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An analytic description for the yield, $\mathcal{P}(\mathbf{p})$, of high-energy electrons ionized from an atom by a short (few-cycle) laser pulse is obtained quantum mechanically. Factorization of $\mathcal{P}(\mathbf{p})$ in terms of an electron wave packet and the cross section for elastic electron scattering (EES) is shown to occur only for an ultrashort pulse, while in general $\mathcal{P}(\mathbf{p})$ involves interference of EES amplitudes with laser-field-dependent momenta. The analytic predictions agree well with accurate numerical results.

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The process of above-threshold ionization (ATI) by a short (few-cycle) laser pulse is highly sensitive to the parameters of the pulse, whose vector potential $\mathbf{A}(t)$ (for the case of linear polarization) may be parameterized as

$$\mathbf{A}(t) = \hat{\mathbf{z}}A(t), \qquad A(t) = f(t)\sin(\omega t + \phi), \tag{1}$$

where $f(t)$ is the pulse envelope (with its maximum at $t = 0$), ω is the carrier frequency, and ϕ is the carrier-envelope phase (CEP). The first ATI experiments with CEP-stabilized short pulses [1] found a significant CEP dependence of the electron yield and differences in the energy extent of the ATI plateau for electrons with negative and positive momentum projections $p_\parallel = \mathbf{p}\cdot\hat{\mathbf{z}} = p\cos\theta$. More detailed measurements [2] found CEP-dependent interference fringes (differing for electrons with $p_\parallel < 0$ and $p_\parallel > 0$) in angle-resolved ATI spectra produced by different half-cycles of a few-cycle pulse. These peculiarities have been confirmed by numerical solutions of the time-dependent Schrödinger equation (TDSE) and explained within the improved strong field approximation, in which the atomic potential $U(r)$ is taken into account perturbatively, in a Born-like approximation [3]. However, recent experiments [4] show that a perturbative treatment of $U(r)$ is inadequate to extract from ATI spectra information on atomic dynamics, such as the field-free differential cross section (DCS) for elastic electron scattering (EES) from the potential $U(r)$. The phenomenological factorization of the ATI yield in terms of an electron wave packet (EWP) and the *exact* (non-Born) DCS for EES [5,6] is very useful for analyzing signatures of atomic dynamics in ATI spectra. For a monochromatic field, this factorization was justified theoretically in Ref. [7] [cf. also Ref. [8] in which this factorization was introduced heuristically (as the authors state in a later paper [9])]. For a one-dimensional zero-range potential model, analytic derivations of the ATI yield for an arbitrary shape of $A(t)$ have been performed in Ref. [10] using an adiabatic approach. However, the validity of a factorized formula for the ATI yield for a short pulse with stabilized CEP, suggested in Ref. [11], remains unjustified theoretically, and is a challenge for theory.

In this Letter we present an analytic description of ATI by a few-cycle, CEP-stabilized laser pulse. Our closed-form analytic formulas show that the photoelectron yield, in general, cannot be factorized into the product of an EWP and the DCS for EES, but involves a sum of DCSs with different (pulse-shape-dependent) electron momenta as well as interference between corresponding EES amplitudes. Only in the ultrashort pulse case (in which only electrons ionized by a *single optical cycle* of the pulse contribute significantly to the photoelectron yield) do our results reduce to factorized form. For the H and He atoms, our TDSE results confirm the high accuracy of our analytic description of the high-energy ATI plateau.

To describe ATI by a short laser pulse, we generalize our analytic description of ATI plateau spectra produced by a monochromatic field [7] in a way similar to that used to describe harmonic generation by a short pulse [12]. The key idea is to consider first ATI by an infinite train of short pulses (1) separated in time by $\mathcal{T}$ with $\mathcal{T} > \tau$, where τ is the duration of the single short pulse (1) whose ATI spectrum we seek. Owing to the periodicity in time of the pulse train, we can employ the quasistationary quasi-energy state approach [13] to obtain an *ab initio* formulation for the differential n-photon ionization rates $\Gamma(\mathbf{p}_n) \equiv d\Gamma(\mathbf{p}_n)/d\Omega_{\mathbf{p}_n}$ in a periodic field of frequency $\omega_\tau = 2\pi/\mathcal{T}$, where $\mathbf{p}_n$ is the photoelectron momentum. The total ionization probability for the period $\mathcal{T}$ is

$$P = \mathcal{T}\Gamma = \frac{2\pi}{\omega_\tau}\sum_{n>n_0}\int \Gamma(\mathbf{p}_n)d\Omega_{\mathbf{p}_n}, \tag{2}$$

where n_0 is the minimum number of photons for ionization from a bound state of energy $E_0 = -\hbar^2\kappa^2/(2m)$. In the limit $\mathcal{T} \to \infty$ ($\omega_\tau \to 0$), the sum over n in Eq. (2) can be replaced by an integral over the electron's momentum $p_n \equiv p$ or energy $E = p^2/(2m)$. The result we obtain is

$$P = \iint \mathcal{P}(\mathbf{p}) dE d\Omega_{\mathbf{p}},$$

where the doubly differential ionization probability, $\mathcal{P}(\mathbf{p})$, for a *single short pulse* has the following form:

$$\mathcal{P}(\mathbf{p}) \equiv \frac{d^2P}{dEd\Omega_{\mathbf{p}}} = \lim_{\omega_\tau \to 0} \frac{2\pi}{\hbar\omega_\tau^2} \Gamma(\mathbf{p}). \tag{3}$$

For an electron in a short-range potential $U(r)$, the rate $\Gamma(\mathbf{p})$ can be obtained (using time-dependent effective range theory [14]) in analytic form in the tunneling limit. This latter result can then be straightforwardly generalized to the case of an active atomic electron, as in Ref. [7]. Our analysis shows that the ATI amplitude $\mathcal{A}(\mathbf{p})$ for a short pulse can be presented as a sum of partial amplitudes, $\mathcal{A}_j(\mathbf{p})$, describing electrons ionized at each jth ($j = 1, 2, \ldots, 2N$) optical half-cycle $T/2 = \pi/\omega$ of the N-cycle pulse (1). In the low-frequency limit ($\hbar\omega \ll |E_0|$), these amplitudes can be estimated using a modified saddle-point analysis, as done similarly in Ref. [7]. As a result, the amplitudes $\mathcal{A}_j(\mathbf{p})$ depend on tunneling ionization $[t_i^{(j)}]$ and rescattering $[t_r^{(j)}]$ times for the jth *half-cycle* [where $t_r^{(j)}$ lies in the $(j+1)$th half-cycle], which satisfy the system of classical equations:

$$\begin{aligned} &A(t_i^{(j)}) - \frac{1}{t_r^{(j)} - t_i^{(j)}} \int_{t_i^{(j)}}^{t_r^{(j)}} A(t) dt = 0, \\ &2F(t_r^{(j)}) + \frac{1}{c} \frac{A(t_r^{(j)}) - A(t_i^{(j)})}{t_r^{(j)} - t_i^{(j)}} = 0, \end{aligned} \tag{4}$$

where $\hat{\mathbf{z}}F(t)$ is the electric field of the pulse $[F(t) = -(1/c)dA/dt]$. The desired solutions $[t_i^{(j)}, t_r^{(j)}]$ of the system (4) are those real solutions that ensure the shortest return time, $\Delta t_j = (t_r^{(j)} - t_i^{(j)}) < T$, and the maximum classical energy, $\mathcal{E}_{\max,j}^{(\mathrm{cl})}$, gained by an electron from the laser field over the time Δt_j. With known $t_i^{(j)}$ and $t_r^{(j)}$, the amplitude $\mathcal{A}_j(\mathbf{p})$ can be approximated in a way similar to that for a monochromatic field [7]. Moreover, for positive (or negative) $p_\parallel$ only those partial amplitudes $\mathcal{A}_j(\mathbf{p})$ contribute for which $F(t_i^{(j)}) < 0$ [or $F(t_i^{(j)}) > 0$].

Omitting technical details, we focus here on the final analytic result for $\mathcal{P}(\mathbf{p})$, which involves two terms:

$$\mathcal{P}(\mathbf{p}) = \mathcal{P}_{\mathrm{dir}}(\mathbf{p}) + \mathcal{P}_{\mathrm{int}}(\mathbf{p}). \tag{5}$$

The first ("direct") term is the sum of partial rates $\Gamma_j(\mathbf{p})$:

$$\mathcal{P}_{\mathrm{dir}}(\mathbf{p}) = \frac{2\pi}{\hbar\omega^2} {\sum_j}' \Gamma_j(\mathbf{p}), \tag{6}$$

where the prime on the sum means that the summation is taken over j of the same (even or odd) parity depending on the sign of $p_\parallel$. The rate $\Gamma_j(\mathbf{p})$ describes photoelectrons created by the jth half-cycle of the pulse and can be represented as a product of three factors similar to that for a monochromatic field [7]:

$$\Gamma_j(\mathbf{p}) = \mathcal{I}_j \mathcal{W}_j \sigma(\mathbf{p} - \Delta\mathbf{p}_j), \qquad \Delta\mathbf{p}_j = -|e|\mathbf{A}(t_r^{(j)})/c. \tag{7}$$

The tunneling factor $\mathcal{I}_j$ describes the tunneling of an active atomic electron at the moment $t_i^{(j)}$:

$$\mathcal{I}_j = \frac{m}{\pi\hbar\kappa} \tilde{\gamma}_j^2 \Gamma_{\mathrm{st}}(\tilde{F}_j), \tag{8}$$

where $\tilde{F}_j = |F(t_i^{(j)})|$, $\tilde{\gamma}_j = \hbar\omega/(|e|\tilde{F}_j\kappa^{-1})$ is an effective value of the Keldysh parameter for the jth half-cycle, and $\Gamma_{\mathrm{st}}(\tilde{F}_j)$ is the tunneling rate for a bound atomic electron in an effective static electric field $\hat{\mathbf{z}}F(t_i^{(j)})$ [15]. The factor $\mathcal{W}_j$ in Eq. (7) describes the propagation of the electron in the laser-dressed continuum between the tunneling and rescattering events and involves the Airy function $\mathrm{Ai}(x)$:

$$\mathcal{W}_j = \frac{p}{\hbar} \frac{\mathrm{Ai}^2(\xi_j)}{\zeta_j^{2/3} \Delta t_j^3 \omega_{\mathrm{at}}^2}, \qquad \xi_j = \frac{\Delta E_j}{\zeta_j^{1/3} E_{\mathrm{at}}}, \tag{9}$$

where $E_{\mathrm{at}} = \hbar\omega_{\mathrm{at}} = e^2/a$, a is the Bohr radius,

$$\Delta E_j = \frac{(\mathbf{p} - \Delta\mathbf{p}_j)^2}{2m} - \mathcal{E}_{\max,j}^{(\mathrm{cl})} + 2|E_0| \frac{F(t_r^{(j)})}{F(t_i^{(j)})},$$

$$\mathcal{E}_{\max,j}^{(\mathrm{cl})} = \frac{e^2[A(t_r^{(j)}) - A(t_i^{(j)})]^2}{2mc^2},$$

$$\zeta_j = \frac{1}{F_{\mathrm{at}}^2} \left\{ -\frac{\dot{F}(t_r^{(j)})}{2|e|} \left[p_\parallel + \frac{|e|}{c} A(t_i^{(j)}) \right] + F^2(t_r^{(j)}) \left[4\frac{F(t_r^{(j)})}{F(t_i^{(j)})} - 3 \right] \right\},$$

$$F_{\mathrm{at}} = \frac{|e|}{a^2}, \qquad \dot{F}(t_r^{(j)}) \equiv \frac{dF(t)}{dt} \bigg|_{t = t_r^{(j)}}.$$

The factor $\sigma(\mathbf{p} - \Delta\mathbf{p}_j)$ in Eq. (7) is the field-free DCS for EES from the atomic core with energy $E_r = (\mathbf{p} - \Delta\mathbf{p}_j)^2/(2m)$ and scattering angle $\Theta = \pi - \theta_r$, where

$$\cos\theta_r = |(\mathbf{p} - \Delta\mathbf{p}_j) \cdot \hat{\mathbf{z}}|/|\mathbf{p} - \Delta\mathbf{p}_j|. \tag{10}$$

For the H atom, $\sigma(\mathbf{p} - \Delta\mathbf{p}_j)$ is known analytically,

$$\sigma(\mathbf{p} - \Delta\mathbf{p}_j) = \frac{m^2 e^4}{(\mathbf{p} - \Delta\mathbf{p}_j)^4} (1 + \cos\theta_r)^{-2}, \tag{11}$$

while for other atoms experimental or theoretical data for $\sigma(\mathbf{p})$ should be used, substituting there $\mathbf{p} \to (\mathbf{p} - \Delta\mathbf{p}_j)$.

The term $\mathcal{P}_{\mathrm{int}}(\mathbf{p})$ in Eq. (5) originates from the interference between the half-cycle ionization amplitudes $\mathcal{A}_j(\mathbf{p})$ and $\mathcal{A}_{j'}(\mathbf{p})$ having the same parity of j and j'. It thus involves their phase difference $\Phi_{j,j'}$:

$$\mathcal{P}_{\rm int} = \frac{2\pi}{\hbar\omega^2}\sum_{j\neq j'}{}' s_{j,j'}\sqrt{\Gamma_j(\mathbf{p})\Gamma_{j'}(\mathbf{p})}\cos\Phi_{j,j'}(\mathbf{p}), \quad (12)$$

$$\Phi_{j,j'} = \varphi_j - \varphi_{j'} + \psi(\mathbf{p} - \Delta\mathbf{p}_j) - \psi(\mathbf{p} - \Delta\mathbf{p}_{j'}), \quad (13)$$

$$\hbar\varphi_j = S_{\mathbf{p}}(t_r^{(j)}) - \int_{t_i^{(j)}}^{t_r^{(j)}}[\mathcal{E}(t, t_i^{(j)}) - E_0]dt, \quad (14)$$

$$S_{\mathbf{p}}(t_r^{(j)}) = \int^{t_r^{(j)}}\left(\frac{[\mathbf{p} + |e|\mathbf{A}(t)/c]^2}{2m} - E_0\right)dt,$$

$$\mathcal{E}(t, t_i^{(j)}) = \frac{e^2}{2mc^2}\left[A(t) - A(t_i^{(j)})\right]^2,$$

where $s_{j,j'} = \mathrm{sgn}[\mathrm{Ai}(\xi_j)\mathrm{Ai}(\xi_{j'})]$ $(= \pm 1)$, and $\psi(\mathbf{p})$ is the phase of the EES amplitude $f(\mathbf{p})$:

$$f(\mathbf{p}) = |f(\mathbf{p})|e^{i\psi(\mathbf{p})}. \quad (15)$$

In Figs. 1 and 2 we compare our analytic predictions for the probability $\mathcal{P}(\mathbf{p})$ with numerical TDSE results for the case of a pulse (1) having a $\cos^2$-shaped envelope:

$$f(t) = -\frac{cF}{\omega}\cos^2\left(\frac{t\pi}{\tau}\right), \qquad t \in [-\tau/2, \tau/2], \quad (16)$$

where $\tau = 2\pi N/\omega$. The peak intensity of the pulse is defined as $I = cF^2/(8\pi)$. The 3D TDSE for the H atom was solved using two different numerical algorithms, which provide the same results for the ATI spectra. (For details of the numerical solution of the TDSE for ATI, see Refs. [16,17].) For He, we used the single active electron approximation with the same one-electron potential as in Ref. [18]. [This potential was also used to calculate $f(\mathbf{p})$ and $\sigma(\mathbf{p})$ for He.] The result (5) for $\mathcal{P}(\mathbf{p})$ agrees well with the TDSE results, as shown in Figs. 1, 2(a), and 2(b) for the H atom [for pulses with $N = 4$ and 6, whose full widths at half maximum (FWHM) of the intensity are 6.3 and 9.5 fs, with $T = 4.3$ fs] and in Figs. 2(c) and 2(d) for He for a six-cycle pulse (with FWHM of 5.8 fs, $T = 2.67$ fs).

Both the momentum distribution $\mathcal{P}(p_\parallel, p_\perp)$ in Fig. 1(a) and the ATI spectra in Figs. 1(b) and 2 exhibit a left-right asymmetry [3], which in our analysis originates from different contributions to $\mathcal{P}(\mathbf{p})$ of half-cycles with $F(t) < 0$ and $F(t) > 0$. Indeed, electrons with $p_\parallel > 0$ are created by half-cycles with $F(t) < 0$, while those with $p_\parallel < 0$ by half-cycles with $F(t) > 0$. Moreover, due to the pulse-shape and CEP dependences of $A(t)$ and $F(t)$, the times $t_i^{(j)}$, $t_r^{(j)}$ and the energies $\mathcal{E}_{\max,j}^{(\rm cl)}$ are different for different j, resulting in different maximal energies of electrons (or plateau cutoff positions), $E_{\rm cut}^{(j)}$, for half-cycles with different j; e.g., for the case $\theta = 0$ or π:

$$E_{\rm cut}^{(j)} = (|\Delta\mathbf{p}_j|/\sqrt{2m} + \sqrt{\mathcal{E}_{\max,j}^{(\rm cl)}})^2. \quad (17)$$

For $p_\parallel > 0$ $(p_\parallel < 0)$ in Fig. 1(b) the major contribution to $\mathcal{P}(\mathbf{p})$ comes from the single half-cycle with $j = 4$ $(j = 5)$, for which $E_{\rm cut}^{(4)} \approx 9.4u_p$ $(E_{\rm cut}^{(5)} \approx 8.0u_p)$, where $u_p = e^2F^2/(4m\omega^2) = 23.7$ eV. Hence, for the ATI spectra in Fig. 1(b), $\mathcal{P}(\mathbf{p}) \approx \mathcal{P}_{\rm dir}(\mathbf{p})$ has a factorized form (cf. Eqs. (6) and (7) with $j = 4$ for $p_\parallel > 0$ and $j = 5$ for $p_\parallel < 0$).

The large-scale interference minima in the ATI spectra in Figs. 1(b) and 2 originate from interference of two (short and long) electron trajectories [that contribute to the partial amplitudes $\mathcal{A}_j(\mathbf{p})$]; they are similar to those for a monochromatic field [7,19]. Besides these "intracycle"

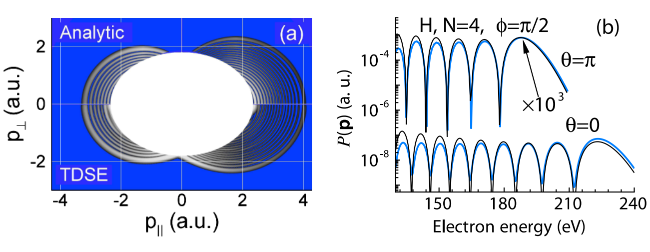

FIG. 1 (color online). (a): The momentum distribution $\mathcal{P}(p_\parallel, p_\perp)$ of electrons ionized from the H atom by a four-cycle $\cos^2$-shaped pulse with wavelength $\lambda = 1.3$ μm, peak intensity 1.5×10^{14} W/cm^2, and CEP $\phi = 0$. Analytic (upper half-panel) and TDSE results (bottom half-panel) are shown for electron momenta at the high-energy end of the ATI plateau, i.e., outside the white ellipse centered at $p_\parallel = p_\perp = 0$. (b) ATI spectra for the laser pulse as in (a) but for $\phi = \pi/2$ and $\theta = 0$ and π. Thin (black) lines: Eq. (5); thick (blue) lines: TDSE results. Data for $\theta = \pi$ have been multiplied by 10^3.

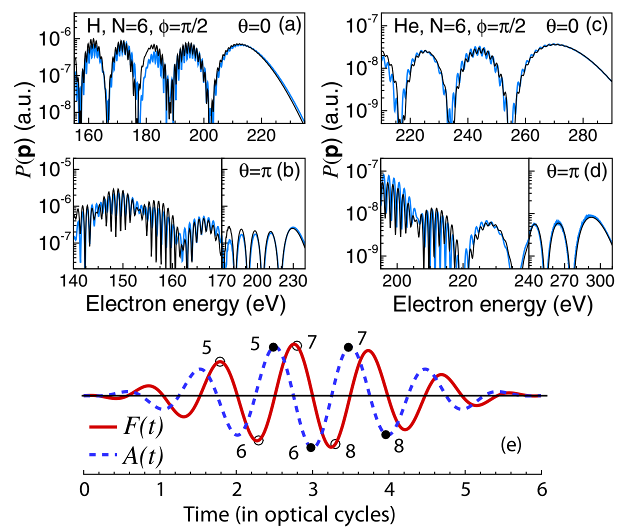

FIG. 2 (color online). ATI spectra produced by a six-cycle $\cos^2$-shaped pulse with $\phi = \pi/2$ for (a),(b) hydrogen (for $\lambda = 1.3$ μm, $I = 1.5 \times 10^{14}$ W/cm^2, $u_p = 23.7$ eV) and (c),(d) helium (for $\lambda = 0.8$ μm, $I = 5 \times 10^{14}$ W/cm^2, $u_p = 29.9$ eV). $\theta = 0$ in (a),(c) and $\theta = \pi$ in (b),(d). Thin (black) lines: Eq. (5); thick (blue) lines: TDSE results. (e) Time evolution of $F(t)$ and $A(t)$ for a six-cycle $\cos^2$-shaped pulse with $\phi = \pi/2$. Open (solid) circles mark the positions of times $t_i^{(j)}$ $[t_r^{(j)}]$, with the numbers marking the index j of the contributing half-cycle.

oscillations, there are fine-scale modulations of $\mathcal{P}(\mathbf{p})$ that have a period ΔE of order $\hbar\omega$ and are characteristic for a short pulse [3], as seen clearly in Fig. 2. These "intercycle" oscillations originate from the interference term $\mathcal{P}_{\rm int}(\mathbf{p})$ in Eq. (5) and become pronounced for pulses with $N > 4$, when two adjacent partial rates, $\Gamma_j(\mathbf{p})$ and $\Gamma_{j+2}(\mathbf{p})$, have different, large magnitudes. In Figs. 2(a) and 2(b) the two are $\Gamma_6(\mathbf{p})$ and $\Gamma_8(\mathbf{p})$ with $E_{\rm cut}^{(6)} \approx 9.7u_p$ and $E_{\rm cut}^{(8)} \approx 6.3u_p$ for $p_\| > 0$, and $\Gamma_5(\mathbf{p})$ and $\Gamma_7(\mathbf{p})$ with $E_{\rm cut}^{(5)} \approx 8.0u_p$ and $E_{\rm cut}^{(7)} \approx 9.0u_p$ for $p_\| < 0$ [cf. Figure 2(e)]. Since for $\theta = 0$ in Fig. 2(a) the cutoff energies and rates $\Gamma_5(\mathbf{p})$ and $\Gamma_7(\mathbf{p})$ have comparable magnitudes, the fine-scale fringes modulate the large-scale oscillations up to the plateau cutoff. However, for $\theta = \pi$, both the cutoff positions and tunneling factors ($I_8 \approx 2I_6$) are rather different, so that fine-scale oscillations in Fig. 2(b) are significant only for electron energies $E \lesssim E_{\rm cut}^{(8)}$, where the rates $\Gamma_6(\mathbf{p})$ and $\Gamma_8(\mathbf{p})$ overlap. The same considerations explain also the intracycle and intercycle modulation features of the ATI spectra for He in Figs. 2(c) and 2(d).

To estimate the period ΔE of fine-scale oscillations analytically, we consider the interference factor $\cos\Phi_{j,j+2}(\mathbf{p})$ in Eq. (12) and approximate the difference $\Delta\Phi(p,\theta) \equiv \Phi_{j,j+2}(p+\Delta p,\theta) - \Phi_{j,j+2}(p,\theta)$ as follows:

$$\Delta\Phi(p,\theta) \approx \frac{d\Phi_{j,j+2}}{dp}\Delta p \approx \frac{d\Phi_{j,j+2}}{dp}\frac{m\Delta E}{p}. \tag{18}$$

On the other hand, since $\Delta\Phi(p,\theta) = 2\pi$ for two adjacent fine-scale peaks, the use of Eqs. (13) and (18) gives

$$\Delta E \approx 2\pi\hbar/\Delta T, \qquad \Delta T = \Delta t_{\rm cl} + \Delta t_{\rm dis} + \Delta t_{\rm W},$$
$$\Delta t_{\rm cl} = t_r^{(j+2)} - t_r^{(j)}, \qquad \Delta t_{\rm dis} = \frac{|e|}{cp}\int_{t_r^{(j)}}^{t_r^{(j+2)}} A(t)dt,$$
$$\Delta t_{\rm W} = \frac{m\hbar}{p}\left[\frac{d\psi(\mathbf{p}-\Delta\mathbf{p}_{j+2})}{dp} - \frac{d\psi(\mathbf{p}-\Delta\mathbf{p}_j)}{dp}\right], \tag{19}$$

where the classical times $\Delta t_{\rm cl}$ and $\Delta t_{\rm dis}$ are the difference between two rescattering times and the laser-induced "displacement" time [3], while $\Delta t_{\rm W}$ has a quantum origin: it is the difference between the Wigner-like time delays [20] for the first and second rescattering events.

Our results for $\mathcal{P}(\mathbf{p})$ are very general and applicable to any atom for which either theoretical or experimental data on the field-free DCS $\sigma(\mathbf{p})$ and the phase $\psi(\mathbf{p})$ of the EES amplitude are available. Since our analytic derivations were carried out in the tunneling regime, the general condition for validity of Eq. (5) for $\mathcal{P}(\mathbf{p})$ is that the Keldysh parameters $\tilde{\gamma}_j$ for all contributing half-cycles should be less than unity ($0.56 \le \tilde{\gamma}_j \le 0.83$ in our results for H, while $0.67 \le \tilde{\gamma}_j \le 0.99$ for He). Our derivations show clearly that $\mathcal{P}(\mathbf{p})$ cannot in general be factorized because the term $\Delta\mathbf{p}_j = -|e|\mathbf{A}(t_r^{(j)})/c$ in Eq. (7) is sensitive to j. [Moreover, owing to the dependence of $\mathcal{P}_{\rm int}(\mathbf{p})$

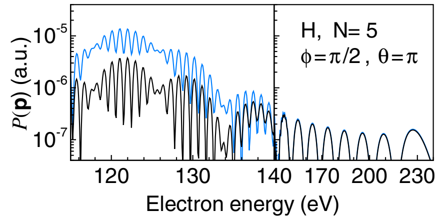

FIG. 3 (color online). H atom ATI spectra for the laser pulse of Fig. 2(b) with $N = 5$. Lower (black) curve: Eq. (5) result. Upper (blue) curve: approximate factorized formula result (cf. text).

on $\psi(\mathbf{p}-\Delta\mathbf{p}_j)$, $\mathcal{P}(\mathbf{p})$ is more sensitive to the atomic dynamics than for a monochromatic field.] Nevertheless, factorization of $\mathcal{P}(\mathbf{p})$ can occur for a few-cycle pulse [as, e.g., in Fig. 1(b)], when only a single rate $\Gamma_j(\mathbf{p})$ contributes predominantly to $\mathcal{P}(\mathbf{p})$. However, in this case, the CEP-dependent "half-cycle" EWPs $w_j = I_j\mathcal{W}_j$ are different for electrons with $p_\| > 0$ and $p_\| < 0$. The factorization postulated in Ref. [11] follows from our results upon replacing $\Delta\mathbf{p}_j$ in $\sigma(\mathbf{p}-\Delta\mathbf{p}_j)$ in Eq. (7) and in the phase $\psi(\mathbf{p}-\Delta\mathbf{p}_j)$ in Eq. (13) by that for the half-cycle with maximum cutoff energy $E_{\rm cut}^{(j)}$, i.e., with maximum value of $|\mathbf{A}(t_r^{(j)})|$. [After such replacement, Eq. (5) takes a factorized form and provides an explicit expression of the EWP in this case.] However, since the DCS $\sigma(\mathbf{p})$ usually decreases with increasing p, this approximate factorization overestimates the contribution of interfering half-cycles with smaller cutoff energies $E_{\rm cut}^{(j)}$, as the upper curve in Fig. 3 shows; the lower (exact) curve agrees well with TDSE results (not shown). For $E > 140$ eV, the exact and approximate results for $\mathcal{P}(\mathbf{p})$ in Fig. 3 coincide since the *single* ionization amplitude, $\mathcal{A}_{j=4}(\mathbf{p})$, is dominant. Thus in this energy region the result (5) for $\mathcal{P}(\mathbf{p})$ indeed reduces to a factorized form with the EWP $w_4 = I_{j=4}\mathcal{W}_{j=4}$, while for $E < 140$ eV the interference between amplitudes $\mathcal{A}_{j=4}(\mathbf{p})$ and $\mathcal{A}_{j=6}(\mathbf{p})$ becomes significant and such factorization is not possible.

To conclude, we have derived quantum mechanically an analytic result for the ATI probability $\mathcal{P}(\mathbf{p})$ that is valid in the high-energy part of the ATI plateau for a short laser pulse of any shape and duration. These results allow one to describe analytically the left-right asymmetry as well as the large-scale (intracycle) and fine-scale (intercycle) oscillations in ATI spectra. To use our results, only the EES amplitude $f(\mathbf{p})$ for the target atom and the solutions $[t_i^{(j)}, t_r^{(j)}]$ of the classical equations (4) for a given short pulse are needed. Our results agree well with TDSE results and provide an efficient tool for the quantitative description of short-pulse ATI spectra.

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