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An Achievable Rate Region for Imperfectly-Known Two-Way Relay Fading Channels

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***Abstract*—[1] In this paper, achievable rates and resource allocation strategies for imperfectly known two-way relay fading channels are studied. Decode-and-forward (DF) relaying is considered. It is assumed that communication starts with the network training phase in which the users and the relay estimate the fading coefficients, albeit imperfectly. Subsequently, data transmission is performed in multiple-access and broadcast phases. In both phases, achievable rate regions are identified by treating the terms that arise due to channel estimation errors and imperfect interference cancellation as Gaussian distributed noise components. The achievable rate region of the two-way relay channel is given by the intersection of the achievable rate regions of multiple-access and broadcast phases. The impact of several training and transmission parameters (such as training power levels, time/bandwidth allocated to the multiple access and broadcast phases, and relay power allocation parameter) on the achievable rate regions and sum rates is investigated.**

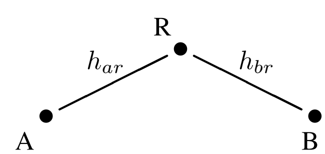
I. Introduction

Cooperative wireless communication schemes have attracted much interest due to their promise of providing higher throughput or increased reliability that can be attained through diversity gains. A spectrally efficient relaying technique named two-way relaying has been proposed in [1] and [2], in which two nodes are able to exchange information via the help of a relay node. Two-way relaying method consists of two phases: the multiple access (MAC) phase in which the source nodes simultaneously transmit their data to the relay, and the broadcast (BC) phase in which the relay forwards the received signal to the sources. One key technique in two-way relaying is interference cancelation in which the source nodes subtract their own forwarded signals from the received signal. However, perfect interference cancelation requires perfect knowledge of the channel conditions and most work on two-way relay channels have assumed the availability of perfect channel side information at the receivers. On the other hand, especially in mobile wireless applications, this assumption is unwarranted as randomly varying channel conditions can be learned by the receivers only imperfectly. Hence, it is of much interest to analyze the performance in two-way relay channels in the presence of imperfect channel knowledge obtained through training and channel estimation. The training design for the two-way amplify-and-forward (AF) relaying was recently studied in [4] and [5]. In [5], the authors derived lower bounds on the training-based individual rates and sum-rate. Given the total transmit power constraint, they investigated the optimal power allocation between the two terminals and the relay.

In this paper, we investigate the training-based achievable rate region of the decode-and-forward (DF) two-way relaying scheme. We note that the DF strategy has certain advantages over AF. In AF, due to the need to estimate the cascade of the channels in non-Gaussian noise, performing minimum mean-square-error (MMSE) estimation is often not feasible and suboptimal linear MMSE estimation is employed. In addition, noise forwarding in AF is a factor that can lead to losses in performance unless the signal-to-noise ratio is high enough. Moreover, degrees of freedom in transmission might be limited in AF schemes since the MAC and BC phases of the transmission are necessarily of equal duration. At the same time, it should be noted that DF requires a more complicated operation at the relay, and training in DF mode takes a duration of three symbols instead of two as required in AF.

II. Channel Model

We consider a three-node two-way relay network which consists of user nodes A and B, and a relay node R.



Channels between A and R, and R and B are modeled as Rayleigh block-fading channels with fading coefficients denoted by h_{ar} and h_{rb}, respectively. We further assume that there is no direct link between user A and user B. Due to the block-fading assumption, the fading coefficients[2] $h_{ar} \sim \mathcal{CN}(0, \sigma_{ar}^2)$, and $h_{br} \sim \mathcal{CN}(0, \sigma_{br}^2)$ stay constant for a block of m symbols before they assume independent realizations for the following block. In this system, user nodes A and B send data to each other with the assistance of the intermediate relay node. It is assumed that none of the nodes has prior knowledge of the instantaneous realizations of the fading coefficients, and the transmission is conducted in two

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[2]$x \sim \mathcal{CN}(d, \sigma^2)$ is used to denote a proper complex Gaussian random variable with mean d and variance σ^2.

phases: network training phase in which pilot symbols are transmitted and the fading coefficients are estimated at the receivers, and data transmission phase. Over these phases, the source and relay nodes are subject to the following average power constraints:

$$\|\mathbf{x}_{a,t}\|^2 + E\{\|\mathbf{x}_a\|^2\} \leq mP_a, \tag{1}$$

$$\|\mathbf{x}_{b,t}\|^2 + E\{\|\mathbf{x}_b\|^2\} \leq mP_b, \tag{2}$$

$$\|\mathbf{x}_{r,t}\|^2 + E\{\|\mathbf{x}_r\|^2\} \leq mP_r, \tag{3}$$

where $\mathbf{x}_{a,t}$, $\mathbf{x}_{b,t}$ and $\mathbf{x}_{r,t}$ are the training signal vectors of users A and B, and the relay R, respectively, and $\mathbf{x}_a$, $\mathbf{x}_b$ and $\mathbf{x}_r$ are the corresponding data transmission vectors.

III. Training and Data Transmission Phases and Achievable Rate Regions

A. Network Training Phase

Each block transmission starts with the training phase. In the first symbol period, user A transmits a pilot symbol to enable the relay to estimate channel coefficient h_{ar}. In the average power limited case, sending a single pilot is optimal because instead of increasing the number of pilot symbols, a single pilot with higher power can be used. The signal received by the relay is

$$y_{ar,t,} = h_{ar}x_{a,t} + n_r. \tag{4}$$

Similarly, in the second symbol period, user B transmits a pilot symbol to enable the relay to estimate channel coefficient h_{br}. The signal received by the relay is

$$y_{br,t,} = h_{br}x_{b,t} + n_r. \tag{5}$$

In the third symbol period, relay transmits a pilot symbol to enable user A to estimate the fading coefficient h_{ra} and user B to estimate h_{rb}. The signals received at A and B, respectively, are

$$y_{a,t} = h_{ra}x_{r,t} + n_a, \text{ and} \tag{6}$$

$$y_{b,t} = h_{rb}x_{r,t} + n_b. \tag{7}$$

In the above formulations, $n_r \sim \mathcal{CN}(0, N_0)$, $n_a \sim \mathcal{CN}(0, N_0)$ and $n_b \sim \mathcal{CN}(0, N_0)$ represent independent Gaussian noise samples at the relay and the user nodes. Notice also in (6) and (7) that we have denoted the fading coefficients experienced when the relay transmits to the users as h_{ra} and h_{rb} rather than h_{ar} and h_{br}, which are the fading coefficients when the users transmit to the relay. It is important to note that although we implicitly assume channel reciprocity and consider that statistically the same fading is experienced in the uplink (user-to-relay) and downlink (relay-to-user) transmissions, this assumption is not required in the analysis and different fading conditions can be considered in the downlink and uplink. Hence, for more generality, we opted to choose different notations for the fading coefficients.

In the training process, it is assumed that the receivers employ minimum mean-square error (MMSE) estimation. Let us assume that the user A allocates δ_a of its total power for training, user B allocates δ_b of its total power for training while the relay allocates δ_r of its total power for training. As described in [6], the MMSE estimate of h_{ar} is given by

$$\hat{h}_{ar} = \frac{\sigma_{ar}^2\sqrt{\delta_a m P_a}}{\sigma_{ar}^2\delta_a m P_a + N_0} y_{ar,t}, \tag{8}$$

where $y_{ar,t} \sim \mathcal{CN}(0, \sigma_{ar}^2\delta_a m P_a + N_0)$. We denote by $\tilde{h}_{ar}$ the estimate error which is a zero-mean complex Gaussian random variable with variance

$$var(\tilde{h}_{ar}) = \frac{\sigma_{ar}^2 N_0}{\sigma_{ar}^2\delta_a m P_a + N_0}. \tag{9}$$

Similarly, we have

$$\hat{h}_{br} = \frac{\sigma_{br}^2\sqrt{\delta_b m P_b}}{\sigma_{br}^2\delta_b m P_b + N_0} y_{br,t},$$
$$y_{br,t} \sim \mathcal{CN}(0, \sigma_{br}^2\delta_b m P_b + N_0), \tag{10}$$

$$var(\tilde{h}_{br}) = \frac{\sigma_{br}^2 N_0}{\sigma_{br}^2\delta_b m P_b + N_0}. \tag{11}$$

$$\hat{h}_{ra} = \frac{\sigma_{ra}^2\sqrt{\delta_r m P_r}}{\sigma_{ra}^2\delta_r m P_r + N_0} y_{a,t},$$
$$y_{a,t} \sim \mathcal{CN}(0, \sigma_{ra}^2\delta_r m P_r + N_0), \tag{12}$$

$$var(\tilde{h}_{ra}) = \frac{\sigma_{ra}^2 N_0}{\sigma_{ra}^2\delta_r m P_r + N_0}, \tag{13}$$

$$\hat{h}_{rb} = \frac{\sigma_{rb}^2\sqrt{\delta_r m P_r}}{\sigma_{rb}^2\delta_r m P_r + N_0} y_{b,t},$$
$$y_{b,t} \sim \mathcal{CN}(0, \sigma_{rb}^2\delta_r m P_r + N_0), \tag{14}$$

$$var(\tilde{h}_{rb}) = \frac{\sigma_{rb}^2 N_0}{\sigma_{rb}^2\delta_r m P_r + N_0}. \tag{15}$$

With these estimates, the fading coefficients can now be expressed as

$$h_{ar} = \hat{h}_{ar} + \tilde{h}_{ar}, \tag{16}$$

$$h_{br} = \hat{h}_{br} + \tilde{h}_{br}, \tag{17}$$

$$h_{ra} = \hat{h}_{ra} + \tilde{h}_{ra}. \tag{18}$$

$$h_{rb} = \hat{h}_{rb} + \tilde{h}_{rb}. \tag{19}$$

B. Data Transmission Phase

The practical relay node usually cannot transmit and receive data simultaneously. Thus, we assume that the relay works under half-duplex constraint. As discussed in the previous section, within a block of m symbols, the first three symbols are allocated for channel training. In the remaining duration of $m-3$ symbols, data transmission takes place. As usual, two-way relaying can be divided into two phases. The first one is usually called the multiple access (MAC) phase in which the users simultaneously transmit their messages to the relay. The second phase is called the broadcast phase (BC) in which the relay transmits to both users. We introduce the MAC transmission parameter α and assume that a duration of $\alpha(m-3)$ symbols is allocated for users' transmission to the relay. Hence, α can be seen as the fraction of total time (or bandwidth) dedicated to the MAC phase. The remaining duration of $(1-\alpha)(m-3)$ symbols is to be used in the broadcast phase.

1) Multiple Access Phase: In the multiple access phase of the bidirectional relaying protocol, nodes A and B simultaneously transmit independent messages m_a and m_b with rates R_a and R_b to the relay node. Thereby, the message m_a from node A is intended for node B and vice versa for message m_b. Then, the input-output relation in the multiple access channel is given by

$$\begin{aligned}\mathbf{y}_r &= h_{ar}\mathbf{x}_a + h_{br}\mathbf{x}_b + \mathbf{n}_r && (20)\\ &= \hat{h}_{ar}\mathbf{x}_a + \hat{h}_{br}\mathbf{x}_b + \tilde{h}_{ar}\mathbf{x}_a + \tilde{h}_{br}\mathbf{x}_b + \mathbf{n}_r && (21)\end{aligned}$$

where the data transmission vectors $\mathbf{x}_a$ and $\mathbf{x}_b$ are assumed to be composed of independent random variables with equal energy. Hence, the corresponding covariance matrices are

$$E\{\mathbf{x}_a\mathbf{x}_a^\dagger\} = P_a'\,\mathbf{I} = \frac{(1-\delta_a)mP_a}{(m-3)\alpha}\,\mathbf{I}, \quad (22)$$

$$E\{\mathbf{x}_b\mathbf{x}_b^\dagger\} = P_b'\,\mathbf{I} = \frac{(1-\delta_b)mP_b}{(m-3)\alpha}\,\mathbf{I}. \quad (23)$$

Using the same techniques described in [3], we can show that capacity lower bounds can be obtained when the channel estimation error is assumed to be another source of Gaussian noise. This is due to the fact that Gaussian noise is the worst uncorrelated noise for the Gaussian model. Now, we can write the new noise vector as

$$\mathbf{z}_r = \tilde{h}_{ar}\mathbf{x}_a + \tilde{h}_{br}\mathbf{x}_b + \mathbf{n}_r. \quad (24)$$

The covariance matrix of this noise vector can be expressed as

$$E\{\mathbf{z}_r{\mathbf{z}_r}^\dagger\} = \sigma_{z_r}^2\mathbf{I} = \sigma_{\tilde{h}_{ar}}^2 E\{\mathbf{x}_a\mathbf{x}_a^\dagger\} + \sigma_{\tilde{h}_{br}}^2 E\{\mathbf{x}_b\mathbf{x}_b^\dagger\} + N_0\mathbf{I}. \quad (25)$$

Using the approach employed in [3], we can obtain the worst-case achievable rate region of the MAC phase as follows:

$$\mathbb{R}_{MAC} := \{[R_a, R_b] \in \mathcal{R}_+^2 : R_a \le R_a^m, R_b \le R_b^m,\\ R_a + R_b \le R_\Sigma^{MAC}\} \quad (26)$$

with the individual and sum-rate upper bounds given by

$$R_a^m = E\left[\frac{\alpha(m-3)}{m}\log\left(1+\frac{P_a'|\hat{h}_{ar}|^2}{\sigma_{z_r}^2}\right)\right] \quad (27)$$

$$R_b^m = E\left[\frac{\alpha(m-3)}{m}\log\left(1+\frac{P_b'|\hat{h}_{br}|^2}{\sigma_{z_r}^2}\right)\right] \quad (28)$$

$$R_\Sigma^{MAC} = E\left[\frac{\alpha(m-3)}{m}\log\left(1+\frac{P_a'|\hat{h}_{ar}|^2}{\sigma_{z_r}^2}+\frac{P_b'|\hat{h}_{br}|^2}{\sigma_{z_r}^2}\right)\right] \quad (29)$$

where $\frac{P_a'|\hat{h}_{ar}|^2}{\sigma_{z_r}^2}$ and $\frac{P_b'|\hat{h}_{br}|^2}{\sigma_{z_r}^2}$ are given on the next page in (30) and (31) in which we have defined $w_{ar} \sim \mathcal{CN}(0,1)$ and $w_{br} \sim \mathcal{CN}(0,1)$. Since $\mathbb{R}_{MAC}$ is a pentagon, it can be completely described by five vertices. The two vertices where the individual rate constraints intersect with the sum-rate constraint are

$$v_{a\Sigma} := [R_a^m, R_b^{a\,\Sigma}] \text{ and } v_{b\Sigma} := [R_a^{b\,\Sigma}, R_b^m] \quad (32)$$

where

$$\begin{aligned}R_b^{a\,\Sigma} &= R_\Sigma^{MAC} - R_a^m && (33)\\ &= E\left[\frac{\alpha(m-3)}{m}\log\left(1+\frac{P_b'|\hat{h}_{br}|^2}{\sigma_{z_r}^2+P_a'|\hat{h}_{ar}|^2}\right)\right], && (34)\\ R_a^{b\,\Sigma} &= R_\Sigma^{MAC} - R_b^m && (35)\\ &= E\left[\frac{\alpha(m-3)}{m}\log\left(1+\frac{P_a'|\hat{h}_{ar}|^2}{\sigma_{z_r}^2+P_b'|\hat{h}_{br}|^2}\right)\right]. && (36)\end{aligned}$$

2) Broadcast Phase: In the succeeding BC phase of duration $(1-\alpha)(m-3)$ symbols, the relay forwards the previously received message m_a to node B and message m_b to node A. Similarly as for the source transmission vectors, we assume that the relay vector $\mathbf{x}_r$ has independent components with equal energy. Hence, the covariance matrix of the relay transmission vector is

$$E\{\mathbf{x}_r\mathbf{x}_r^\dagger\} = P_r'\,\mathbf{I} = \frac{(1-\delta_r)mP_r}{(m-3)(1-\alpha)}\,\mathbf{I}. \quad (37)$$

In this paper, we consider the superposition encoding strategy. Therefore, the messages m_a and m_b are separately encoded as for the point-to-point Gaussian channel. Then, the vector transmitted from the relay node obtained with superposition encoding can be expressed as

$$\mathbf{x}_r = \mathbf{w}_a + \mathbf{w}_b, \quad (38)$$

where the vectors $\mathbf{w}_a$ and $\mathbf{w}_b$ correspond to the codewords of the messages m_a and m_b, respectively. Note that $E\{\|\mathbf{x}_r\|^2\} = E\{\|\mathbf{w}_a\|^2\} + E\{\|\mathbf{w}_b\|^2\}$. Let β_1 and β_2 denote the proportion of relay transmit power P_r' used for the codewords w_a and w_b, respectively. Hence, $E\{\|\mathbf{w}_a\|^2\} = \beta_1 P_r'$ and $E\{\|\mathbf{w}_b\|^2\} = \beta_2 P_r'$. Then, the simplex

$$[\beta_1, \beta_2] \in [0,1]\times[0,1] : \beta_1 + \beta_2 \le 1 \quad (39)$$

characterizes the set of feasible relay power distributions that satisfy the relay transmit power constraint.

Now, the signals received at nodes A and B can be expressed as

$$\begin{aligned}\mathbf{y}_k &= h_{rk}\mathbf{x}_r + \mathbf{n}_k \qquad \text{for } k = a, b && (40)\\ &= \hat{h}_{rk}\mathbf{w}_a + \hat{h}_{rk}\mathbf{w}_b + \tilde{h}_{rk}\mathbf{x}_r + \mathbf{n}_k. && (41)\\ &= \hat{h}_{rk}\mathbf{w}_a + \hat{h}_{rk}\mathbf{w}_b + \mathbf{z}_k && (42)\end{aligned}$$

where we have defined

$$\mathbf{z}_k = \tilde{h}_{rk}\mathbf{x}_r + \mathbf{n}_k \quad (43)$$

as the effective noise vector with covariance matrix

$$E\{\mathbf{z}_k{\mathbf{z}_k}^\dagger\} = \sigma_{z_k}^2\mathbf{I} = \sigma_{\tilde{h}_{rk}}^2 E\{\mathbf{x}_r\mathbf{x}_r^\dagger\} + N_0\mathbf{I}. \quad (44)$$

Note that the user nodes A and B know their own transmitted codewords $\mathbf{w}_a$ and $\mathbf{w}_b$, respectively. Moreover, through the

$$\frac{P_a'|\hat{h}_{ar}|^2}{\sigma_{z_r}^2} = \frac{\delta_a(1-\delta_a)\sigma_{ar}^4 m^2 P_a^2(\sigma_{br}^2\delta_b m P_b + N_0)|w_{ar}^2|}{\sigma_{ar}^2 N_0(1-\delta_a)mP_a(\sigma_{br}^2\delta_b m P_b + N_0) + +\sigma_{br}^2 N_0(1-\delta_b)mP_b(\sigma_{ar}^2\delta_a m P_a + N_0) + N_0(m-3)\alpha(\sigma_{ar}^2\delta_a m P_a + N_0)(\sigma_{br}^2\delta_b m P_b + N_0)} \tag{30}$$

$$\frac{P_b'|\hat{h}_{br}|^2}{\sigma_{z_r}^2} = \frac{\delta_b(1-\delta_b)\sigma_{br}^4 m^2 P_b^2(\sigma_{ar}^2\delta_a m P_a + N_0)|w_{br}^2|}{\sigma_{ar}^2 N_0(1-\delta_a)mP_a(\sigma_{br}^2\delta_b m P_b + N_0) + \sigma_{br}^2 N_0(1-\delta_b)mP_b(\sigma_{ar}^2\delta_a m P_a + N_0) + N_0(m-3)\alpha(\sigma_{ar}^2\delta_a m P_a + N_0)(\sigma_{br}^2\delta_b m P_b + N_0)} \tag{31}$$

network training phase, they are equipped with the channel estimate $\hat{h}_{rk}$. Hence, they can suppress the interference due to their own messages, and the signals at nodes A and B can now be expressed, respectively, as

$$\mathbf{y}_a = \hat{h}_{rk}\mathbf{w}_b + \mathbf{z}_k, \text{ and} \tag{45}$$

$$\mathbf{y}_b = \hat{h}_{ra}\mathbf{w}_a + \mathbf{z}_k. \tag{46}$$

It should also be noticed that due to the presence of channel estimation errors, self-interference cannot be canceled perfectly. The residual interference components $\tilde{h}_{rk}\mathbf{w}_a$ at node A and $\tilde{h}_{rk}\mathbf{w}_b$ at node B are incorporated into the noise term $\mathbf{z}_k$.

Now, assuming superposition encoding at the relay and self-interference suppression at the receiver nodes, and regarding the noise component, which includes the residual interference terms and the background noise, as Gaussian distributed, we can easily see that the worst-case achievable rate region of the BC phase is given by

$$\mathbb{R}_{BC} := \{[R_a, R_b] \in \mathcal{R}_+^2 : R_a \le R_a^b(\beta_1), R_b \le R_b^b(\beta_2)\} \tag{47}$$

where

$$R_a^b = E\left[\frac{(1-\alpha)(m-3)}{m}\log\left(1 + \frac{P_r'\beta_1|\hat{h}_{rb}|^2}{\sigma_{z_b}^2}\right)\right] \tag{48}$$

$$R_b^b = E\left[\frac{(1-\alpha)(m-3)}{m}\log\left(1 + \frac{P_r'\beta_2|\hat{h}_{ra}|^2}{\sigma_{z_a}^2}\right)\right] \tag{49}$$

with

$$\frac{P_r'|\hat{h}_{rb}|^2}{\sigma_{z_b}^2} = \frac{\delta_r(1-\delta_r)\sigma_{rb}^4 m^2 P_r^2|w_{rb}^2|}{\sigma_{rb}^2 N_0(1-\delta_r)mP_r + N_0(m-3)(1-\alpha)(\sigma_{rb}^2\delta_r m P_r + N_0)}$$

$$\frac{P_r'|\hat{h}_{ra}|^2}{\sigma_{z_a}^2} = \frac{\delta_r(1-\delta_r)\sigma_{ra}^4 m^2 P_r^2|w_{ra}^2|}{\sigma_{ra}^2 N_0(1-\delta_r)mP_r + N_0(m-3)(1-\alpha)(\sigma_{ra}^2\delta_r m P_r + N_0)}.$$

Above, $w_{ra} \sim \mathcal{CN}(0,1)$ and $w_{rb} \sim \mathcal{CN}(0,1)$.

On the boundary of the BC achievable region $\mathbb{R}_{BC}$, we have $\beta_1+\beta_2 = 1$. Let us set $\beta_1 = \beta$ and $\beta_2 = 1-\beta$. Now, any point on the boundary can be achieved by varying β from 0 to 1. Of particular interest is the value of β that achieves the maximum sum rate $R_{\sum}^b := \max_{[R_a,R_b]\in\mathbb{R}_{BC}} R_a + R_b$ in the broadcast phase. In general, it is difficult to analytically determine the sum-rate-maximizing value of β for the cases in which β is kept fixed by the relay for different channel realizations. On the other hand, if the relay knows the channel estimates $\hat{h}_{ra}$ and $\hat{h}_{rb}$ of the source nodes, then it can adapt β to these estimates in each block. For this case, we can find the optimal β^* value, which maximizes the sum rate, in closed-form as follows:

$$\beta^* = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{1}{2P_r'}\left(\frac{\sigma_{z_a}^2}{|\hat{h}_{ra}|^2} - \frac{\sigma_{z_b}^2}{|\hat{h}_{rb}|^2}\right) < 0 \\ \frac{1}{2} + \frac{1}{2P_r'}\left(\frac{\sigma_{z_a}^2}{|\hat{h}_{ra}|^2} - \frac{\sigma_{z_b}^2}{|\hat{h}_{rb}|^2}\right) & \text{if } 0 \le \frac{1}{2} + \frac{1}{2P_r'}\left(\frac{\sigma_{z_a}^2}{|\hat{h}_{ra}|^2} - \frac{\sigma_{z_b}^2}{|\hat{h}_{rb}|^2}\right) \le 1 \\ 1 & \text{if } \frac{1}{2} + \frac{1}{2P_r'}\left(\frac{\sigma_{z_a}^2}{|\hat{h}_{ra}|^2} - \frac{\sigma_{z_b}^2}{|\hat{h}_{rb}|^2}\right) > 1 \end{cases}$$

3) Achievable Rate Region for Two-Way Relay Channel: The worst-case achievable rate region of the two-way decode-and-forward relaying scheme considered in this paper is given by the intersection of the rate regions of the multiple-access and broadcast phases:

$$\mathbb{R}(\alpha) := \mathbb{R}_{MAC} \bigcap \mathbb{R}_{BC}. \tag{50}$$

IV. Numerical Results

The achievable rate regions obtained in the previous section depend on several parameters, such as the fractions of power allocated to training δ_a, δ_b, and δ_r; the fraction of time allocated to the MAC phase α; the relay power allocation parameter β; the coherence block length m; and the fading variances σ^2. Other than some special cases as seen in the discussion of the sum-rate-maximizing value of β above, finding closed-form expressions for the optimized values of training and data transmission parameters seems unlikely in general scenarios. For this reason, we resort to numerical methods in order to identify the impact of these parameters.

In Figure 1, we plot the achievable rate regions of the multiple access and broadcast phases of two-way relaying for different values of α when the other parameters are $P_a = P_b = P_r = 1, m = 50, \sigma_{ra} = \sigma_{ar} = \sigma_{br} = \sigma_{rb} = 1, \delta_a = \delta_b = \delta_r = 0.1$. It can be easily seen that the MAC region expands and the BC region shrinks, as expected, as the value of α is increased. Hence, for small values of α, MAC region dictates the achievable rate region of two-way relaying while the BC region does so for larger α. In Figs. 2 and 3, we plot the sum rate of users A and B as a function of α for relatively high and low SNR values, respectively. In both cases, the optimal α value is around 0.55, indicating that when sum rate is

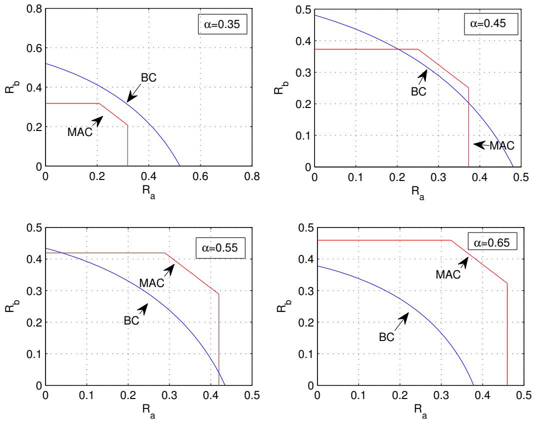

Fig. 1. Achievable Rate Region for different values of α when $P_a = P_b = P_r = 1, m = 50, \sigma_{ra} = \sigma_{ar} = \sigma_{br} = \sigma_{rb} = 1, \delta_a = \delta_b = \delta_r = 0.1$.

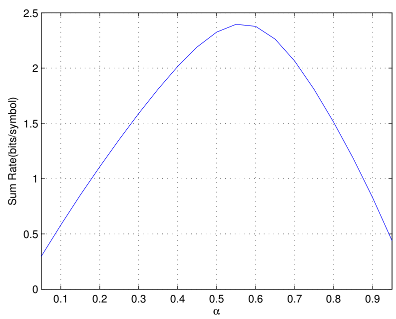

Fig. 2. Sum rate vs. α with $P_a = P_b = P_r = 10, m = 50, \sigma_{ra} = \sigma_{ar} = 1, \sigma_{br} = \sigma_{rb} = 2, \delta_a = \delta_b = \delta_r = 0.1$.

concerned, equal time/bandwidth allocation between multiple access and broadcast phases is not necessarily optimal.

Next, we investigate how much power needs to be spent on training to maximize the sum rate. For simplification, we assume all nodes spend the same ratio of power for training, i.,e. $\delta_a = \delta_b = \delta_r = \delta$. In Fig. 4, sum rate is plotted as a function of this common δ value. We observe that the

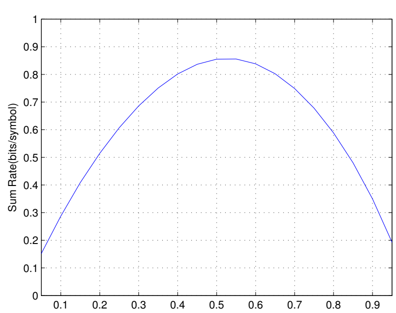

Fig. 3. Sum rate vs. α with $P_a = P_b = P_r = 1, m = 50, \sigma_{ra} = \sigma_{ar} = 1, \sigma_{br} = \sigma_{rb} = 2, \delta_a = \delta_b = \delta_r = 0.1$.

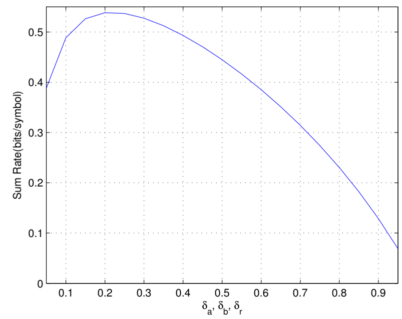

Fig. 4. Sum rate vs. $\delta_a, \delta_b, \delta_r$ with $P_a = P_b = P_r = 1, m = 50, \sigma_{ra} = \sigma_{ar} = \sigma_{br} = \sigma_{rb} = 1, \alpha = 0.55$.

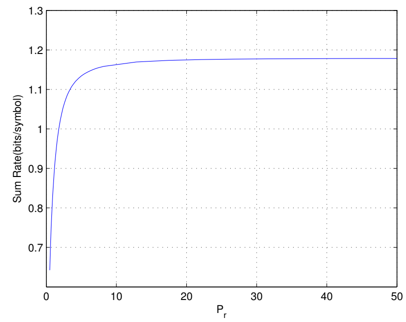

Fig. 5. Sum rate vs. P_r with $P_a = P_b = 1, m = 50, \sigma_{ra} = \sigma_{ar} = 1, \sigma_{br} = \sigma_{rb} = 2, \delta_a = \delta_b = \delta_r = 0.1, \alpha = 0.55$.

optimal fraction of power allocated for training is around 0.2. Further increase in training power leads to a decrease in the overall throughput as it diminishes the available power for data transmission.

Finally, in Fig. 5, we provide the sum rate curve as a function of the relay power P_r. We see that the sum rate saturates as the relay power is increased beyond some threshold. This is mainly because of the fact that MAC phase becomes the bottleneck of the whole system for large relay power levels.

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