections can be enhanced by more than two orders of magnitude. To describe LARA analytically, we use time-dependent effective range (TDER) theory, which provides a means to account for the short-range potential $U(r)$ in LARA non-perturbatively.

To describe electron-atom collisions in a monochromatic field with electric vector $F(t) = eF \cos \omega t$ (where $F$ and $\omega$ are the field amplitude and frequency) using the electric dipole approximation, the quasienergy (or Floquet) approach [12] is most appropriate. Within this approach, the laser-dressed scattering state of an electron with momentum $\mathbf{p}$ and energy $E = p^2/(2m)$ in the potential $U(r)$ has the form

$$
\Psi_{e, p}(r, t) = e^{-i\epsilon t/\hbar} \Phi_{e, p}(r, t),
$$

$$
\Phi_{e, p}(r, t) = \Phi_{e, p}(r, t + T), \quad T = 2\pi/\omega, \quad (1)
$$

where $\epsilon$ is the quasienergy, $\epsilon = E + u\epsilon'$, where $u = e^2F^2/(4ma^2)^2$ is the mean quiver energy of an electron in the field $F(t)$. For $F(t) = 0$, the quasienergy state $\Psi_{e, p}(r, t)$ reduces to the scattering state $\psi_{e}(r)$ of the recombining electron in the potential $U(r)$. In the field $F(t)$, the bound (final) state $\psi_{e}(r)$ with energy $E_0$ evolves to the quasistationary quasienergy state (QQES), $\Psi_{e, p}(r, t)$, which also has the form (1), but with the complex quasienergy $\epsilon = E_0 + \Delta E_0 - i\Gamma/2$, where $\Delta E_0$ and $\Gamma$ are the field-induced Stark-shift and width (or total decay rate $\Gamma/h$) of the state $\psi_{e}(r)$ [13].

We consider the LARA/LARR process as a dipole transition between initial and final states $\psi_{e}(r)$ and $\Psi_{e, p}(r, t)$ with emission of a spontaneous photon, whose energy differs from the field-free energy $\hbar\Omega = E - E_0$. Within the QQES approach, the LARA/LARR cross section $\sigma(\Omega)$, integrated over the directions of emission and summed over polarizations of the spontaneous photon, can be written as [cf [7]]

$$
\sigma(\Omega) = \frac{4m\Omega^3}{3\hbar c^3} |d(\Omega)|^2, \quad (2)
$$

$$
d(\Omega) = \frac{1}{T} \int_0^T dt \int d\mathbf{r} \tilde{\Psi}_e^* (\mathbf{r}, t) d\Psi_{e, p}(\mathbf{r}, t) e^{i\Omega t}, \quad (3)
$$

where $d = |d\epsilon |$ and $\Omega$ is the frequency of the spontaneously emitted photon:

$$
h\Omega = \epsilon + nh\omega - Re \epsilon.
$$

The function $\Psi_{e, p}(r, t)$ in (3) is the so-called dual function to $\Psi_{e}(r, t)$. If $F(t)$ is linearly polarized and $\psi_{e}(r)$ is a bound $s$-state, then $\Psi_{e}(r, t)$ is defined as [14–16]

$$
\Psi_{e}(r, t) = e^{-i\epsilon t/\hbar} \Phi_{e}(r, -t), \quad (4)
$$

Since the QQES wavefunctions $\Phi_{e, p}(r, t)$ diverge asymptotically as $r \to \infty$ (since they describe the ionization of a bound state $\psi_{e}(r)$ in the field $F(t)$), the use of dual functions as bra-vectors in the QQES approach is necessary to ensure proper normalization of the wavefunctions $\Phi_{e, p}(r, t)$ and the regularization of matrix elements involving these functions (cf [14–16] for further details). To describe resonant LARR or LARA processes, we note first that the wavefunction $\Psi_{e, p}(r, t)$ can be obtained as a residue of the scattering state $\Psi_{e, p}(r, t)$ in the complex plane of $\epsilon$ at $\epsilon = \epsilon + \mu \hbar \omega = Re \epsilon + \mu \hbar \omega - i\Gamma/2$ [17]:

$$
\text{Res} \Psi_{e, p}(r, t) \bigg|_{\epsilon = \epsilon + \mu \hbar \omega} \sim e^{i\varphi_{e, p}(r, t)} \psi_{e}(r, t), \quad (5)
$$

where $\mu$ is an integer. Therefore, for $\epsilon = \epsilon_{\mu}$, the scattering state $\Phi_{e, p}(r, t)$ can be approximated by a sum of potential (non-resonant) and resonant parts [17]:

$$
\Phi_{e, p}(r, t) = \Phi_{e, p}(r, t) + B(p_{\mu}) \frac{\Phi_{e}(r, t)}{E - E_0 + i\Gamma/2}, \quad (6)
$$

where $E_\mu = p^2_{\mu}/(2m) = Re \epsilon + \mu \hbar \omega - u_p$ is the resonant electron energy and the coefficient $B(p_{\mu})$ is proportional to the amplitude for stimulated $\mu$-photon recombination or attachment (cf (30)). Substituting (5) into (3), the amplitude $d(\Omega)$ can also be presented as a sum of potential and resonant terms:

$$
d(\Omega) = d^{(p)}(\Omega) + B(p_{\mu}) \frac{\tilde{d}(\Omega)}{E - E_\mu + i\Gamma/2}, \quad (7)
$$

where the potential term $d^{(p)}(\Omega)$ is given by (3) upon substituting there $d^{(p)}(p_{\mu})$, while the resonant term involves the dual dipole moment, $\tilde{d}(\Omega) = d(\Omega)e_\jmath$, which determines the rate $R(\Omega)$ for the generation of a harmonic of the field $F(t)$ with frequency $\Omega = (n + \mu)\omega$ by a bound electron in the s-state $\psi_{e}(r)$ [16]:

$$
R(\Omega) = \frac{\Omega^3}{2\pi \hbar c^3} |\tilde{d}(\Omega)|^2, \quad (8)
$$

The parameter $\Omega$ involves only two vectors, $e_\jmath$ and $\hat{p}$ (mutually oriented at an angle $\theta$), the vector $d^{(p)}(\Omega)$ lies in the plane $(e_\jmath, \hat{p})$ and can be presented as

$$
d^{(p)}(\Omega) = d^{(p)}_{\parallel}(\Omega)e_\jmath + d^{(p)}_{\perp}(\Omega)e_\perp, \quad (9)
$$

where $e_{\parallel} = [\hat{p} \times e_{\perp}]$. Using (6) and (8), we obtain the general parametrization for the LARA/LARR cross section (2) near a $\mu$-photon resonance

$$
\sigma(\Omega) = \sigma^{(p)}(\Omega) + \sigma_{\perp}^{(p)}(\Omega) \frac{2(Re q - \delta |Im q| + |q|^2)}{\delta^2 + 1}, \quad (10)
$$

where $q = 2(\epsilon - E_\mu)/\Gamma$, $\delta = -2i(p_{\mu})$, $d(\Omega)/(\Gamma d^{(p)}_{\parallel}(\Omega))$, and $\sigma^{(p)}(\Omega)$ and $\sigma_{\perp}^{(p)}(\Omega)$ are given by (2) upon substituting there $d(\Omega) \to d^{(p)}(\Omega)$ or $d(\Omega) \to d^{(p)}(\Omega)e_\perp$. The parametrization (9) simplifies for parallel geometry, $\hat{p} \parallel e_{\parallel}$, in which case $\sigma_{\parallel}^{(p)} = \sigma^{(p)}$:

$$
\sigma(\Omega) = \frac{\sigma^{(p)}(\Omega)}{\delta^2 + 1} \frac{(Re q + 1)^2 + (Im q - \delta)^2}{\delta^2 + 1}. \quad (11)
$$

Results (9) and (10) show that (for a given $n$) $\sigma(\Omega)$ as a function of $\Omega$ is asymmetric with respect to the resonance energy $E_{\mu}$. For small $\Gamma$ (i.e. taking into account only terms $\sim 1/\Gamma^2$), the result for the cross section $\sigma(\Omega)$ at the resonance, $\delta = 0$, is

$$
\sigma(\Omega) \approx \frac{32\pi m |B(p_{\mu})|^2}{3p_{\mu} \Gamma^2} R(\Omega). \quad (12)
$$

Since $B(p_{\mu})$ does not depend on the number of absorbed photons, the shapes of resonant LARA/LARR spectra as
functions of \( n \) replicate the shapes of the corresponding bound state HHG spectra.

To present quantitative results for laser-induced resonance phenomena in LARA, we use TDER theory to describe both the incident continuum (\( \Phi_{\text{inc}}(t) \)) and final bound (\( \Phi_{\text{f}}(t) \)) field-dressed states of the active electron. This theory assumes that the interaction of an electron with a short-range potential \( U(r) \) (having only a single bound state \( \psi_{\text{E}}(r) \) with angular momentum \( l \)) is described by the \( l \)-wave scattering phase \( \delta_l(E) \) that is parameterized by the scattering length \( a \) and the effective range \( r_e \), which are parameters of the problem. For simplicity, we consider the case of a bound s-state \( \psi_0(r) \) of energy \( E_0 = -(\hbar^2/2m) \). For this case, the TDER wavefunctions \( \Phi_{\text{inc}}(r, t) \) and \( \Phi_{\text{f}}(r, t) \) are expressed in terms of one-dimensional integrals [18, 19]:

\[
\Phi_{\text{inc}}(r, t) = e^{i\mathcal{P}(t) - S(t)} e^{iE_0r/\hbar} f(r, t),
\]

where

\[
f(r, t) = -\frac{2\pi\hbar^2}{mk} \int G(r, t, 0, t') g_e(r') e^{iE_0(t-t')/\hbar} dt',
\]

\[\Phi_{\text{f}}(r, t) = -\frac{2\pi\hbar^2\sqrt{k}}{m} \int G(r, t, 0, t') g_e(r') e^{iE_0(t-t')/\hbar} dt',\]

where \( G(r, t, 0, t') \) is the retarded Green function and \( S(p, t) \) is the classical action of an electron in the field \( F(t) \),

\[S(p, t) = \int \left[ P^2(t')/(2m) - E \right] dt',\]

\[P(t) = \text{the canonical momentum,}\]

\[P(t) = p - eE \omega_0 \sin \omega_0 t,\]

and \( f_e(p, t) \) and \( g_e(t) \) are dimensionless periodic functions,

\[f_e(p, t) = \sum_k f_k(p) e^{-i\kappa_0 t}, \quad g_e(t) = \sum_k g_k e^{-i\kappa_0 t}.\]

The Fourier coefficients \( f_k(p) \) and \( g_k \) as well as the complex quasienergy \( \varepsilon \) can be found from a system of homogeneous (for \( f_k(p) \)) or homogeneous (for \( g_k \) and \( \varepsilon \)) linear equations:

\[
\sum_{k} \mathcal{M}_{k,k'}(\varepsilon) f_k(p) = c_k(p),
\]

\[
\sum_{k} \mathcal{M}_{k,k'}(\varepsilon) g_{k'} = 0,
\]

where

\[
\mathcal{M}_{k,k'}(\varepsilon) = R(\varepsilon + k\hbar\omega_0)\delta_{k,k'} + \mathcal{M}_{k,k'}(\varepsilon),
\]

\[
R(\varepsilon) = \frac{1}{\omega_0} - \frac{m r_0 \varepsilon}{\hbar^2 k^2} + i \frac{\varepsilon}{|E_0|},
\]

\[
c_k(p) = \iota^{-k} \sum_{l=0}^{\infty} J_{k+2l}(\frac{2\varepsilon F p}{m\hbar^2 \omega_0}) J_{l-\frac{1}{2}}(\frac{u_p}{2\hbar}),
\]

where \( a_0 \) and \( r_0 \) are the scattering length and the effective range, \( p = (e \cdot p) = p \cos \theta \), and \( f_k(x) \) is a Bessel function. The matrix elements \( M_{k,k'}(\varepsilon) \) are non-zero only if the difference \( k - k' \) is even and have the form

\[
M_{k,k'}(\varepsilon) = \sqrt{\frac{\iota^{-k-k'}\hbar^2}{8\pi i|E_0|}} \int_0^\infty \frac{d\tau}{\tau^{3/2}} e^{i(2\varepsilon/|E_0| + k-k')\tau} \times \left\{ e^{i\lambda(\tau)} J_{k-k'}/[z(\tau) - \delta_{k,k'}] \right\},
\]

where

\[
\lambda(\tau) = \frac{2u_p}{\hbar \omega_0} \left( \frac{\sin^2 \tau}{\tau} - \tau \right),
\]

\[z(\tau) = \frac{u_p}{\hbar \omega_0} \left( 2\sin^2 \frac{\tau}{\tau} - \sin^2 \tau \right).\]

From the explicit form of \( M_{k,k'}(\varepsilon) \) follow the symmetry relations

\[
M_{k,k}(\varepsilon) = M_{k',k}(\varepsilon), \quad M_{k,k}(\varepsilon + \hbar\omega) = M_{k+k',k'+k}(\varepsilon).
\]

As shown in [19], the function \( g_k(t) \) (as well as the system of equations (15)) includes only coefficients \( g_k \) with even \( k \). The complex quasienergy \( \varepsilon \) is given by that root of the transcendental equation, \( \text{Det} \mathcal{M}_{k,k}(\varepsilon) = 0 \), which becomes \( E_0 \) when \( F \to 0 \). For \( F(t) = 0 \), the matrix elements \( M_{k,k}(\varepsilon) \) are zero and coefficients \( f_k(p) \) and \( g_k \) reduce to

\[
f_k(p) = R^{-1}(E) \delta_{k,0}, \quad g_k = C_k \delta_{k,0},
\]

where \( C_k \) is a dimensionless coefficient in the asymptotic form of \( \psi_0(r) \) for \( r \gg k^{-1} \):

\[
\psi_0(r) \approx C_k \sqrt{k/(4\pi)} r^{-1} \exp(-kr).
\]

With the use of (12) and (13), the analytic evaluation of \( d(\Omega) \) in (3) involves the spatial integration of two Green functions and a threefold integration over time. The spatial integration and two of the temporal integrations can be performed analytically (as done in [16]) and the final result for \( d(\Omega) \) can be presented as

\[
d(\Omega) = d_{pw}(\Omega) + d_{\text{esc}}(\Omega),
\]

\[
d_{pw}(\Omega) = \sum_{p=\pm 1} \frac{\mathcal{L}_{n+p}}{\omega_0 + \Omega + p} - \frac{2i\mathcal{L}_n(p)}{\omega_0 |F|},
\]

\[
\mathcal{L}_n(p) = \sum_{k=\pm \infty} g_k c_{k+n}(p), \quad C = \frac{\pi e^2 \hbar F k^{1/2}}{m^2 \omega_0 k^2},
\]

\[
d_{\text{esc}}(\Omega) = \frac{\lambda_0 C}{\pi} \sum_{k,k'} f_k(p) W^n_{k,k'} g_k g_{k'},
\]

where the matrix elements \( W^n_{k,k'} \) are non-zero only if the difference \( n - k \) is odd:

\[
W^n_{k,k'} = \mathcal{D} \int_0^{\infty} \frac{d\tau}{\tau^{3/2}} e^{i(\varepsilon + \varepsilon')/(\hbar \omega_0) + k-k'\tau + \lambda(\tau)} \times \left\{ j_-(\tau) J_{k-k'}(\tau) - j_+(\tau) J_{k-k'}(\tau) \right\},
\]

\[
j_{\pm}(\tau) = \sin \tau \sin(\Omega \tau/\omega) - \Omega \sin(\Omega \tau/\omega \pm 1)\tau, \quad s = n-k+k' - 1, \quad \mathcal{D} = \frac{1}{2} \left( \frac{\hbar \omega_0}{2\pi|E_0|} \right)^{1/2}.
\]

Result (22) for \( d(\Omega) \) is exact within the TDER theory and valid for both resonant and non-resonant electron energies \( E \). The term \( d_{pw}(\Omega) \) originates from the first term in (12) and corresponds to the first-Born (or plane wave) approximation in the potential \( U(r) \) for the scattering state. This term is smooth at the resonant energy \( E = E_0 \) and contributes only to the potential part of the
LARA amplitude (6). Moreover, for an intense low-frequency \((\hbar \omega \ll |E_\mu|)\) field \(F(t)\), the coefficients \(g_k\) with \(k = 0\) are small compared to \(g_\omega\) (cf [19]). Approximating in (24) \(g_k = g_\omega \delta_{k,0}\) and \(g_\omega = C\) (cf (21)), \(d_{\text{res}}(\Omega)\) yields an exact (i.e. without using saddle-point methods for its evaluation) TDER result for the LARA amplitude in the SFA [3, 4].

Resonant phenomena are described by the second (rescattering) term, \(d_{\text{res}}(\Omega)\), in (22). This term originates from the integral term in (12) and, within the TDER theory, ensures an exact account of the effects of the potential \(U(r)\) on the field-dressed initial and final states of the attaching electron (through the coefficients \(f_{\mu}(p)\) and \(g_k\) in (25)).

To extract from (25) the resonant part of the amplitude (6) in an explicit form, we solve the system (14) for \(f_{\mu}(p)\) near the resonance, i.e. for \(\epsilon = (\epsilon + \mu)\hbar \omega\). Expanding matrix elements in (14) up to the linear term in \(\Delta \epsilon = \epsilon - \epsilon - \mu \hbar \omega = E - E_\mu + i\Gamma/2\), approximating \(c_\mu(p) \approx c_\mu(p_\mu)\) and employing the symmetry relations (20), we obtain

\[
\sum_{k} M_{k',k}(\epsilon) f_{k' - \mu}(p) + \Delta \epsilon \sum_{k} M_{k',k}(\epsilon) f_{k - \mu}(p) = c_{k - \mu}(p_\mu),
\]

(27)

where \(M_{k',k}(\epsilon) = \partial M_{k',k}(\epsilon)/\partial \epsilon\). In the lowest resonant approximation \((\Delta \epsilon \to 0)\), the coefficients \(f_{k' - \mu}(p)\) in (27) are proportional to \(g_k\): \(f_{k' - \mu}(p) = a(p) g_k\). To find \(a(p)\), we multiply the system (27) by \(g_k\) and then sum over \(k\). Taking into account the symmetry relations (20) and the equality \(\sum_{k} g_k M_{k',k}(\epsilon) = 0\) (cf (15)), the system (27) reduces to a single equation

\[
\Delta \epsilon \sum_{k} g_k M_{k',k}(\epsilon) f_{k' - \mu}(p) = \sum_{k} g_k c_{k - \mu}(p_\mu),
\]

(28)

from which \(a(p)\) is easily obtained upon substituting \(f_{k' - \mu}(p) = a(p) g_k\). The resulting resonant approximation for \(f_{k' - \mu}(p)\) is

\[
f_{k' - \mu}(p) = \frac{g_k}{E - E_\mu + i\Gamma/2} \sum_{s} g_s c_{s - \mu}(p_\mu) g_s.
\]

(29)

Changing in (25) the summation index \(k\) to \(k - \mu\) and substituting there the result (29) for \(f_{k' - \mu}(p)\), the resonant term in \(d_{\text{res}}(\Omega)\) can be presented in the same form as in (6), where the explicit form for the dual dipolemoment \(d(\Omega)\) in the TDER theory (in terms of \(g_k/\sqrt{C}\)) and the matrix elements \(W_{n'k'k}(\mu)\) is given in [20]. The coefficient \(B(p_\mu)\) in (6) is related to the amplitude \(A_\mu(p_\mu)\) for \(n\)-photon laser-stimulated attachment:

\[
A_\mu(p_\mu) = \sum_{k} g_k^* c_k - \mu(p_\mu),
\]

(30)

\[
B(p_\mu) = \frac{2\pi \hbar^2 \sqrt{\epsilon}}{m} A_\mu(p_\mu).
\]

The potential part \(d^{(0)}(\Omega)\) of \(d(\Omega)\) in (6) is given within TDER theory by (22)–(25) in which we set \(p = p_\mu\) and replace \(f_{k' - \mu}(p)\) in (25) by \(f_{k' - \mu}(p)\) in (25).

Key features of resonant LARA cross sections are shown in figure 1. (Qualitatively, resonant LARR features are similar.) Results for \(\epsilon = H\) attachment with the formation of the H\(^+\) ion are shown for both non-resonant \((E)\) and resonant \((E_\mu)\) incident electron energies for the cases of odd \(\mu\) in (a) and even \(\mu\) in (b). (TDER parameters for this case are (cf, e.g., [19]) \(|E_\mu| = 0.755 eV, C_\mu = 2.304, a_\mu = 6.16 a_0, r_0 = 2.64 a_0\) where \(a_0\) is the Bohr radius.) In Figure 1(a), the laser parameters and energy \(E\) are the same as in a recent analysis of nonresonant LARA processes [9]. Resonant effects are more pronounced in figure 1(b) for \(I = 1.35 \times 10^{11}\) W cm\(^{-2}\) and \(h\omega = 0.0453 eV\). Figure 1 exhibits several qualitative features: (i) a two or three-photon LARR maximum for \(\mu\) and \(\sigma\) coincides with that for high harmonics generated by the H\(^+\) ion; (ii) the extent of the high-energy plateau in the resonant process (\(\approx |E_\mu|\)) is larger than for high harmonics generated by the H\(^+\) ion; (iii) the extent of the high-energy plateau in the resonant process (\(\approx |E_\mu|\)) is larger than for high harmonics generated by the H\(^+\) ion; and (iv) enhancements occur only for those numbers \(n\) of absorbed photons whose parity is opposite to that of \(\mu\) (cf also Figure 2). This last result is a simple consequence of the fact that the resonant cross section (11) involves the rate for emission of the \((n + \mu)\)th harmonic of the field \(F(t)\). As is well known, an atom can emit only odd harmonics of a monochromatic field (owing to electric dipole selection rules), so that \(n + \mu\) must be odd.

Figure 2 shows the energy dependence of the partial \((n\text{-photon})\) LARA cross sections in the resonance region (as well as our general parametrization for \(\sigma_n(E)\) for the

Figure 1. LARA spectra for \(\epsilon = H\) attachment in a linearly polarization field \(F(t)\) (with \(e = \|p\) having (a) intensity \(I = 3.75 \times 10^{11}\) W cm\(^{-2}\), \(h\omega = 0.098 |E_\mu| = 0.074 eV\) or (b) \(I = 1.35 \times 10^{11}\) W cm\(^{-2}\), \(h\omega = 0.06 |E_\mu| = 0.0453 eV\). Thick (blue) and thin (red) solid lines: exact TDER results for the resonant \((E_\mu)\) and non-resonant \((E)\) electron energies shown in each panel. Dashed (green) lines: HHG spectra of the H\(^+\) ion (in arbitrary units) for the same field parameters \(I, \omega\) as in (a) and (b). Vertical lines in (b) mark photon numbers \(n = 403, 417\) (cf figure 2).
laser parameters of Figure 1(b), giving a Stark-shift and width of the H\(^-\) ground state \(\psi_0(r)\) of \(E_0 = \text{Re } \epsilon - E_\infty = -6.250 \times 10^{-3} \text{ eV}\) and \(\Gamma = 8.335 \times 10^{-4} \text{ eV}\). Owing to the complexity of the parameter \(q\) (i.e. the ratio of the resonant and potential parts of the amplitude (6) at \(E = E_\infty\)), the asymmetric resonance profile shape as well as the positions of the maxima and minima in \(\sigma(E)\) (for given \(n\) and laser parameters) are sensitive to both the absolute value of \(q\) and the relation between \(\text{Re } q\) and \(\text{Im } q\).

Figure 2 shows examples of resonance profiles for both odd (\(\mu = 236\)) and even (\(\mu = 237\)) values of \(n\). In terms of \(\delta\), the positions of the maxima (\(\delta^+\)) and minima (\(\delta^-\)) of \(\sigma_n(E)\) can be obtained from (9) or (10) by equating to zero the derivative of \(\sigma(\Omega)\) with respect to \(\delta\):

\[
\delta_\pm = \frac{|q + 1|^2 \mp |q(q + 2)| - 1}{2 \text{Im } q}.
\]  

(31)

Since \(\delta_\pm\) are the roots of a quadratic equation, the following relations are valid:

\[
\delta_+ \delta_- = -1, \quad \delta_+ - \delta_- = -|q(q + 2)|/\text{Im } q.
\]  

(32)

Relations (32) show that for \(\text{Im } q < 0\) (or \(\text{Im } q > 0\)) the maximum occurs at \(E > E_\infty\) (or \(E < E_\infty\)), while the location of the minimum has the opposite behavior. These results agree with those in Figures 2(a) and (d) (in which \(\text{Im } q < 0\)) and 2(b) and (c) (in which \(\text{Im } q > 0\)). The “window resonance” behavior of \(\sigma_n(E)\) (as in Figure 2(d)) occurs whenever \(|\delta_-| \rightarrow 0\) or, equivalently, the parameter \(\Delta \rightarrow 1\), where

\[
\Delta = \frac{\delta_+ - \delta_-}{\delta_+ + \delta_-} = \frac{|q(q + 2)|}{1 - |q + 1|^2}.
\]  

(33)

The parameter \(\Delta\) is positive when \(|q + 1| < 1\), and this latter inequality is fulfilled for negative \(\text{Re } q\) (with \(\text{Re } q\) \(-\infty \rightarrow -2\) when \(|\text{Im } q| \rightarrow 0\)). The limiting case \(\Delta = 1\), i.e. \(|q(q+2)| = 1 - |q + 1|^2\), is realized when \(\text{Im } q = 0\). Therefore, a “window resonance” in \(\sigma_n(E)\) appears for negative \(\text{Re } q\) when (i) \(|\text{Im } q| \ll |\text{Re } q|\) and (ii) \(|q + 1| < 1\).

For example, \(\text{Re } q = -0.612\), \(\text{Im } q = -0.207\) and \(\Delta = 1.124\) for the results in figure 2(d).

As is clear from our general considerations, the continuum resonance phenomena shown in Figures 1 and 2 disappear in any theory that does not account for the influence of the atomic potential \(U(r)\) on the scattering states (such as, e.g., the Born approximation or even an improved SFA [1, 7]). However, an accurate non-perturbative account of the interaction of a recombining electron with both a laser field and an atomic potential leads to complicated results even for a potential \(U(r)\) supporting only a single bound state, as in the TDER theory (cf. the result (25)). Nevertheless, the results simplify for a low-frequency \((\hbar \omega \ll |E_\infty|\)) field \(F(t)\), in which case the system (14) can be solved iteratively, taking into account nondiagonal matrix elements \(M_{\ell,k}(\epsilon)\) perturbatively [21]. In the lowest approximation, neglecting nondiagonal matrix elements, the coefficients \(f_k(p)\) take the following form:

\[
f_k(p) \approx f_k^{(1)}(p) = \frac{c_k(p)}{M_{\ell,k}(\epsilon)} = \frac{c_k(p)}{M_{0,0}(\epsilon + k\hbar\omega)}.
\]  

(34)

Also, as noted above, the coefficients \(g_{k,n,0}\) are small for low frequencies, so that we can make the approximation \(g_k \approx C_{\delta_{1,0}}\). As is seen in figure 2, the approximations (34) and \(g_k = C_{\delta_{1,0}}\) reasonably describe the resonant phenomena. The resonant structures originate from the matrix element in the denominator of (34), which has zeros in the complex plane of \(\epsilon\) at \(\epsilon = \tilde{\epsilon} - k\hbar\omega\), where \(\tilde{\epsilon}\) is the complex quasienergy \(\epsilon\) in the low-frequency approximation [19].

Finally, we note that resonant phenomena in LARA/ LARR processes cannot be described in the low-frequency Kroll-Watson approximation (KWA) for the scattering state wavefunction [22]. Within the TDER theory, the KWA is formulated in terms of the function \(f_\ell(p, t)\) in (12) [23]:

\[
f_\ell^{\text{KWA}}(p, t) = \frac{e^{-i S(p, t)/\hbar}}{R(\mathcal{E}(t))}, \quad \mathcal{E}(t) = \frac{p^2(t)}{2m},
\]  

(35)

where \(R(\mathcal{E})\) (cf. (17)) is related to the partial s-wave amplitude \(f_\ell(E)\) of elastic electron scattering from the potential \(U(r)\) in the TDER theory at \(F(t) = 0\): \(f_\ell(E) = [x R(E)]^{-1}\) [19]. The shortcoming of the KWA is that in this semiclassical approximation, the quantization of the photon energy is completely neglected so that both \(S(p, t)\)
and $R(\hat{c}(t))$ in (35) depend only on the classical energy $\hat{c}(t)$ of an electron in a laser field, which is always positive. Therefore, although the amplitude $f_0(E)$ has a pole at negative energy $E = E_0$ (since $\hat{R}(E_0) = 0$ [19]), the function $R(\hat{c}(t))$ in (35) has no zeros, i.e. the KWA result (35) fails to describe resonant effects (cf Figure 2). (For this reason, the resonant effects disappear also in the KWA for LAES [10]).

In conclusion, we have analyzed the key features of resonant phenomena in LARA/LARR processes that occur for electron energies corresponding to $\mu$-photon laser-stimulated attachment/recombination. For such energies, we find that the spectra of spontaneously emitted photons in the high-energy parts of the LARA/LARR plateaus coincide with the harmonic generation spectra of the bound systems. Owing to the significant enhancement of resonant cross sections versus non-resonant ones, we expect that our findings should facilitate experimental observation of the resonant modification of radiative electron attachment/recombination in a laser field and the emission of high-order harmonics in laser-assisted collision processes.

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References