Precision Top-Quark Mass Measurement in the Lepton+Jets Topology in $pp\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

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explain the large range of quark and lepton masses. Within the context of the standard model of particle physics, the top-quark mass is related to the masses of the $W$ boson and the Higgs boson, the latter object being the key to our understanding of the origin of mass [1]. Precision measurements of the top-quark and $W$ boson masses test the consistency of the standard model, and in particular the top quark is the heaviest known elementary particle with a mass approximately 40 times that of the next-heaviest quark or lepton. Because of this comparatively large mass, top-quark studies provide insight into our understanding of mass in general, and test theories that explain the large range of quark and lepton masses. Within

\[ M_{\text{top}} = 173.2^{+2.4}_{-2.2} \text{(stat)} \pm 3.2 \text{(syst)} \text{ GeV}/c^2 \text{ or } 173.2^{+4.4}_{-4.0} \text{ GeV}/c^2 \).

The second method reconstructs a top-quark mass in each event using the measured invariant mass of the hadronically decaying $W$ boson to constrain the jet energy scale to obtain a value for $M_{\text{top}}$ of $173.5^{+3.7}_{-3.6} \text{(stat)} \pm 1.3 \text{(syst)} \text{ GeV}/c^2$ or $173.5^{+3.9}_{-3.8} \text{ GeV}/c^2$. We take the latter, which is more precise, as our result.

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Higgs mechanism. A precision measurement of the top-quark mass is therefore a main goal of the experiments at the Fermilab Tevatron collider.

In this Letter we present two measurements of the top-quark mass in the lepton + jets decay channel. We use a sample of $t\bar{t}$ decays corresponding to 318 pb$^{-1}$ of proton-antiproton collisions at $\sqrt{s} = 1.96$ TeV and collected using the Collider Detector at Fermilab (CDF II) between February 2002 and August 2004. In the lepton + jets channel, $t\bar{t}$ pair production is followed by the decay of each top quark to a $W$ boson and a $b$ quark, the hadronic decay of one $W$ boson, and the leptonic decay of the other. This decay channel has the largest branching fraction with good signal-to-background ratio, allowing accurate top-quark mass measurements. Events in this channel contain an electron or muon and a neutrino from the leptonic $W$ boson decay, two quark jets from the hadronic $W$ boson decay, and two $b$-quark jets.

We select events consistent with this decay topology and analyze them using two complementary methods. The first method uses an event-by-event likelihood analysis employing the leading order matrix element for $t\bar{t}$ production and decay to extract a joint likelihood as a function of the top-quark mass, $M_{\text{top}}$. This technique, known as the “dynamical likelihood method” or DLM, was developed by the CDF Collaboration [2] and is similar to that used by the D0 Collaboration to make the previous most precise measurement of the top-quark mass [3]. The second method, developed by the CDF Collaboration [4], reconstructs a top-quark mass, $m_{\text{top}}^{\text{reco}}$, in each event and compares the distribution of $m_{\text{top}}^{\text{reco}}$ with template distributions derived from model calculations to estimate $M_{\text{top}}$. We have improved this “template method” by making further use of the fact that the hadronically decaying $W$ boson daughters should form a final state whose invariant mass is consistent with the known $W$ boson mass and width. This allows us to constrain the jet energy scale, an important uncertainty in the earlier measurements. These two methods have a top-quark mass accuracy 30% greater than earlier results, and have different statistical and systematic uncertainties.

The CDF II detector [5] is a general-purpose charged and neutral particle detector located at the Tevatron collider. We employ cylindrical coordinates where $\theta$ and $\phi$ are the polar and azimuthal angles, respectively, with respect to the proton beam, and pseudorapidity is $\eta = -\ln \tan(\theta/2)$. Transverse energy and momentum are $E_T = E \sin \theta$ and $p_T = p \sin \theta$, respectively, where $E$ and $p$ are energy and momentum. The detector comprises a solenoidal charged particle spectrometer, consisting of an eight-layer silicon microstrip detector array and a cylindrical drift chamber immersed in a 1.4 T magnetic field, a segmented sampling calorimeter with acceptance up to pseudorapidity $|\eta| = 3.6$, and a set of charged particle detectors outside the calorimeter used to identify muon candidates.

<table>
<thead>
<tr>
<th>Source</th>
<th>Expected background</th>
<th>DLM sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W + \text{jets}$</td>
<td>$19.6 \pm 2.4$</td>
<td>$5.3 \pm 1.1$</td>
</tr>
<tr>
<td>Multijet</td>
<td>$4.7 \pm 0.7$</td>
<td>$3.1 \pm 1.0$</td>
</tr>
<tr>
<td>Other</td>
<td>$2.3 \pm 0.2$</td>
<td>$0.8 \pm 0.1$</td>
</tr>
<tr>
<td>Total</td>
<td>$26.6 \pm 3.0$</td>
<td>$9.2 \pm 1.8$</td>
</tr>
</tbody>
</table>

Events for these analyses were selected by requiring an electron or muon candidate with transverse momentum $p_T > 20$ GeV/$c$ and $|\eta| < 1$ and missing transverse energy exceeding 20 GeV, corresponding to a high-energy neutrino candidate. The signal-to-background ratio was improved by requiring in each event the presence of four or more jets with $|\eta| < 2.0$. To reduce backgrounds further we required either (a) at least four jets with transverse energy $E_T > 21$ GeV or (b) at least three jets with $E_T > 15$ GeV and a fourth jet with $E_T > 8$ GeV with at least one jet with $E_T > 15$ GeV identified as a $b$-quark candidate through the presence of a displaced vertex within the jet arising from the decay of the long-lived bottom hadron ($b$ tag). This selection resulted in 165 events that, based on our background estimates, are primarily $t\bar{t}$ events. The methods used to estimate the backgrounds are detailed in [6].

The DLM analysis uses a 63-event subset of those data defined by requiring exactly four jets with $E_T > 15$ GeV where at least one of the jets has a $b$ tag. We have estimated the various sources of background contamination in this sample, summarized in Table I, to be $9.2 \pm 1.8$ events. The template method divides the 165 events into four non-overlapping subsamples with different expected $m_{\text{top}}^{\text{reco}}$ distributions and background levels. Ordered by decreasing statistical power, the subsamples are (1) events with at least four jets with $E_T > 15$ GeV and one $b$-tagged jet (“1-tag Tight” sample with 63 events), (2) events with two or more $b$-tagged jets (“2-tag” sample with 25 events), (3) events with a fourth jet with 8 GeV $< E_T < 15$ GeV and one $b$-tagged jet with $E_T > 15$ GeV (“1-tag Loose” sample with 33 events), and (4) events with four jets with $E_T > 21$ GeV and no $b$-tagged jets (“0-tag” sample with 44 events). The estimated background levels in the samples with a $b$ tag are summarized in Table I. The background level in the 0-tag sample is determined in the subsequent fit.

Both analyses use calibrated jet energies, based on a combination of instrumental calibration and analysis of data control samples [7]. The uncertainty $\sigma_c$ on the jet

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energy scale is the major source of uncertainty on \( M_{\text{top}} \). For jets in the \( t\bar{t} \) sample, \( \sigma_c \) is approximately 3% of the measured jet energy, depending on the \( \eta \) and \( p_T \) of the jet. The parameter \( \Delta_{\text{JES}} \) is defined as the difference, averaged over all jets in the sample, between the true jet energy and our measured jet energy after calibration in units of \( \sigma_c \).

The DLM technique, described in detail in [8], defines a likelihood for each event based on the differential cross section per unit phase space volume of the final state partons, \( d\sigma_{\text{M}}/d\Phi \), as a function of \( M_{\text{top}} \). Detector resolution effects are accounted for using \( t\bar{t} \) events generated by the HERWIG Monte Carlo program [9] and full detector simulation to derive a transfer function (TF). The TF relates the transverse energies of the final state quarks, denoted by \( x \), and the observed jets. For a given event, a Monte Carlo integration is performed over the possible \( t\bar{t} \) final state kinematics in the following way: we first generate a random value for the virtual mass squared of the \( W \) boson in the leptonic channel, \( s_W \), according to the Breit-Wigner form. We identify the momentum of the electron or muon daughter with the measured value, and the neutrino transverse momentum with the measured missing transverse energy. We then generate random values for the momenta of final state quarks according to the TF probabilities. We determine the \( z \) component of the neutrino momentum, with a twofold ambiguity, using \( s_W \) as a constraint. Thus, for a given set of \( x \) and \( s_W \), we fully determine the event kinematics, and the event likelihood as a function of \( M_{\text{top}} \) is given by

\[
L(M_{\text{top}}) = N \sum_{I_{\gamma}} \sum_{I_{\nu}} \frac{d\sigma_{\text{M}}}{d\Phi}(M_{\text{top}}; x, s_W),
\]  

where the normalization factor \( N \) is independent of \( M_{\text{top}} \) for a given event, and the indices \( I_{\gamma} \) and \( I_{\nu} \) run over the parton-jet assignments and the two neutrino solutions, respectively. The event likelihood is obtained by numerically integrating over \( x \) given by the TF and \( s_W \) given by the Breit-Wigner distribution.

Figure 1 shows the distribution of the top-quark mass value at the point of maximum likelihood in each event compared with the expectation from simulated events. An inset shows the joint log likelihood as a function of \( M_{\text{top}} \), formed by multiplying the likelihoods of the individual events together. We account for the presence of background events by evaluating the shift of \( -1.4 \) GeV/c\(^2\) they make in the measured top-quark mass. From the joint likelihood we infer \( M_{\text{top}} = 173.2^{+2.6}_{-2.4} \) (stat) GeV/c\(^2\), where the uncertainty is only statistical. The systematic uncertainty due to the jet energy scale is estimated as the shift in \( M_{\text{top}} \) arising from a \( \pm 1\sigma_c \) change in jet energies, and is 3.0 GeV/c\(^2\).

The template method is described in detail in [10]. We perform a \( \chi^2 \) minimization to fit the parton momenta from the \( t\bar{t} \) daughters and determine \( m_T^{\text{reco}} \) for each event, assuming that the final state arises from the decay of a \( t\bar{t} \) pair into \( W \) bosons and \( b \) quarks. We use only the four leading jets in the mass reconstruction. In the \( \chi^2 \) fit, both sets of \( W \) decay daughters are constrained to have the invariant mass of the \( W \) boson, and both \( Wb \) states are constrained to have the same mass. The ambiguity arising from the different ways of assigning the jets to the four quarks is resolved by selecting the assignment with the lowest \( \chi^2 \), taking into account the \( b \)-tagging information. We construct a histogram of \( m_T^{\text{reco}} \) for each subsample, discarding events with \( \chi^2 > 9 \), corresponding to poorly reconstructed or background events.

The parameter \( \Delta_{\text{JES}} \) is determined within this event sample by removing the \( W \) boson mass constraints, and

![FIG. 1 (color online). The value of the top-quark mass at the maximum of the DLM likelihood is plotted for each event. Data events (points) are compared to an expected distribution (histogram) comprising simulated \( t\bar{t} \) (\( M_{\text{top}} = 172.5 \) GeV/c\(^2\)) and background events. The last bin includes events with masses > 305 GeV/c\(^2\). The inset shows the joint log likelihood for the 63 events, before accounting for the presence of background.](022004-5)
fitted curves. We obtain agreement between the observed data distributions and the show the background contributions. In all cases, we see distributions.

differences in predicted signal and background mass
is determined to be their uncertainties. The background level in the 0-tag sam-
extrinsic jet energy calibrations, and we constrain the

Loose samples to the estimated background rates within
background rates in the 2-tag, 1-tag Tight, and 1-tag

identifying for each event all pairs of jets that would be consistent with the W boson final state. We form histograms of the invariant masses of these jet pairs for each of the four event subsamples and compare these with what we expect given the precisely known W boson mass [11].

We use these eight histograms to measure simultaneously \( M_{\text{top}} \) and \( \Delta_{\text{JES}} \). An unbinned likelihood fit is performed to parametrized signal templates taken from simulated \( t\bar{t} \) events generated using different values of \( M_{\text{top}} \) and \( \Delta_{\text{JES}} \), and background templates derived from studies of the relevant background processes. We include in the fit a Gaussian constraint (\( \Delta_{\text{JES}} = 0 \pm 1\sigma_c \)) from the extrinsic jet energy calibrations, and we constrain the background rates in the 2-tag, 1-tag Tight, and 1-tag Loose samples to the estimated background rates within their uncertainties. The background level in the 0-tag sample is determined to be 15.7 \( \pm 3.0 \) (stat) events by the fit using the differences in predicted signal and background mass distributions.

The four reconstructed top-quark mass distributions and the results of the fit are shown in Fig. \( 2 \), where we also show the background contributions. In all cases, we see agreement between the observed data distributions and the fitted curves. We obtain \( M_{\text{top}} = 173.5 \pm 3.7 \) (stat) \( \text{GeV/c}^2 \), where the uncertainty is statistical and incorporates the uncertainty due to the jet energy scale, which we estimate contributes \( \sim 2.5 \) \( \text{GeV/c}^2 \). Figure 3 shows the likelihood in the \( M_{\text{top}} - \Delta_{\text{JES}} \) plane. If we do not constrain \( \Delta_{\text{JES}} \) to the nominal value of zero, we obtain \( \Delta_{\text{JES}} = -0.25 \pm 1.22\sigma_c \), which indicates our nominal jet energy calibrations are in good agreement with information provided by the W boson mass peak in the \( t\bar{t} \) decay. This also demonstrates that the constraint on \( \Delta_{\text{JES}} \) from the W boson decay has comparable precision to the jet energy calibration.

There are a number of additional systematic uncertainties that affect both analyses: initial state and final state radiation uncertainties (ISR/FSR), uncertainties arising from the parton distribution functions (PDFs), and uncertainties arising from modeling of the background processes, the choice of event generators, and \( b \)-jet fragmentation, decays, and color connections (modeling) [12,13]. Table II summarizes these uncertainties.

The DLM method has additional uncertainties that arise from the use of transfer functions and from the procedure that corrects the measured mass for the presence of background (method). Together with the jet energy scale and other common sources noted above, the systematic uncertainty on the DLM mass measurement is 3.2 \( \text{GeV/c}^2 \).

The template method has additional uncertainties arising from the statistical precision of the templates themselves and approximations made in treating \( \Delta_{\text{JES}} \) as a single parameter affecting all jets coherently (method). The total systematic uncertainty on the template mass measurement is 1.3 \( \text{GeV/c}^2 \).

In summary, we have presented two new measurements of the top-quark mass. The analysis using the DLM method results in \( M_{\text{top}} = 173.2 \pm 2.6 \) (stat) \( \pm 3.2 \) (syst) \( \text{GeV/c}^2 \); the analysis using the template technique results in \( M_{\text{top}} = 173.5 \pm 3.7 \) (stat) \( \pm 1.3 \) (syst) \( \text{GeV/c}^2 \). There is a large statistical correlation between these measurements given the common data sample, so that we quote as a result only the more accurate measurement, the template method result of \( M_{\text{top}} = 173.5 \pm 3.8 \) \( \text{GeV/c}^2 \). This provides the most precise single measurement of this important physical parameter. In comparison, the previous most precise measurement was \( M_{\text{top}} = 180.1 \pm 5.3 \) \( \text{GeV/c}^2 \) [3] and the world average was \( M_{\text{top}} = 178.0 \pm 4.3 \) \( \text{GeV/c}^2 \) [14].

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science, and Technology of Japan; the Natural Sciences and Engineering Research

![Figure 3](color online) Contours of the template method likelihood are shown in the \( M_{\text{top}} - \Delta_{\text{JES}} \) plane for the combined fit to all four subsamples. The crosshair shows the best fit point. Contours are given at intervals of \( \Delta \ln L \), the change in log likelihood from its maximum.

<table>
<thead>
<tr>
<th>Systematic</th>
<th>DLM ( \Delta M_{\text{top}} ) (( \text{GeV/c}^2 ))</th>
<th>Template ( \Delta M_{\text{top}} ) (( \text{GeV/c}^2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet energy scale</td>
<td>3.0</td>
<td>([\sim 2.5]^a)</td>
</tr>
<tr>
<td>ISR/FSR</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>PDFs</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Modeling</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Method</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Total</td>
<td>3.2</td>
<td>1.3</td>
</tr>
</tbody>
</table>

\( ^a \)The uncertainty due to the jet energy scale is included in the uncertainty reported by the likelihood fit.

TABLE II. The systematic uncertainties for the two analyses.
Council of Canada; the National Science Council of the Republic of China; the A. P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean Science and Engineering Foundation and the Korean Research Foundation; the Particle Physics and Astronomy Research Council and the Royal Society, UK; the Russian Foundation for Basic Research; the Comisión Interministerial de Ciencia y Tecnología, Spain; in part by the European Community’s Human Potential Programme under Contract No. HPRN-CT-2002-00292; and the Academy of Finland.