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Elementary School Teachers' Interpretation and Promotion of Creativity in the Learning of Mathematics: A Grounded Theory Study

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ELEMENTARY SCHOOL TEACHERS’ INTERPRETATION AND PROMOTION
OF CREATIVITY IN THE LEARNING OF MATHEMATICS: A GROUNDED
THEORY STUDY

by

YINJING SHEN

A DISSERTATION

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Creativity is important for young children learning mathematics. Comparing the investment theory of creativity and national standards and principles for early mathematics shows that doing mathematics is more than applying rules and procedures; rather, learning mathematics takes a lot of creativity. However, much literature claimed that creativity for young children in the learning of mathematics was not adequately supported by teachers in the classroom due to teachers’ poor college preparation in mathematics content knowledge, teachers’ negativity towards creative students, teachers’ occupational pressure, low quality curriculum, and the like. The purpose of this grounded theory study was to generate a model that explains how teachers make sense of creativity in the learning of mathematics and how teachers promote or fail to promote it in the classroom. In-depth interviews with 30 Kindergarten to Grade-3 teachers, participating in a graduate mathematics specialist certificate program in a medium-sized Midwestern city were conducted. These teachers were also asked to draw a picture to represent their understanding of
creativity for young students in the learning of mathematics. A theoretical model was
developed describing: 1) the central phenomenon of how teachers interpret
mathematical creativity; 2) the strategies teachers use to promote creativity in the
learning of mathematics; and 3) the consequences of how different aspects of
mathematical creativity are promoted by different strategies in different degrees. The
findings challenge the popular notion that teachers do not view mathematics in early
grades as requiring creativity and that they are not supporting enough creativity in
the learning of mathematics in the classroom. Instead, this study finds that teachers
from the graduate certificate program have a well-developed concept of
mathematical creativity and that they are also resourceful about how to promote
creativity in the learning of mathematics. This study provides researchers and
teacher educators information on how to assist teachers to facilitate creativity and
strong mathematics capability for children from an early age.
“From the Tao, the One is created;

from the One, Two;

from the Two, Three;

from the Three, ten thousand things.”

(Lao Tzu, 600 BC)
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Before leaving China, my grandma asked me why I chose to do a Ph.D. program abroad. I told her because I can take cool courses where students can eat snacks and jump in to talk; because I can work with the smartest scholars in the world for five years long; because I can ask questions that no one has an answer before and design studies to find the answer and explain it to everyone I meet with; and because I will graduate with a hundreds of pages long dissertation all in another language! How cool it is to be a Ph.D. student!

However, now, I find being a doctoral student means more than the above exciting moments. I learned how to search for the meaning of life with patience and curiosity; how to calm myself down to make sense of the world and reflect on “who I am”; and how to extend and stretch myself little by little to embrace the stories of the others. Doing a Ph.D. is an emergent process of incubating new views from the old ways of interpreting the world.

Today, I am surprised to see myself become a young woman who has made noticeable progress to accomplish many missions seemed impossible to me before. I want to thank my dearest family, advisors, and friends who have always been strongly supportive as I develop into who I am today. This dissertation is a static product, but it reflects the dynamic improvements in all kinds of abilities in me. All my capabilities—research and life skills, language and social communication skills, flexible and reflective thinking styles, self-motivation and persistence, creativity and curiosity, confidence and independence, and domain-specific and general
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CHAPTER 1

STATEMENT OF THE PROBLEM

1.1. Defining Creativity in Mathematics for Adults and Children

Creativity is important to mathematics. For professional mathematicians doing advanced mathematics, creativity involves the creation and testing of new theories and hypotheses. Without creative discovery, mathematics never moves forward. Johann Carl Friedrich Gauss, for instance, could not have proved the law of quadratic reciprocity which was a great contribution to number theory; Luogeng Hua could not have developed the additive prime number theory; and Jingrun Chen who was inspired by this number theory could not have achieved the best result of Goldbach’s conjecture which was significant progress for the number theory.

Moreover, creative thinkers in many fields depend on the creativity of mathematicians, as advances in mathematics underlie many breakthroughs and advances in all of the scientific disciplines. For example, social science researchers benefit from advances in the field of statistics. Mathematical creativity at the professional level can be defined as “(a) the ability to produce original work that significantly extends the body of knowledge, and/or (b) the ability to open avenues of new questions for other mathematicians” (Sriraman, 2005, p. 23).

For young children, however, who are still in the beginning stages of learning mathematics, mathematical creativity must be defined in a different way. It seems intuitively inappropriate to apply the professional definition to them, because they must first master the fundamentals of mathematics before they can reach the stage of
inventing original and influential theories. Yet, children still can show creativity of another kind. According to Sriraman (2005), creativity for school learners can be defined as “(a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination” (p. 24).

Obviously, creativity by Sriraman’s definition does not require higher level invention of new mathematical theorems and/or proving of fancy hypotheses. Nevertheless, the ability to produce unusual or insightful solutions and pose new questions or possibilities requires children to go beyond mechanically following procedures. Instead, they must draw on their own inner resources to understand mathematics problems, abstract and represent the problems, discern internal structures and patterns, make generalizations, draw analogies and connections, take alternative perspectives and think about the numbers flexibly, apply in real world scenarios, endure and conquer struggles and confusions, keep motivated and passionate in the face of mistakes, find personal and/or unconventional solutions, examine and evaluate the solutions, reflect and revise the solutions, communicate and explain the solutions to others, and get feedback and make use of the feedback to polish the solutions.

Indeed, children need to solve numerous problems and propose many questions everyday, including mathematical ones. The inner resources children apply in mathematics are emphasized in various national and state standards and principles
for mathematical proficiency, including standards and principles from the National Council of Teachers of Mathematics (NCTM, 2000a), mathematical practice standards from the Common Core State Standards for Mathematics (CCSS-M, CCSS Initiative, 2010), and five strands for mathematical proficiency from the National Research Council (NRC, Kilpatrick, Swafford, & Swindell, 2001). As I discuss later, there are many overlaps among these standards and principles and childhood creativity in mathematics. Creativity is essential for good learning of mathematics. Creativity is important not only to professional mathematics, but also to school learners.

1.2. Creativity in the Classroom

While the importance of creativity for professional mathematicians is widely recognized and appreciated, the same cannot be said about creativity for young children in mathematics. Creativity is not emphasized enough in the classroom, or in discussions about classroom mathematics for young children. The NCTM (2000b) advocates that teachers should support students to think creatively about mathematical concepts and solve mathematical problems flexibly. However, teachers are often unprepared to design activities that promote creativity in the learning of mathematics, owing to the lack of prior experience or adequate content preparation in college (Shriki, 2009).

Indeed, knowledge of mathematics is an important capacity when teachers promote creativity. Teachers need to understand student thinking and the creativity in it before they can promote it. Teaching requires the understanding of both the subject
content matter and the students. The content knowledge is of particular importance because, in order to foster creativity in student thinking, teachers need to help their students acquire a strong foundation of content knowledge and skills (Baer & Garrett, 2010).

However, teacher preparation systems often fail to lay a strong foundation in mathematical knowledge for teaching. Teacher education cannot meet all the needs for effective teaching of mathematics to young children. The National Mathematics Advisory Panel (NMAP, 2008) recommends that:

Mathematics preparation of elementary and middle school teachers must be strengthened as one means for improving teachers’ effectiveness in the classroom. This includes preservice teacher education, early career support, and professional development programs. A critical component of this recommendation is that teachers be given ample opportunities to learn mathematics for teaching. That is, teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach. (p. 38)

In fact, given that 77% of students have not achieved “proficient” level of mathematics right before they enter college (U.S. Department of Education, 2005), there is strong demand for remedial mathematics education for incoming college students (NMAP, 2008). These courses are designed for students who lack mathematical skills necessary to perform in college at the level required by the
institution (Parsad, Lewis, & Greene, 2003). However, degree-granting postsecondary institutions offer only 2.5 such courses on average, with some colleges not offering them at all (Parsad et al., 2003).

Moreover, for students who are going to be teaching mathematics in primary schools, they are often not well-prepared in the subject of mathematics. As a principal from an elementary school in Vermont described, “Teachers coming from college today are typically taking one or two mathematics content courses. They’ve memorized some formulas but they don’t have a conceptual understanding of how mathematics works” (Teachers College Columbia University, 2009, p. 32). Weak knowledge foundation impedes not only teachers’ self-confidence in teaching but also their students’ mathematics achievement (NMAP, 2008). Ginsburg, Lee, & Boyd (2008) claim that teachers today are not ready to teach; the poor training they receive before teaching makes them afraid of mathematics, underestimate the importance of teaching mathematics, teach mathematics in low quality, or not teach at all. Only a small number of primary school mathematics educators in the States are well educated in this subject (Ball, 1997).

In addition, teachers’ attitudes and beliefs may also be an issue. Though most teachers claim that they value creativity (Runco & Johnson, 2002), their implicit attitudes indicate negativity towards creative students (Runco & Johnson, 1993; Westby & Dawson, 1995). In Westby and Dawson’s (1995) study, teachers’ least favorite students showed more characteristics that were identified by experts as creative, than teachers’ favorite students did. In fact, this study showed that teachers
interpreted creativity differently from expert definitions. Though experts identified characteristics such as nonconformity, emotionality, impulsivity, and trying to do what others think was impossible as typical creative traits, teachers rated them as the least creative traits (Westby & Dawson, 1995). Similarly, teachers in Scott’s (1999) study also showed negative attitudes towards creative students. Those teachers perceived creative students as disruptive (Scott, 1999). Even among prospective teachers, it was found that these teachers had already developed an opinion that students’ unique and novel responses in classroom discussions were potential distractions (Beghetto, 2007). Moreover, compared to teachers teaching other subjects, the negative attitude towards creativity was even stronger among teachers teaching mathematics (Beghetto, 2007).

Such negative attitudes could be related to teachers’ philosophical view of mathematics. Many primary school teachers may not have developed an appropriate understanding of the nature of mathematics. Zoltan P. Dienes, a distinguished mathematician who stands alongside influential figures like Jean Piaget who have had tremendous impact on mathematics education, claimed that, “the real problem occurs when one doesn’t understand what mathematics is about in the first place and then tries to teach it” ((in Sriraman & Lesh, 2007, p. 62). Instead, mathematics is more than a utility or tool or some simple tricks; rather, it is a way of thinking of how structures relate to one another (Sriraman & Lesh, 2007). Lockhart (2002/2008) described mathematics in “A Mathematician’s Lament” that:

To do mathematics is to engage in an act of discovery and conjecture,
intuition and inspiration; to be in a state of confusion— not because it makes no sense to you, but because you gave it sense and you still don’t understand what your creation is up to; to have a breakthrough idea; to be frustrated as an artist; to be awed and overwhelmed by an almost painful beauty; to be alive, damn it. (p. 8)

Unfortunately, Lockhart (2002/2008) believes that very few teachers know enough about this subject to provide more than just data transmission or passive reception of information without joyful creation of new ideas. Take operating fractions as an example. “If adding fractions is to the teacher an arbitrary set of rules, and not the outcome of a creative process and the result of aesthetic choices and desires, then of course it will feel that way to the poor students” (Lockhart, 2002/2008, p. 11). Such an image of mathematics could be left from teachers’ own previous schooling. Indeed, teachers tend to teach the way they were taught in school (Pehkonen, 1997).

Moreover, many teachers today are under occupational pressures that force them to adopt a less creative and more authoritarian style of teaching (Besancon & Lubart, 2008). First, they are under acute schedule pressure that makes time a precious commodity. Elementary teachers in the U.S. are required to teach all subjects and they spend significantly more time in the classroom than teachers in Asian countries who are specialized in one or two subjects do (Niu & Zhou, 2010). Teachers in the U.S. have less time and opportunity to develop expertise in each subject being taught. It is demanding for them to devote much time and energy after
class to think deeply to improve the teaching just for mathematics.

Second, teachers tend to follow the curriculum closely when teaching mathematics. Teachers are teaching under the pressure of accountability and testing (Baer, 1999; Baer & Garrett, 2010; Beghetto & Plucker, 2006). Rather than promoting creativity in the classroom, primary school teachers prioritize traditional structured curriculum and test scores (Plucker & Dow, 2010). In fact, mathematics has long been a subject associated with textbooks and curriculum (Remillard, 2005). Even when creativity is encouraged in the classroom, it is often exclusive to subjects like arts and literacy (Diakidoy & Phtiaka, 2002). In Sosniak and Stodolsky’s (1993) study, for example, the same elementary teachers would try to enrich the textbook for literacy, yet choose to rigidly follow the mathematics curriculum. Such a phenomenon could be related to the social norm, the nature of the subject content, and the comfort level of teaching it (Remillard, 2005). Even for pre-service teachers, they feel little freedom to go off track in their mathematics curriculum (Beghetto, 2007). To keep the pace, teachers often give limited opportunities for children to explore something not in the textbook. The development of creativity in the learning of mathematics takes time and experience. However, if teachers often rush through materials only in the textbook, it is doubtful they could promote creativity.

In addition, the typical mathematics curricula used in schools today are usually shallow and lack coherence (Ball, 1997; Burns, 1998; Drake, 2009). Take the typical mathematics curriculum for K-3 children as an example, it usually covers a wide range of contents such as “number and operation, shape, space, measurement,
and pattern” that require young children to process their thinking, develop corresponding skills and strategies, and memorize as well (Ginsburg et al., 2008). Such a curriculum is “a mile wide and an inch deep” and is filled with ill-connected knowledge and information not supportive for deep learning (Teachers College Columbia University, 2009, p. 31). These curricula are not helpful for young children to obtain sophisticated understanding of key concepts (Teachers College Columbia University, 2009), nor supportive for teachers to promote creativity in the learning of mathematics.

1.3. Summary of the Problem

The research problem to be explored arises from the situation that creativity in the learning of mathematics is important for young children, but it is not adequately supported by teachers in the classroom. Mathematics is more than just applying rules or following procedures. Instead, it involves a lot of mental activities and flexibilities in dealing with mathematics problems. Children’s creativity plays a critical part in the learning of mathematics as is reflected in the standards and principles from the NCTM, the CCSS-M standards, and the National Research Council. Teachers need a solid knowledge foundation to understand and foster creativity in student thinking, but the college preparation may not have left them well-prepared. In addition, teachers’ beliefs may further impede their ability to teach for creativity. They may hold negative implicit attitudes towards creative students; they may favor students with non-creative characteristics yet view students with creative personality traits as disruptive. Moreover, teachers may lack appropriate
understanding of the nature of mathematics. They may typically fail to see the creative process involved in mathematics. Influenced by previous schooling, they may have an image of the learning of mathematics as receiving and memorizing facts. Last but not least, teachers are also under occupational pressures that force them to teach with less flexibility, which could lead to their discouraging of mathematical creativity in children. Teachers teach long hours and teach all other subjects in addition to mathematics which leaves them limited time to work hard to improve the instruction of mathematics. They tend to follow the curriculum and keep pace, which leaves little time for students to work on important ideas and exercise creativity. However, the curriculum they are given to follow may be too wide and shallow and lacking in connections, which makes it hard for students to develop deep understanding of key concepts and conduct creative thinking in mathematics. Thus, research is needed to explore how teachers interpret creativity of K-3 students in the learning of mathematics and how teachers promote or fail to promote creativity in the learning of mathematics.

1.4. Purpose of the Study

This dissertation explores how K-3 elementary school teachers interpret and promote or fail to promote creativity in the learning of mathematics. To be specific, in this study, I try to understand how elementary school teachers define creativity in the learning of mathematics; how much they value creativity in the learning of mathematics; how they attempt to support creativity, if they do, in the learning of mathematics; how much they feel they are able to promote creativity in the learning
of mathematics, and if not, why; and what obstacles they see as standing in the way of supporting children’s mathematical creativity in the classroom.

The research questions are: (1) how do teachers understand and define mathematical creativity? (2) How do teachers support mathematical creativity? And (3) what are the support and obstacles for teachers to promote mathematical creativity?
CHAPTER 2
LITERATURE REVIEW

The first part of this chapter begins with an illustration of the connection between creativity and mathematics. Investment theory is introduced to explain the six resources for creativity to develop: (1) intellectual abilities; (2) thinking styles; (3) personality; (4) motivation; (5) knowledge; and (6) environment. I discuss in this chapter the triarchic theory of intelligence (Sternberg, 1985), standards and principles from the National Council of Teachers of Mathematics (NCTM, 2000a), mathematical practice standards from the Common Core State Standards for Mathematics (CCSS Initiative, 2010), five strands for mathematical proficiency from the National Research Council (NRC, Kilpatrick, Swafford, & Swindell, 2001), and the concept of Mathematical habits of mind (HOMs) in an effort to describe these resources and illustrate their importance for mathematical proficiency.

The second section of the chapter starts with a discussion of teaching methods that promote the development of creativity in the learning of mathematics. Teacher characteristics, curriculum and physical environment can interact with teaching methods to become obstacles in supporting mathematical creativity. These implementation barriers teachers encounter in promoting mathematical creativity in the classroom are discussed after the explanation of teaching methods.

2.1. Creativity and Mathematics

There is a question remaining to be answered: why (or how) is creativity important to K-3 mathematics? Before giving a decent explanation, we need to
understand the nature of mathematics and the relationship between creativity and the learning of mathematics. In other words, we need to comprehend how creativity plays a part in early mathematical proficiency in a classroom. In the first part of this chapter, I illustrate the essential role of creativity in the learning of mathematics.

The NCTM (2000b) advocates that students solve problems creatively and resourcefully, but they do not include a clear definition of mathematical creativity in their report. In fact, a consensus definition is lacking, both for creativity in general (Lau, Hui, & Ng, 2004; Sawyer, 2003, 2006) and creativity in the specific area of mathematics (Sriraman, 2005; Sriraman & Freiman, 2007). Researchers use different models and theories to describe and define creativity, including: (a) mystical approach that defines creativity as divine inspiration; (b) pragmatic approach that targets at the promotion of creativity; (c) psychodynamic approach that studies the tension between conscious reality and unconscious drives; (d) psychometric approach that aims at quantifying creativity; (e) cognitive approach that focuses on mental process; and (f) social-personality approach that emphasizes motivation, personality and socio-cultural environment (Sternberg & Lubart, 1999).

The currently accepted lenses through which to view creativity stem from a confluence of perspectives, such as the investment theory of creativity (Sternberg & Lubart, 1999) and the systems model of creativity (Csikszentmihalyi, 1990, 1999). The investment theory of creativity is based on an inclusive framework in which different aspects of and contributors to creativity are under consideration. The investment theory is used as the theoretical framework for creativity in this study.
The following is the description of the investment theory and the six creative resources claimed in this theory.

2.1.1. The Investment Theory of Creativity

In the investment theory of creativity, Sternberg and Lubart (1995) propose that a creative person is an investor who shall “buy low and sell high” for his ideas. In buying low, he “generate[s] and promote[s] ideas that are novel and even strange and out of fashion” (p. 2). Usually, at first, these ideas are not accepted or favored by others, but the creative person should persist despite the discouragement and resistance, and finally he can sell high when these ideas are recognized and appreciated (Sternberg & Lubart, 1995).

According to the investment theory, creativity requires six different resources to develop, including intellectual abilities, thinking styles, personality, motivation, knowledge, and environment (Sternberg & Lubart, 1996). Intellectual abilities refer to the ability to view things in novel ways, evaluate the ideas, communicate and promote the ideas to others, and utilize outside feedback; whereas, thinking styles refer to “how one utilizes or exploits one’s intelligence. They [thinking styles] are not abilities but rather ways in which one chooses to engage and use those abilities” (p. 7). A person who has the intellectual ability to come up with new solutions may not do so if the person does not enjoy utilizing that ability but prefers regular solutions in problem-solving. Thus, thinking style is “whether and how one uses that ability…it is needed to help complete the circuit; to ‘switch on’ abilities that otherwise might lie dormant” (p. 7).
Personality refers to “a preferred way of interacting with the environment” (p. 205). A creative person usually shows a series of personality traits, including perseverance in the face of obstacles, willingness to take sensible risks, willingness to grow, tolerance of ambiguity, openness to experience, and belief in oneself and the courage of one’s own convictions (Sternberg & Lubart, 1995). These traits are relatively stable, but can also change with time and environment.

Motivation refers to “the driving force or incentive that leads someone to action…[or] the nature and strength of your desire to engage in an activity” (p. 236). For knowledge, it can be formal or informal. Formal knowledge refers to “facts, principles, aesthetic values, opinions on an issue, or knowledge of techniques and general paradigms” (p. 150). Such knowledge can be learned from printed materials, speeches and lectures, and other direct instructions” (p. 150). Informal knowledge, in contrast, is “the knowledge you pick up about a discipline or a job from time spent in that arena…[It] is rarely explicitly taught and often isn’t even verbalized” (p. 150). Both types of knowledge do not equate to intelligence; instead, they are the raw materials for intellectual processes. Finally, environment refers to “a setting that stimulates creative ideas, encourages them when presented, and rewards a broad range of ideas and behaviors” (p. 10).

The six resources for creativity are embodied in standards and principles of mathematical proficiency, including the NCTM Process Standards and Principles, the CCSS-M Mathematical Practice Standards, the claims by National Research Council, and the concepts of Habits of Mind. The illustration of the correspondences
between creative resources and mathematical proficiency standards and principles are given in Table 1. In the following passages, the six resources are explained in more detail followed by an illustration of the standards and importance of the six resources for mathematical proficiency.

Table 2.1. The Correspondence Between Creativity and Mathematical Proficiency.

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**Resource 1. Intellectual Abilities and the Triarchic Theory**

Intellectual abilities, as elaborately expanded in the triarchic theory of intelligence (Sternberg, 1985) are very important for creativity (Sternberg & Lubart, 1996). Such abilities refer to three types of skills: (a) experiential ability (i.e., unconventional thinking and information processing in dealing with novel problems and demands); (b) componential ability (i.e., monitoring which ideas are valuable
and which are not); and (c) contextual ability (i.e., promoting a fit between one’s idea and the environment). A person must employ all three of them in problem solving to be genuinely creative (Sternberg & Lubart, 1995). With only experiential ability (otherwise known as synthetic ability), one can produce new and original ideas, but those ideas may elude the inspection process required to make those ideas feasible. With only componential ability (otherwise known as analytical ability), one can reason and analyze critically, but not creatively. With only contextual ability (otherwise known as practical-contextual ability), one can spread ideas and persuade others successfully, not because of the quality of the ideas but because of the powerful presentation. The intellectual abilities are emphasized in this study because they are an important focus in the standards of mathematical proficiency (this is discussed later).

**A. Experiential ability.** In doing creative work, experiential ability is reflected both in the novelty in task understanding (i.e., projecting the awareness of novelty in the conceptual system of solving a problem) and in three psychological processes in performing tasks: (1) selective encoding (i.e., sorting through information to separate the relevant from the irrelevant resources); (2) selective combination (i.e., putting originally isolated pieces of information into one combinative unit); and (3) selective comparison (i.e., associating new input of information with existing knowledge and resources) (Sternberg, 1985).

Let us consider a concrete example in mathematics to illustrate experiential ability. When learning to solve addition and subtraction problems, students are given
two representations of a clock face and are asked to compute the time difference between them. One student makes a connection between the time problem and the number line he learned earlier (*selective comparison*). To deal with the difference between the scale on a clock face and the scale on the number line, he mentally stretches out the round scale of the clock into a straight line, and matches the increments to the scale of the number line. In other words, he selects and encodes useful information by ignoring irrelevant differences and keeping relevant similarities between the clock face and number line (*selective encoding*). Now, the pieces of information about hour and minute on the two different representations are transformed into two points, each representing a simple number, on one single continuous number line (*selective combination*). He calculates the interval between the two time points represented by the clock faces in a creative way and solves the problem.

**B. Componential ability.** In conducting creative and intelligent performance, componential ability is reflected in three kinds of mental processes: (1) meta-components (i.e., a series of higher-order executive processes such as *planning*, *monitoring*, and *decision making*); (2) performance components (i.e., a series of processes in executing a task, such as encoding, combining and comparing, and responding); and (3) knowledge-acquisition components (i.e., a series of processes in acquiring new information such as selective encoding, selective combination, and selective comparison) (Sternberg, 1985). Among the three mental processes, meta-components are the foundation on which performance and
knowledge-acquisition components are based. Meta-components include the following: (1) decision on what the problem is that awaits to be solved; (2) selection of lower-order components; (3) selection of representation(s) or organization(s) for information; (4) selection of strategies to combine lower-order components; (5) decision on the allocation of attentional resources; (6) monitoring of solution(s); and (7) sensitivity to external feedback (Sternberg, 1985).

Let us consider a concrete example in mathematics to illustrate componential ability. In the computing of a three-digit number subtraction, 345 - 123, one student gets an answer of 122 by mistake. Fortunately, she is also monitoring her solution. She records, reflects on, and checks her answer. She compares the two numbers, 345 and 123, digit by digit. She finds that the number on each of the three digits of 345, that is, 3 on the hundreds place of 345, 4 on the tens place of 345, and 5 on the ones place of 345, are bigger than the numbers on the corresponding places of 122, so she feels that the answer should not be less than the difference of subtracting 100 from 300, that is, 200. Then she goes back to redo the problem and gets the answer 222 correctly. Now she thinks that she should check the answer again (the process of planning). However, simply comparing the magnitude of the number on each digit does not ensure the correctness of the answer. She feels that this strategy is not sensitive enough to check the answer now (the process of monitoring). She then draws from her mental database the knowledge of reversal relationship between addition and subtraction, that is, if a - b = c, then b + c = a, or a - c = b. She compares the two strategies and decides that using reversal relationship is a better
one than comparing numbers digit by digit (the process of *decision making*). Thus, she checks her answer by computing $222 + 123$ to see if she gets 345, and by computing $345 - 222$ to see if she gets 123. Now, she is more confident about her answer.

**C. Contextual ability.** Creative and intelligent behavior cannot be separated from the larger sociocultural context where it takes place. Contextual ability enables a person to deal with the context more successfully to be creative. It involves the following processes: (1) adaptation to the present environment; (2) selection of a more ideal environment than the present one; and (3) shaping of the present environment to improve the fit with one’s skills, interest, and values (Sternberg, 1985). Such ability enables one to *communicate* and *understand* others’ critiques, *justify* and *revise* one’s ideas, and *transmit* and *sell* these creative ideas.

Let us consider a concrete example in mathematics to illustrate contextual ability. In the learning of solving addition problems, students are asked to compute 7 + 8. Rather than adopting a conventional way, one student creatively breaks down the numbers and quickly solves the problem. Here is how he approaches the problem: $7 + 8 = 5 + 2 + 8 = 5 + 10 = 15$. The teacher asks him to explain his solution to the class. By simply reading the above equations, many students fail to get the reasoning of breaking down and recombining of numbers. They wonder where the 10 comes from. This student receives this feedback and detects what other students’ doubts are (he is showing the ability to *understand* others’ critique). To address their doubts and convince them of the effectiveness of the strategy, this student provides further
evidence to support his reasoning. He explains his decomposing and recomposing by referencing the associative property of addition, that is, \((a + b) + c = a + (b + c)\). He then rewrites the solution and adds one more step to make his reasoning visible: \(7 + 8 = (5 + 2) + 8 = 5 + (2 + 8) = 5 + 10 = 15\) (he is showing the ability to revise and justify). Then he continues, “so in this case, 7 is decomposed into 5 and 2. Then, 2 and 8 are recomposed and calculated first. And that’s how I get the 10” (he demonstrates the ability to communicate). In this way, he successfully justifies and promotes his creative solution (he demonstrates the ability to transmit and sell ideas).

**Resource 2. Creative Thinking Styles**

As is described above in the investment theory, there are other resources important for creativity, such as thinking style, motivation, and personality. With regard to thinking style, there are several styles associated with creativity. For example, “the legislative style is the single style most conducive to a creative mode of thought” (Sternberg & Lubart, 1995, p. 180). People with this style usually like to plan and do things their own way; they prefer problems with little structure; they enjoy exploring and discovering how to solve a problem rather than being told to follow rules and steps. It is also claimed by the investment theory that legislative style is often correlated with liberal styles. A liberal style refers to the preference to “go beyond existing rules and procedures… [A person with this style] prefers novelty, likes to maximize change, and seeks ambiguous situations” (p. 195). In addition, a creative person also tends to be more global than local. A global style
refers to the preferences for the big picture rather than the details. People with such style like to think abstractly; they sometimes ignore the small details. “If you were crossing a jungle, you would take crude tools like a machete and an axe, rather than a fine tool like a clarinet screwdriver” (p. 192). Solving real world problems are like crossing a messy jungle. The problems are often ill-defined without a clear clue pointing to one standard answer. Thinking in a global style is walking with a crude tool while fighting ones way out is the top priority. Thus, it is advised that one should often redirect oneself and zoom out from details to the big picture to come up with a creative solution. In contrast, a local style refers to the preference for details. People with this style “tend to be more pragmatic, concrete, and often down-to-earth… [and sometimes, they] are susceptible to not seeing the forest for the trees” (p. 192). Thinking in a local style is walking with refined tools. After one fights his way out of the forest, he can always switch to more elaborate tools and go back to redefine the depth and width of the path he already cut out. According to the investment theory, the ideal image of somebody who is creative is a mixture of both styles, with more global than local style (Sternberg & Lubart, 1995).

The thinking style resources proposed in the investment theory overlaps with and relates to some other creative thinking styles supported by many empirical studies in the field of creativity, including preferences for thinking metaphorically, being flexible, making independent judgments, thinking logically, breaking conventional mind-set, finding order in chaos, creating internal visualizations, using wide categories and images, building new structures, asking “why?” and questioning
norms and assumptions, being alert to novelty and gaps in knowledge, utilizing existing knowledge as base for new ideas, valuing originality and creativity, and others (Tardif & Sternberg, 1988).

**Resource 3. Creative Personality**

According to the investment theory, a creative person often has several personality traits that support creativity, including (1) perseverance in the face of obstacles, (2) willingness to take sensible risks, (3) willingness to grow, (4) tolerance of ambiguity, (5) openness to experience, and (6) belief in yourself and the courage of your convictions (Sternberg & Lubart, 1995).

When talking about obstacles, people meet with obstacles from time to time in doing creative work. According to the investment theory, these obstacles can come from both external sources (such as negative feedback from other people) and internal sources (such as intellectual difficulties and concern for going against the rules). However, a creative person should be able to live with these pressures and still be persistent in the work.

When it comes to taking risks, risks refer to “the chance of a loss, and losses are indeed possible when one is taking gambles with… ideas” (p. 44). According to the investment theory, “in order to do something really creative, and something that makes a difference to the world, you have to take that risk” (p. 213). “Just as nobody ever got rich or even well off by placing their money in low-interest passbook bank savings accounts, so has no one ever gotten rich with ideas by always going for the safest options” (p. 214).
When it comes to continuous growth, to stay creative, a person has to fight against pressures that keep him or her staying with just one single creative idea. According to the investment theory, people who refuse to grow are usually under pressure such as the fear for failure (Sternberg & Lubart, 1995). They desire to maintain reputation after one success; they worry about “statistical regression toward the mean,” that is, the next idea tends to be more mediocre and can never be as good as the first creative idea. In addition, people are likely to get so immersed in all the praise and rewards for the first successful creative idea that they lose the motivation to work on the next creative idea. Moreover, they may also feel pressure from others if they make some change in their solutions. Others may prefer the way things used to be that seem to be more familiar. “In the world of work it is often quite difficult to establish yourself in a new endeavor once you have become well known in another” (Sternberg & Lubart, 1995, p. 221), such as the stereotyping of role for actors.

When speaking of ambiguity, tolerating such feeling means to “withstand the uncertainty and chaos that result when a problem is not clearly defined or when it is unclear how the pieces of the solution are going to come together” (p. 223). Encountering uncertainty is quite common in creative work, and such feeling of ambiguity makes people uncomfortable and anxious so that they want to get rid of it. However, many times people cannot tolerate the ambiguity long enough for the ideas they are producing to get fully ready. These ideas could have been fantastic in creativity, but end up as mediocre or even undesirable. “[To] optimize your creative potential, you (and others) need to be able to tolerate the discomfort of an ambiguous
situation long enough so that what you produce is the best or close to the best of which you are capable” (p. 224).

When it comes to experience, creative people are always curious about the world and they seek new experiences that they can later return to for inspiration. Lastly, a creative person would have self-belief and courage. New and creative ideas usually challenge the status quo so that they are often disagreed with or unsupported by others, and it is commonplace that creative people get discouraged from time to time. Thus, it becomes very important for a creative person to believe in herself and have the courage to stand against the crowd. “The question is not whether you have failures but whether you believe in yourself, have enough courage of your convictions, and are able to bounce back from failures” (p. 229).

These personality traits discussed above overlap with and relate to some other personality traits supported in many empirical studies in the field of creativity. For example, it is found that creative people often demonstrate willingness to confront hostility and take intellectual risks; they are perseverant, curious and inquisitive, and open to new experiences; they value the freedom of spirit that rejects limits imposed by others; they have a high degree of self-organization to set their own rules; they are reflective and internally preoccupied; they often have an impact on people surrounding them; they can tolerate ambiguity; and they tend to play with ideas (Tardif & Sternberg, 1988).

**Resource 4. Motivation for Creativity**

According to the investment theory, motivation is “the driving force or
incentive that leads someone to action. Basically, it’s the nature and strength of your desire to engage in an activity” (Sternberg & Lubart, 1995, p. 236). There are two basic types of motivation: extrinsic motivation and intrinsic motivation. Extrinsic motivation refers to the motivators other than the task itself. Usually, the work one person is doing is just a means for this person to reach something else ultimately. What this person can gain has nothing to do with what the person is working on at the moment. For example, a child who makes his own bed in the morning and does gardening every weekend to earn some pocket money is extrinsically motivated. In contrast, intrinsic motivation is the work itself that motives. Usually, people who work on something because of pure enjoyment of the task, personal satisfaction, or the meaning of the work per se are motivated intrinsically. According to the investment theory, intrinsic motivation is most important for creativity because it keeps a person focused on the task (Sternberg & Lubart, 1995). Many researchers find support in their studies that intrinsic motivation is conducive to creativity (Collins & Amabile, 1999; Rolen, 1995).

Intrinsic motivation is often linked with creativity, yet extrinsic motivation can also facilitate creative work. According to Collins and Amabile (1999), extrinsic motivators can be divided into two types: synergistic and nonsynergistic. The former refers to the motivators which “provide information or enable the person to better complete the task and which can act in concert with intrinsic motives” (Collins & Amabile, 1999, p. 304). And the latter “lead [s] the person to feel controlled and are incompatible with intrinsic motives” (Collins & Amabile, 1999, p. 304). The
synergistic type of extrinsic motivators can facilitate intrinsic motivation to promote creativity (Collins & Amabile, 1999). For example, parents who use reward and feedback in terms of recognizing children’s competence and providing important information on further improvement are utilizing synergic extrinsic motivation. These synergic motivators positively contribute to creativity particularly in compensating for the lack of intrinsic motivators in the stage of work that requires less novelty, such as the evaluation and validation stage (Collins & Amabile, 1999; Torrance, 1963). Deadlines and the promise of rewards, for example, are less likely to hurt in such stage, and instead, they may help keep the creator involved in the work (Collins & Amabile, 1999). In short, as is claimed in the investment theory, “intrinsic and extrinsic motivation are often highly interactive, and can work together rather than in opposition to each other” (Sternberg & Lubart, 1995, p. 243). These motivational conditions proposed in the investment theory also overlap with and relate to what is supported in many empirical studies in the field of creativity, including having a driving absorption, having discipline and commitment to one’s work, having high intrinsic motivation, and being task-focused (Tardif & Sternberg, 1988).

**Resource 5. Knowledge for Creativity**

According to the investment theory, both formal knowledge and informal knowledge are important for creativity. With regard to formal knowledge, one indeed needs to know something within a formal discipline in order to be creative. According to this theory, preparation in formal knowledge promotes creativity by
helping one invent something original rather than reproducing something already existent; offering one a good understanding of the field so that one can think against the common trend; assisting one in elaborating an idea into a complete work; providing one with a solid foundation so that one can get focused on the new idea rather than the basic knowledge; and by making one sensitive to little hints that inspire creative ideas (Sternberg & Lubart, 1995). A good mastery of domain specific knowledge seems to be critical for creativity (Csikszentmihalyi, 1990; Howe, 1999).

In the area of mathematics, a good conceptual understanding of the content is important for creative ideas. For example, let us consider an addition problem “97 + 99 + 95 = ____”. A regular way to solve the problem is by lining up the numbers vertically, adding 7, 9, and 5 in the ones place to get 21, writing down 1 and carrying the 2 to the tens place, adding the three 9s and the 2 in the tens place to get 29.

Children may use their fingers to count or retrieve their memories of basic facts to solve this problem in a standard way. However, for a child who has developed a good number sense, it may take him only seconds to calculate. Knowing that 97 is just 3 away from 100, 99 is 1 away from 100, and 95 is 5 away from 100, he can quickly get the answer by adding three 100s and subtracting the sum of 3, 1 and 5, which is 300 – 9 = 291. In contrast, for a child who lacks a good number sense, he may have to rely on the standard way of adding the three numbers. He can also get the correct answer because he has practiced millions of times how to count on to add, but the process of how he has solved the problem involves limited creativity. It is
likely that this child can count from one to 100 with no difficulty, but he does not understand the decimal system, see the numbers as both ordinal and cardinal, or understand the relationship among the numbers.

Let us consider another example of a multiplication problem involving fractions, “4 × 2 ½ = ____”. Students who memorized multiplication table may not be able to solve this problem if they do not understand what multiplication means. They may be able to tell you that “4 × 2” means four, two times, or adding two fours together. But they may say they cannot do four, two and a half times. However, if a young child understand the commutative law of multiplication, she can see that “4 × 2 ½” is the same as “2 ½ × 4”. Then she knows she can add 2 ½ four times to get the answer. Moreover, with good understanding of the correspondence between fractions and decimal numbers, this child can use another way to solve the problem by converting 2 ½ into 2.5. Then she can even make an analogy with money (four quarters make a dollar) to solve the problem quickly.

However, knowledge may also prevent one from seeing things in a new way and thus restrict creativity. For experts in an area, too much knowledge may restrain their thinking in a standard way and therefore other possibilities or creative alternatives elude them (Frensch & Sternberg, 1989). For young children, too much knowledge is also likely to harm imagination or limit the creation of new ideas (Lubart & Geogsdottir, 2004; Soh, 1999). There is a popular anecdote that happened in the year of 1968. A mother living in Nevada accused her daughter’s prekindergarten of suppressing creativity by teaching the English alphabets. Her
daughter Edith used to think of all kinds of things to represent a round shape: it can be the sun, donuts, eggs, and apples. But then, she could only name it as the letter “O”. The story is a little dramatic, but it clearly reflects the respect for young children’s creative thinking without the burden of knowledge. In fact, not only the mother won the lawsuit, but the state government of Nevada also revised their Education Protection Act afterwards (Chen, 2007). The negative impact of knowledge on creativity can happen to everyone. However, as is suggested in the investment theory, one can avoid or reduce such impact by alternating one’s routine, asking for feedback, or keeping learning new and different things.

When it comes to informal knowledge, it sometimes plays an even more important role than formal knowledge in decision making (Sternberg & Lubart., 1995). For example, children in school should know in which occasion or what type of creativity is appropriate to show and will be expected and appreciated. Informal knowledge helps children to “most effectively… use their creativity so that it would benefit rather than hurt them” (p. 172). However, one needs to conform to the expectation and exceed the expectation as well. Children should know not only how to adapt to the routines but also how to detach from the rules to create something new.

**Resource 6. Environment for Creativity**

According to investment theory, the environment influences how creative one can be. An environment that feels relaxing and cheerful and rich in cues can facilitate creativity. Task constraints affect people by either restricting or promoting creativity.
If one has little previous knowledge with a task, giving some rules and limits would help and inspire creativity. However, if the task is already very familiar, extra information and limits may make the task too easy and thus kill the creativity. “In any case helping people realize the extent of their freedom to create is likely to facilitate creativity, whereas impinging on this freedom is likely to impede creativity” (p. 259). When it comes to evaluation from others, investment theory suggests that when one perceives it as threatening, it would harm creativity. However, if one knows in advance the criteria of the evaluation, one usually would do better.

Competition is also a double-edged sword. Competition brings pressure. An appropriate amount of pressure can boost creativity, but too much pressure may interfere with creativity. How much pressure one feels depends on the difficulty of the task and one’s arousal level. Cooperation also has contradictory effects. To fully develop a creative idea into a complete production, one always needs different kinds of cooperation and support from others. However, as is claimed in the investment theory, “members of a professional group will accept and support work only if it conforms to the group’s norms” (p. 264). If the group shows a strong wish to cooperate, one’s idea may not be highly creative because “when highly creative people seek to ignore or violate the norms of their peer groups, they may find these groups to be distinctly noncooperative” (p. 265).

Home climate also influences creativity. Home climate that fosters independency and intellectual development would help promote creativity. If parents are performing themselves as creative role models, they let children observe and
imitate, which helps them develop creativity. When it comes to the school climate, teachers can influence student creativity. To promote student creativity, teachers need to value creative personal attributes in students; they may even model creativity to their students. The school climate tends to reflect the values of society. When the society favors beautiful test scores and perfect memorization of facts, it is likely trading off the creative use of knowledge. Though an open education that allows students to do more exploration and design their own curriculum can be very helpful for creativity development, it might seems less desirable compared to traditional education that favors academic achievement.

In this section, I explained how the six resources in the investment theory of creativity can apply directly to the domain of mathematics. The question arises: how is creativity important to mathematical proficiency? In the next section, I explain how each of the six resources is embodied in quality learning of mathematics, as laid out by professional standards and guidelines. The particular standards and guidelines I have chosen are NCTM Process Standards and Principles, CCSS-M mathematical practice standards, NRC report, and the concept of Mathematical HOM.

2.1.2. Creativity in the Learning of Mathematics

**Resource 1. Creative Intellectual Abilities**

As described above, there are three intellectual abilities associated with creativity in the investment theory: experiential, componential, and contextual. Each of these abilities intersects with several of the standards and principles of mathematical proficiency. First, let us consider the NCTM Process Standards.
Creative intellectual abilities and the NCTM Process Standards.

According to the NCTM (2000a), there are five Process Standards: (1) problem solving; (2) reasoning and proof; (3) communication: (4) connections; and (5) representation (NCTM, 2000a). With regard to the experiential ability, these standards can involve such creative ability, that is, process of mathematical thinking applied under new and novel situations. For example, with regard to the first standard, problem solving, the NCTM advocates that students should be able to apply learned strategies to new and unfamiliar problems and situations either inside or outside the mathematics classroom context. With regard to the second standard, reasoning and proof, the NCTM advocates that students should be able to select among and utilize a variety of types of reasoning and methods of proof. With regard to the fourth standard, connections, the NCTM advocates that students should be able to recognize and make use of connections among mathematical ideas, to understand how different ideas interconnect, and to build the idea on one another to construct a unified whole. With regard to the last standards, representation, the NCTM advocates that students should be able to select among and apply mathematical representations to solve problems.

With regard to the componential ability, the NCTM process standards call for the application of such ability. For example, in the process standard of problem solving, it is advocated by NCTM that students should be able to monitor and reflect on the process of problem-solving. In the process standard of reasoning and proof, it is advocated by NCTM that students should be able to evaluate mathematical
arguments and proofs. In the process standard of communication, it is advocated by NCTM that students should be able to *analyze* and *evaluate* mathematical thinking and strategies of *others*. In the process standard of representation, it is advocated by NCTM that students should be able to create and use *representations* to *organize*, record, and communicate mathematical ideas, and to *select*, apply and translate among mathematical *representations* to solve problems.

With regard to the contextual ability, the NCTM process standards call for the application of such ability. For example, in the process standard of communication, it is advocated by NCTM that students should be able to *communicate* mathematical thinking *coherently* and *clearly* to peers, teachers, and others, and to use the *language of mathematics* to express mathematical ideas *precisely*. In the process standard of representation, it is advocated by NCTM that students should be able to create and use representations to *communicate* mathematical ideas.

**Creative intellectual abilities and CCSS-M mathematical practice standards.** The advocacy for experiential, componential, and contextual abilities is also emphasized in the eight mathematical practice standards from CCSS-M (CCSS Initiative, 2010). The eight standards are: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure; and (8) Look for and express regularity in repeated reasoning. Take the
first standard as an example, it is required that students should be able to analyze the given problem, monitor and evaluate progress, check answers in a different method, and always ask if it makes sense or not; these skills involve componential ability.

Take the third standard as an example, it is required that students should be able to understand and use stated assumptions, definitions, and previous results to construct arguments, make conjectures and build logical progression of statements, justify conclusions, communicate to others, and respond to others’ arguments (contextual ability). Take the seventh standard as an example, it is required that students should be able to discern a pattern or structure, step back for an overview, shift perspectives, see complicated things as single objects or as composed of several objects (experiential ability).

Each standard does not speak to only one kind of intellectual ability. Instead, a combination of two or three intellectual abilities is reflected in each standard. For example, in the fourth standard, it is required that students should be able to apply math to solve problems in everyday life, society, and workplace different from the ones in a classroom, make assumptions and approximations to simplify a complicated situation, and realize these may need revision (these skills involve experiential ability). It is also required that students should be able to identify quantities in a practical situation, use tools to map relationships, and analyze relationships and draw conclusions (these skills involve componential). In addition, students are also expected to reflect on whether the results make sense and improve the model if it fails to serve its purpose (these skills involve a mixture of
componential ability and contextual ability).

Creative intellectual abilities and the National Research Council’s claim for mathematical proficiency. The development of CCSS-M mathematical practice standards is partly based on a report by the National Research Council in which mathematical proficiency is regarded as an organic composition of five strands (Kilpatrick, Swafford, & Swindell, 2001). They are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Three of them—procedural fluency, strategic competence, and adaptive reasoning—speak to the importance of creative intellectual abilities. Conceptual understanding refers to the “comprehension of mathematical concepts, operations, and relations” (p. 5). Procedural fluency refers to the “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (p. 5). Strategic competence refers to the “ability to formulate, represent, and solve mathematical problems” (p. 5). And adaptive reasoning refers to the “capacity for logical thought, reflection, explanation, and justification” (p. 5). Each of the strands involves one or more of the three creative intellectual abilities: experiential; componential; and contextual abilities.

Resources 2, 3 and 4. Creative Thinking Styles, Personality and Motivation

As described above, there are certain thinking styles, personality and motivation traits associated with creativity in the investment theory. In my view, each of these inner resources intersects with the concept of mathematical habits of
mind (HOMs) and the guidelines from NRC. First, let us consider the mathematical HOMs.

**Creative thinking style, personality and motivation and mathematical habits of mind.** Mathematical HOMs reflect the nature of doing mathematics, and they represent the ways of discovering in approaching mathematics. According to Cuoco, Goldenberg and Mark (1996), Mathematical HOMs refer to a way of doing mathematics. It resembles a process of conducting research during which students think like mathematicians. They claim that students today are learning specific mathematical results rather than the process of doing mathematics. Students usually have little experience with how mathematics is created or applied in other settings. However, the HOMs by which mathematicians create mathematical results bear much more importance than a collection of the results per se (Cuoco, Goldenberg, & Mark, 1996). It is an urgent need that students should learn to think like a mathematician; they should study mathematics through creating, inventing, conjecturing, and experimenting, which alkies the process of doing research (Cuoco, Goldenberg, & Mark, 1996). They proposed some general habits of mind for students, including being pattern sniffers, experimenters, describers, tinkerers, inventors, visualizers, conjecturers, and guessers (Cuoco, Goldenberg, & Mark, 1996). For young children, HOMs refer to “curiosity, imagination, inventiveness, risk-taking, creativity, and persistence…[viewing] mathematics as sensible, useful and worthwhile and… themselves as capable of thinking mathematically… [and appreciating] the beauty and creativity that is at the heart of mathematics” (Clements
et al., 2004, p. 57).

How do creative thinking style, personality and motivation relate to the learning of mathematics? These creative characteristics correspond to the mathematical HOMs. The creative characteristics mentioned earlier can be considered to be synonyms with mathematical HOMs. For example, the HOMs of being sensitive to patterns as proposed by Clements et al. (2004), or being a “pattern sniffer” who has delight in finding hidden patterns and short-cuts as proposed by Cuoco et al. (1996), resembles the creative characteristic of finding order in chaos and being alert to novelty. The HOMs of being willing to experiment as proposed by Clements et al. (2004), or being an “experimenter” who plays with available strategies to solve problems, gives evidence to support the answers, is skeptical for the results, and is aware of the limitations of approaches, as proposed by Cuoco et al. (1996), resembles the creative characteristic of questioning norms and assumptions, utilizing existing knowledge as base for new ideas, tendency to play with ideas, and being reflective and internally preoccupied. The HOMs of being curious, imaginative, inventive, and persistent as proposed by et al. (2004) are also identified as important characteristics for creativity.

Creative thinking style, personality and motivation and National Research Council’s claims for mathematical proficiency. Among the five strands for mathematical proficiency as reported by the National Research Council (Kilpatrick, Swafford, and Swindell, 2001), one of them—productive disposition—corresponds to the mathematical HOMs. Productive disposition refers
to the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 5). This strand is one component of the basis upon which CCSS-M practice standards are developed (CCSS Initiative, 2010).

Resource 5. Knowledge

As described above, knowledge is associated with creativity in the investment theory. In my view, this creative resource, knowledge, intersects with several of the standards and principles of mathematical proficiency. First, let us consider the NCTM Content Standards.

Knowledge and the NCTM’s claim for mathematical conceptual understanding. Another resource claimed by the investment theory as important for creativity is knowledge. Knowledge is indispensable for creativity as I have discussed above. How does knowledge play a part in mathematical creativity? The NCTM confirms that knowledge is critically important for doing mathematics creatively, by adopting or exercising those resources, abilities, skills, or attributes described in the theories earlier. For example, knowledge supports children in dealing with novel problems (experiential abilities), monitoring and reflecting on solutions (componential abilities), being flexible (thinking styles), having self confidence and perseverance (personality traits), and developing eagerness to solve mathematical problems (motivational attributes). According to the NCTM (2000a), “[c]onceptual understanding is an essential component of the knowledge needed to deal with novel problems and settings… [And] a major goal of school mathematics
program is to create autonomous learners, and learning with understanding supports this goal” (p. 20). The vision of the NCTM principles and standards is built upon student understanding in the learning of mathematics (NCTM, 2000a). In their words, “conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility… [W]ithout understanding … learning is often quite fragile” (p. 20).

According to the NCTM standards, there are five content standards specifying what content knowledge students should obtain to be proficient in mathematics: (1) Number and operations (students should understand numbers and their representations and relationships, comprehend the meanings of operations, and compute fluently based on conceptual understanding); (2) Algebra (students should understand patterns, relations, and functions, use algebraic symbols to represent and analyze mathematical situations, use mathematical models to represent quantitative relationships, and comprehend the concept of change); (3) Geometry (students should understand geometric shapes and structures to analyze their characteristics and relationships, comprehend and represent relative positions and locations, and develop the knowledge of transformation, symmetry, visualization, spatial reasoning, and geometric modeling to analyze and solve problems); (4) Measurement (students should develop the understanding that objects have measurable attributes, comprehend the units, systems, and processes of measurement, and apply techniques to determine a measurement); and (5) Data analysis and probability (students should develop understanding of data, formulate questions based on data, collect, organize,
and present data to answer questions. During this process, students should understand statistical methods and can choose appropriate ones to analyze data, understand, develop and evaluate statistical inferences based on data, and understand and utilize basic concepts of probability) (NCTM, 2000a).

**Knowledge and NRC’s claim for conceptual understanding.** Among the five strands for mathematical proficiency reported by the National Research Council, the very first of them—conceptual understanding—speaks to the importance of knowledge (Kilpatrick, Swafford, and Swindell, 2001). Conceptual understanding refers to the “comprehension of mathematical concepts, operations, and relations” (p. 5). This strand is one component of the basis upon which CCSS-M practice standards are developed (CCSS Initiative, 2010).

**Knowledge and the CCSS-M Standards for Mathematical Content.** The significance of knowledge is clearly described in the CCSS-M Standards for Mathematical Content. Creative intellectual abilities cannot work alone without knowledge of the domain; similarly, the eight mathematical practice standards from CCSS-M cannot be achieved without solid mathematical knowledge preparation. According to CCSS-M (CCSS Initiative, 2010):

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word ‘understand’ are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they
may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. (p. 8)

**Resource 6. Environment**

As described above, environment is associated with creativity in the investment theory. This creative resource, environment, intersects with the NCTM principles.

**Creative environment and the NCTM principles.** The last resource claimed by the investment theory as important for creativity is environment. Environment is indispensable for creativity as I have discussed above. How does environment play a part in mathematical creativity? The NCTM (2000a) describes several important things in the learning environment in their principles for school mathematics. As is discussed earlier in the sixth resource for creativity, school climate can influence child creativity. In the principles proposed by the NCTM (2000a), students learning mathematics can be influenced by curriculum, teaching, assessment, and technology. First, according to the NCTM (2000a), a mathematical curriculum determines what students are given the opportunity to learn. An effective curriculum that builds on true understanding should demonstrate coherence among
different concepts, place emphasis on important ideas, and articulate clearly the progression of ideas as the grade level goes up. Second, effective mathematical teaching has several requirements for teachers: (a) teachers should be equipped with good understanding of mathematical content knowledge, student knowledge and pedagogical strategies; (b) they should advocate for and nurture a challenging and supportive learning environment with the provision, analysis and explanation of meaningful and inspiring mathematical tasks; and (c) they should reflect on and improve teaching practices. Third, assessment should be based on the purpose to inform students and improve their learning; it should also be a tool that supports teaching with insights. Assessment should be a regular activity in the classroom to support rather than interrupt the movement of learning. Fourth, technology should be used appropriately to solidify student understanding in the learning of mathematics and to improve mathematical instruction as well. New technology enables the teaching of what used to be inaccessible, which facilitates learning in these areas and creates connections among them.

2.1.3. Summary

In summary, this section describes the close relationship between creativity and mathematics. The six resources for creativity proposed by the investment theory, that is, intellectual abilities, thinking styles, personality traits, motivational attributes, knowledge, and environment, are emphasized and advocated by the NCTM, the CCSS-M, and the NRC in their standards and principles as critical for young children to learn mathematics proficiently. Therefore, it is meaningful to study
creativity in the context of early learning of mathematics in this dissertation project. The remainder of this chapter introduces the context of teaching to discuss the problems and obstacles teachers face in interpreting and promoting creativity in the learning of mathematics.

2.2. Strategies to Promote Creativity

Teachers can struggle in promoting each of the six resources in mathematical creativity. Teaching involves the interaction with other elements in the environment, and these elements can challenge teachers in fostering creativity. The following section begins with an illustration of obstacles to teaching for creativity, in terms of three elements that influence creativity: (A) teacher characteristics (i.e., knowledge and personality and beliefs); (B) curriculum (i.e., content, organization, and assessment goals); and (C) physical environment (i.e., classroom setup, availability of opportunity, and quality of supportiveness).

With regard to teacher characteristics, how teachers teach can be influenced by their knowledge of teaching the subject of mathematics. Teaching also varies among teachers with different personalities and beliefs about creativity. The second element, curriculum, influences teaching for creativity via the content (how well the big ideas are provided, presented, and connected), organization (whether and how “psychology” or methodology of mathematics subject is taught, and how schedule is arranged), and assessment goal and vision (whether it is teaching to the test, speed, and memorization or creative thinking and deep understanding). The third element, physical environment, influences teaching for creativity by classroom setup (whether
it allows for movement and interaction), availability and opportunity of experience (whether there are rich materials, tools, and technology for students to explore with), and the quality of supportiveness (whether students see the potential usage of these tools and materials and are attracted to manipulate them).

Table 2 describes the three elements and their corresponding obstacles to teaching for creativity. After the discussion of the three elements, I introduce and describe the teaching methods to promote mathematical creativity. Potential obstacles in using these methods are also discussed.

Table 2. Elements that influence teaching and corresponding obstacles to developing creativity in the learning of mathematics.

<table>
<thead>
<tr>
<th>Elements that Influence Teaching for Mathematical Creativity</th>
<th>Obstacles to Developing Creativity in the Learning of Mathematics</th>
</tr>
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<tbody>
<tr>
<td>A. Teacher Characteristics</td>
<td></td>
</tr>
<tr>
<td>① Knowledge</td>
<td>• Teachers’ lack of math content knowledge and therefore lack of profound understanding</td>
</tr>
<tr>
<td>② Personality and Beliefs</td>
<td>• Teachers’ lack of personality traits that support creativity, such as flexibility, openness, high energy level, optimism, and etc.</td>
</tr>
<tr>
<td>B. Curriculum</td>
<td></td>
</tr>
<tr>
<td>① Content</td>
<td>• Curriculum with lack of connections across different levels of understanding</td>
</tr>
<tr>
<td></td>
<td>• Curriculum missing key or important ideas</td>
</tr>
<tr>
<td></td>
<td>• Curriculum that is a “mile wide and an inch deep”</td>
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<tr>
<td></td>
<td>• Curriculum that lacks good challenge problems</td>
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<tr>
<td>Elements that Influence Teaching for Mathematical Creativity</td>
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<tr>
<td>-------------------------------------------------------------</td>
<td>---------------------------------------------------------------------</td>
</tr>
<tr>
<td>② Organization</td>
<td>• Curriculum that fails to structure mathematics contents and knowledge systematically in its presentation and delivery</td>
</tr>
</tbody>
</table>
| ③ Goal and Assessment                                        | • Curriculum that relies on standardized testing to measure learning  
|                                                             | • Curriculum that forces teaching to focus on simple procedural learning, speed, and memorization |
| ① Classroom Setup for Multiple Learning Styles               | • Environment that neglects children’s different learning styles  
|                                                             | • Environment that lacks space for free movements of children and their interaction with materials and other people |
| C. Physical Environment                                      | • Environment that lacks rich variety of materials, tools, and technologies  
|                                                             | • Environment without inviting presentation of materials, tools and technologies, or that restricts children’s access or exploration of them |

2.2.1. Teacher Characteristics

**Teacher knowledge.** Teachers need to have knowledge to teach for creativity. According to Renzulli & De Wet (2010, p. 34), knowledge of the discipline is significant because: first, “through such content mastery and personal involvement… teachers…develop the kinds of appreciation for within-discipline thinking that improves the guidance of learning” (p. 34); and second, teachers should have “understanding of general research methodologies [within this discipline] and a repertoire of managerial skills that allow them to guide students through investigative activities” (p. 34). In other words, teachers’ knowledge of the discipline
is more than just facts and theories; it also involves an understanding of the bigger picture including methodology and the art and process of guiding students through the application in personally meaningful problems in the real world, which is the true meaning of creativity (Renzulli & De Wet, 2010). However, it is a common problem that teachers are often underprepared in knowledge, as has been noted. The course design for teacher preparation programs and professional development training does not cover all necessary contents; and the opportunities for learning are insufficient (NMAP, 2008).

The lack of proficient understanding of the discipline could be an obstacle for teachers to promote creativity in the learning of mathematics. According to Ma (1999) in a study comparing primary mathematics teachers in China and in U.S., U.S. teachers often lack a profound understanding of fundamental mathematics. According to Ma (1999), the U.S. teachers who participated failed to have deep, vast, and thorough knowledge foundation of doing mathematics. A profound understanding usually requires connectedness, multiple perspectives, basic ideas, and longitudinal coherence (Ma, 1999). With connectedness, a teacher would avoid isolated learning but instead guide students to learn a connected body of knowledge across operations and sub-domains; with multiple perspectives, a teacher would appreciate different approaches to a problem and encourage students to reason flexibly; with basic ideas, a teacher would emphasize basic yet powerful concepts and principles in mathematics and lead students to real mathematical activity; and with longitudinal coherence, a teacher would have a good understanding of the
curriculum across all elementary levels, and when teaching a certain grade, the
teacher would be ready to review key ideas taught previously and to establish a good
foundation for future learning as well (Ma, 1999).

Proficiency in these four areas (i.e., connectedness, multiple perspectives,
big ideas, and longitudinal coherence) can support teachers in promoting student
creativity in the learning of mathematics. For example, connectedness across
operations and sub-domains supports children’s *selective combination* ability;
teachers who appreciate multiple perspectives create an encouraging *environment* for
children to exercise *liberal thinking style*; basic ideas prepare children with
*conceptual understanding and knowledge* necessary for mathematical creativity; and
longitudinal coherence across grade levels fosters children’s *selective comparison*
ability.

Besides profound understanding of the discipline, teachers also need to have
good knowledge about how students learn. Before they can promote creativity, they
should be able to understand student thinking and detect the creativity within that
thinking. According to Shulman (1986), teachers need to have pedagogical content
knowledge so that they can use various ways to represent and formulate the subject
to make it understood by the students; at the same time, such knowledge also
endows them with the awareness of positive and negative influence of students’
conceptions and preconceptions. Ball, Thames and Phelps (2008) elaborate on
Shulman’s concept of pedagogical content knowledge that they specify a particular
kind of knowledge: knowledge of the content and the students. Such knowledge
refers to a combined understanding of the students and mathematics, such as predicting students’ common thoughts about mathematics, identifying the contents they are more likely to be puzzling, and understanding and interpreting students’ emerging thinking. Without such understanding of the content and the students, teachers may be insensitive to student thinking, miss the teachable moment for creativity, or fail to give effective and appropriate support for creativity.

**Teacher personality and beliefs.** Teachers with certain personalities are more effective in teaching for creativity (Renzulli & De Wet, 2010). These characteristics include “flexibility, openness to experience and new ideas, a high energy level, optimism, commitment to excellence, and enthusiasm for living” (Renzulli & De Wet, 2010, p. 35). Teachers with such personality traits may not only be more likely to motivate students and appraise their creative ideas but also more likely to seek for creativity in themselves (Renzulli & De Wet, 2010, p. 35).

However, these personal characteristics may be lacking in some teachers. Though they can be cultivated and developed from experience, training and exercises, they are not easily changed and usually take a long time and much experience to grow. For example, a teacher may not feel comfortable being flexible with student strategies in solving mathematics problems if she is just starting her career without much in-service practice or knowledge about student thinking. It is also very likely that sustaining in a high energy level over time is quite demanding in managing a classroom, which requires multitasking. Stuck in a basic mode of teaching mathematics to survive, teachers may find it hard to remain optimistic and
to make a commitment to excellence. In addition, even though they have a
creativity-friendly personality, pressure from the curriculum may still push them
away from how they like to teach. Curriculum pressure is discussed in more detail
later in this chapter.

Teachers may also hold incorrect beliefs about creativity, which keep them
from cultivating creativity. For example, some myths are: creativity is an innate
characteristic that some students are born creative and some uncreative; creativity is
time sensitive that it is only for the young children; creativity cannot be generalized
from one area to other areas; creativity is a blurred and soft construct that cannot be
grasped concretely or enhanced with scientific intervention; creativity cannot be
developed for individual children but only in a group; and constraints, such as
guidelines and directions, can only hinder children’s creativity (Pucker & Dow, 2010,
p. 373).

Inappropriate attitudes may prevent teaching for creativity. Teachers often do
not like to go off track of their original plan or routines; they may view new and
unusual creative behaviors as disturbing and not be willing to accept ideas different
from their own (Esch, 2010, p. 117). When a creative student adopts a novel
approach that does not conform to the teacher’s plan, the teacher may not tolerate or
encourage the idea but regard this child as making undesirable troubles due to
nonconformity (Esch, 2010, p. 117).

To foster creativity effectively and consistently, teachers should also be aware
of their own view on creativity (Esch, 2010; Plucker & Dow, 2010). It is necessary
for them to reflect on their own values and goals for creativity to avoid devaluing creativity or neglecting creativity in the classroom. Such reflection and awareness also support teachers to become creative role models.

2.2.2. Curriculum

Curriculum content. The content of curriculum plays an important role in the promotion of creativity. A good curriculum shows in its content the emphases on the interrelationship among different levels of knowledge and the provision of open-ended challenge problems. With regard to interrelationship among different levels of knowledge, according to Renzulli and De Wet (2010), the selection of content material should follow the hierarchy of different dimensions of knowledge, including “facts, conventions, trends, criteria, principles and generalizations, and theories and structures” (p. 32). Learning should be a spiraling process upwards so that the lower level dimensions of knowledge, such as facts and conventions, could always be referred to and comprehended in relation to higher level dimensions of knowledge, such as theories and structures. Similarly, the NMAP (2008) also calls for a focused and coherent curriculum:

A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. Any approach that continually revisits topics year after year without closure is to be avoided. By the term focused, the Panel means that the curriculum must include (and engage with adequate depth in) the most important topics underlying success in school algebra,
particularly the Critical Foundations of Algebra. By the term coherent, the Panel means that the curriculum is marked by effective, logical progressions from earlier, less sophisticated topics into later, more sophisticated ones. (p. 22)

However, there is the dilemma of what to cover in a curriculum. Too much content is a problem. A mild wide and an inch deep is certainly not going to work because the connection between ideas may be too few to weave into a quality fishing net to catch and hold understanding and inform more advanced learning in higher grades. However, too little content is also problematic. It is dangerous to narrow curriculum by cutting too many side topics away. Though curriculum is accelerated, “the shape of the mathematics education of a typical student is tall and spindly …[and it] reaches a certain height above which its base can support no more growth, and there it halts or fails” (Thurston, 1990, p. 845). Thus, an ideal curriculum should seek for the balance of what and how much content to cover and how to flow smoothly and meaningfully. It is an important decision to make about the big mathematical ideas, the scope and depth of ideas, the layering of difficulty, and the flowing quality.

With regard to challenge problems, different types of mathematics problems vary in how much creativity is allowed. Open-ended novel problems encourage creative thinking. Such problems require more than just recall and application of regular algorithms (Carpenter & Moser, 1983; NCTM, 2000a; Schoenfeld, 1992), but allow for multiple approaches, which provide opportunities for creativity to grow
(Chamberlin & Moon, 2005). According to McLeod (1988, 1994), problems can be divided into routine versus non-routine ones according to children’s past experience and familiarity with the problems. Another classification is focused on the structure of problems: well-structured versus ill-structured (Jonassen, 1997). The former are usually clearly expressed with all the information that points to one single correct answer. When solving well-structured problems, students are supposed to exercise a learnt procedure to get the answer which is the main purpose of such type of problems. In contrast, the ill-structured problems are more like real world problems. These problems lack clear organization and require students to find extra information. Students often need to think divergently to approach such problems, and there might be more than one correct answer (Jonassen, 1997). However, textbooks today often do not provide enough opportunities for students to explore challenge problems (Ball, 1997; Burns, 1998; Drake, 2009; Ginsburg et al., 2008; Thurston, 1990). Teachers may have to find supplementary materials and outer resources themselves that contain such problems.

**Curriculum organization.** Organization refers to the way content knowledge is systematically structured, presented and delivered. The structure of the content presents the “psychology” of the subject, that is, “how to think in the discipline” (Renzulli & De Wet, 2010, p. 30). The organization of the curriculum reflects the “systematic way of thinking about a body of knowledge-its forms and connections, its unsolved problems, its methods of inquiry, its aspirations for improving mankind, and the special way it looks at phenomena” (Renzulli & De Wet,
2010, p. 30). By learning the “psychology” of a subject, students get to know that they are supposed to be a “firsthand inquirer in a field rather than … mere assimilator of information” (p. 31). Students should believe that they have the right and ability to “question, criticize, and more important, add their own interpretations and contributions to existing knowledge” (p. 31). A systematic way of mathematical thinking is more important than the knowledge students already know or are expected to obtain.

A good curriculum should support teachers to teach such thinking and students to use and develop such thinking. The thinking process is a set of “enduring skills that form the cognitive structures and problem-solving strategies that have the greatest transfer value” (Renzulli & De Wet, 2010, p. 32). The thinking process and content knowledge should be introduced to students as one integrated unit that “the acquisition of a scheme for acquiring, managing, and producing information [is conducted] in an organized and systematic fashion” (p. 32). In other words, teaching thinking process is also teaching methodology which helps students develop an “understanding of and appreciation for the application of methods to the… problems that are the essence of particular fields of knowledge…[through] thinking, feeling, and doing” (p. 32). However, curriculum nowadays may not present the methodology of mathematics very well. The process behind developing, exploring, and producing the rules and procedures may very likely be neglected or ignored. Students may not see the enduring and transferrable skills that can be applied flexibly in solving different problems.
Curriculum goal and assessment. The vision of curriculum is important in the promoting of creativity. If the writers of curriculum assume that teachers have the role of catering to standardized testing, it is very likely that teachers will not get enough support to promote creativity in the learning of mathematics. Unfortunately, today’s curriculum often pushes teaching to the test. According to Thurston (1990):

[H]igher test scores are often treated as the goal. Legislators, newspapers and parents put superintendents and school boards under pressure, superintendents and school boards put principals under pressure, principals put teachers under pressure; and teachers put students under pressure to raise scores. The sad result is that many mathematics courses are specifically designed to raise scores on some standardized test. (p. 3)

With so much reliance on conventional tests, creativity is very likely to be ignored in the teaching and learning of mathematics. Students are expected to produce the single right answer regardless of many other possibilities, and such type of close-ended choice making does not encourage divergent thinking (Sternberg, 2010). In fact, this is a problem not only in system-wide standardized tests, but may also exist in the daily quizzes and tests given by individual teachers (Sternberg, 2010). It may sound uninspiring, but many teachers tend to teach mathematics as steps and procedures (Ginsburg et al., 2008). They may be in a rush to have students arrive at the correct answers without stopping to consider alternative ways and other possibilities in the journey of choosing, forming, comparing, and examining among solutions. Thus, teachers may find it common and easier to trade creativity, deep
thinking and conceptual understanding for speed, memorization and procedural learning. Under such curriculum goals and assessment pressures of teaching to the test, it is not easy for teachers to stand up for creativity in the learning of mathematics.

2.2.3. Physical Environment

Classroom setup for multiple learning styles. Learning is more than just listening to the teacher. Children have a multitude of ways of thinking and learning. Learning style refers to “the preferred way in which an individual approaches a task or learning situation” (Cassidy, 2004, p. 421). For example, some children can be wholistic learners (i.e., learners who prefer processing information as a whole) while some can be more analytic learners (i.e., learners who prefer breaking down information into smaller parts to process); some children may prefer to be verbalizers (i.e., learners who prefer representing information as words) while some are more like imagers (i.e., learners who prefer representing information as images) (Riding & Cheema, 1991). Moreover, children vary in the preference for the learning environment, such as the degree of instruction, auditory and mobility predilection, and the choice between independent versus group work (Renzulli & De Wet, 2010). Their different learning styles should be allowed, respected, and encouraged in the environment. A creativity-friendly classroom enables students to move around, communicate, interact, practice, and exercise all their ways of studying mathematics. For young children, physical movement is especially important. Physical activities elicit creativity in children (You, 2010). The classroom should be set up in a way that
space is provided for easy movement and communication. By moving around, students are given opportunities to go find and interact with probes and tools in the classroom. It is also easier for them to start social communication with peers that may promote creative thinking. Freedom to move around can facilitate students to try out understanding, manipulate tools, compare strategies, and communicate with peers and the teacher, all of which provide them with more practice of creative thinking in solving mathematics problems.

The Reggio Emilia approach to early education emphasizes classroom setup that allows for different learning approaches of different children. Children’s individual needs and interests are respected and valued by Reggio Emilia educators. This Italian approach has been receiving much attention all over the world since it was identified as the best preschool by Newsweek magazine (1991). In this approach, there is a special emphasis on early creativity development that can be facilitated by appropriate environment (Edwards, 2009). The Reggio Emilia educators’ philosophy is that creativity is not “sacred” or “extraordinary”, but that it “emerge[s] from daily experience”; creativity is not a “separate mental faculty,” but “a characteristic of … thinking, knowing, and making choices” (Malaguzzi, 1998, p. 75). A creativity-friendly environment “protects originality, subjectivity, and differences” (Rinaldi, 1998, p. 115); it not only encourages the process of “revisiting, reflecting, and interpreting…, [but also] supports the children’s memory…, [let] them…retrace their own processes…, find confirmation or negation, and…self-correct” (Rinaldi, 1998, p. 122)
Malaguzzi is the founder of this world famous Reggio Emilia approach. He claimed in a metaphor, that children are using “hundreds” of ways to learn. Schools and the culture may seem to deprive children of all their hundred languages in the process of focusing on the standard. Instead, he advocated that learning is not only using the head, but also takes the whole body to explore, try, and grow. In a poem, he wrote (2011):

The child

is made of one hundred.

The child has

a hundred languages

a hundred hands

a hundred thoughts

a hundred ways of thinking

of playing, of speaking.

A hundred always a hundred

ways of listening

of marveling, of loving

a hundred joys

for singing and understanding

a hundred worlds

to discover

a hundred worlds
to invent

a hundred worlds

to dream.

The child has

a hundred languages

(and a hundred hundred hundred more)

but they steal ninety-nine.

The school and the culture

separate the head from the body.

They tell the child:

to think without hands

to do without head

to listen and not to speak

to understand without joy

to love and to marvel

only at Easter and Christmas.

They tell the child:

to discover the world already there

and of the hundred

they steal ninety-nine.

They tell the child:

that work and play
reality and fantasy

science and imagination

sky and earth

reason and dream

are things

that do not belong together.

And thus they tell the child

that the hundred is not there.

The child says:

No way. The hundred is there. (p. 3)

**Access to rich variety of materials, tools, and technologies.** Rich variety of materials, tools and technologies can provide opportunities for creative learning experience in the classroom. As is claimed in the CCSS-M practice standards, students should be able to choose appropriate tools and use them strategically to solve problems (CCSS Initiative, 2010). These tools and materials include but are not limited to a pencil and a piece of paper, a ruler, a protractor, a compass, some manipulatives and tokens, a calculator, a spreadsheet, a geometrical ball, a set of jigsaw puzzle pieces, a decimal system model, a computer application of addition and subtraction problems, an online resource website, and otherwise.

Some of the above tools and materials are based on advanced digital technologies. To develop a good mastery of the tools, one may take some time to learn to operate. However, these new technologies can foster creativity because they
offer new ways of knowing and unique connections between knowledge (Baile & Johnson, 2010).

In Reggio Emilia, children are developing creative thinking when using complex computer programs, such as Photoshop (a graphic software) and spectrograph (an audio program) (Forman, 2011). For example, by using the spectrograph to transform the sound children recorded into audiograms, children reinvented codes and marks to represent sounds. Such new connection between sound and graph helps children understand their own voices in a new way. In a mathematics course, say, geometry, computer programs can also be very useful in presenting and manipulating three dimensional structures. For example, the teacher and students can rotate the structures, combine or divide the structures, select and control one dimension while manipulating and observing other dimensions, and map onto a diagram. Computer programs enable such operations so handy and easy that graphs drawn on two dimensional traditional materials can hardly compare.

After reexamining a large amount of existing research studies, the NMAP (2008) confirmed that “uses of educational technology can make a significant, positive contribution to students’ learning of mathematics” (p,142). It is recommend by the panel (NMAP, 2008) that:

Computer programming be considered as an effective tool, especially for elementary school students, for developing specific mathematics concepts and applications, and mathematical problem solving abilities. Effects are larger if the computer programming language is designed for learning (e.g.,
Logo) and if students’ programming is carefully guided by teachers so as to explicitly teach students to achieve specific mathematical goals. (p. 52)

Traditional educational tools, materials, and technologies are still useful. Traditional and digital tools have different strengths in utility and effectiveness. When students have both types of tools, they can compare them, decide which one to use on which occasion, how to combine and complement each other, and in which order to use them. The knowledge and ability to examine the limitations and strengths of tools and pick suitable ones for the problems they want to solve are very important for creative intellectual abilities. Teachers should make these tools available for students to interact with. However, these opportunities are not always available in the classroom. Time can be another problem that limits the use and exploration with tools and technologies.

Even if students are given access to these tools, materials, and technologies, simple provision of physical tools does not ensure the promotion of creativity. Students should feel the connection or attraction to these tools. Whether and how much students obtain support from the physical environment partly depends on the quality of the tools. Tools that are attractive, inviting, and stimulating are more likely to be picked up by students. For example, hands-on manipulatives and fancy computer programs might be appealing to students. However, teachers can promote the attractiveness of materials by demonstrating their potential usage and powerful functions. For example, even a pencil and a piece of paper can support and facilitate visual thinking if students draft ideas, conceive strategies, and examine models by
doodling quickly and freely on the paper. A piece of paper can also become a three-dimensional representation tool by folding it. Teachers can also create a humorous and relaxing environment that encourages students to use different tools to try out, develop, verify, and compare different strategies they come up with themselves or inspired by others.

Teachers sometimes have limited knowledge and experience with tools so they fail to provide enough quality support for students. In some cases, even though teachers have access to the facilities and tools, they may not develop the habits of using them regularly. For example, according to a large survey study among 743 teachers from 258 different schools who have sufficient access to computers and other educational tools, these teachers reported that they used technological tools less than once a week on average (NMAP, 2008). Around one-third of them had no experience with the graphing calculator before, and they used manipulatives only occasionally (NMAP, 2008).

2.2.4. Strategies to Promote Mathematical Creativity

In what follows, I explain teaching methods that promote creativity in the learning of mathematics. I also discuss the obstacles associated with the above three elements (teacher characteristics, curriculum, and classroom environment) afterwards.

Teaching methods that encourage and emphasize different aspects in learning can promote different creative resources: (1) the first creativity resource, intellectual abilities, can be promoted by encouraging students to define and redefine problems,
question and analyze assumptions, welcome and learn from mistakes, generate ideas, cross-fertilize ideas, identify and overcome obstacles, collaborate with others, take others’ perspectives, and sell ideas; (2) the second creativity resource, thinking styles, can be promoted by encouraging students to reexamine problems, question assumptions, and generate new ideas; (3) the third creativity resource, personality, can be promoted by encouraging students to take sensible risks, tolerate ambiguity, take responsibility, develop self-regulation, and delay gratification; (4) the fourth creativity resource, motivation, can be promoted by encouraging students to build self-efficacy and find interest and passion; (5) the fifth creativity resource, knowledge, can be promoted by encouraging students to understand the pros and cons of knowledge; and (6) the sixth creativity resource, a creativity-friendly environment, can be promoted by teacher role modeling creativity, allowing time for creative thinking, instructing and assessing creativity, and rewarding creativity (Sternberg, 2010, p. 403; Sternberg & Williams, 1996). The details of and obstacles to exercising the above instructional activities in promoting mathematical creativity are discussed in detail below.

Redefining problems is a process of “extricating oneself from the box…which is the synthetic [or experiential] part of creative thinking” (Sternberg, 2010, p. 402). Students should be given the opportunities to make their own choices even if these choices may be not always good. They can make mistakes, but they can analyze and learn from the mistake and then redefine their choices. However, many times teachers may make the decisions for students. When students make a mistake,
teachers redefine the problems for them. They may lecture students right away on
the standard solution, which deprives students of the valuable opportunity for
creativity. This could be the result of the teachers’ knowledge (e.g., not knowing
enough about student competency in thinking and learning; not knowing how to
scaffold students other than teacher leading), teacher personality (e.g., impatience),
curriculum content and organization (i.e., no emphases or focus on problem defining
and redefining), assessment and goals (i.e., aiming for memorization and quick recall
instead of longer time of reasoning), and opportunity and support (i.e., tools and
materials that support re-examining of problems are not available or not being used
effectively).

Teachers who promote questioning and analyzing of assumptions are
supporting student analytical thinking which is a type of componential ability for
creativity as claimed in the investment theory of creativity (Sternberg, 2010).
Though knowing how to answer questions is always a goal for students who are
learning in the classroom, knowing how to ask good questions is often neglected.
Teachers can make questioning as part of the daily routine. They can model the
questioning of worthwhile assumptions, encourage students to evaluate their own
questioning, and inspire them to put emphasis on how they think rather than what
they think. However, questioning might not be encouraged enough in a classroom
under pacing pressure, nor is it likely to happen when the teachers do not have good
understanding in mathematics to support, guide, and develop student questioning.

Teachers can help students identify and overcome obstacles by informing
them that negative reactions from others are possible. Teachers can even share their own stories of meeting obstacles and, at the same time, they can encourage students to lower their expectations regarding others’ acceptance and value for the idea. However, teachers themselves may have similar difficulties dealing with others’ non-appreciation of their approaches to mathematics problems. Some teachers may become defeated and thus devalue creativity themselves, and it would be hard for them to encourage students to exercise creativity in the learning of mathematics. It is also likely that teachers are not encouraged to develop creativity in their schooling and thus had few experience and knowledge about how to deal with creative ideas.

In addition, if the goal and assessment of the curriculum only focus on finding the correct answer, finding multiple and different solutions may not be encouraged in the first place, then the struggles associated with such exploration can hardly be recognized.

When it comes to the selling of ideas, teachers could encourage students to communicate their thinking to peers and persuade them the goodness of the idea. However, other people may often hold a suspicious and doubtful view for creative ideas because these ideas are so different from what they prefer or used to think (Sternberg, 2010). In a classroom, peer students may prefer traditional and standardized solutions taught in class. A creative student needs to not only make his or her nonstandard idea understandable to other students, but also demonstrates the advantage over traditional approaches displayed in their textbooks or by the teacher. This requires the student to think from others’ perspectives. A creative student should
also be able to recognize the merits in others’ thinking and reasoning. Through understanding, appreciating and interacting with others’ ideas, this student can improve his or her own idea by collaborating with others’. However, selling ideas requires that the student have good mastery of language skills and cognitive understanding of others’ thinking to be able to receive and send out messages clearly and effectively. This could be a challenge especially for students in early grades. Moreover, the teacher may not see the creativity if she has a strong mindset for traditional solutions. Even though the teacher perceives the creativity, time and test pressure might push her to explain the solution for the student instead of letting the student do the explanation and persuasion.

Teachers should support students to generate ideas by encouraging a legislative thinking style in creating new ideas. Legislative style is when one enjoys discovering a new way to solve a problem rather than following standard rules. (Sternberg, 2010). The process of exploring new ways can be circuitous and some ideas may seem unattractive in initial. However, teachers should not just criticize those ideas that seem to be less appealing. Instead, teachers should find the creativity embodied in children’s discovery and encourage children to transform and modify their unique ideas into workable solutions. It is bad for teachers to quickly deny students’ underdeveloped ideas. Instead, if teachers are able to see and recognize the potential of many initially unappealing thoughts, they can support students to develop those thoughts into creative and attractive strategies or solutions. However, teachers often do not have enough time for idea generation and development. They
may be in a hurry to instruct the big topics in the curriculum rather than having students explain their thinking, let alone giving them support in improving and modifying ideas.

When it comes to the understanding of the pros and cons of knowledge, teachers should recognize the indispensable role of knowledge in creative thinking but also be aware of how knowledge shapes and limits one’s thinking and strategies for choosing and approaching problems. Teachers should regard “teaching-learning…[as] a two-way process” (Sternberg, 2010, p. 405). Teachers have more knowledge, but students can be more flexible in thinking, and it is worthwhile for teachers to learn from them. However, teachers sometimes may regard themselves as the dominating source of knowledge in the classroom and their role is to impart knowledge. And thus they may ignore the merits in student thinking. They might also allow less freedom for students to explore in multiple ways.

When speaking of taking sensible risks, teachers should encourage students to learn to assess risks and take intellectual risks in class and study (Sternberg, 2010). However, both students and the teacher might be fearful of taking risks. They are often under the pressure of curriculum, tests, school requirements, and social expectation that compel them to seek for good scores and decent performances on standardized tests. Teachers might teach in a safe mode that they give close-ended problems in assignments and assess children through traditional activities (Cohen, 1988).

Teachers can support students in tolerating ambiguity by encouraging
students to live with the uncomfortable feeling of uncertainty before the full realization of creative ideas. There is always a period of “feeling scattered and unsure…[or even doubting whether one is] on the right track” (Sternberg, 2010, p. 407) before the person totally figures his or her creative idea out. It takes time for creative ideas to become fully polished. Teachers should be willing to accept the temporarily imperfection and messiness of the ideas and teachers should encourage students to recognize the feeling of uncertainty before their ideas are mature. Sometimes, the uncomfortable feeling may push students to another ordinary or traditional approach. Therefore, when teachers have the experience and knowledge of the unsettling period of creativity, they can be more likely to recognize such process in students and give students encouragement. Teachers may allow students more time to conquer the struggles during the uncertainty period. However, time can be an issue in class.

When it comes to building self-efficacy, teachers should encourage students to believe in their creative potential and take responsibility to develop self-regulation. As I have discussed above, creative people can go through a rather long period of uncomfortable uncertainty. During this period, they may receive discouragement from others. They may feel that no one trusts or values their ideas, and they may even doubt their own ability. Students should take learning as an internal process that belongs to their own responsibility. They should have confidence to regulate themselves to endure such a vague and hard period of time. They should also believe in their ability to handle it. To support students, teachers should avoid giving
students negative comments on their capability, but provide them with encouragement and recognition of their endeavors. However, sometimes, teachers themselves might not see the potential in students and thus unconsciously give feedback that undermine students’ creative potential.

Teachers can encourage students to delay gratification by advising students that rewards do not always come right after creativity. In fact, it is very likely that a person works on an idea for very long time during which there is no immediate reward at all. Teachers should encourage persistence that students do not give up so easily without getting quick success and recognition. Teachers can even share with students their life experience of a long period of silence and hardship. However, assignments and traditional tests usually bring short-term rewards, which may not be helpful for developing delaying gratification. Long-term projects can be beneficial, but the curriculum often does not support such an approach (Ginsburg et al., 2008; Thurston, 1990).

When it comes to interest and passion, teachers should help students find what attracts them instead of what teachers are excited about or what teachers wish the students will be excited about; teachers should help students find the match among their abilities, interest, and opportunities (Sternberg & Williams, 1996, p. 44). Solving mathematics problems can be very intriguing if students view such a process as discovering a puzzle little by little. Teachers can encourage passionate exploration by providing appropriate problems, giving students interesting hints and enough time, discussing and explaining the problems from different perspectives, and appraising
students for using uncommon and flexible strategies. Teachers should be facilitators and motivators when students find their interest in trying out their intellectual abilities. However, the curriculum might not provide challenge problems or time to do such problems. The teachers also may not be knowledgeable to design open-ended mathematics problems, give appropriate probes, understand student thinking, and explain from different perspectives.

Teachers could support cross-fertilizing ideas by encouraging students to expand their thinking from within one subject to cross disciplines (Sternberg & Williams, 1996). Knowledge in other science classes can inform the learning of mathematics, and vice versa. Students’ experiences in personal life can also bring insights to understanding mathematics problems. At the same time, mathematics may also inform creative problem-solving in daily life. However, the curriculum may impress the teachers and students that knowledge in different subjects is separate from one another, and subject learning in school has nothing to do with everyday life. Even when teachers are aware of the connections, they may not be able to detect student prior knowledge to inform current learning of a mathematics topic.

When speaking of making mistakes, teachers should encourage students to welcome and learn from mistakes. It is almost impossible for students to always come up fast, good, and correct solutions. They sometimes think in a new way and it can be problematic. But the value is that the problematic approach can be the basis for better and more creative ideas. Sometimes, a strategy appears to be problematic just because it does not meet the teacher’s expectation. When teachers are only
teaching to find the correct answers, teachers might cut off students and correct them right away if their ideas are different from the answer key. Students may get discouraged from trying anything new. They may choose to play safe to avoid making any mistakes. This is not good for learning mathematics that requires a lot of flexible thinking, uncommon approaches, trial and errors, and creativity.

When teachers instruct and assess creativity, they should do more than just promoting quick recalls or multiple choices. Based on the Torrance Test of Creative Thinking, teachers can instruct and assess student creativity by emphasizing cognitive abilities: fluency (the ability to produce different ideas); originality (the ability to produce uncommon ideas); elaboration (the ability to work on ideas with more details); flexibility (the ability to create connection between different categories of ideas), and resistance to premature closure (an open mind for future modification and development) (Kim, 2006, 2011; Torrance, 1963, 1967). In the learning of mathematics, teachers can design nontraditional assignments, such as asking students to deduce and develop a theory and invent word problems to exercise their fluency, originality, elaboration, flexibility and open-mindedness. Teachers can even go beyond written tasks and make full use of other tools in the classroom. For example, teachers can block out a time each day for small group discussion on one particular challenge problem. Students can gather around the blackboard and demonstrate, share, and discuss their solutions on the board. When teaching geometry, teachers can encourage students to bring in shapes or photos and share their knowledge of shapes they experience outside of the school. Teachers can
create opportunities for children to connect mathematics with their previous experience, and encourage them to apply mathematics in life flexibly. For instance, they can challenge students to calculate the surface area or volume of big architectures, such as their home, a motive theatre downtown, or a large shopping mall. Students are encouraged to use personal strategies to solve the problems, such as different ways of dividing the space and using the knowledge they know of classic shapes. However, teacher knowledge, teacher creativity, curriculum and timing could be issues that can interfere with such teaching for creativity.

2.2.5. Summary

Teaching for creativity takes a lot of strategies to value, encourage, support, and reward students in their mathematical creativity. The learning environment should welcome creative thinking so that students feel safe and free to try out their creative ideas. Students are supported by teachers to redefine problems, question and analyze assumptions, identify and overcome obstacles, collaborate with others and take others’ perspectives, sell their own ideas, generate ideas, understand the pros and cons of knowledge, take sensible risks, tolerate ambiguity, build self-efficacy, delay gratification, find interest and passion, cross-fertilize ideas, welcome mistakes, and develop creative thinking from teacher role modeling and instruction and assignments for creativity. However, teacher knowledge and skills, personality, curriculum content, problems and assignments, time pressure, standardized assessment, physical environment can be potential obstacles to promoting mathematical creativity. It is worthwhile to find out what strategies teachers are able
to use and what aspects of mathematical creativity they are able to promote and fail
to promote, which is explored in this dissertation.
CHAPTER 3

METHODOLOGY

In this chapter, I (1) discuss the constructivist paradigm; (2) introduce the grounded theory methodology and its strength; and (3) explain the constructivist approach to the study and its procedures (i.e., research questions, research settings and procedures, participants, data collection strategy, and data analysis and verification procedures).

3.1. Constructivist Paradigm

This study is based on a constructivist paradigm (also known as interpretivist paradigm). A constructivist worldview is a philosophical standpoint that requires researchers to spend an extensive amount of time in the field, pay attention to multiple worldviews people hold, and try the best to understand and present these worldviews with subtleties and details (Creswell, 2013; Hatch, 2002; Lincoln, Lynham, & Guba, 2011; Neuman, 2011). Constructivists tend to generate meaning inductively (Creswell, 2013). They tend to avoid playing relatively distanced roles, but seek for active engagement in the field through interaction with participants and co-construction of the reality (Hatch, 2002). When asking questions, constructivist researchers are open to multiple modes of meanings so that ideas and feelings are exchanged and meaning made visible; they also look for specific information to understand the background of the participants (Creswell, 2013). They acknowledge that experience and context influence not only participants’ belief system but researchers’ interpretation of the findings as well (Creswell, 2013; Lincoln et al.,
Generally speaking, they avoid detaching interpretation from the setting or arbitrary aggregation of participants’ answers and behaviors (Creswell, 2013; Lincoln et al., 2011), and take time to carefully examine the meaning underneath the surface that may have been taken for granted (Lincoln et al., 2011).

This study on mathematical creativity was conducted under a constructivist theoretical worldview. To explore elementary school teachers’ definition of creativity in the learning of mathematics, I respected each individual teacher’s subjective understanding, interacted and exchanged information with the teachers, conducted deep conversations, and collected details to reveal teacher understanding behind common teaching practices. In effect, the teacher participants and I were conducting the meaning-making process together to co-construct a theory.

Teachers differed in their mastery of mathematics content knowledge, beliefs and awareness of creative problem-solving, and access to training and focus on mathematical creativity, not to mention the differences in expectations and pressures from the district and the school, and the strategies used to deal with them. I explored each of the teacher participants’ personal stories to understand their situations. I sought to explore a series of processes through a grounded theory design to interpret teachers’ understanding, advocacy, and struggles when they promote creativity in the learning of mathematics. An overview of the grounded theory methodology (GTM) is given below.

3.2. Grounded Theory Methodology

3.2.1. Definition of Grounded Theory Methodology
Grounded theory methodology is an approach by which researchers discover a theory that explains a social process experienced by the participants (Bryant & Charmaz, 2007; Creswell, 2013; Glaser & Strauss, 1967). The theory, grounded in the data, can be utilized to inform future studies (Creswell, 2013; Glaser & Strauss, 1967). Grounded theory methodology is characterized by theoretical sampling, constant comparison of data, and theoretical saturation of categories, which distinguishes itself from other generic qualitative approaches (Hood, 2007).

Theoretical sampling is purposeful sampling, but not vice versa. A purposeful sample can be preplanned. In contrast, a theoretical sample in grounded theory methodology is not chosen ahead of time, but is gradually enrolled as data analysis goes on. During the process of data analysis, researchers find the emerging conditions that may potentially influence the process under study, and thus, participants who show diversity in those conditions will be enrolled for on-going data collection (Hood, 2007). At the same time, data are constantly compared to previous codes within categories (Hood, 2007). Unlike generic qualitative studies in which data collection stops when substantive findings are accumulated, grounded theory data collection keeps going until theoretical saturation is reached, that is, no new theoretical categories can be established from additional data (Hood, 2007).

When it comes to generalization, generic qualitative studies are more likely to focus on application to similar cases, while in GTM, the generated theory is expected to be applied across diverse situations and disciplines in which people experience similar inner processes (Hood, 2007).
3.2.2. Strength and Uniqueness of Grounded Theory Methodology

Grounded theory methodology provides flexibility allowing researchers to gradually trace their research interests and questions and narrow down focus along with data collection (Charmaz, 2006). Moreover, the on-going data collection also allows for the addition of more information to develop a theory as researchers dig deeper into the phenomenon or process (Charmaz, 2006; Creswell, 2013; Hood, 2007). Because humans are highly dependent on the use of symbols such as verbal language, written characters, and body movements, what lies beneath the surface may not be consistent with what is shown explicitly on the surface. Grounded theory researchers aim to discover these hidden assumptions not being recognized or appreciated. Thus, the generated theory is verisimilar that it finds and presents the unseen core values behind a phenomenon.

Grounded theory researchers depict a rich well-rounded picture without a heavy emphasis on a rigid preconception (Charmaz, 2006). A grounded theory design enables a detailed and deeper understanding of a phenomenon or process in context. In a grounded theory study, researchers often aim to develop an understanding and interpretation of the causes, contextual conditions, and the consequences across situations in regard to a process of change or interaction (Strauss & Corbin, 1990, 1998). This makes grounded theory a unique design different from other generic qualitative studies because the generated theory applies to different settings in which participants have very different experiences but share common feelings (Hood, 2007). In the following section, the approach to grounded
theory methodology that was used in this study is discussed.

3.3. Approach and Procedures

3.3.1. Charmaz’s Constructivist Approach to Grounded Theory Methodology

Several different approaches to collecting, analyzing, and interpreting data exist in GRM. According to Babchuk (2010), the four most well-known approaches to grounded theory methodology include Glaser’s (1967) traditional approach, Strauss and Corbin’s (1990, 1998) systematic approach, Clark’s (2005) situational analysis, and Charmaz’s (2000, 2006) constructivist approach. Charmaz’s approach was used because of her constructivist worldview. She emphasizes the co-construction of knowledge (Babchuk, 2008, 2011; Charmaz, 2006; Creswell, 2013); stresses the rapport with participants to co-construct the hidden structures (Charmaz, 2006); and puts emphasis on the feelings and views of the participants rather than the pure methods per se (Charmaz, 2006; Creswell, 2013). According to Charmaz, grounded theory methods are systematic but flexible as well. “Readers and researchers’ perspectives, purposes, and practices influence how they will make sense of a method” (Charmaz, 2006, p. xi). As claimed by Charmaz (2006), “…neither data nor theories are discovered. Rather, we are part of the world we study and the data we collect. We construct our grounded theories through our past and present involvements and interactions with people, perspectives, and research practices” (p. 10). She encourages researchers to enter participants’ situations to see from the inside because “what outsiders assume about the world you study may be limited, imprecise, mistaken, or egregiously wrong” (p. 14).
Charmaz’s approach to grounded theory methodology fits this research study on teacher interpretation and support of mathematical creativity because teachers might take creativity for granted—they often agree that creativity is important in learning and problem solving—but it is challenging for them to explicitly explain and justify their assumptions of the significance of creativity and what creativity looks like in the learning of mathematics. Such vagueness may also affect the accuracy and effectiveness of communication among teachers and between teachers and other scholars and policy makers because they may imply different things when talking about mathematical creativity. Teachers can have different understandings of the term creativity when reading NCTM standard in which it is contained but not clearly defined. I facilitated teachers to reflect on their belief and pedagogy by asking relevant questions; attempted to understand teachers’ feelings and views; and looked at teachers’ mental process through their eyes. The teachers and I used this information to interpret teachers’ definition, value, and teaching process regarding creativity in the learning of mathematics. They made visible the meaning and process of teacher interpretation that remained silent before.

**3.3.2. Research Questions**

The purpose of this grounded theory study was to explore (1) how a group of teachers from a public school system in the Midwest interpret creativity in K-3 mathematics learning and (2) how these teachers promote or fail to promote creativity for students in the learning of mathematics. To be specific, the research questions were framed into three aspects outlined below.
The first question was related to teachers’ definition of mathematical creativity: How do teachers make sense of mathematical creativity in young children? For example, is mathematical creativity a series of cognitive traits in children? Is mathematical creativity part of personality and habits? Is mathematical creativity related to children’s styles of thinking and reasoning? How much knowledge does it require for children to be creative in mathematics? Whether and how is mathematical creativity environmentally sensitive? How important is creativity to mathematics? Does the learning of mathematics require creativity in children at all?

The second question of interest was: how do teachers support creativity in children? For example, can creativity be promoted and what have teachers been doing to promote it? How do teachers describe their role in promoting children’s creativity: a leading role where teachers are the primary providers of information who pass down knowledge to children, versus a secondary role where teachers are facilitators who step aside and let children take more initiatives?

The third question of interest was: what support and obstacles do teachers meet in promoting children’s creativity in the learning of mathematics? For example, how do teachers perceive the following things as advantages or challenges: their own knowledge background and personality; requirements from national standards, district, and school; professional development programs; expectations from curriculum, students, and parents; and classroom environment? And how do they take advantages of or conquer these influences?

3.3.3. Research Settings and Procedures
Thirty teachers from a public school system in a medium-sized Midwestern city were interviewed from January through March 2013. All interviews took place in teachers’ own classrooms after school except for two teachers (one was interviewed in a coffee shop close to the school, and the other was interviewed in my office on the university campus). Each of the interviews lasted from 30 to 45 minutes. The interview questions were semi-structured. These questions were piloted among four teachers before the formal study. Interviews with the four teachers were in-depth, ranging from 45 minutes to 90 minutes each. One teacher was interviewed twice for a total of 180 minutes. Some questions were then revised after the pilot study before they were used in the formal study. More details about the interview questions can be found in the data collection strategy section of this chapter. Coding and analyzing data were conducted simultaneously with data collection. Constant comparison and contrast of the data were conducted throughout the study. More details are discussed in the data analysis and verification procedures section of this chapter. A theory was generated to explain teachers’ views and practices in regard to creativity in the learning of mathematics.

3.3.4. Participants

Participants in this study were 30 K-3 teachers from a public school system. All of them are female teachers, which is representative of K-3 teachers in general. These teacher participants have attended a professional development program called Primarily Mathematics (PM), a six-course (18 credit hour), 14-month program for current elementary school teachers leading to a K-3 mathematics specialist.
certificate. PM is part of NebraskaMATH, a five-year, $9.2 million Math Science Partnership grant funded by the National Science Foundation. NebraskaMATH is a P-16, from preschool (P) through college (16), statewide partnership and its overall goal is to improve achievement in mathematics for all students and to narrow achievement gaps of at-risk populations.

This PM program offered three courses focused on mathematics and three on pedagogy and child development. The three mathematics courses were centered on number and operations, and geometry and algebraic thinking to develop teachers’ mathematical habits of mind (HOMs). The other three courses on pedagogy and child development were to help teachers become increasingly intentional, planful, observant, and reflective practitioners. Teachers studied about young children’s learning trajectories in mathematics, read quality research on effective pedagogy, developed and applied strategies in lesson planning and teaching diverse learners, and examined and reflected on mathematics lessons. Teachers were supported to situate their individual lesson planning within the mathematical ideas of the curriculum, giving particular attention to creating coherence and connections to the learning trajectories of children. Teachers adopted specific strategies to facilitate student learning in mathematics, including how to space learning over time, enhance productive math talk, use tasks of high cognitive demand, choose a variety of representations, create thought-provoking questions and explanations, and support individual needs by communicating with families. This PM program, however, did not have a special focus on promoting creativity in the learning of mathematics for
young children in any kind.

Teachers who graduated from PM either remained in the classroom as generalists, took math-intensive positions (teaching two or more math classes), or moved to mathematics coaching roles at the building or district level. This program included an optional seventh course focused on moving teachers into leadership roles. This program also included ongoing support in the form of study groups (a form of a professional learning community) involving teachers for the two years after completing PM coursework.

The 30 teachers who graduated from the PM program were chosen for this study because compared to other teachers in general who did not participate in this program, these teachers were likely to have more training and thus have deeper reflection on national and state level standards for mathematical proficiency, as was required in the PM coursework. In fact, a key purpose of PM courses was to help teachers become more reflective practitioners and to develop a stance of inquiry (Cochran-Smith & Lytle, 2009) into mathematics education. Standards and principles from the NCTM and CCSS-M were frequently referred to throughout the program. As was described earlier in the second chapter of literature review, these standards are closely associated with creativity in the learning of mathematics. Teacher participants were likely to be prepared to verbalize their understanding of creativity in the learning of mathematics with clear and correct language found in national and state level standards.

In this grounded theory study, theoretical sampling was used. However,
according to Charmaz (2006), conducting theoretical sampling too early may restrain the quality of categories. Thus, in the beginning of the study, initial sampling was used. The three basic initial sampling criteria were: PM teacher participants; teaching K-3 grades; and teaching mathematics.

Categories emerged through the process of data collection and analysis. As more clues became available in categories and codes, these clues provided valuable insight for me to detect which area(s) to dig deeper into. At this point, theoretical sampling was initiated to include participants who fit into the categories being studied. By using theoretical sampling, I was able to “fill out the properties of a category…[to] create an analytic definition and explication…[and] demonstrate links among categories” in the later stages as well (Charmaz, 2006, p. 107). Theoretical sampling guided me analytically in terms of specifying properties of the categories, improving category preciseness, moving from description to analysis, enhancing generalizability, making inductions data-driven, connecting among categories, and making the theory succinct and concise (Charmaz, 2006, p. 105; Hood, 2007).

3.3.5. Data Collection Strategy

In this study, the primary data were collected from interviews. A secondary set of data were teachers’ drawings. In Charmaz’s words, interviewing is a process of, “reconstruction… of the reality” (2006, p. 27). The reason I used interviewing as the main data collection strategy was because interviewing was “a flexible, emergent technique; ideas and issues emerge during the interview and interviewers can immediately pursue these leads” (Charmaz, 2006, p. 29). I focused on interpreting
teachers’ feelings and views to collect in-depth and rich data for theory-development purpose.

Supported by Charmaz’s (2006, p. 30) interview guidelines, I developed interview questions for this study (see Appendix A), which was semi-structured. Questions were added, modified, and removed as the study progressed. Additional questions were asked when teachers expressed interesting and meaningful ideas worthy of additional exploration. I would pause, point out the idea, and invite the teachers to further elaborate and give examples. Teachers varied among themselves and some had less experience or thoughts in regard to one or more questions, and therefore these questions were removed or reframed into more general ones. The interviews were audiotaped and transcribed verbatim by me.

Teachers were also asked to use drawing to represent their theory of creativity for students in the learning of mathematics. Teachers were provided with different sizes of papers and a variety of drawing tools, including a package of 50-color oil sticks, a box of 24-color crayons, a bunch of color pencils, two packages of rainbow color markers, and a package of 10-color gel pens. Teachers were then asked to draw a model of creativity in the learning of mathematics with their choice of materials provided. Teachers were then asked to explain the drawing orally. Teachers’ verbal explanations were also tape-recorded and transcribed. The picture data were collected after each interview to compare with, enrich, and assist the analysis of the interview data.

**3.3.6. Data Analysis and Verification Procedures**
MAXQDA (11.0) was used for coding and analysis. According to Charmaz’s (2006) approach, coding data happens concurrently with data collection. Coding was conducted after each interview to inform the following interviews. For example, the second interview was conducted after the first one was coded. In this way, some areas of potential interest in the earlier interview were expanded on in the upcoming interviews. After the second interview was transcribed and coded, the first two sets of codes were compared. Commonalities and differences were identified. Such comparison kept throughout the study to reveal the need for information on certain ideas with potential value. Interesting ideas were explored to a greater extent in subsequent interviews. Teachers were also contacted after interview to verify some of their ideas or provide more details for their ideas. As the study progressed, constant comparison and frequent revisit and reflection with teachers continued until theoretical saturation was reached.

In line with Charmaz’s (2006) approach, the first round of coding was open coding during which the codes stayed close to the data, were kept simple and precise, were compared with each other, and were coded in quick speed. Though coding word by word and line by line over a long transcription took a long period of time, it urged me to closely read the data and make constant comparison and contrast. The strategy of focusing on words or phrases of actions was also beneficial and effective to extract the essence of data.

After open coding, focused coding was conducted by sifting through the large amount of codes across interviews to verify their frequency and significance and
therefore determining their adequacy (Charmaz, 2006). The resulting codes were more conceptual and selective. Potential categories and subcategories were raised from these codes. Then, these categories and subcategories were compared through theoretical coding to examine their relationship. The bond between categories and subcategories was thus reestablished. Theoretical coding was a process of elaborating and specifying a larger category by merging relevant categories or subcategories. This is a process that joins different pieces to form a coherent story.

While coding, I also kept a memo. Besides keeping track of words, phrases, and sentences, as Charmaz recommended, this memo also included visual notes, such as circles, squares, and arrows that mapped the relationships among codes, categories, and themes. By jotting down quick notes and flashing thoughts, I was able to examine and verify if these notes were confirmed by the data as the coding and analysis proceeded.

In the study, I used several forms of verification, including member checking, peer review, and document analysis to strengthen the findings. In member checking, I shared the data analysis with all the teacher participants to ensure that the interpretation was accurate, credible, adequate, clear, and true to their thoughts (Creswell, 2013). Member checking allowed the teacher participants and myself to revisit and reflect together on their reasoning and interpretations. In peer review, I showed the analysis to and received feedback from professors, graduate students, and scholars whose specialties were in early childhood education, developmental psychology, mathematics education, teacher education or related areas. In addition,
other document data were also analyzed, including drawings and math stories. The drawings of teachers’ interpretation of mathematical creativity were collected. Teachers’ verbal explanations of the drawings were audio-taped and transcribed. Both the picture data and text data were analyzed to identify main themes in teachers’ understanding of creativity for students in the learning of mathematics. These themes were used to enrich and verify the findings from the interview data. In the PM program, teachers were asked to write math stories as a reflection on their experience of mathematics. They wrote an essay in response to the questions including: their goals for students in the learning of mathematics; their interpretation of how students could reach mathematical proficiency; their self-identification of strengths and weakness as a mathematics teacher, and their description of challenges in their mathematics life. Analyzing these artifacts gave me a better understanding of the background of each individual teacher and her thoughts and understanding on mathematics.
CHAPTER 4

FINDINGS

The purpose of this grounded theory study was to generate a theory that describes the process of teachers promoting student creativity in the learning of mathematics in K-3 classrooms. Creativity is critical for learning mathematics in that it helps students develop deep understanding, take ownership in mathematics, and explore all kinds of possibilities and connections. This project explored: 1) teachers’ interpretation of creativity in the learning of mathematics; 2) how teachers promote mathematical creativity and their strategies; and 3) support and obstacles teachers meet with in promoting children’s mathematical creativity.

In the first section of this chapter, I discuss the data analysis framework that reveals the spiraling process of analysis with grounded theory’s technique of constant comparison. A visual diagram is given to show the structure of the findings, including the central phenomenon, strategies, and consequences, which are discussed in the following sections of this chapter. Analysis of the drawings is also given to describe the central phenomenon. Teachers did not mention any major obstacles in promoting children’s mathematical creativity, but teachers referred to their experience in the Primarily Math (PM) program and the new curriculum Math Expressions adopted by the district as supporting them in promoting mathematical creativity. This is described in the last section of this chapter.

4.1. Data Analysis Framework

Starting with open coding, I kept simple and precise codes that stayed close
to the data. These codes were assigned word by word and line by line when I read through the transcripts. The codes were constantly compared and contrasted within and across transcripts, as I describe in fuller detail below. This process helped me grab the essence of the data and set up the stage for focused coding. Then, through the process of focused coding, I sifted through the large amount of codes across interviews to verify their frequency and significance to decide which ones were adequate to keep. The codes selected to be kept in this round of coding were more conceptual. Categories and subcategories were found from these codes. At the same time, the central phenomenon was identified. Later on, these categories and subcategories were then compared during theoretical coding. Their relationships were examined and these categories and subcategories were reorganized to construct a more reasonable story. To better contour and connect concepts, I further compared to merge relevant categories and subcategories, took an existing subcategory and developed it into a new and bigger category, and relocated codes and subcategories among categories to achieve most reasonable classification.

Analyzing the data was a spiraling process. With help of writing memos, I was able to keep reflecting and revising throughout the study. The visual diagram (see Figure 4.1.1) shows the framework of the findings. The framework was initiated early in the analysis stage and modified many times before it was final. I constantly went back to the data to compare and move the categories and sub codes around, and sometimes to combine or decompose categories. Both the diagram and analysis guide and inform each other to yield findings that are clear and true to the data. In
the diagram, the readers can find the central phenomenon, strategies, and context.

The central phenomenon was teacher interpretation of mathematical creativity in students. After the establishment of central phenomenon, strategies of how teachers promote mathematical creativity were explored. Different emphases on the aspects of mathematical creativity promoted by the teachers were then identified.

Figure 4.1.1 Visual diagram of findings on central phenomenon, strategies, and consequences.

Analyzing with a grounded theory approach takes a lot of courage and flexibility. I was using Charmaz’s (2006) constructivist approach, and I started without a strong and concrete framework or rubric to lead the way. The analysis proceeded in a messy process like crossing a river, bare-handed, for the first time, in
the darkness. To reach the wonderland on the other side of the river, I had to test the depth of the water and feel the riverbed each step of the way. It was hard to know where to put the next footstep because what information from the data should be kept and what information can be left behind were hard decisions to make. I stepped back and forth during the crossing to find a good pathway to the land. I interpreted the findings to represent the views of the teachers as precisely, organically, and lively as possible. Let me use the development of themes to describe teachers’ interpretation of mathematical creativity as an example to illustrate the process of data analysis. Codes regarding similar content were grouped together. Groups dealing with the same concept were put under the same category. Different groups in one category showed different aspects or levels of the same concept. Categories that were fully developed finally upgraded into themes. At first, many codes were spread out among different groups. Some of the groups share similar concepts, such as "students solve problems in different ways and students find strategies other than the standard way," both of which refer to the concept of flexible thinking style. And thus the two groups of codes were combined into one category. Sometimes, codes in one group emphasized different aspects of the same concept. In this case, the group was decomposed and replaced with several more refined groups of codes. For example, in the category regarding flexible thinking style, one segment was coded from the perspective of the quantity of strategies, that is, “students decompose numbers in several different ways,” while another segment was coded with an emphasis on self exploration with one’s own unique way different from other students’ solution, that
is, “students use a very different personal way.” The two types of codes were fine to be put in one group at the beginning, but as more segments were coded, the two concepts needed to be divided into two new groups, one is quantity of strategies and the other is self exploration of strategies. Both of the groups still belong to the same category. This category later developed into the theme, if and NOT only if, which is explained later in this chapter.

Sometimes, codes in a group were associated with concepts in multiple themes, and thus these codes needed to be rearranged. For example, in a group labeled “making mistakes,” one type of codes leaned towards the knowledge gained from figuring out mistakes, another type of codes described the willingness to take risks and make mistakes (be ok to be wrong), and a third type of codes dealt with teachers asking questions to support students to reflect and fix mistakes in a non-threatening way. The three types of codes all talked about something related to making mistakes, but they were actually dealing with different concepts: development of knowledge and understanding; personality; and teaching strategy. Therefore, these codes belonged to three different themes that deals with knowledge (i.e., the theme of capacity, which is explained later in this chapter), personality (i.e., the theme of false conjecture, which is explained later in this chapter), and teachers’ strategies, respectively.

As I was sifting through the huge amount of data, categorizing the data, collapsing the categories, establishing new categories, and moving around the codes back and forth, the structure emerged. Everything seemed to come together: I found
that teacher interpretation of mathematical creativity could be deciphered as a six-aspect concept.

I named the six aspects from the inspiration of the terms used in mathematics according to the meaning each of the terms represents. The acronym of the concept is DCIFER (pronounced as “decipher”). The six aspects of DCIFER are:

D—dynamic equation; C—capacity; I—if and NOT only if; F—false conjecture; E—exterior angle; and R-remainder. A description of DCIFER is given in the next section of this chapter (see 4.2).

Figure 4.1.2 Visual diagram of findings. Details of central phenomenon provided.

When I deciphered teachers’ interpretations of mathematical creativity, I was ready for the next step: developing the themes for teachers’ strategies. However,
even though the themes of the central phenomenon DCIFER were formed, the themes still underwent several revisions during the analysis of teacher strategies before they were final.

In the first stage of developing themes to describe teachers’ strategies to promote mathematical creativity, I classified, labeled, and defined the strategies teachers reported to utilize to promote creativity in the learning of mathematics. The five types of strategies are: 1) *reviving curriculum*; 2) *cognitive scaffolding*; 3) *reinforcement and encouragement*; 4) *nurturing environment*; and 5) *one step back*. Description of the strategies is given in the fifth section of this chapter (see 4.5).

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**Central Phenomenon**
(DCIFER)
D—Dynamic Equation
C—Capacity
I—If and NOT Only If
F—False Conjecture
E—Exterior Angle
R—Remainder

**Strategies**
- Reviving Curriculum
- Cognitive Scaffolding
- Reinforcement and Encouragement
- Nurturing Environment
- One Step Back

**Consequences**
Differentiated Promotion of Mathematical Creativity

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Figure 4.1.3 Visual diagram of findings. Details of strategies provided.
The categories went through a lot of adjustments. As I was enriching the content of each category by deciding what codes/categories to be included and in which category, I was able to contrast and compare across the categories and redefine the categories more appropriately. When a code seemed to fit two different categories, I needed to compare and further contrast two categories. For example, the code, *teachers provide open ended problems*, can either be classified as *reviving curriculum* or *nurturing environment*. On one hand, open ended problems are something from the environment that stimulates creativity. On the other hand, giving challenging problems is an added activity that shows the teacher is using the curriculum flexibly and in a lively way. After encountering several similar cases, I decided that *nurturing environment* referred to the physical space and social atmosphere, while *reviving curriculum* was associated with the mathematical content and specific learning activities. Thus, for the code above, *reviving curriculum* is where it belongs. However, the definition of strategies was not final and the themes were getting more and more organized as constant comparison and contrast continued.
After developing the strategy rubric to some extent, I found it possible to map strategies to DCIFER, that is, to find out the specific types of strategies used for each of the six aspects of mathematical creativity. More description can be found in the sixth section of this chapter (see 4.6). For the rest of the work, though still involving a lot of moving back and forth and combing and collapsing groups, there was a much clearer direction. I was not wandering in the dark anymore. I was able to proceed with more efficiency and confidence.

When around two-thirds of the interviews were done, a fairly clear structure of themes was found. The rest of the interviews were conducted to enrich the categories and subcategories. When I was doing the last couple of the 30 interviews,
a saturation point was reached. At that point, I found the six aspects of DCIFER and the five teaching strategies were well developed and no more popular themes came up in the interviews. The 30 teacher interviews were adequate to reach theoretical saturation for this study. To fully explore the six aspects of DCIFER, I adjusted the interview questions along the way. For example, I purposefully asked teachers to explain more about flexible thinking and give examples to illustrate. Some teachers were using the same or similar words, but their emphases were different aspects of the concept. For example, to some teachers, flexible thinking meant students using different strategies; while to some other teachers, flexible thinking was when students were taking an unconventional route to approach a problem. Asking teachers to clarify themselves helped to establish and enrich subcategories.

Not all the six aspects of DCIFER and five teaching strategies were covered by all 30 teachers simultaneously. Teachers varied in their focus, but teachers usually covered most of the themes. For example, among the six aspects of DCIFER, three of them were most recurrent among the teachers, and the other three were less popular among the teachers. This is explained in more detail in the discussion chapter.

4.2. Central Phenomenon: DCIFER

This section explains the findings of teacher definition of mathematical creativity in students. Based on the teacher interviews, mathematical creativity is deciphered as a six-aspect concept. The acronym of the concept is DCIFER. The labels of the six aspects of DCIFER are created with inspiration from mathematical
language. The six mathematical terms are chosen because their meanings in mathematics resemble the six concepts implied in the interview data. D stands for dynamic equation; C stands for capacity; I stands for if and NOT only if; F stands for false conjecture; E stands for exterior angle; and R stands for remainder. Each of the six aspects of DCIFER is explained in terms of the following aspects: 1) meaning of the label (and its synonym); 2) definition of the theme; 3) subthemes, if any; 4) example quotes; and 5) contrast and comparison with other themes, when necessary.

Table 4.2

Central Phenomenon (DCIFER)

<table>
<thead>
<tr>
<th>Code</th>
<th>Explanation</th>
<th>Sample Quotation</th>
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<tbody>
<tr>
<td>Dynamic Equation</td>
<td>Working on mathematics problems is a dynamic and evolving process that students use intellect to collect information, retrieve strategies, compare ideas, choose among approaches, develop strategies, revise solutions, and communicate and promote strategies. Students can always compare and change something in the strategy to make it a different solution, and thus the problem solving is a lasting process instead of a one-time shot.</td>
<td>“…[mathematical creativity is] not only solving in many different ways but as a way kind of checking yourself, so being able to… if you solve an equation, and being able to solve it with a picture or a diagram, or a math knot, or something like that. So having that flexibility and being able to see when all these different ways add to the one and then make that choice of which one is better for you, which one works for you.”</td>
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<tr>
<td>Code</td>
<td>Explanation</td>
<td>Sample Quotation</td>
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<tr>
<td>Capacity</td>
<td>Capacity is more than a large volume to accumulate knowledge; it is a strong capability based on deep understanding to connect, retrieve, and expand between and among mathematics concepts a student is learning and or has already learned.</td>
<td>“I think creativity in math is probably the exploration of concepts...going deeper into those concepts, really getting the meaning behind the concepts.”</td>
</tr>
<tr>
<td>If and NOT Only</td>
<td>There is more than one standard way to solve a mathematics problem. Mathematical creativity enables students to view a problem from multiple perspectives. Students can explore all kinds of possibilities and be open to the fact that their ideas may be distinct from what the textbook says, the teacher teaches, or the other students come up with. Everyone has different preconceptions and preferred ways of approaching problems. Students are encouraged to explore and follow their own way with individuality. Mathematics is a personal exploration in countless possible ways.</td>
<td>“I would say in terms of math, what I see creativity from students would be, being able to represent or show things in a variety of ways... some students were able to show it in multiple ways, so I felt like they kind of took that creativity or that freedom to express it in multiple ways.” “I would say that children need to be able to explore their own methods of problem solving.”</td>
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<tr>
<td>If</td>
<td></td>
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<tr>
<td>Code</td>
<td>Explanation</td>
<td>Sample Quotation</td>
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<tr>
<td>False Conjecture</td>
<td>Students are ready to make mistakes and face mistakes in a proper way. They welcome mistakes and are willing to learn from mistakes with the awareness that mistakes can lead to success. They put more emphasis on the process of trying and exploring rather than the correctness of the answer. Thus, they are not afraid of taking risks to try something that may end up being wrong.</td>
<td>“…they are not afraid of making a mistake. The kids who are really afraid of making mistakes are very damp[\textendash]… They can’t be creative very well because they’re so afraid. They cannot try anything.” “And I think these kids... they are ok with making a mistake. And they are open to looking at their strategy and say what went well and what didn't go so well. They’re willing to take [risks]… and let peers teach them. They’re willing to let me teach them.”</td>
</tr>
<tr>
<td>Exterior Angle</td>
<td>People and objects in the context influence students in the learning of mathematics with creativity. The textbook, working problems, curriculum schedule, tools and materials, physical environment in the classroom, and even parent involvement can nurture, oppress, or be neutral to mathematical creativity in students.</td>
<td>“Since we adopted the Math Expressions program a couple of years ago, I think that program has helped us see the importance of that deep understanding, um, and in our old curriculum, we kind of moved very quickly through concept, and if it seemed like they [students] had it, we moved on to the next thing. And we introduced many new things very quickly.”</td>
</tr>
<tr>
<td>Remainder</td>
<td>A remainder is somebody who has strong motivation to explore mathematics. The motivation can be either the positive emotion associated with mathematics itself or social rewards and recognition from without. This person is often willing to remain in the struggle if stuck, keep pushing through by switching perspectives, and experience great joyful emotions both during exploration and figuring out mathematics in a creative way.</td>
<td>“The kids always want to share, and if someone has already shared a way, they want to share a different way. So they’ll solve a problem, and show in different, in a couple, in a few different ways on that white board, and then hoping that they’ll get a chance to show one of their ways.”</td>
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4.2.1. Dynamic equation.

*Dynamic equation* (or dynamic problem solving) refers to a concept similar with the intellectual ability in the investment theory of creativity. In mathematical physics, with the information we have of an object at the current moment, a dynamic equation gives us the new location and speed of this object at another time point. The object keeps moving, and at every new time point, we can always get something different from the equation. Solving mathematics problems with creativity is a dynamic process just like operating the equation. A student can always change and modify her solution a little bit to achieve different or better results.

The concept represented by *dynamic equation* highlights the evolving process of creative problem solving with intellect. In this process, students progress in a dynamic process during which they find, try out, and select among strategies; they apply, adjust, and check solutions; and they express, refine, and promote their ideas. Problem solving is not a one-time shot, but a dynamic period for students to manipulate and develop ideas, which was mentioned by 28 of the 30 teacher participants (93%).

Teachers had different foci associated with different stages of problem solving. For example, there were 19 teachers (63%) whose focus was on the initial development of the strategy. Many teachers recognized the importance of retrieving previous knowledge to start with. For example, one teacher stated, “I guess maybe applying knowledge that they have and using it in a new way to solve a problem...so taking what they know and then expand on that.” There were 18 teachers (60%) who
focused on comparing and adjusting strategies. Many teachers described the importance of choosing and checking strategies. For example, one teacher said:

[Mathematical creativity is] not only solving in many different ways but as a way kind of checking yourself… if you solve an equation, and being able to solve it with a picture or a diagram, or a math knot, or something like that… so having that flexibility and being able to see when all these different ways add to the one and then make that choice of which one is better for you… which one works for you.

There were 18 teachers (60%) whose focus was on promoting and communicating the ideas. Many teachers emphasized the ability to explain the thinking process. For example, one teacher said, “[Mathematical creativity is] students expressing their math processes, in ways that make sense to them, with accuracy and precision.”

4.2.2. Capacity.

Capacity (or comprehension) refers to a concept similar to the knowledge in the investment theory of creativity. In mathematics, *capacity* is the amount that something can hold. Here, capacity means more than the amount of knowledge a student can hold. Learning mathematics with creativity requires more than simple accumulation of knowledge; ideas should be able to connect with deep level comprehension. Learning mathematics with creativity needs knowledge just like building a brick house needs bricks. However, a house is not secured if there is no cement between the bricks. Creativity cannot happen in mathematics if there is no
profound comprehension of the knowledge to give students a solid mental capacity to connect different pieces of knowledge. In short, **capacity** refers to not only a large volume to add in and store knowledge, but also a strong capability to connect, retrieve, and expand what students learn or already learnt.

The importance of deep understanding of knowledge was mentioned by 17 teacher participants (57%) when they described their definition of mathematical creativity. For example, one teacher said, “I think creativity in math is probably the exploration of concepts...going deeper into those concepts, really getting the meaning behind the concepts.” Some teachers contrasted deep understanding with fluency to show they respected understanding more than the procedures. For example, one teacher stated,

It’s more about students making sense of the math, more so than trying to find the correct answer or procedure that fits the mode…the students are trying to analyze, generalize, make connections, and show relationships among the concepts that they're learning, versus just trying to get that answer, or just trying to figure what is it that the teacher is teaching or what correct response that I need to put on my paper.

### 4.2.3. If and NOT only if.

*If and NOT Only If* (or infinite possibilities for individualization) refers to a concept similar with the thinking style in the investment theory of creativity. In mathematics, *if and only if* is a biconditional statement that consists of a conditional (P) and its converse (Q). P and Q can only be true or false simultaneously. Q is
necessary and sufficient condition for P; P is equivalent to Q; and P precisely if
given Q. However, in mathematics, a problem and its solution(s) are usually not in a
one-on-one relationship. Mathematics creativity has more than one possibility. Using
the number line is an approach to solve a time difference problem, but it is not the
only solution. In short, if and NOT only if reflects teachers’ emphasis on different
strategies based on individual learners’ different preconceptions and their original
and personal way of approaching a problem. It is no longer “if and only if” the
standard way of doing mathematics, but a lot of diversity and individualization in
solving a mathematics problem.

All of the 30 teacher participants (100%) stated their appreciation for
different strategies. However, the appreciation can be further divided into three
different types or levels. There were 23 teachers (77%) who put an emphasis on the
quantity of strategies (basic level). Many teachers expected to find mathematical
creativity in students’ multiple ways of solving a problem. Teachers valued students’
ability to think of a variety of strategies or approach a problem from various
perspectives. However, the quality of the ideas was irrelevant. For example, one
teacher said:

I would say in terms of math, what I see creativity from students would be,
being able to represent or show things in a variety of ways... some students
were able to show it in multiple ways, so I felt like they kind of took that
creativity or that freedom to express it in multiple ways.

Another teacher used a student anecdote to describe her recognition for the
number of strategies in defining mathematical creativity, “I have a girl who always solves problems in different ways. I just mean she’s not lazy, she just likes to keep going and doing things, and so. She'll keep herself going.”

There were 21 teachers (70%) whose focus was on personal exploration of ideas, which is the medium level of if and NOT only if. These teachers respected individual exploration in the learning of mathematics. They allowed and tolerated differences and diversity among students. These teachers believed that student learners should take ownership in learning in order to develop mathematical creativity. For example, one teacher stated, “I would say that children need to be able to explore their own methods of problem solving.” Teachers valued individuality and independency in their descriptions of mathematical creativity. For example, one teacher expected students to make personal sense of a mathematics problem and explore in a way that is different from the rest of the class, “We want them to be independent… and we want to teach independent thinking. We want to teach self...um...gratification. And I think that’s hard to feel good about yourself and feel confident when everybody else is doing the same thing.” Another teacher described the importance of personal exploration with the mention of open-mindedness:

And one other thing I do encourage is you can do it in whatever way you want, you can do it with an equation, you can do it with a picture, and so this student just choose to do all of these ways on their own...they don’t feel like ok this is the only way that I can solve the problem, they’re really thinking about the problem and then, you know what makes sense to [them].
Some teachers described, from the instructors’ side, that teachers themselves should be open to students’ personal ways of doing mathematics, even if these ways were not the same as what was taught in the textbook or demonstrated by the teacher. For example, one teacher stated, “I mean it’s more open-ended. They don’t have to do the way I’m doing it. If they want to try a different way, they can, and I don’t stop them. I usually try and follow what they’re doing.”

There were 13 teachers (43%) who relied on unconventional strategies, which is the advanced level of if and NOT only if. These teachers advocated for using a non-standard way of thinking in the learning of mathematics. For example, one teacher described atypical thinking in terms of a student anecdote:

I have a little boy... He is very creative. He often thinks out of the box, like, he's the one we can often call on to show us...he’ll do it in a different way than the other kids do. He doesn't go straight to what you would see the answer should be. He would take different approaches to solve it. So we often ask him to share. And he always kind of does unique ways of doing things. He sees it differently, you know...And of course we give them models of doing it, but how he’ll decompose a number, or how he’ll...he’s just a little bit different.

In summary, on the basic level of if and NOT only if, teachers put stress on the number of strategies per se without consideration of the quality of these strategies. On the advanced level, the quality of the strategies is concerned in terms of uniqueness, unconventionality, and goodness. The medium level, personal
exploration, is a bridge that connects the other two levels. A large amount of strategies on the basic level does not guarantee uncommon strategies on the advanced level. By exploring in one’s own way, students may transform some of the ideas on the basic level to the advanced level. In short, all the three levels of if and \textit{NOT only if} require open-mindedness to the endless possibilities in mathematics, but the degree of originality and desirability of ideas is stratified across the three levels. Teachers in this study had varied demands among the three levels. Compared to the number of teachers emphasizing unconventional strategies (advanced level), there were more teachers who were focused on the number of strategies (basic level) and personal exploration of ideas (medium level).

\subsection*{4.2.4. False conjecture.}

\textit{False conjecture} (or fault-making) refers to a concept similar to personality in investment theory of creativity. In mathematics, conjecture is an educated guess or proposition that is very likely to be true. A conjecture that is disproved by a counterexample renders itself as a false conjecture. Conjectures are always expected to be true by mathematicians. However, in the learning of mathematics with creativity, students should be able to face mistakes. They should not only be ready for being wrong, but also welcome mistakes and false conjectures, which was supported by 9 teachers (30\%). For example, one teacher said, “I’ve seen kindergarteners be very adventurous about numbers. They are not concerned if they make a mistake. They try anything, (and) they want to share what they’re thinking.” Another teacher said, “…they are not afraid of making a mistake. The kids who are
really afraid of making mistakes are very damp[en]. They can’t be creative very well because they’re so afraid. They cannot try anything.”

These teachers advocated that students should not be afraid of taking risks and making mistakes, but instead be willing to grow and learn from mistakes. For example, one teacher said:

And I think these kids... they are ok with making a mistake. And they are open to looking at their strategy and say what went well and what didn't go so well. They’re willing to take [risks]… and let peers teach them. They’re willing to let me teach them.

It was implied by the teachers that what really mattered for students was the process of trying without too much concern about the final answer. Even mathematicians’ conjectures can be proved wrong. Making mistakes sometimes are important stepping stones to a successful creative solution. So the teachers cared more about the process of trying and exploring the unknown than the exact answer. For example, one teacher stated, “They are more willing to risk ....they are all about the work and not necessarily the answer. And I think that’s where you show real growth...it’s kind of enjoying the journey, you know, until when you get there.”

4.2.5. Exterior angle.

*Exterior angle* (or external conditions) refers to a concept similar with the environment factor in the investment theory of creativity. In geometry, an *interior angle* is the angle formed by one side of a polygon and its adjacent side. If a line is extended from the adjacent side, the first side and the extended line forms another
angle, and this angle is called an **exterior angle**. The **internal angle** and the **external angle** on the same vertex always sum up to 180 degrees.

The term **exterior angle** is borrowed to indicate people and things in the context that would influence students in the learning of mathematics with creativity. If a child is an interior angle, the context becomes the exterior angle for the child. How the exterior angle adjusts its degree would affect the change of the value of the child angle, and vice versa. There were 26 teachers (87%) who emphasized the role of environment in mathematical creativity. Some teachers mentioned home environment and parent involvement that parents may impose their own philosophy on their child when they supervise their child doing homework. Parents may allow less freedom or flexibility in the way of problem solving and insist on the “standard” way they were taught before rather than recognizing a creative shortcut the child creates. For example, one teacher said, “I think that for our kids, many parents were not taught the way that they [students] are being taught math. Many parents were taught just standard algorithms and this is the way you do it and this is the way everybody does it.”

Many teachers were focused on the curriculum content and structure. For example, one teacher commented that the textbook supported students to achieve deep understanding:

Since we adopted the *Math Expressions* program a couple of years ago, I think that program has helped us see the importance of that deep understanding, um, and in our old curriculum, we kind of moved very
quickly through concept, and if it seemed like they [students] had it, we moved on to the next thing. And we introduced many new things very quickly. And then as they got older, they wouldn’t really understand as they got to work with bigger numbers, wouldn’t really understand place values, wouldn’t really understand the value of each digit. And so we’re trying to work more with that in a younger level. And really make sure they understand that before we move on quickly. And then, and so far it seems like it’s helping.

Another teacher commented on the time for creativity to grow:

Time is a huge component because in order to be creative, you have to have thinking time. In order to be creative, you have to have kids to have down time, they have to able to have conversations. They have to be able to work in groups, and support each other.

These teachers also mentioned manipulatives and other learning tools in the classroom that played a part in a child’s creative learning process. These materials can boost and facilitate creativity if used appropriately. One teacher commented on the lack of time of exploring with manipulatives:

There really isn’t that time and, trying to build that in, it’s hard to get the, you know even 10 or so minutes to get that in. And the curriculum does [give some time] every once in a while when there is a new manipulative, it will be like, oh, they get free exploration for seven minutes. Well that’s not really enough for them to really dig deep with the materials that come up, you know,
like pull it together with something else they were familiar with, to make those connections.

Another teacher described that materials facilitate thinking about complex problems. She said:

And if they are good at drawing, especially in drawing pictures, or even using some manipulatives or something to build what it is they’re thinking about, they can figure out complex problems, just with those materials. Even though it's not been taught to them yet, they worked it [the problem] out. We have to allow them to have those opportunities.

4.2.6. Remainder.

Remainder (or remaining in the exploration) refers to a concept similar to the motivation in the investment theory of creativity. In mathematics, remainder is the amount left over after division. This term is borrowed here to indicate a situation in which students remain engaged in solving mathematics problems. Imagine in a classroom where all the kids went out for recess but only one kid who was left in the classroom alone, struggling but still trying to figure out a challenge problem. This student is a remainder who is patiently trying, curiously exploring, and passionately persevering to find the “a-ha” moment.

A remainder is somebody who has strong motivation to force himself to keep pushing along on the creativity road to mathematics. There were 16 teachers (53%) who discussed the importance of motivation. Some of the teachers emphasized motivation from inner drives, such as the positive emotion associated with doing
mathematics that motivated students. For example, one teacher stated, “If you’re creative, you enjoy life… looking at it in different ways, and you don’t have to have one way of doing things or one thing that makes you happy, but there’s lots of different things...” Some teachers emphasized motivation that came from outer resources, such as rewards, praise, and public recognition. For example, one teacher talked about sharing unique strategy in public was attractive to students:

The kids always want to share, and if someone has already shared a way, they want to share a different way. So they’ll solve a problem, and show in different, in a couple, in a few different ways on that white board, and then hoping that they’ll get chance to show one of their ways.

Among the six aspects of DCIFER, three of them were most frequent when teachers described their definition of creativity in the learning of mathematics. The three aspects were: \textit{if and NOT only if}; \textit{dynamic equation}; and \textit{exterior angle}. These three aspects were also the three big themes found in the drawings. The descriptions of the drawings are given below.
Figure 4.2 Visual diagram of DCIFER: More frequently mentioned and less frequently mentioned aspects.

4.3. Teacher Drawings of Mathematical Creativity

Themes found in teachers’ drawings fell into one or more of the six aspects of DCIFER. The drawings often had one major theme that was most salient, as shown in the visual image per se and in teachers’ oral explanation of the drawing as well. Three themes were found as major themes in the drawings: if and NOT only if; dynamic equation; and exterior angle. Some minor themes were also found in the drawings, which were one or more of the six aspects of DCIFER. In the following section, I describe several sample drawings for each main theme.

The most popular theme in teachers’ drawings was if and NOT only if. There were 20 drawings showing the value for exploring different ways to solve mathematics problems. Two sample drawings along with teachers’ explanations are given below. For example, one teacher drew different pathways students took to
solve a mathematics problem. *If and NOT only if* was clearly shown as the main theme of this drawing (see Figure 4.3.1). The teacher also made a note in her comment that being able to monitor the problem-solving process by comparing and choosing among different strategies was also an important aspect of creativity in mathematics. This comment showed a minor theme: *dynamic equation*. In short, for this drawing, *If and NOT only if* is the main theme, while *dynamic equation* was the minor theme. This teacher described her drawing in this way:

So when I think about teaching kids how to learn math, my goal is that when they see a problem, they don’t see like an immediate solution that are arrived at the answer, but that they see there are lots of different ways they could take to try and get there. And I think they’re really creative students [who] have more ways, or they’re more… you know, they don’t feel a box in which they have to choose certain strategy, but they feel they can pick the way that sounds the most appealing to them at the moment. And they are able to recognize…they might even be able to recognize there are some ideas that are dead ends that they can think about a strategy and they realize that won’t get them where they need to be. And I think a creative learner learns there are shorter ways to the answer and there are really roundabout ways. And it doesn’t mean that they may not ever try this one, but as they get more experienced as a mathematician, eventually your goal is for them to choose the most efficient way. But I’m more happy with kids that are able approach it in any way, you know, that you can think of.
Many teachers drew concrete examples of students using multiple ways to do mathematics. For example, one teacher showed in her drawing different ways students represent and decompose the number five. In the drawing (see Figure 4.3.2), the teacher put down the number “5”, a hand with five counting fingers, five tokens, a five-cent nickel, and different ways to decompose the number five. She described her drawing:

We are working on five, so we could [represent five] with this symbol [“5”], then with that set [five circles], and the partners of it [3 and 2, 4 and 1, 1 and 4, and 2 and 3], or it could be a nickel, and we did a lots of five on our hand. So when they’re trying to solve problems, they can use any of these models for thinking about numbers, and then verbally, hopefully, tell that, or [by using] models, manipulatives or something. So that wouldn’t be just ‘5’, but it could be all these other ways too.
The second most popular theme in teachers’ drawings was *dynamic equation*, which was shown in 7 drawings. Two sample drawings along with teachers’ explanations are given below. For example, one teacher showed the evolving process of solving a mathematics problem in her drawing. Students started with thinking about the problem in their head, and underwent several different stages, such as asking for more information, trying alternative ways, and talking about ideas, and maybe went back and forth between these steps, and finally got to share their ideas. As clearly shown from the drawing (see Figure 4.3.3), the teacher emphasized on the dynamic process of solving a mathematics problem, which is the main theme of this drawing. A minor theme of this drawing is *exterior angle* because the teacher mentioned time and tools when she described the long-lasting process of problem-solving. Another minor theme is *remainder* because the teacher mentioned working through struggles and frustrations in her description. The teacher explained:
This is them [students] thinking the math (pointed to top left figure) so they are thinking up there, and then I think it moves to, they want to talk about it. [If they don’t understand,] they want to ask more because they don’t know what you [the teacher] are asking or where should I [the student] start. And I think a lot of times when they say nothing, they’re really in this inquiry mode (pointed to the figure on the top right). But they don’t know what should they ask, or, I think they get stuck there. But then I think if you give them some time, then they’ll try something, so that they’ll try, depends on what we were doing or what materials they had. They will try (pointed to the next figure on bottom left) unless they stuck over here. I think this is a hard part. They are frustrated, they may not know exactly what to do or try something, but then if they work through that, then there might need to be another step in here (pointed in between the two figures on the bottom) of more talking, or going back here (pointed to the figure on the top right) to talk for some more understanding or ask questions, but then eventually they get to the ‘I know’, and most of the time ‘and I want to share it’ or ‘I want to show it in some way’.
Another teacher represented the process of problem-solving in her drawings in terms of thinking of the problem, collecting some thoughts, making and correcting some mistakes, and explaining the solutions (see Figure 4.3.4). She stressed that creativity was shown when a student took the process slowly by pondering and evaluating the ideas rather than writing down an answer in a quick snapshot. This teacher elaborated on her drawing:

At first, it's kind of questioning, like ‘I wonder how to solve this, what could I do’, and kind of getting the light bulb, ‘oh I’ll try it this way’. And then maybe having a mistake and cross[ing] it out, and being able to kind of explain at the end. I think creative thinkers are more able to slow themselves down and take themselves to the process rather than, some thinkers are just like ‘oh the answer is two’ and they write down; whereas a more creative
person is like ‘oh, ok, wait a minute’ they will kind of think it through and try something and get an idea and then evaluate it. Because I’m thinking from a first grader standpoint of course, I still think most of our kids are in the concrete stage and so I think a lot of kids are like ‘oh you have 14 and you got 8 more, well that’s 6 because 14 - 8 =6’. They’ll write it. And whereas a more creative thinker would be ‘ok, well, it was 14 and then there’s 8 more, so it has to be a bigger number’ and then maybe they’ll draw the picture out, maybe they’ll do 14 candy bars, so they’ll draw the 14 candy bars, and 8 more. You know, they’ll just think of those and they’ll evaluate it, not just like ‘oh here’s the answer’.

Figure 4.3.4 Teacher drawing four.

The third most popular theme in teachers’ drawings was exterior angle, which was reflected in 3 drawings. Two sample drawings along with teachers’
explanations are given below. One teacher described that time and opportunity, learning materials, and adults were playing roles in the learning environment for students to develop creativity in mathematics (see Figure 4.3.5). This teacher explained her drawing in this way:

For creativity in math in young children, they [young children] need to have the opportunity to converse. So a lot of talking has to take place, so the talking and the thinking (the teacher pointed to talking bubbles and thinking clouds). Question marks [represent] thinking about what that person is thinking about, and in a light bulb like, ‘oh I understand’. And then having it go back and forth. Lots of time needs to be devoted to being able to allow kids to develop that understanding of creativity. A tool box (the teacher pointed to the bag on the bottom to the left) there’s a lot of manipulatives and tools that kids need, so that they have access to all the materials. And then, kind of a facilitator (the teacher pointed to the person to the right on the bottom), like someone to help kids, to help push their thinking, whether that’s a teacher or a parent, or a community member.
Another teacher described the importance of standards, learning opportunities, and tools for students to develop and keep developing creativity in mathematics (see Figure 4.3.6). She elaborated on her drawing:

I think we have to start with the standards and processes, and then, in order for learning to happen, the teaching has to be focused on where we start with our standards, we have to combine it with some stories and experiences to make it real for our students. And then we provide them with opportunities to show their own interpretation through pictures and models. And instead of a finish line, we just keep going, we do it again.
A complete display of drawings is provided in the Appendix C.

4.4. Comparison across Groups

Given the exploratory nature of this study, I tentatively compared teachers’ interpretation of mathematical creativity according to their professional roles. One way of dividing teacher participants was between regular classroom teachers and teachers with coaching/leadership roles in mathematics. As was described earlier, some teachers from the Primarily Math PD program became mathematics coaches, and some took leadership roles as an instructional coordinator or curriculum leader. Another way of dividing teacher participants was based on the grade level of teaching. Teacher participants in kindergarten were a group. Teacher participants in grades 1-3 were in another group.

Teachers in different groups interpreted DCIFER in similar ways except for false conjecture and remainder. Teachers with coaching/leadership roles were more...
likely to stress *false conjecture* than regular classroom teachers, $X^2 (1, N = 30) = 7.645, P < .01$. Teachers teaching 1-3 grade levels were more likely to stress *false conjecture* than teachers in kindergarten group, $X^2 (1, N = 30) = 3.09, P < 0.1$.

Teachers teaching grade levels 1-3 were also more likely to stress *remainder* than kindergarten teachers, $X^2 (1, N = 30) = 6.467, P < 0.02$. Table 4.4.1, 4.4.2, and 4.4.3 show the group differences in terms of the number of teachers in different groups responding to *false conjecture* and *remainder*.

Table 4.4.1

*Chi-Square Test on False Conjecture by Regular Teachers versus Teachers with Coaching/Leadership Roles.*

<table>
<thead>
<tr>
<th>Teacher Roles</th>
<th>False Conjecture</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Regular Teachers</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>Coaches and Leaders</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>21</td>
</tr>
</tbody>
</table>

Chi-Square 7.654**

*Note: **p < .01.*
Table 4.4.2

Chi-Square Test on Remainder by Grade Levels of Teaching.

<table>
<thead>
<tr>
<th>Grade Levels</th>
<th>Remainder</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Kindergarten</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Grades 1-3</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>

Chi-Square 3.09**

Note: **p < .01.

Table 4.4.3

Chi-Square Test on False Conjecture by Grade Levels of Teaching.

<table>
<thead>
<tr>
<th>Grade Levels</th>
<th>False Conjecture</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Kindergarten</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Grades 1-3</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>21</td>
</tr>
</tbody>
</table>

Chi-Square 3.09*

Note: *p < .02.

4.5. Strategies to Promote Mathematical Creativity

This section explains the findings concerning teaching strategies to promote mathematical creativity in students. Based on the teacher interviews, teachers’ strategies fell into five themes: 1) reviving curriculum; 2) cognitive scaffolding; 3)
reinforcement and encouragement; 4) nurturing environment; and 5) one step back.

The five types of strategies are explained in the first part of this section, including
the definition of the themes and their subthemes, if any. Also discussed are the
thematic comparisons. According to the data, one strategy often facilitates different
aspects of DCIFER, and each aspect of DCIFER is facilitated by more than one type
of strategies. Thus, in the next part of this section, I further explain these strategies
by the aspects of DCIFER promoted. Sample quotations are given.

Table 4.5

Strategies.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Sample Practice</th>
<th>Sample Quotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reviving Curriculum</td>
<td>Find time and stretch time for mathematics; demonstrate different ways of approaching problems more than the ones on the textbook; selectively choosing content to teach; design learning activities with flexibility; and arrange “math talk” for the whole class.</td>
<td>“I kind of am a happy medium… I...I...and I love my curriculum, please [let me] remind you, I really see wonderful things happen with my students, but I’ll extend onto what they have, and that’s how I found a way to put myself in my students’ needs into learning. I’ll do the activities, I’ll do what’s asked, but I may add on some more questioning. I may let them struggle with that longer than the book tells me to. If they say 15 minutes, we might work 20 minutes on it.”</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Theme</th>
<th>Sample Practice</th>
<th>Sample Quotation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cognitive Scaffolding</strong></td>
<td>Ask questions to facilitate students’ understanding; listen to and check in student thinking; adjust level of complexity to match students’ cognition; and show examples to facilitate problem solving.</td>
<td>“I think some kids need more scaffolding and they need...asking them questions I guess. So if they’re totally stuck, you know, you can ask them a question to help them... probe them along to trying something, or to think about something in a different way. So questioning is really important.”</td>
</tr>
<tr>
<td><strong>Reinforcement and Encouragement</strong></td>
<td>Praise and recognize student thinking verbally or with physical awards; encourage and invite students to try all kinds of possibilities; and inspire students to open up minds.</td>
<td>“They need a lot of praise for taking small steps. They need to be recognized in front of the class when they do something correctly, or when they just put forth effort. So they may not reach the correct answer of the problem, but you can still find a way to praise that effort they’ve made in their steps they took. When you continue to do that with children, they start to grow confidence in math, whether or not...even if they’re not really [getting the correct answer] you know.”</td>
</tr>
<tr>
<td><strong>Nurturing Environment</strong></td>
<td>Create safe environment for students to take risks and make mistakes; establish a learning community that allows students to try different ways; set up a classroom that welcomes and accepts mistakes as routines; introduce stimulating tools and materials; arrange physical environment that invites and allows creativity to flow; and extend creativity-friendly environment into students’ homes,</td>
<td>“I would encourage them to...let them discuss their methods of math problem solving. So to allow discussions among peers, discussion with their teachers, or in front of their peers, to explain problem solving and be able to talk about differences in problem solving, maybe two or three or multiple different ways.”</td>
</tr>
</tbody>
</table>
4.5.1. Reviving curriculum.

Reviving curriculum refers to a classroom status where the teacher gives life to the curriculum to promote creative learning of mathematics by flexible design and redesign of learning activities, which was mentioned by 26 out of 30 teacher participants (87%). Teachers’ responses fell into several categories: 1) teachers find and stretch time for mathematics activities; 2) teachers select content purposefully to include in the lesson; 3) teachers provide open ended challenge problems; 4) teachers demonstrate different ways of solving a problem; 5) teachers arrange “math talk” in the classroom (i.e., let students explain her strategy to the class; students talk about their strategies with peers, in a group, or with the teacher; and 6) teachers expose students to other peers’ different ideas. In this theme, teachers’ focus is on the flexibility of carrying out the curriculum, with the purpose to open up students’ minds and help students achieve deeper understanding and better learning outcomes.

4.5.2. Cognitive scaffolding.

Cognitive scaffolding refers to the way a teacher supports students cognitively to become more creative in learning mathematics, which was mentioned
by 21 teachers (70%). Teachers’ responses fell into several categories: 1) teachers model creative problem solving, such as using multiple strategies; 2) teachers facilitate students to solve mathematics problems creatively, such as by asking questions, checking in student understanding, giving hints and showing examples, and instructing and re-teaching; 3) teachers adjust level of complexity to match student capability; and 4) teachers analyze mistakes with students to help them achieve success.

In this theme, teachers’ focus is on facilitating students’ cognition to bring them up to the next level. In *reviving curriculum*, the emphasis is on teacher instruction and activity design at the whole class level with a special stress on teaching with flexibility. In contrast, cognitive scaffolding relies on the individual students. Teachers adjust strategies to meet the individual needs of students to let them fly higher. For example, “math talk” in the *reviving curriculum* appears to overlap “teacher asking questions” in the *cognitive scaffolding* because both of them involve dialogues about mathematical thinking. However, the former is more regular classroom activity of sharing mathematics ideas, while the latter is more often a talk between the teacher and an individual student on a specific issue that the student is dealing with. The purpose is to provide individualized help to support students’ understanding and creativity. Such one-on-one talk is more personal than regular classroom talk.

### 4.5.3. Reinforcement and encouragement.

*Reinforcement and encouragement* refers to the way teachers motivate
students to learn mathematics creatively, which was mentioned by 19 teachers (63%). Teachers’ responses fell into several categories: 1) when students show creativity in learning mathematics, teachers use praising or other positive expressions to recognize them; 2) when students struggle with mathematics, teachers encourage and invite them to use creativity to do mathematics; and 3) teachers encourage students to be open-minded to try different possibilities.

In this theme reinforcement and encouragement, teachers broaden students’ vision on particular problems in order to inspire strategies from multiple perspectives. In contrast, although making students open to alternative approaches is also the essence of reviving curriculum, such openness is included in regular classroom activities instead of for a special occasion. For example, a teacher introduces two other ways to solve a problem that are not in the textbook when she is teaching the class how to calculate time differences. Let us compare the two themes: reinforcement and encouragement and cognitive scaffolding. Though encouraging students to learn mathematics with creativity is also a purpose for cognitive scaffolding, this purpose is achieved by more specific strategies, such as teacher asking questions and showing examples. However, strategies in reinforcement and encouragement are less concrete but more general as is shown in the teachers’ wording.

4.5.4. Nurturing environment.

Nurturing environment refers to a teacher working with the class to establish a safe and stimulating physical and social learning space for mathematical creativity,
which was mentioned by all 30 teacher participants (100%). Teachers’ responses fell into several categories: 1) teachers help to create a safe social environment for students to take risks and be okay to make mistakes; 2) teachers introduce making mistakes as a routine for daily learning of mathematics; 3) teachers advocate for a classroom that allows for all kinds of approaches and free exploration and trying; 4) teachers provide physical tools, materials and space, and digital technology to stimulate student thinking; and 5) teachers work with parents to extend mathematical creativity into home environments.

In this theme nurturing environment, teachers’ focus is on the physical space and the invisible social atmosphere, such as sense of acceptance and security when students take risks. Teachers can design activities and provide opportunities to achieve this purpose. However, this theme is not related to the content of the curriculum. In contrast, reviving curriculum has a special association with the mathematics content. Teachers design activities to facilitate learning of a specific content. For example, a challenge mathematics problem belongs to the physical environment. However, it is provided by the teacher to facilitate student understanding of the mathematics content and exercise student creative thinking, and thus the challenge problem pertains to reviving curriculum instead of to nurturing environment. Let us compare reinforcement and encouragement and nurturing environment. The former emphasized the individual teacher’s role in encouraging and allowing for student creativity in mathematics. In contrast, the latter puts more emphasis on the whole classroom environment—it can be initiated by the teacher,
but takes the whole class to realize—that gives permission and shows passion for creativity.

4.5.5. One step back.

One step back refers to the teacher taking secondary role to let students take more responsibility in learning mathematics, which was mentioned by 22 teachers (73%). Teachers’ responses fell into several categories: 1) teachers release control to students so that students are taking ownership in learning mathematics creatively; 2) teachers listen to students about their exploration in mathematics instead of teacher dominating the instruction; and 3) teachers allow students to make connections to their real life to make personal sense of mathematics concepts. In this theme, teachers’ focus is on taking a back seat and observing in the classroom where students are the leading actors performing up front. In contrast, teachers take more initiative when using the strategies in cognitive scaffolding to support each individual learner. However, stepping back and stepping up are not contradicting strategies. Instead, teachers often rotate to use the two types of strategies. Teachers let students explore and listen to the students to check student understanding, and then teachers can decide what questions to ask and what materials to provide to facilitate students to realize their creative ideas and even take the ideas to the next level.

4.6. Consequences: Differentiated Strategies for DCIFER

In this section, I further describe the strategies teachers utilized to promote different aspects of DCIFER (see Table 4.6). Please note that some teachers might
not necessarily describe a certain aspect of DCIFER when they explained their interpretation of mathematical creativity among young children, but these teachers utilized and talked about their strategies to promote this aspect of DCIFER.

Table 4.6

*Number of Teachers Using Strategies to Promote DCIFER.*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Equation (28)</td>
<td>12</td>
<td>8</td>
<td>3</td>
<td>13</td>
<td>6</td>
<td>23</td>
</tr>
<tr>
<td>Capacity (17)</td>
<td>20</td>
<td>15</td>
<td>2</td>
<td>9</td>
<td>3</td>
<td>26</td>
</tr>
<tr>
<td>If and NOT Only If (30)</td>
<td>22</td>
<td>8</td>
<td>14</td>
<td>23</td>
<td>17</td>
<td>30</td>
</tr>
<tr>
<td>False Conjecture (9)</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Exterior Angle (26)</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>30</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>Remainder (16)</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

*Notes:* Numbers in the first column are the numbers of teachers who described a certain aspect of DCIFER when they were asked to explain their understanding of mathematical creativity among young children; Numbers in the last column are the total numbers of teachers who mentioned any one or more among the five strategies to promote a certain aspect of DCIFER; Numbers from the second to the sixth columns are in boldface when the numbers of teachers who described a certain strategy are equal to or more than half of the total number of teachers who mentioned one or more strategies to promote the corresponding aspect of DCIFER.
4.6.1. Most overarching strategies.

*Nurturing environment* and *reviving curriculum* are the two most popular strategies teachers brought up. For *nurturing environment*, all the 30 teacher participants described some strategies of creating a safe and stimulating space, both socially and physically, for students to explore and create in mathematics. This strategy was largely used to promote five aspects of DCIFER: *dynamic equation; if and NOT only if; false conjecture; exterior angle; and remainder.* *Nurturing environment* is the most popular strategy across all the five aspects (percentage of teacher nominators ranged from 57% to 100% among the five aspects). It is interesting to find that learning atmosphere seemed to be a bigger focus than cognitive support when teachers promoted mathematical creativity. It seems to the teachers that mathematical creativity is more of a habit that takes the environment to form than a highly developed intelligence that calls for much cognitive support in the learning of mathematics content.

There were 26 teachers who shared the strategy of *reviving curriculum*, such as flexible choice of content, rearrangement of time, and redesign of activities. This strategy was largely used for four aspects of DCIFER: *dynamic equation; capacity; if and NOT only if; and remainder.* *Reviving curriculum* is a popular strategy across all the four aspects (percentage of teacher nominators ranged from 50% to 77% among the four aspects) It is interesting to find that teachers were able to tailor the curriculum to the needs of the class. Rather than complaining about time, pacing pressure and the quality of the textbook content, teachers managed to handle these
challenges in one way or another to revive the curriculum, which is quite inspiring.

4.6.2. Specially-focused strategies.

The other three strategies – cognitive scaffolding, reinforcement and encouragement, and one step back— are directed at one or two specific aspects of DCIFER. Cognitive scaffolding was mostly used for capacity (58%). For example, teachers would ask questions and adjust the complexity level to match the cognitive level for each individual student so that teachers can facilitate each student to understand mathematics concepts to a deep level.

Reinforcement and encouragement was mainly applied in two aspects of DCIFER: remainder (63%) and if and NOT only if (47%). On one hand, teachers encouraged students to explore different possibilities and take alternative perspectives if students were not open-minded yet. On the other hand, teachers recognized and praised students if students already bravely tried uncommon approaches. Through reinforcement and encouragement, teachers not only fostered personal exploration in mathematics with flexible thinking but also provided motivators for students to endure struggles and play with unexpected difficulties.

One step back was specially used for if and Not only if (57%). Rather than giving more guidance, teachers preferred taking a step back and releasing control to students to lead themselves in their exploration. Compared to teacher-directed scaffolding, taking one step back was a much popular choice among teachers (27% versus 57%). It seemed many teachers were aware of the importance of students taking ownership and responsibility in exploring mathematics in their own way.
However, taking one step back was not a common strategy for teachers to use in regard with the rest five aspects of DCIFER. In other words, teachers took initiatives in most parts of the learning of mathematics.

To sum up, Figure 4.6 shows varied degrees of popularity of teachers’ five strategies to promote creativity in the learning of mathematics for young children. *Reviving curriculum* and *nurturing environment* are the two popular ones utilized to promote most of the five aspects of DCIFER; while *cognitive scaffolding*, *reinforcement and encouragement*, and *one step back* are the three strategies that are aimed at promoting one or two aspects of DCIFER.

![Figure 4.6. Visual diagram of teaching strategies: Most overarching strategies and specially focused strategies.](image)

In the following section, for each aspect of DCIFER, I further describe how these five teaching strategies were used differently by teachers when they promoted different aspects of mathematical creativity (DCIFER) in children.
4.6.3. Dynamic equation.

There were 23 out of 30 teacher participants (77%) who described strategies directed at promoting dynamic equation, or dynamic problem solving abilities in students. The two strategies mentioned most often by teachers were *nurturing environment* and *reviving curriculum*, with 13 teachers (57%) and 12 teachers (52%) out of the 23 for each of them, respectively. One sample quote by a teacher regarding *nurturing environment* is to allow the discussion of all kinds of ideas in class. The teacher used this strategy to help students communicate about and comparing solutions with the goal to promote the dynamic process of problem solving. She said:

I would encourage them to…let them discuss their methods of math problem solving. So to allow discussions among peers, discussion with their teachers, or in front of their peers, to explain problem solving and be able to talk about differences in problem solving, maybe two or three or multiple different ways.

An example for *reviving curriculum* is a quote by a teacher who described how she used math talk to help students compare and revise solutions. She said:

And I’ve talked to my kids through math talk about agreeing and disagreeing, and if you disagree with someone, then before you move to another partner, you just talk about why you disagree and come to a conclusion together, so, one of you is going to have to understand what the other one is saying and come to a conclusion of what your answer is going to be.

*Cognitive scaffolding* is the third most often mentioned strategy under
dynamic equation, which was stated by 8 teachers (27%). For example, one teacher talked about asking questions to facilitate students who are stuck in a mathematics problem. She said:

I think some kids need more scaffolding and they need… asking them questions I guess. So if they’re totally stuck, you know, you can ask them a question to help them… probe them along to trying something, or to think about something in a different way. So questioning is really important.

Another teacher gave a student anecdote, in which the teacher adjusted the difficulty level for the student to facilitate her in problem solving. She said:

If it was just a written problem, she could probably cry because it was just overwhelming for her. So we break it down into the ten sticks and the one sticks. And then, she’s awake like … and then it's fun to watch her...if it’s addition she’s grouping the tens, and ungrouping if it's subtraction.

4.6.4. Capacity.

There were 26 out of 30 teacher participants (87%) who had strategies directed at promoting capacity, or comprehension in students. The most often mentioned strategy was reviving curriculum, which was mentioned by 20 teachers out of the 26 (77%). For example, one teacher explained her flexibility in carrying out the curriculum to build in more things to expand student capacity in being creative in learning mathematics. She said:

I kind of am a happy medium…and I love my curriculum, please [let me] remind you, I really see wonderful things happen with my students, but I’ll
extend onto what they have, and that’s how I found a way to put myself in
my students’ needs into learning. I’ll do the activities, I’ll do what’s asked,
but I may add on some more questioning. I may let them struggle with that
longer than the book tells me to. If they say 15 min, we might work 20
minutes on it.

*Cognitive scaffolding* was mentioned by 15 teachers of the 26 (58%) as being
part of *capacity*. For example, one teacher described the strategy of asking deeper
level questions to help student make more connections in mathematics to achieve
better understanding. She said:

I also say ask lots of questions to kids, not ones of what do you do next, not
necessarily about the steps, but more about ‘how do you solve this, why do
you think it worked, do you think it will always work’, and you know... or
even another good one is having them compare their answers with each other.
And in a lot of times, they agree with their answers, but compare how they
solve it. So, ‘did you do...did you solve it the same way as John? Talk about
what you did the same or what you did differently’. Because that allows them
to make that connections and see how those ideas relate and ...just how that
deep understanding of what they’re doing.

4.6.5. If and NOT only if.

All of the 30 teacher participants (100%) described strategies directed at
promoting *if and NOT only if*, or open-mindedness to infinite possibilities in students.
The two most often mentioned strategies by teachers were *nurturing environment*
(23 teachers, 77%) and reviving curriculum (22 teachers, 74%), followed by one step back strategy, which was mentioned by 17 teachers (57%). For nurturing environment, many teachers talked about giving permission to students to approach mathematics problems in alternative ways. For example, one teacher said:

I just think a lot of them at this age are very black and white, they are very concrete…[for example, students would ask,] ‘I need to solve it this way, right?’ [But I would ask them,] ‘how else could you solve that?’ [Then students would be opened up and say,] ‘oh…’ you know, when you gave them permission to solve it another way, then they would want to [solve it in another way].

Another teacher said:

So I think the teachers really need to… provide a classroom where kids feel safe to share creative ideas… [The teacher should] not ever put a value on [the strategy by commenting] ‘oh this strategy is better than that one’ from the teacher’s point of view, but let the kids say ‘oh that one is better because it’s more efficient or fast or was more accurate than [the other one]’.

For reviving curriculum, many teachers talked about building in time and activities for students to make mathematics a personal exploration in which students are free to follow their own trace of thoughts even if it is different from what was given by the textbook or taught by the teacher. For example, one teacher said:

[We can promote creativity] maybe [by] giving the time [for] being creative… and how you use your time…and maybe being able to use one
[lesson] time… maybe one part of this [lesson]… [and make it] more like an exploratory lesson. Maybe [in the lesson, the teacher] not saying right away what you [students] are going to do, but let them [students] discover and figure it out, and then teach them [students] the rules and algorithms instead of beginning [the lesson] with it and saying ‘this is how we’re going to do’, [but instead] waiting to see when they [students] come up with that ‘ah-ha’ on their own.

Another teacher said:

You know, it’s kind of a balance. I know what pacing has to do, but I take time everyday to make sure they have a chance to do their own problem solving, then I go on to the lesson where they are taught one strategy or taught in a different way. But I make sure every day they have at least a little bit of time where they get to do it their way...and show their way. So, just trying to balance, like I said with... I had to keep it moving on, so trying to understand when it’s time to move on and when it would be a better time [to let students explore their own way].

For one step back strategy, rather than focusing on the number of different solutions, many teachers used this strategy to facilitate students to take more ownership in exploring ideas, which is the medium level of the theme if and NOT only if. For example, one teacher said, “We started the year off that way, with less of me giving strategy but more of them coming up with their own strategy. And I think that really makes a huge difference with the students.”
4.6.6. False conjecture.

There were 10 teacher participants (33%) who had strategies directed at promoting false conjecture, or mistake-making in students. The most mentioned strategy was nurturing environment, which was mentioned by 7 teachers out of the 10 (70%). Teachers commented that creating a safe environment would help students keep trying be willing to make mistakes and learn from mistakes. For example, one teacher explained, “So part of it is creating a...they need to be in an environment where they feel safe to try something even if doesn’t end up working.” Another teacher said:

We’re trying to create a very safe environment in the classroom where kids are not afraid of taking risks and they’re willing to talk about math and talk about their work, and they know that it’s ok if somebody else disagrees with them because it happens all the time, just that...this is a respectful environment, and so everybody is encouraged to try.

4.6.7. Exterior angle.

All 30 teacher participants (100%) spoke about strategies directed at promoting exterior angle, or environment, to nurture creativity in the learning of mathematics. The most mentioned strategy is nurturing environment, which was mentioned by all the teachers (100%). Teachers talked about all kinds of ways to set up a nourishing environment that allows for and encourage creativity in the learning of mathematics. For example, one teacher said, “I think it goes back to how you set up your classroom, how you set up your routines, and how you set up your
environment that is going to let that creativity comes through.”

Many teachers mentioned extending nourishing environment into students’ homes. For example, one teacher said:

I think, like every year we have a curriculum night where we talk about the new math program, and we explain the strategy, we send out letters home from the math book, showing them the strategies their child are going to be taught. But I think just continual education about the philosophy will help [parents be more open to allow their kids use strategies different from what parents may expect].

4.6.8. Remainder.

There were 8 teacher participants (27%) who described strategies directed at promoting remainder, or motivation in students. The two most mentioned strategies were *nurturing environment* and *reinforcement and encouragement*, both asserted by 5 teachers out of the 8 (63%), followed by *reviving curriculum* mentioned by 4 teachers out of the 8 (50%). To create *nurturing environment*, teachers provided different tools and technology to motivate students, such as laptops, Ipads and Ipods. For *reinforcement and encouragement*, teachers motivated students to keep trying by recognizing and rewarding student interaction with mathematics. For example, one teacher described her way of publicly recognizing students to boost their confidence level. She said:

They need a lot of praise for taking small steps. They need to be recognized in front of the class when they do something correctly, or when they just put
forth effort. So they may not reach the correct answer of the problem, but you can still find a way to praise that effort they’ve made in their steps they took. When you continue to do that with children, they start to grow confidence in math, whether or not...even if they’re not really [getting the correct answer] you know.

Another teacher talked about rewarding strategies with physical prizes. She had students with special needs from different classrooms coming to her room to get extra mathematics support during special time. She motivated those students to approach mathematics problems in a number of ways even more than their ordinary student peers could think of. She described:

And then I always tell them [the students], one more, tell me one more way. So the other day, I said we’re going to add double digit, and for every different way you tell me, you’ll all get a blue note. Here are the blue notes, these are gold…it’s a piece of paper, they’re gold. And they [the students] came up with seven different ways to do a double digit addition problem. So I was really proud of them, but I think it has to go to them, intrigue them, hook them, and then have them start working. First the [classroom] teachers were angry when they [the students] went back and had seven coupons, and then I said no, I explained why and why, and then they [the students] explained it, and then the teachers said oh, ok, ok. Well most of them [ordinary students] only had two or three ways, so here they had seven!

For reviving curriculum, teachers advocated for the art of teaching that
motivates students. For example, one teacher described her way of gradually raising demands to keep students engaged in pursuing mathematics with creativity. She said:

I think it [mathematical creativity] be focused on one area to start with. You [the teacher] can really want to get good at having kids explain why they did what they did, or give them multiple opportunities, you know, focus on one thing that they are trying to get really good at, and then maybe when they feel like they're doing a good job with that, then add one more thing. And slowly set the pace.

Another teacher said:

Math is creativity, don’t you think? I mean I do. There has to be way to them [the teachers] to make it creative enough to bring them [the students] to the challenge, that they [the students] want the challenge, that they [the students] are not afraid of the challenge.

4.7. Support and Obstacles

Teachers rarely named any major obstacles in promoting mathematical creativity in children. Only a few teachers mentioned time, students’ personality and previous training, and students’ family environment as something they sometimes need to be aware of. In contrast, many teachers commented that the Primarily Math (PM) program and the new curriculum *Math Expressions* greatly supported them in promoting mathematical creativity in children.

Many teachers commented that the PM program helped them to better understand the nature of mathematics that mathematics takes a lot of creativity.
Teachers took away from the PM program that mathematics was an interesting and creative process rather than just applying rules to get that correct number for the answer. There are many smart and creative ways to solve a problem. Through the PM program, many teachers deepened their understanding of the learning of mathematics that it should be a customized exploration for students. For example, one teacher said:

Creativity is just letting the kids explore, that kind of open exploration of concepts. A lot of times we say use your creativity in art, and those areas. But I think it’s also—after Primarily Math especially—being able to do it in math.

Teachers also commented that their new curriculum Math Expressions supported deep understanding in mathematics and allowed students to make their own decision in exploring mathematics. For example, one teacher commented that the worksheets provided in the new curriculum were focused on key concepts to promote deep understanding rather than scattered around a range of problems for replication of procedures. She said:

I think the worksheets are so much better than they were in the past. It’s more like quality versus quantity. It used to be like 25 problems to practice, and now it's only like a couple problems to practice, but they have to more explain their thinking...We spend more time on discussing and talking about it...letting kids do a problem [and] spending maybe 20 minutes on a problem, rather than just doing a series of problems... Doing one problem and doing it
five different ways… It’s a lot better.

Another teacher described that it was encouraged in *Math Expressions* that students should explore mathematics in their own ways. She said:

A lot of the units start with the kids figuring out [the problem] first and they have a chance to explore with story problems, especially with adding and subtracting. And they’ve given some of the different ways to look at it, like drawing the picture or writing an equation. I think that’s pretty open to them, and it’s not always ‘you must do it this way’, it’s ‘solve it how you can solve it and then prove it, how you can prove it’.

Another teacher pointed out explicitly that in this new curriculum, students were allowed to try alternative approaches other than the commonly used way to develop creativity. She said:

That creativity piece... to look at numbers in different ways… that’s a much stronger piece in this curriculum. So I think you know, [doing mathematics] in a different way… is a more meaningful exploration, and more meaningful way to incorporate creativity.

4.8. Summary

Teachers from this study interpreted students’ creativity in the learning of mathematics as a six-aspect concept (DCIFER) that indicates: (1) D—*dynamic equation* (i.e., problem solving in mathematics as an evolving process); (2) C—*capacity* (i.e., developing mathematics knowledge with deep understanding and connections among concepts and ideas); (3) I—*If and NOT only if* (i.e., opening to
all kinds of possibilities in mathematics and taking personal trips to explore the rich and endless world of mathematics); (4) F—false conjecture (i.e., willing to run the risks of being unsuccessful with new and unfamiliar things in mathematics and welcome and ready to learn from mistakes); (5) E—exterior angle (i.e., a creativity-friendly and stimulating environment that provides social support and physical materials for the leaning of mathematics); and (6) R—remainder (i.e., motivation to stay in struggles in exploring mathematics).

Teachers managed to promote mathematical creativity by means of: (1) reviving curriculum (i.e., teachers design and conduct learning activities with purpose and flexibility to make the curriculum lively); (2) cognitive scaffolding (i.e., teachers adjust instruction to support individual students on different levels or with different needs); (3) reinforcement and encouragement (i.e., teachers give positive feedback and encouragement to motivate and inspire students); (4) nurturing environment (i.e., teachers advocate for a safe, stimulating, and respectful learning community); and (5) one step back/taking a back seat (i.e., teachers release control for students to take lead and ownership in exploring mathematics). These strategies were used in different degrees to promote different aspects of DCIFER, as I describe in fuller detail in the discussion chapter.

Teachers did not stressed on any major obstacles in promoting mathematical creativity in children. Instead, teachers pointed out that their experience in Primarily Math (PM) program and the newly adopted curriculum Math Expressions supported them in promoting children’s creativity in the learning of mathematics.
CHAPTER 5
DISCUSSION

The purpose of this grounded theory study was to explore (1) how a group of teachers from a public school system in the Midwest interpret creativity in K-3 mathematics learning and (2) how these teachers promote or fail to promote creativity for students in the learning of mathematics. I found in this grounded theory study that K-3 teachers interpreted mathematics as more than just memorizing facts, applying rules and following procedures. Instead, the learning of mathematics involves a lot of creativity as teachers described in the six aspects of DCIFER. Teachers were resourceful in promoting mathematical creativity and their strategies fell into five big categories: reviving curriculum; cognitive scaffolding; reinforcement and encouragement; nurturing environment; and one step back.

Teachers did not name many major obstacles in promoting mathematical creativity. Rather, teachers pointed out that their experience in the Primarily Math (PM) program and the new curriculum *Math Expressions* were helpful for teachers in promoting children’s creativity in the learning of mathematics.

It is claimed in much literature that teachers often do not favor students’ creativity and teachers teach mathematics for fluency in procedures without much care for comprehension. Mathematics seems to be something not from within but from without: the mathematical rules are already there and students are just memorizing and applying these rules. There is only one correct answer for a problem, and therefore no personality or individuality seems to be involved. However, on the
contrary, this study shows that teachers in the sample did value personal involvement in learning mathematics. They encouraged students to launch personal exploration to make sense of mathematics. They believed that students should take ownership to comprehend concepts and connect different pieces of knowledge. They knew that students could arrive at the same answer but their approaches can vary dramatically among individual students, depending on their ways of knowing.

In the first part of this chapter, I compare the six aspects of DCIFER with the six resources in the investment theory of creativity, the national standards for mathematical proficiency and mathematical habits of mind, respectively, in the descending order of the number of teacher participants who nominated the aspects of DCIFER. The investment theory of creativity was used to illustrate the significance of creativity in mathematics (as was discussed earlier in Chapter 2) instead of a framework to guide the coding process of DCIFER. The six aspects of DCIFER and the six resources in the investment theory of creativity share some similarities but they have many differences too. Then, a discussion of different emphases on mathematical creativity by teachers in different professional roles (i.e., regular classroom teachers versus teachers with coaching/leadership roles, and teachers in charge of different grade levels) follows.

In the second part of this chapter, I talk about the strategies teachers selected to promote different aspects of DCIFER and the varied frequency of these strategies. In the third part of this chapter, I discuss why teachers met with more support than obstacles in promoting mathematical creativity. A deeper investigation into the
Primarily Math (PM) professional development program is given. In the last section of this chapter, I summarize the implications followed by a statement of strengths and weaknesses of this study and future directions.

5.1. Decipher Mathematical Creativity (DCIFER)

In this section, each of the six aspects of DCIFER is discussed in comparison to the corresponding creative resource claimed in the investment theory of creativity. For each aspect of DCIFER, teacher quotations are cited to illustrate the connection with the creative resource claimed in the investment theory of creativity. Then, a discussion on how various national standards on mathematical proficiency and the mathematical habits of mind (HOMs) are embodied in the six aspects of DCIFER follows.

5.1.1. If and NOT only if: a personal journey to unlimited possibilities.

Teachers spoke forcefully about the idea that students should be open to all kinds of possibilities in mathematics that students can use their own way to discover, explore, and make sense. All the 30 teacher participants in this study (100%) asserted the importance of if and NOT only if in their definition of mathematical creativity. If and NOT only if indicates the infinite possibilities of individualization in how students learn mathematics. If and NOT only if reflects teachers’ emphasis on different strategies based on individual learners’ different preconceptions and their original and personal ways of approaching a problem.

If and NOT only if is similar to the concept of thinking style in the investment theory of creativity. Thinking style refers to the flexibility to view things from
multiple perspectives. Thinking style is a choice regarding how to apply one’s intellect. Choosing a creative thinking style is like pushing an internal switch to power on open-mindedness. In mathematical creativity, it is a choice of directing oneself to explore the richness in mathematics. Teachers’ description of if and NOT only if overlaps with professional scholars and researchers’ definition of thinking styles for creativity, including legislative thinking style, flexible thinking style, independent thinking style, convention-breaking thinking style, and thinking with existing knowledge (Sternberg & Lubart, 1995). For example, people with legislative thinking style like to do things in their own way and they enjoy exploring and discovering how to solve a problem rather than being told steps one, two, and three (Sternberg & Lubart, 1995). Teachers shared anecdotes of their students with legislative thinking style. For instance, one teacher described one of her students who did not use the standard strategy to solve a division problem but took initiative in exploring a unique way himself. This student used a pictorial representation to accomplish division through repeated subtraction. The teacher described this scenario that:

When we teach strategies, he comes up with his own. So for example, it was 48 divided by 8… so [here’s] how he did it. He made ten sticks and then he put dots (the teacher drew 4 sticks and 8 dots on a piece of paper), and then he crossed off each of the ten sticks and put a "2" (the teacher crossed off the 4 sticks and put a number "2" underneath each of the sticks). So he took out 8 from the 10 and had 2 left... And I was like, oh, he got the answer.
Another teacher described one of her female students who always liked to take ownership in exploring other ways of solving problems. She said that:

I have a little girl. She’s very bright. Before we even really show them how to do it, anything like math drawings or things like that to solve problems, like you could tell that she already had ideas on how to ...you know, ‘oh I could draw a picture to do this, or I could use my fingers, or I could…’, you know, like think of different ways that she could [do] to solve a problem, not just one that I had shown her. She wanted to come up with other ways that she could do it on her own.

When teachers explained their understanding of if and NOT only if, many of them mentioned that they always asked students to work like mathematicians who adopted habits of mind (HOMs) in exploring mathematics. Teachers believed that students should be able to see the beauty of creativity in mathematics when they explore with HOMs, such as curiosity, imagination, inventiveness, risk-taking, and persistence. Mathematics is not a cold subject but it becomes sensible, valuable and personal to students. Mathematics is intimate and each student can have a personal relationship with mathematics. It was inspiring to me as an interviewer to hear teachers talking about making mathematics a personal trip in which students explored in mathematics like mathematicians. For example, one teacher described the way students learned like mathematicians who found their own solutions that made sense to them:

Your broader goal is to… foster mathematicians who can function in those
process standards. And so when that comes to creativity… when you’re showing students a story problem to solve… the goal is not that they all get the right answer—that’s a secondary goal— but your primary goal is that they learn how to work on a story problem, and they learn the process that they need to go through to understand the problem, and come up with a method that gets them into the solution and then solve it. So that is a bigger goal than whether or not they get the right answer, and that has to happen first.

National standards for mathematical proficiency are reflected in teachers’ advocacy for *if and NOT only if.* Teachers seemed to understand the standards and the teachers were teaching to meet these guidelines. For example, in one of the strands for mathematical proficiency by the National Research Council (NRC, Kilpatrick, Swafford, & Swindell, 2001), students are expected to develop a productive disposition. This productive disposition occurs when students view doing mathematics as a sensible and valuable pursuit that never separates itself from personal exploration. This is also what teachers were advocating for in *if and NOT only if.*

**5.1.2. Dynamic equation: think, talk, and extend the math.**

The second most popular aspect of mathematical creativity among teachers in this study is *dynamic equation,* or the evolving nature of problem solving (28 out of 30, 93%). Solving a mathematics problem is not a one-time event only to get that right answer. Instead, it is an evolving process going from brainstorming and trying
of strategies, comparing and configuring strategies, to explaining and communicating strategies. Dynamic equation represents a concept similar to the intellectual abilities in the investment theory of creativity. Intellectual abilities refer to the ability to view things in novel ways, evaluate the ideas, communicate and promote the ideas to others, and utilize and react to outside feedback.

Intellectual abilities involve three types of skills: (1) experiential ability (i.e., unconventional thinking and information processing in dealing with novel problems and demands); (2) componential ability (i.e., monitoring which ideas are valuable and which are not); and (3) contextual ability (i.e., promoting a fit between one’s idea and the environment through communicating, taking feedback, revising, and selling one’s ideas) (Sternberg & Lubart, 1995). A person must employ all three of them in problem solving to be genuinely creative (Sternberg & Lubart, 1995). With only experiential ability (otherwise known as synthetic ability), one can produce new and original ideas, but those ideas may elude the inspection process required to make those ideas feasible. With only componential ability (otherwise known as analytical ability), one can reason and analyze critically, but not creatively. With only contextual ability (otherwise known as practical-contextual ability), one can spread ideas and persuade others successfully, not because of the quality of the ideas but because of the powerful presentation.

The three stages of dynamic equation, correspond to the three types of skills for intellectual abilities in the investment theory of creativity, that is, (1) experiential ability corresponds with coming up with a strategy for a new problem, (2)
componential ability corresponds with comparing, evaluating, and adjusting strategies, and (3) contextual ability corresponds with expressing ideas, revising ideas according to feedbacks, and promoting ideas. Sternberg and Lubart (1995) claimed in the investment theory of creativity that experiential ability is the most important for creativity. However, there is not a clear preference for the first stage of problem solving among the teacher participants in this study.

When describing the communication of ideas, teachers centered themselves on “math talk”. Math talk is classroom dialogue by which the teacher and students discuss, break down to analyze, and comprehend “ideas, relationships among those ideas, strategies, procedures, facts, mathematical history, and more” (Chapin, O’Connor, & Anderson, 2009, p. 6).

Math talk is encouraged in the NCTM process standards. For example, in the process standard of communication, students should be able to communicate mathematical thinking coherently and clearly to peers, teachers, and others, and to use the language of mathematics to express mathematical ideas precisely (NCTM, 2000a). The way teachers in this study described math talk went beyond the requirements for communication skills given in the NCTM process standards, that is, coherency, clarity and preciseness. Teachers were able to see the essence of math talk: math talk is a way that students explore mathematics with personality, flexibility, and understanding.

One teacher described her emphasis on math talk when giving advice to novice teachers, “so I would just encourage them (novice teachers) to really allow
the time for the math talk especially. And I think when you start with the math talk, that’s where you’ll start to see the creativity.” Another teacher described how math talk inspired creative solutions:

I also think about when I observed their daily math routines… for students like in second grade, sometimes the way students choose to solve their problems is done differently, and so a lot of that comes out through math talk when they sit down and talk to each other, they have different and creative solutions to their problems.

Some teachers elaborated on the specific skills and abilities students developed through math talk, such as explaining and justifying. For example, one teacher said:

We work a lot on math talk in kindergarten, so we’re working getting from that ‘show me’ stage to that ‘tell me’ stage, so kind of getting them from there to being able to explain to another student or to me, how they can justify their answers and how they found their solutions.

Another teacher thought math talk could help students organize thinking:

I like to let them talk to a partner first, and then talk to the whole group, so that they can kind of have a chance to get their thinking organized before they share, and then we kind of decide whether we agree or disagree.

Math talk associates thinking and talking and brings creativity up to a next level. Through math talk, on one hand, a student recalls his reasoning and puts it into words; on the other hand, the student listens to others’ feedback and transforms
others’ words into thinking materials for mental processing. Then the student compares different information, reacts verbally, and reflects on and modifies the original strategy, and finally represents the updated version in a more convincing way. One teacher described how math talk not only inspired students, but also enabled students to reflect on and compare strategies with higher level thinking. She said:

[Students] just share what methods they’ve tried so that those kids that need help in knowing where to go next might get ideas from each other, and those kids who have all sorts of ideas can kind of reflect on it and think of what they’ve done so that they’re not just constantly trying different things but actually thinking about, ‘ok, I did do several different things but which ones were helpful and which ones were maybe not helpful.

5.1.3. Exterior angle: nurturing and stimulating environment. The third most mentioned aspect of mathematical creativity by teachers in this study is exterior angle, or nurturing social and physical environment (26 out of 30, 87%). Students need to have freedom to explore mathematics in a learning community in which they feel safe, respected, and confident. This community should be supportive and stimulating with good supply of nutrients that help creativity grow. Exterior angle is a concept similar to the environment in the investment theory of creativity. Environment refers to “a setting that stimulates creative ideas, encourages them when presented, and rewards a broad range of ideas and behaviors” (Sternberg & Lubart, 1995, p. 10).
Creativity used to be viewed often as a disposition within the individual. However, nowadays, researchers have realized the power of the environment. They tend to view creativity from a confluence of perspectives that include both the individual and the context. It is thus exciting to see teachers paying attention to the environmental contributors to mathematical creativity. Teachers in this study addressed the textbook content and organization, scheduling and timing, home environment, and tools and manipulatives. Most comments were about the curriculum.

As NCTM (2000a) proposed in their principles about the environment, curriculum is one important thing that influences the learning of mathematics. A quality curriculum displays concepts with coherency, places emphasis on important ideas, and clearly articulates the progression of ideas across grade levels. Teacher participants in this study are from a public school system that had a curriculum change a year prior to the school year I collected data. Teachers had many positive comments about it, and therefore the new curriculum seemed to demonstrate a positive influence on mathematical creativity.

5.1.4. Capacity: connect and comprehend.

The fourth most mentioned aspect of mathematical creativity by teachers in this study is capacity, or deep understanding of knowledge (17 out of 30, 57%). Students need to see the connections between mathematical concepts with deep comprehension. Capacity is a concept similar to knowledge in the investment theory of creativity. Knowledge can be formal and informal. Formal knowledge refers to
“facts, principles, aesthetic values, opinions on an issue, or knowledge of techniques and general paradigms” (Sternberg & Lubart, 1995, p. 150). Such knowledge can be learned from printed materials, speeches and lectures, and other direct instructions” (p. 150). Informal knowledge, in contrast, is “the knowledge you pick up about a discipline or a job from time spent in that arena… [It] is rarely explicitly taught and often isn’t even verbalized” (p. 150).

According to the investment theory of creativity, formal knowledge preparation promotes creativity in several ways: helping one invent something original rather than reproducing something already existent; offering one a good understanding of the field so that one can think against the common trend; assisting one in elaborating an idea into a complete work; providing one with a solid foundation so that one can get focused on the new idea rather than the basic knowledge; and making one sensitive and alert to little hints to grasp and inspire creative ideas (Sternberg & Lubart, 1995).

In mathematics, formal knowledge can be the knowledge of standard approaches to solving a problem, and basic skills and strategies that enable students to calculate and organize information. With formal knowledge in mathematics, a student is able to avoid repeating the standard solution, get inspiration from what she already knows, find a unique approach, and develop the idea into a complete work. However, capacity is more than knowledge preparation. Capacity has a special emphasis on the depth of understanding of mathematical concepts reflected in the ability to connect and apply. For example, one teacher described her understanding
of capacity that:

I also feel that… it’s [mathematical creativity is] more about students making sense of the math… more so than trying to find the correct answer [or] procedure that fits the mode… that the students are trying to analyze, generalize, make connections, and show relationships among the concepts that they’re learning, versus just trying to get that answer, or just trying to figure what it is that the teacher is teaching or what correct response that I need to put on my paper.

Deep understanding of mathematical concepts and their relationship, or conceptual understanding, is a big idea supported by NRC, NCTM, and CCSS, unanimously. It is described in the NRC document regarding the importance of deep understanding in the strand of conceptual understanding that students should have good comprehension of mathematical concepts, operations, and relations (Kilpatrick, Swafford, & Swindell, 2001, p. 5).

It is also claimed by NCTM (2000a) that conceptual understanding is essential for the accumulation of knowledge; learning is not solid if conceptual understanding is missing. Without conceptual understanding, students can hardly practice mathematics because they rely too much on procedures, and lack the ability to make analogies, give representations, develop justifications, think of alternatives, conduct applications, and present explanations (CCSS Initiative, 2010).

Informal knowledge plays a role in decision making (Sternberg & Lubart, 1995). Informal knowledge helps children to “most effectively… use their creativity
so that it would benefit rather than hurt them” (p. 172). In short, it is a kind of knowledge students perceive from the environment whether their creativity would be accepted and respected before they explore something new. However, in this study, teachers did not include students’ ability to decide whether to explore or not as a kind of informal knowledge. Instead, teachers seemed to view such decisions more as environment-driven, that is, whether the learning community provided students the safe feeling for them to explore different ways and make mistakes.

It is interesting to find out that, overall, teachers in this study did not feel their knowledge in mathematics was an obstacle for them to promote deep understanding in their students. According to the literature, teachers’ own deficiencies in content knowledge of mathematics and pedagogical knowledge of teaching mathematics are impeding quality teaching for many elementary school teachers. According to this view, teachers lack solid foundation in mathematics largely due to surface understanding of mathematics concepts in grade school education and insufficient college remediation and preparation. However, knowledge was not reported as an obstacle for the teacher participants in this study.

There can be several possible reasons for this particular finding. Two new experiences may have contributed to this unexpected finding: the experience in the PM program and the implementation of a new curriculum, *Mathematics Expressions*. The PM program not only helped teachers gain content knowledge in mathematics, but also improved their understanding of what mathematics was. In this program, teachers were recognized as mathematicians and encouraged to explore mathematics
with HOMs through a lot of challenging problems. These problems were given for teachers after and during each class. First, teachers applied HOMs and explored like mathematicians individually. Then they discussed in pairs or in groups. And finally, these problems were dissected and explained to the teachers by the instructors in a sense-making way. According to teachers’ test scores on the Mathematical Knowledge for Teaching (Hill, Rowan, & Ball, 2005) instrument as well as their self-reported written reflections, their competence and confidence in mathematics significantly improved throughout their experience in the PM program. At the beginning of the PM program, the majority of the teachers identified in their math stories that they did not have much positive experience with mathematics and they would not label themselves as mathematically-gifted or mathematics-lovers. Many of them said that they feared mathematics. While they were themselves students, what teachers learned most were the procedures without deep understanding. However, after the PM program, almost all of the teachers reported in their math stories large gains in mathematics knowledge and pedagogy as well as their self-confidence in teaching mathematics, and many of them even became mathematics coaches for the school or the district after they completed the program. Thus, it is possible that PM program helped these teachers be more confident of their capability to promote mathematical creativity that they do not perceive their knowledge foundation as a weakness.

Another possible reason for the finding that teachers in this study did not view their knowledge in mathematics as an obstacle to promote deep understanding
in students is the new textbooks. *Math Expressions* may have supported teachers in promoting conceptual understanding of mathematical knowledge in children in terms of the provision of learning materials focused on understanding of and connection among mathematical ideas. *Math Expressions* stresses a few key concepts and presents them with connection. Teachers are able to promote deeper understanding in students when the worksheets are not just problems for students to repeat the same strategies over and over again. There are fewer mathematics problems on the worksheet each day, but there are more questions asking students to explain their thinking. These types of questions challenge students to explore and reach deeper comprehension. 

Teachers in this study had positive comments on *Math Expressions*, such as the focus on different strategies, big ideas/key concepts, quality instead of quantity, challenge problems, and tools and manipulatives. These aspects collectively provide a foundation that can enable deep understanding and inspire creativity. This is a promising finding given the situation that the curricula in the U.S. have been widely criticized as lacking connections, and sometimes too wide and too shallow to foster deep understanding. For more details about how *Math Expressions* support conceptual understanding, see Houghton Mifflin Harcourt (2012).

This new curriculum is only available to K-2 grades due to the limited budget of the district. However, the district-level math coaches were tasked with significantly revising the curricular materials for grades 3-5 to bring in the most positive and effective aspects of the Math Expressions curriculum that the district
was able to adopt for grades K-2. The worksheets were adjusted and new classroom activities were added to replace the older ones. Such change provided extra support for higher grades teachers in this study. From what the 3rd grade teachers in this study reported, they were quite satisfied with the remediation of the existing curriculum. For example, they reported that the district created some extra materials to replace the problem sets in the old textbooks. These problems looked more like the ones in the new textbooks for lower grades students. The problems were reduced in quantity to focus students on just a few carefully-chosen problems to develop conceptual understanding. There are also other online resources for teachers to use in class, such as hands-on materials to learn shapes, and puzzle problems to exercise HOMs.

A third possible reason for the finding that teachers did not view their knowledge in mathematics as an obstacle is that teachers were aware that it is not always the case that the more knowledge the better. According to many research studies (Lubart & Georgsdottir, 2004; Soh, 1999), knowledge can be a double-sided sword. Too much knowledge can keep students from thinking flexibly or originally in mathematics. Teachers may want to give students more space and freedom to explore creatively without the restriction from previous knowledge.

To sum up, teachers were aware of the importance of conceptual understanding of mathematical ideas. Teachers were confident of their mastery of content knowledge in mathematics, and the PM program and the new textbooks *Math Expressions* may have helped teachers to build a solid knowledge foundation.
in mathematics.

5.1.5. **Remainder: stay in productive struggles.**

There were 16 out of 30 (54%) teachers who mentioned *remainder*, or motivation to stay with their struggles and keep pushing through. Teachers believed that students should be able to endure the uncomfortable feeling during unsuccessful and ambiguous moments, and keep trying until they find their way out. *Remainder* is a concept similar to the motivation in the investment theory of creativity. Motivation is the driving force to action. Intrinsic motivation is the work itself that motivates students. Usually, people who work on something out of pure enjoyment of the task, personal satisfaction, or the meaning of the work per se, are motivated intrinsically. According to the investment theory, intrinsic motivation is very important for creativity because it keeps one focused on the task (Sternberg & Lubart, 1995). However, extrinsic motivation can also facilitate creative work. For example, synergistic motivation is an extrinsic motivation that facilitates one’s creative work through the provision of information helpful for the current task.

In this study, teachers put both types of motivation into consideration. They expected that students make sense of mathematics concepts to create personal relationship with what they are learning. In this way, students can enjoy mathematics and develop intrinsic motivation. At the same time, teachers also recognized the power of extrinsic motivation, both tangible and intangible. For example, a tangible motivator would be blue start tickets as rewards for creative ideas. An intangible example would be the public recognition from peers and the teacher when a student
shares creative ideas with the whole class. At the same time, public sharing also allowed other peers to provide feedback to the student which would facilitate the student to go further in thinking.

It is interesting to find that kindergarten teachers put less emphasis on motivation, compared to their colleagues who were teaching higher grades (29% versus 75%). As was described by many teacher participants, they found younger children to have less concern about self image but more curiosity in learning in general, and thus teachers believed that younger children were often brave to explore all kinds of possibilities. Teachers might have taken kindergarteners’ motivation for creative exploration in various kinds of learning for granted, and thus the teachers understated the importance of motivation to learn mathematics creatively for kindergarteners.

5.1.6. False conjecture: stress-free mistakes and risks.

There were 9 teachers (30%) teachers who mentioned false conjecture, or taking risks to try new ways regardless of being wrong. Teachers believed that students needed to be willing to try things without too much concern about the correctness of the answer. Students should welcome mistakes and appreciate the learning opportunities from making mistakes. False conjecture is a concept similar to personality in the investment theory of creativity. A creative person usually shows a series of personality traits, including perseverance in the face of obstacles, willingness to take sensible risks, willingness to grow, tolerance of ambiguity, openness to experience, and belief in oneself and the courage of one’s own
Among all the personality traits associated with creativity, teachers in this study mostly focused on risk taking and willingness to grow. Some teachers also mentioned openness to exploration and self confidence and courage. An interesting finding is that more teachers in higher grades emphasized false conjecture more than did lower grades teachers (44% versus 14%). It is possible that younger students are more open to trying new unfamiliar things and are less fearful of making mistakes. Lower grades teachers might take it for granted that young children would bravely approach mathematics problems in new ways and feel free to be wrong, and thus teachers might understate the contribution of such open-mindedness to mathematical creativity in young children. For example, one second and third grade teacher said:

I think students like to create the context [for a mathematics problem] better than having the context giving to them, especially when they are very little. When they are older… I see older students, like 2nd and 3rd grade students, being afraid to make their own context. They get nervous. They just want to know what the problem is and so they can solve it. They stop thinking creatively about numbers…I've seen kindergarteners be very adventurous about numbers. They are not concerned if they make a mistake. They try anything. They want to share what they’re thinking. And some of that is how they’re socially too. Kindergarteners are not very exclusive socially; they are very open socially. Everyone is my friend, you know, I like everyone, which is a very great time. And as kids get older, they do start to notice differences
between each other, they start to learn social habits from their parents, they
start to learn, you know, they start to… bullying starts to come into play,
some of those certain things, and so they become less open to sharing what
they’re thinking, which I think that shut[s] down creativity sometimes.
One kindergarten teacher described:
I think kindergarten kids come in with the creativity already in them… And I
think the older you get to, the more nervous you get to take risks… Five, six
years olds pretty much try anything you ask them to do, you know.
Teachers with coaching/leadership roles were more likely to emphasize false
conjecture than regular classroom teachers (58% versus 11%). It is likely that the
former had experience with more teachers and students so that teachers with
coaching/leadership roles came to realize the virtue of students being prepared to fail
and learn from failures.

5.2. Promoting Mathematical Creativity

In this section, I begin with a comparison of the five types of strategies in
terms of how differently they are used to promote different aspects of DCIFER. Then,
I discuss how teachers can start with something basic and simple when applying
these strategies to promote mathematical creativity. Lastly, I talk about how these
strategies can be used by preschool teachers to help young children develop
mathematical creativity even before they enter grade school.

5.2.1. Most and least supported aspects of mathematical creativity.

First, let us discuss about the most frequently mentioned aspects of DCIFER.
If and NOT only if and exterior angle were both supported by all the 30 teachers (100%), followed by capacity (87%) and dynamic equation (77%). Though these four aspects of DCIFER were all supported by the majority of teachers, the number of different strategies teachers utilized to promote these aspects varied. For example, if and NOT only if was the center of teacher support. Four out of the five strategies (except for cognitive scaffolding which was used by 27% of the teachers) were extensively used by teachers to promote if and NOT only if. In contrast, exterior angle was extensively supported by the strategy of nurturing environment only (100%). Both capacity and Dynamic equation had two big supporters: reviving curriculum and cognitive scaffolding for capacity; and reviving curriculum and nurturing environment for dynamic equation.

As was discussed earlier in this chapter, if and NOT only if, exterior angle, and dynamic equation are the three big concepts in DCIFER. These three aspects of DCIFER were emphasized by most teacher participants. In contrast, capacity, remainder, and false conjecture were the three less frequently mentioned by teacher participants. However, when it comes to the strategies to promote DCIFER, capacity, a less frequently mentioned aspect, was promoted in some way by the majority of teachers. The popularity of promoting capacity in terms of teacher nominators of strategies (87%) even surpassed another popular aspect, dynamic equation (77%). It is interesting to find out that although teachers failed to give much credit to capacity when they defined mathematical creativity, teachers seemed to still treasure capacity in terms of teaching strategies.
False conjecture and remainder were not as popular as capacity. Neither of them received much attention from teachers in terms of promoting strategies. There were only 10 (33%) and eight (27%) teachers who had strategies for false conjecture and remainder, respectively. Among the teachers who shared their strategies for false conjecture, most of them mentioned nurturing environment (70%) as their approach. Remainder was promoted mainly by three types of strategies: reinforcement and encouragement, nurturing environment, and reviving curriculum. It is interesting to note that remainder was nominated by more than half of the teacher participants when they defined mathematical creativity; while false conjecture was only mentioned by about one-third of the teacher participants. However, when it comes to teaching strategies to promote DCIFER, remainder dropped behind false conjecture (27% versus 33% in teacher nomination). It is interesting that although teachers valued remainder to some extent when they defined mathematical creativity, teachers seemed to ignore the promotion of remainder in their teaching to some extent.

To sum up, figure 5.2.1 shows differentiated promotion of different aspects of DCIFER. Dynamic equation, if and NOT only if, exterior angle, and capacity were promoted more; while false conjecture and remainder were promoted less.
5.2.2. Start with something small.

Teachers vary in creative dispositions and developed skills, and some teachers may view themselves as more creative than other teachers. However, a teacher who defines herself as less creative may be worried about her ability to promote creativity in her students. She might wonder: do I have to be creative to help my students to be creative in mathematics? What if I do not have a lot of fancy strategies in teaching for creativity?

Many teachers in this study had interesting and insightful responses to this question. For example, to advise new teachers who are panicked about promoting mathematical creativity in the classroom, one teacher suggested that:

I think they (new teachers) can [promote creativity] if they keep it simple. I
think, I know, when I heard the word ‘creativity,’ it makes me go ‘Oh, I’m not doing enough of it.’ Like I instantly say, ‘Oh my gosh, I’m not being creative enough. I’m not letting them [students] be creative enough.’ You know, and it makes me panic a little bit, and I’m sure they (new teachers) feel the same way where creativity, they think, means it has to be really complicated or has to do with a lot of materials, or it has to be some fabulous amazing lesson that they’ve never been able to think of. It doesn’t have to be like that. It can be just as simple as taking five minutes at the beginning of or at the end of a lesson.

Creativity promotion can start with something really simple and small. It is not mandatory that teachers have to be extraordinarily creative to promote mathematical creativity in their students. There are a lot of ways for teachers to promote mathematical creativity if teachers know their students and the math. The five strategies found in this study are perfect sample strategies to promote mathematical creativity. It does not take tremendous creativity for a teacher to use the strategies effectively, but instead, the teacher may just need an open mind, a pair of listening ears, a couple of observing eyes, and two soft and steady hands to find, recognize, and nourish students’ creative practices in mathematics.

For example, teachers can make choices for the curriculum and adjust learning activities to facilitate deep understanding when the teachers detect the needs of the students (e.g., teachers can give more time for students who are still exploring challenge problems, and teachers can encourage dialogues for idea exchange among
students to inspire each other); teachers can observe students and provide cognitive support when necessarily (e.g., teachers can ask deep questions or give little hints to help students who are struggling to conquer the difficulties and achieve better understanding); teachers can praise and recognize creative ideas (e.g., teachers listen to students, encourage students to be open to different and unexpected options, ask students to share, and reward students’ creative ideas); teachers can provide tools and welcome risk taking in mathematics (e.g., teachers can view mistakes as valuable learning materials and teachers can invite students to focus on the reasoning behind the procedures); and teachers can leave more space for students to take control and explore themselves (e.g., teachers can allow students to choose their own unique way to make sense of mathematics in which students’ personal experiences are used to create connections among ideas).

Of course, if teachers themselves are creative in the teaching processes, they may be more capable of creating activities that are original and engaging. Creative teachers may be more open to accept different strategies from students even if these strategies are not normally expected. Creative teachers may also find it easier to model creative practices so that students have a role model in the classroom. Creative teachers may have experienced and thus understand better the puzzled and unresolved feelings associated with the exploring stage of problem-solving and thus can give better suggestions to students when students get stuck. Creative teachers may get excited and whole-heartedly appreciate the completion of a creative idea and passionately celebrate the joyful moment with students. However, what we are
discussing in this study is how to teach for mathematical creativity, not how to teach creatively. In other words, we expect teachers to teach in a way or in several ways to promote creativity in the learning of mathematics, not necessarily teachers teaching in a creative way. But as long as teachers are practicing to the suggestions above, it is likely that our teachers may adopt more creative characteristics personally and become more open to accept alternative ideas or feel more comfortable encouraging mathematical creativity in students and even experiencing creative practices with students. As one teacher described in the interview, promoting creativity is an evolving process that teachers may end up being more capable of promoting creativity in the learning of mathematics:

I think anytime you’re doing something new, you have to take baby steps.

You have to start with what you have, you know, whether it’s the curriculum, or whatever. And then you think of one or two goals of what you want to work on because no, you can’t take everything all at once. You won’t be good at it. And so I think you could definitely start with creativity and make that your goal. Or when you think about setting goals, you can think about what’s my ending: do I really want to be a creative teacher, and do I really want to be a problem solver? Then you might start with, ok, what question can I ask, and focus on the questions I’m asking them. Not just yes or no questions, but making sure they’re in deeper level… the ones that have them (students) explain… and then [I] realize that, ok, I feel comfortable questioning, then what can I do next to shoot to my end goal of creativity or whatever it would
be.

5.2.3. How to promote mathematical creativity for preschool children?

The No Child Left Behind (NCLB, 2003) Act of 2001 seemed to many people to transfer academic pressure to younger children too early too soon. Preschool educators sometimes feel compelled to have young children learn materials that used to be taught in higher grades to get ready for school. It is not a bad idea to let children learn more things at an early age, but educators should make sure that the learning materials make sense and are developmentally appropriate. Indeed, learning mathematics is not purely accumulating and memorizing number facts. Instead, learning mathematics should be holistic and fun for young children.

Inspired from this study, preschool educators may want to reconsider what “getting ready for school” means. The appropriate anticipation should be that preschool education is enhancing young children’s creativity in mathematics rather than preparing young children with just basic facts to be ready for school mathematics. Preschool teachers can help develop early number sense and interest in mathematics. For example, children’s real life experiences can be shared in class and used as materials for mathematics problem solving. Preschool teachers can also arrange math talk among young children to exercise verbal expression of the steps in their reasoning and to inspire, cross-fertilize, and improve ideas among peers. Preschool teachers can also encourage young children to take challenges and risks in bringing in and following up uncommon thoughts. In addition, preschool teachers may also want to motivate children who are struggling learning counting skills and
other early mathematical concepts.

Children's development in the early years is best viewed in a holistic way, including such components as the development of creativity. Creativity develops not only in mathematics but also in other areas as well, such as intellectual development, social emotional development, literacy development, and personality development. These areas are developed as a whole especially before grade school, as the curriculum is not yet divided up for different academic subjects and goals. According to DCIFER, creativity in mathematic takes more than mathematics content knowledge to develop; mathematical creativity also needs support from other areas, such as literacy skills for communicating mathematical thinking, motivation and perseverance in struggles, self-confidence and braveness to take risks, flexible and open thinking style, and the like.

Preschool teachers should not feel forced to uplift young children’s mathematics knowledge to match the level expected for students in grade schools in order to be ready for school. Given that “match” is often proved mainly by good performance on standardized tests, a more natural way with respect for children’s development and maturation level should be advocated. It would benefit children when they enter elementary school if their preschool teachers value different aspects of mathematical creativity. Preschool teachers can promote these aspects by pointing out different mathematics ideas in real life for young children, nurturing young children’s interest in exploring mathematics themselves, helping young children to build up self-confidence and persistence in solving hard problems, supporting young
children to develop effective communication with peers, inviting young children to take alternative perspectives to consider what happens in daily life, encouraging young children to maintain curiosity to explore the unknown, and assisting young children in tolerating differences and unconventionality.

In short, for preschool education, “getting ready” may mean more than just a rush for memorizing mathematics facts. Instead, young children get ready by developing creativity in the learning of mathematics in a nutritious environment where preschool teachers encourage children to take their own experience and understanding and form good habits to explore mathematics in life themselves. No children should be left behind in developing creativity in mathematics.

5.3. Support from Primarily Math

Why were the findings of this study so different from past work that has suggested many limitations in teachers’ competence to understand and foster creativity in the early childhood classroom? Why did teachers in this study rarely mention any major obstacles but instead shared many useful strategies they utilized to promote mathematical creativity? Perhaps earlier studies have not used interviewing techniques that access the depth and breadth of teachers’ understanding of mathematical creativity and capabilities in cultivating mathematical creativity; or perhaps teachers today are more skillful and competent than those studied previously. Another possibility is that the teachers in this study had been supported in understanding and promoting mathematical creativity indirectly, from their professional development in the Primarily Math (PM) program.
In the three pedagogy and child development courses in PM, for example, teachers learned specific skills and strategies that would help promote creativity in mathematics, even though these were not discussed in terms of “creativity.” They learned about how to sustain productive math talk, provide cognitively challenging tasks, incorporate various ways of representation, create thought-provoking questions and explanations, support individual needs of young children, and place young children in the center of learning mathematics. These practices were utilized by teachers to promote mathematical creativity for young children as teachers described in their five strategies.

Moreover, in the three mathematics courses in the PM program, teachers learned to understand more deeply the nature of mathematics and view themselves and their students as mathematicians. Teachers were exposed to the ideas of mathematical habits of mind (HOMs), and were modeled student-centered teaching by the instructors. Even though these experiences were not labeled as “creativity,” teachers may have generalized or extended their new knowledge to better understand and value mathematical creativity in their students. For example, when describing the activities where creativity was often observed, one teacher commented that she was able to recognize and allow for more creative practices to happen in mathematics after she went through the PM program:

Probably [I would observe creativity] most during specials, when they go to art and computer and a little bit media. [But] I have to say, they do more [creative practices], since I was in PM program, in math, and solving their
[math] problems. I mean it’s more open-ended. They don’t have to do the way I’m doing it. If they want to try a different way, they can, and I don’t stop them. I usually try and follow what they’re doing.

Many teachers commented that this PM program helped them with content knowledge, boosted their confidence in mathematics, and transformed their philosophy of mathematics (now teachers view mathematics as fun, flexible, personal, and creative) and of people doing mathematics (now teachers believe that they can be mathematicians, and their students can be mathematicians too). Better understanding of the nature of mathematics as well as growth in content knowledge and teaching knowledge seem to be greatly helpful for teachers to promote mathematical creativity. One teacher described what she took away from PM program:

I feel like PM [program] helped me really to understand the importance of deep thinking and allowing math talk and multiple solutions, and even though our math curriculum now also supports that. I don’t think I would understand it as much had I not been in that program. And I see colleagues that maybe haven’t taken it [PM program], and [they] don’t seem to have the level of deep understanding for math, or the importance of the deep understanding than maybe I do. And so just trying to share that with them… the importance that it (deep understanding) has… and [I] hope that they also feel the same way.

The HOMs teachers learned in the PM program are connected with the six
aspects of DCIFER. *Dynamic equation* is associated with the following HOMs: sensitivity to patterns, real world application, reflectiveness, learning from mistakes, and communication. *Capacity* is associated with the HOM of finding connections. *If and NOT only if* is associated with the following HOMs: flexibility, tolerance for uncertainty, openness to the new and unfamiliar; self-direction and ownership in learning, and inventiveness and imagination. *False conjecture* is associated with the following HOMs: risk-taking and willingness to struggle and making mistakes. *Exterior angle* is associated with the following HOMs: using tools and materials and feeling safe in the classroom environment to explore and share ideas. And *remainder* is associated with the following HOMs: perseverance, curiosity and interest, motivation and joyful emotion.

The national standards and principles teachers discussed and reflected upon in the PM program were also closely related to the six aspects of DCIFER. For example, according to the NCTM process standard of problem solving, students should be able to apply learned strategies to new and unfamiliar problems and situations either inside or outside the mathematics classroom context (2000a). This advocacy corresponds to *dynamic equation*. In the NCTM standard of connections, students should be able to recognize and make use of *connections* among mathematical ideas, to understand how different ideas *interconnect*, and to build the idea on one another to construct a *unified whole* (2000a). This standard corresponds to *capacity*. According to the National Research Council (Kilpatrick, Swafford, & Swindell, 2001), a productive disposition is advocated, such that students should
adopt “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 5). This advocacy corresponds to remainder. The findings are consistent with a conclusion that the PM program enriched teachers’ interpretation and promotion of mathematical creativity by deepening teachers’ understanding of the nature of mathematics and supporting teachers to be intentional practitioners who catch teachable moments and address students’ individual needs.

Some teachers commented that better knowledge of standards supported them to promote mathematical creativity. For example, one teacher shared her experience with the NCTM process standards in terms of the using of tools:

The mathematical process standards, I think that does [encourage mathematical creativity]! And I think if you follow those eight process standards to reach the mathematical standards, then you will have creativity. For example, using a variety of mathematical tools, we talked a lot about that, especially in kindergarten, you have to set them up to be able to do that with [tools], and still be productive, because it would be too easy for you to say ‘go pick something in the room’ and they just run crazy and get things that don’t help them. So we will slowly introducing different types of tools, make a chart of all the ones we’ve used in the past and then say, ‘Ok here are all your choices, now you can pick’, so they can pick something productive. So you know we were using the process standards, we were using a variety of tools to be creative to meet the standards I think.
Another teacher talked about her understanding of the process standards in terms of motivation and persistence:

At the beginning, there are process standards that define how a classroom should be functioning or what kinds of thinking or approaching that kids should be encouraged in… There’s one that talks about stamina… they talk about like the ability to stick with a task… when it’s frustrating or when they’re meeting failure… Persistence—that’s what it is. That has to do a lot with creativity. But even I mean reasoning and proof is another process standard, and I think that has to do with creativity because they’re (students are) having to explain their thinking in their process that they went through.

Many teachers said they learned much deeply about the process standards from PM program. For example, one teacher compared her feeling for the standards to somebody who had not been in the program, “I think, you know, myself and another teacher could both feel like we’re teaching the standards, but one could be at a very surface level, and one could promote that deeply.”

Another teacher commented her gratitude to PM that:

If I had never taken the classes that I took, I don’t think I would know how to use those [standards]... I mean it’s just pick the words on paper for normal classroom teachers, until they actually go through the process of learning how to use those standards…They [PM program personnel] need to continue that grant or whatever they need to do to help classroom teachers. Huge, it’s a huge help. And that’s a lot of work that teachers are doing [in PM program]
because they just want to be better teachers. So they [PM program personnel] have to find a way to continue that grant.

Based on these words from teachers, it seems beneficial to teachers if professional development helps deepen teachers’ understanding of standards. Teachers need to spend time not only reading the standards but also making sense of the key ideas and connecting the ideas with classroom practice. And then they can internalize and apply these standards in the classroom in a nutritious way. A good professional development may strive to support teachers to associate the standards with the curriculum and the students.

5.4. Implications

To draw out implications for researchers, teachers, and teacher educators, I summarize what can be learned from this study. In the following paragraphs, I present five common myths about promoting mathematical creativity based on the literatures I referenced in chapter two. For each of the myths, I present what I found in this study to correct or revise the statement. Next, I propose four suggestions regarding the promotion of mathematical creativity for young children.

5.4.1. Myths for promoting mathematical creativity.

*Rich interpretation of mathematical creativity.* It was beyond my expectation that teacher participants in this study would have such a developed understanding of mathematical creativity. Given that many elementary school teachers lack a good understanding of the nature of mathematics (Ball, 1997; Ginsburg et al., 2008; Lockhart, 2002/2008; Sriraman & Lesh, 2007), it is hard for
them to see creativity in mathematics. However, it turned out to be quite opposite in this study. Although creativity was not a popular word teachers would use in teaching mathematics, teachers did have a well-developed definition of mathematical creativity. Moreover, teachers’ definition of mathematical creativity was a good match with the professional definition of creativity in the investment theory of creativity (Sternberg & Lubart, 1995).

**Myth 1. Wrong philosophy of mathematics?** According to the literature (Ball, 1997; Beghetto, 2007; Ginsburg et al., 2008; Lockhart, 2002/2008; Scott, 1999; Sriraman & Lesh, 2007; Westby & Dawson, 1995), teachers often lack appropriate understanding of the nature of mathematics; many of them may fail to see the creative process involved in mathematics. Influenced by previous schooling, they may have an image of the learning of mathematics as receiving and memorizing facts (Pehkonen, 1997).

In this study, some teachers did talk about their previous views of mathematics as memorizing facts. However, by the time of the interviews, teachers had already deepened their understanding of learning mathematics as an exploring process. Many of them mentioned PM program from which they benefited a lot. In regard to the nature of mathematics, many teachers advocated for flexibility and personality (i.e., *if and NOT only if*) in the learning of mathematics because mathematics is more than just applying rules or following procedures. Mathematics involves exploring and finding the unexpected and unknown.

**Myth 2. Negative views towards creativity?** According to the literature
(Beghetto, 2007; Scott, 1999; Westby & Dawson, 1995), teachers often hold negative attitudes towards creative students. Teachers may favor students who are less creative but regard students who showed more creative characteristics as disruptive.

Though it was not explicitly asked in the interviews whether teachers favored mathematical creativity or not, it was implied in teachers’ responses that they valued creativity in mathematics. In the pilot study, I did ask the question directly, and all of the teacher participants replied with a positive answer. However, such answers did not provoke any further interpretation.

So for this study, I asked teachers for more details about their understanding of mathematical creativity rather than a yes or no question of whether they appreciate creativity in mathematics or not. From teachers’ descriptions of their understanding of mathematical creativity, it seemed clear that teachers did value it. For example, teachers used all kinds of ways to inspire flexible thinking and uncommon solutions; teachers created opportunities for students to share creative ideas; teachers allowed time and release control for students to explore their own way to make sense of the problem; and teachers advocated for a safe and respectful environment for students to take risks and play with mathematical ideas creatively.

In short, teachers were doing their job to promote creativity in the learning of mathematics as something they valued.

**Capability in promoting mathematical creativity.** It was not expected that teacher participants in this study would feel highly capable of promoting
mathematical creativity. It is often believed that elementary school teachers have a lot of struggles when teaching mathematics, and many teachers even disfavor creative students. But this study reveals an alternate picture: teachers do favor mathematical creativity and they are utilizing a number of strategies to promote it.

Myth 3. Impossible to promote mathematical creativity? At first, I was focusing on challenges and obstacles teachers faced when promoting creativity in the classroom. It was originally expected that complaints about the obstacles and disruptions, such as textbook quality, teacher knowledge, and time and pacing pressures, would dominate teacher interviews. However, during the interviews, teachers displayed a much more positive view on creativity. Though they recognized some drawbacks of the curriculum, such as pacing and testing, they shared with me many effective strategies and advices about how to promote creativity in learning mathematics. Thus, my focus in this study switched from the less positive/help-seeking side to the strength-based/competent side. In other words, the original questions about the challenges teachers were facing transformed into questions about the successful strategies teachers were using to promote mathematical creativity.

All the teachers described their effective strategies to promote mathematical creativity. There were 22 out of the 30 teachers who responded that it was possible even for new teachers who had considerable occupation pressure to promote mathematical creativity in the classroom. It is worth noticing that there were some specific suggestions these teachers wanted to give to novice teachers. For example,
teaching is not always about the teacher's performance but rather about letting students take the leading roles in the classroom; this is always a good alternative in teaching. Some teachers advised new teachers to pair up or team up; new teachers can form a group to get support for how to promote mathematical creativity because it is better to do it with somebody else rather than doing everything all by oneself. New teachers may also want to join veteran teachers in professional development or training groups. It is good to have ambitions, but new teachers can take on the task of promoting creativity slowly by starting with something small. It can be overwhelming for teachers who are just starting their teaching career or just switched to a new grade level to combine a lot of strategies in teaching, but adding a little thing each time to promote creativity is not impossible. For example, a new teacher can just focus on one issue in the first semester, such as letting students explore mathematics for a while before the teacher intervenes and instructs. This small change in activity can be effective to promote mathematical creativity, and doing just this one thing at a time does not necessarily put the new teacher under timing pressure. When the teacher and students feel comfortable about these creative activities, the teacher can add one more piece of strategy, and this process can go on and on. Mathematical creativity is approachable. It can start very simple and continue to grow.

Myth 4. Overwhelmed by occupational pressure? According to the literature (Baer, 1999; Baer & Garrett, 2010; Beghetto & Plucker, 2006; Besancon & Lubart, 2008; Niu & Zhou, 2010; Plucker & Dow, 2010; Remillard, 2005), elementary
school teachers are often under occupational pressures that force them to teach with less flexibility. They teach long hours and teach all other subjects in addition to mathematics which leaves them limited time to work hard to improve the instruction of mathematics. They tend to follow the curriculum, often times under district pacing pressures, and move their children along at a rapid pace, which leaves little time for students to work on important ideas or exercise creativity.

However, teachers from this study managed to build flexibility into their learning activities, which prevented them from feeling too much pressure. These teachers did have a curriculum to follow, but they were also able to incorporate a lot of personal decisions and extra activities into their teaching to revive the curriculum. For example, they were able to stretch time for mathematics, and choose what content to teach for a longer amount of time and what to teach for a shorter amount of time.

**Myth 5. Shallow curriculum?** According to the literature (Ball, 1997; Burns, 1998; Drake, 2009; Ginsburg et al., 2008), the curriculum for elementary school mathematics is often wide and shallow and lacking connections, which makes it hard for students to develop deep understanding of key concepts or practice mathematics creatively.

However, for the teachers from this study, their district had adopted a new curriculum program which turned out to be quite satisfactory. The new curriculum promoted deep understanding, supported multiple perspectives, and encouraged personal exploration. Fluency in procedures is no longer the only focus. Teachers
were able to make use of this new curriculum to facilitate the fostering of mathematical creativity.

5.4.2. Advices for promoting mathematical creativity.

Advice 1. A good package of knowledge. The package of knowledge includes deep understanding in: 1) the nature of mathematics (such as the mathematics content knowledge and the understanding of mathematical habits of mind (HOMs)); 2) the creativity in mathematics (according to DCIFER and the investment theory of creativity, mathematical creativity is a combined product of intellectual abilities, conceptual understanding, personal exploration and multiple thinking styles, risk taking, stimulating environment, and motivation); 3) the national standards and principles for mathematical proficiency (such as the NCTM process standards, the CCSS-M, and the NRC standards); and 4) the children and the pedagogical strategies (such as the knowledge of children’s developmental trajectory in developing number sense, the knowledge of the five strategies to promote creativity in the learning of mathematics for young children, and the knowledge of making connections between the two types of knowledge to inform teaching practice). This package of knowledge can be gained from college preparation, practicum experiences, in-service teaching, professional development programs, reading books and articles, communicating with veteran teachers and the like. College preparation and professional development programs are advised to stress on these types of knowledge for potential and in-service teachers to help them promote mathematical creativity in the classroom.

Advice 2. A good curriculum. A good curriculum should be coherent in
content to present mathematical concepts with connections. It should not be too wide or too shallow. Challenging problems should be available for students to explore with teacher scaffolding if needed. The quality of problems matters more than the quantity of problems. For example, rather than solving 20 problems purely by repeating the same strategy, students can be asked several deep and thought-provoking questions in one problem to allow students time for exploring other strategies, developing connections among ideas, deepening conceptual understanding, explaining strategies to peers, revising strategies, getting exposed to other ideas and the like.

**Advice 3. Start with something little.** It is not necessary for elementary school teachers to adopt all the five strategies in order to promote mathematical creativity. These teachers can start with one or several of them that they feel comfortable with, and then gradually adjusting the strategies, adding new strategies, or removing old strategies, and even developing their own strategies as teachers accumulate more experience in the classroom and according to students’ changing needs. Starting with something basic and then building it up to develop a personal strategy set is a beneficial method not only for new teachers but for teachers in general. On one hand, new teachers are less overwhelmed. On the other hand, every time teachers change a grade level of teaching or start a new academic year, the student body changes and the teachers need to find and experiment different strategies that suit this new class, and starting the strategy set all over again is necessary.

**Advice 4. A stronger image of the teachers.** This advice is for researchers in
particular. Teachers’ voices are often ignored or subjected to researchers’
pre-suppositions. Elementary level mathematics education is often viewed from a
negative lens. It is believed that primary grades teachers often lack solid knowledge
foundation in mathematics, and that the majority do not teach well. Curriculum,
classroom environment, and teacher’s negative attitudes are not considered as being
supportive for mathematics education. It is common for researchers and policy
makers to find fault with early mathematics education in the United States and focus
most of the attention on teachers’ weaknesses and disadvantages rather than teachers’
strengths and capabilities.

In this study, however, I adopted the grounded theory methodology which
allowed me to base the study on the voices of teachers who were participating in an
effective professional development graduate certificate program. I flexibly adjusted
and readjusted my questions constantly according to what the teachers showed to me.
For example, finding the obstacles and challenges in promoting mathematical
creativity was one goal of this study. However, the data revealed a different picture.
Teachers’ descriptions were very positive and strength-based, and therefore, the
focus of the study moved to capture a better and more realistic view accordingly.
Teachers had appropriate understanding of the nature of mathematics. Teachers gave
a well-developed interpretation of mathematical creativity that matched the
professional definition of creativity by scholars and researchers, especially in the
investment theory of creativity. This is an inspiring finding for researchers that
teachers’ understanding of mathematical creativity is comparable with professional
researchers’ interpretation. This may encourage researchers to rely less on their presumptions, especially those less positive ones, and value teachers’ voices more.

In addition, teachers in this study were quite capable of promoting mathematical creativity without any serious concerns about obstacles. This finding reminds researchers to adopt a strength-based perspective to study teachers in elementary mathematics education. Researchers should be open-minded to what they could find from real world classroom teachers who participated in mathematics professional development programs: the findings can be very different, in a good way. Moreover, teachers from this study have given a lot of great suggestions to promote mathematical creativity, which are worthy of being examined and explained further by researchers and then introduced and recommended to a larger population.

5.5. Limitations and Future Directions

Strengths of this study have been described in details in terms of implications for researchers, teachers, and teacher educators. In this section, some limitations of this study are discussed.

This grounded theory study is exploratory and the methodology does not allow for any causal inferences. It is not clear, for example, which aspect of the PM program resulted in particular aspects of teachers’ interpretation of mathematical creativity (DCIFER) or teachers’ five strategies to promote DCIFER. In addition, the data collection methods have limitations. What teachers say in the interviews may not exactly represent their teaching practices in the classroom.

Future analyses will relate the teacher interviews to their actual classroom
behavior, recorded in sample videotapes of mathematics lesson. These videotapes were taken for the PM program, and the teachers were not explicitly required to promote creativity in the learning of mathematics when they took the videos. Therefore, the videos are likely to show how teachers were promoting creativity without deliberate intentions in mind. In the videos, some teachers were constantly using the strategies they talked about to foster mathematical creativity. Some teachers, however, were not effectively demonstrating the strategies they talked about in interviews. Overall, teachers were using the strategies described in the interviews to some degree, but some teachers seemed to have a better mastery of the strategies than the others. The videos of the classroom practices showed more variances in teaching practices. My next step of research is analyzing the videos and comparing these videos with interview data to better understand teachers’ interpretation and promotion of mathematical creativity in young children.

Furthermore, teachers in this study have been through the PM program, and thus one should be cautious when applying the findings to teachers in general. The PM program had a focus on enhancing teachers’ knowledge in the mathematical content, child development and pedagogy. For teachers who have not received similar support in these areas of knowledge, their interpretation of mathematical creativity and strategies to promote it may not be that elaborate or rich. However, this PM program did not provide any intensive training of advanced ideas about mathematical creativity or smart strategies to promote it. From the interviews, the teachers were using ordinary language to express their personal understanding of
mathematical creativity based on their everyday practice in the classroom. And therefore, teachers in general should be able to understand the findings and make some connections to their teaching experience.

Learning mathematics can be a personal and enjoyable process. Instead, if mathematics is presented to children with a focus only on procedures, rules and facts to achieve fluency, then mathematics may scare or bore children instead of engaging and inspiring them. Mathematics is an academic subject, but it is more than that. It is a way of thinking and reasoning that underlies many disciplines, including engineering, the natural sciences, and many parts of the social sciences. When mathematics is presented not just as a set of rules and procedures, or as a narrow academic subject, mathematical creativity may have more possibility to develop and thrive. Creativity in the learning of mathematics is comprehensive: intellect, thinking styles, motivation, personality, knowledge, and environment are all involved. When elementary school teachers in this study adopted such a comprehensive view for mathematical creativity, they were able to describe corresponding strategies to promote it. The findings suggest that it is possible for teachers to nurture mathematical creativity even in the early grades of elementary school; they can demonstrate awareness of what it takes to be a mathematical thinker and the significance of developing creativity in doing mathematics.
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Appendices A: Interview Protocol

Project: Elementary School Teachers’ Interpretation and Promotion of Creativity in the Learning of Mathematics

Time and Date: 
Place: 
Interviewer: 
Interviewee: 

[Introduction]

I want to thank you for taking the time to talk to me today. The purpose of the study is to explore the process of teacher interpretation and promotion of creativity in student learning and to ultimately generate a theory based on the findings. This interview will take about 30-45 minutes. Please feel free to elaborate on any questions and ask for clarification as needed. You do not need to answer any questions if you feel uncomfortable. I will be recording and transcribing what we say today. I will also ask you to review the transcription with some of the notes I make regarding my interpretations of what you say for clarification or accuracy. I want you to review it to make sure I am representing your views. It is possible that I will request another interview. Only the researchers will have access to the data, and the results of the interview will only be used for academic research purposes. Are you ready to start?

[Turn on the audio recorder and test it.]

<table>
<thead>
<tr>
<th>Questions:</th>
<th>Notes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. When I say creativity in general, what does it mean to you? Can you give me an example?</td>
<td></td>
</tr>
<tr>
<td>2. Now, let’s apply creativity to young children, what does it mean to you?</td>
<td></td>
</tr>
<tr>
<td>3. Think about a typical day in your classroom, in what activities do you often find students showing creativity in your classroom?</td>
<td></td>
</tr>
<tr>
<td>4. Tell me a story about a student who you think is creative in solving math problems.</td>
<td></td>
</tr>
</tbody>
</table>
**Probes:**

What’s your feeling about it? (Wait for them thinking)

Were you surprised? Were you amazed? Were you challenged? Were you at all troubled?

What was your role in the story? That is, did you do anything to help? Or did you fail to do anything to help, and why?

Was this story typical for your classroom? Or unusual? Do you see many mathematically creative students? Or just a few?

5. **What helps a student be creative in math?**

**Probes:**

Thinking Processes: What thinking processes are important for a child to engage in math in a creative way in early grades?

Learning and thinking styles: What style(s) of learning or thinking do you find important for a child to engage in math in a creative way in early grades?

Motivation: How would you describe motivation to engage in math in a creative way in early grades?

Personality: Give me some personality traits you think are most important to engage in math in a creative way in early grades?

Knowledge: How much knowledge do you think a child needs to engage in math in a creative way? There are mixed views about knowledge and creativity. Do you think math knowledge promotes or inhibits math creativity?

What about the context?

Environment: How would you describe a classroom environment that supports math creativity, such as its physical setup and social atmosphere?
6. If you are to come up with a definition of creativity in the learning of mathematics among young children, what would you say?

**Probes:**

What does it mean to have young children being creative in learning mathematics?

Think about creativity in relation to math competency and proficiency, what do you think creativity means or brings to the learning of mathematics in grades K-3?

7. What do you think is challenging, if anything, in promoting creativity in the learning of mathematics for young children?

**Probes:**

Could you describe your own knowledge background and to what extent it supports or inhibits teaching for creativity?

Could you describe your own personality and to what extent it supports or inhibits teaching for creativity?

Could you describe your daily schedule and curriculum and to what extent they support or inhibit teaching for creativity in mathematics?

Could you describe the classroom environment (e.g. materials and technology) and to what extent it supports or inhibits teaching for creativity?

Any other things you want to mention that help or discourage you to teach for creativity in mathematical learning?

8. As a teacher, how do you deal with these challenges? What would you suggest to change the situation?
Probes:

Tell me more about it.

9. Think about CCSS, NCTM, and other state or national level standards for mathematical proficiency, do you think they are supporting/guiding you to teach for creativity or discouraging you from expecting students to do math creatively?

10. If you are to give advice to new teachers about what creativity is and how it relates to student learning of mathematics, what would you say to them?

Probes:

Is there anything else you want to add?

11. Some new teachers feel overwhelmed when they first start teaching, do you think it’s possible for them to support students to be creative in math at the beginning?

12. Here’s a piece of paper and some pencils. Please draw a model or picture of creativity in the learning of mathematics.

Observation:

Is there a child included in the picture?
Is there only one child or more children?
Is there a teacher or other adult(s) included in the picture?

[Thank the individuals for their cooperation and participation in this interview. Assure them of the confidentiality of the responses and the potential for future interviews.]
Appendices B: Teacher Demographic Information

1. **Gender**
   A. Female
   B. Male
   C. Other

2. **Birth Year**
   ________________________________________________________________

3. **Race/Ethnicity**
   A. European-American
   B. African-American
   C. Asian
   D. Hispanic
   E. Multi-Racial
   F. Other _______________________________________________________

4. **Education**
   Bachelor’s Degree in _____________________________________________
   Master’s Degree in _______________________________________________
   Doctoral Degree in _______________________________________________
   Others __________________________________________________________

5. **Certificate or Diploma**
   Please specify ____________________________________________________

6. **Teaching Role (Grade level and subjects)**
   Present __________________________ (Years of Teaching______________)
   Before 1 __________________________ (Years of Teaching______________)
   Before 2 __________________________ (Years of Teaching______________)
   Before 3 __________________________ (Years of Teaching______________)

7. **Other Role(s)**
   A. Math Coach
   B. Administrative Role (Please specify ______________________________)
   C. Others__________________________
Appendices C: Teacher Drawings of Mathematical Creativity

[Image of a child's drawing with numbers and a thought bubble showing a calculation: 26 + 74 = ?]

[Image of another child's drawing showing a thought bubble with the text: "I get this answer because ..."

[Image of a child's drawing with a numbered chart: 0 + 0 = 0, 3 + 2 = 5, 0 + 0 = 0, 4 + 1 = 5, 2 + 3 = 5]
One day 36 kids went to the park. Some more joined them. Now there are 48. How many joined them?
Taking your own experiences from what you learn and making your own discovery.

Math

Enjoy Life

Reading

Creativity

(3-2)
\[ \begin{align*}
\circ \ \circ & + \ \circ \circ \circ \circ \circ = 5 \\
2 & + 3 = 5 \\
\text{I have 2 green dots} & \text{and 3 blue dots. I see} \\
\text{5 dots.}
\end{align*} \]
3 + 7 = 10
There were 4 kids in a line.

1 and 3
2 and 2
0 and 4
48 - 8 =

50 - 5 = 45

8 + 4 = 12
11 + 1 = 12

6 + 6

10 + 2

1 ten 2 ones
\[
\begin{align*}
428 + 269 &= 697 \\
400 + 20 + 8 + 200 + 60 + 9 &= 697 \\
600 + 90 + 7 &= 697 \\
\end{align*}
\]

\[
10 + 8 = 18
\]

\[
\begin{array}{c}
18 \\
10 + 8
\end{array}
\]