Post-LHC7 fine-tuning in the minimal supergravity/CMSSM model with a 125 GeV Higgs boson

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The recent discovery of a 125 GeV Higgs-like resonance at LHC, coupled with the lack of evidence for weak scale supersymmetry (SUSY), has severely constrained SUSY models such as minimal supergravity (mSUGRA)/CMSSM. As LHC probes deeper into SUSY model parameter space, the little hierarchy problem—how to reconcile the Z and Higgs boson mass scale with the scale of SUSY breaking—will become increasingly exacerbated unless a sparticle signal is found. We evaluate two different measures of fine-tuning in the mSUGRA/CMSSM model. The more stringent of these, $\Delta_{\text{HS}}$, includes effects that arise from the high-scale origin of the mSUGRA parameters while the second measure, $\Delta_{\text{EW}}$, is determined only by weak scale parameters: hence, it is universal to any model with the same particle spectrum and couplings. Our results incorporate the latest constraints from LHC7 sparticle searches, LHCb limits from a scan over the entire viable model parameter space. We find a $\Delta_{\text{HS}} \approx 10^3$, or at best 0.1%, fine-tuning. For the less stringent electroweak fine-tuning, we find $\Delta_{\text{EW}} \approx 10^2$, or at best 1%, fine-tuning. Two benchmark points are presented that have the lowest values of $\Delta_{\text{HS}}$ and $\Delta_{\text{EW}}$. Our results provide a quantitative measure for ascertaining whether or not the remaining mSUGRA/CMSSM model parameter space is excessively fine-tuned and so could provide impetus for considering alternative SUSY models.

I. INTRODUCTION

The recent spectacular runs of LHC at $\sqrt{s} = 7$ and 8 TeV have led to identification of a Higgs-like boson with mass $m_h \approx 125$ GeV [1,2]. This is in accord with predictions from the minimal supersymmetric standard model (MSSM) which requires that the lighter Higgs scalar mass $m_h \lesssim 130$–135 GeV [3]. Since values of $m_h > M_Z$ are only possible due to radiative corrections, the upper end of the range depends on the masses of third-generation sparticles that one is willing to allow. To achieve $m_h \sim 125$ GeV, either large mixing or several TeV masses are required in the top squark sector. In models such as the much-studied minimal supergravity (mSUGRA) or CMSSM model [4,5], values of trilinear soft breaking parameter $|A_0| \sim (1.5–2)m_0$ are favored, along with top squark masses $m_{\tilde{t}_1} \sim 1$–2 TeV; for positive $A_0$ values, $m_0$ is typically larger than 5 TeV [6,7].

While the measured value of $m_h$ is within the expected range of even the simplest SUSY models, there is at present no sign of SUSY particles at LHC. From LHC data analyses within the mSUGRA model, mass limits of $m_{\tilde{g}} \approx 1.4$ TeV when $m_{\tilde{q}} \sim m_{\tilde{g}}$ and $m_{\tilde{q}} \gtrsim 0.9$ TeV when $m_{\tilde{g}} \gg m_{\tilde{g}}$ have been reported [8,9]. Several groups [10] have updated their fits of the mSUGRA/CMSSM model to various data sets, now including information from LHC7 and LHC8 Higgs-like boson discovery and LHC7 sparticle mass limits. Typically, the best-fit regions have moved out to large values of $m_0$ and $m_{1/2}$ to accommodate the LHC sparticle mass limits and Higgs discovery. Such large $m_0$ and $m_{1/2}$ values lead to sparticle masses in the multi-TeV mass range, thus exacerbating what has become known as the little hierarchy problem: how do such large SUSY particle masses and soft breaking parameters conspire to yield the weak scale typified by the $Z$-boson mass $M_Z \approx 91.2$ GeV. The conflict between the strong new LHC sparticle mass limits and the comparatively low values of $M_Z$ and $m_h$ has intensified interest in the fine-tuning in supersymmetric models [11–15].

To set the stage for this analysis, we begin by reviewing radiative corrections (assumed perturbative) to scalar field masses. In a generic quantum field theory, taken to be the low-energy effective theory whose domain of validity extends up to the energy scale $\Lambda$, the physical mass squared of scalar fields takes the schematic form (at leading order),
In Eq. (1.1), $g$ denotes the typical coupling of the scalar $\phi$, $m_{\phi}$ is the corresponding mass parameter in the Lagrangian, $16\pi^2$ is a loop factor, and $C_i$ are constants that aside from spin, color and other multiplicity factors are numbers $O(1)$. The scales $m_{\text{low}}$ and $\Lambda$, respectively, denote the highest mass scale in the effective theory and the scale at which this effective theory description becomes invalid because heavy degrees of freedom not included in the low-energy Lagrangian become important. For instance, if we are considering corrections to the Higgs sector of the MSSM embedded into a Grand Unified Theory (GUT) framework, $\Lambda \sim M_{\text{GUT}}$ and $m_{\text{low}} \sim M_{\text{SUSY}}$ (or more precisely, $m_{\text{low}}$ is around the mass of the heaviest sparticles that have large couplings to the scalar $\phi$). Finally, the last term in (1.1) comes from loops of particles of the low energy theory, and their scale is set by $m_{\text{low}}$. These terms may contain logarithms, but no large logarithms since effects of very high momentum loops are included in the $C_1$ and $C_2$ terms. These finite corrections provide contributions to that which we have referred to as electroweak fine-tuning in a previous study [14].

If the effective theory description is assumed to be valid to the GUT scale, the $C_1$ term is enormous. Even so it is always possible to adjust the Lagrangian parameter $m_{\phi}$ to get the desired value of $m_\phi^2 \leq m_{\text{low}}$. This is the big fine-tuning problem of generic quantum field theory with elementary scalars. This problem is absent in softly broken supersymmetric theories because $C_1 = 0$. We see from Eq. (1.1) that if the physical value of $m_\phi$ is significantly smaller than $m_{\text{low}}$ (which in the case of the MSSM $\sim m_{\tilde{t}_L}$), we will still need to have significant cancellations among the various terms to get the desired value of $m_\phi$. This is the little hierarchy problem. We also see that in models such as mSUGRA that are assumed to be valid up to very high-energy scales $\Lambda \sim M_{\text{GUT}} - M_p$, the magnitude of the $C_2$ term typically far exceeds that of the $C_3$ term because the logarithm is large, and hence is potentially the largest source of fine-tuning in such SUSY scenarios.

Because the $C_2$ and $C_3$ terms in Eq. (1.1) have somewhat different origins—the $C_2$ term represents corrections from physics at scales between $m_{\text{low}}$ and $\Lambda$, while the $C_3$ term captures the corrections from physics at or below the scale $m_{\text{low}}$—we will keep individual track of these terms. In the following we will refer to fine-tuning from $C_2$ type terms as high-scale fine-tuning (HSFT) (since this exists only in models that are valid to energy scales much larger than $m_{\text{low}}$) and to the fine tuning from $C_3$-type terms as electroweak fine-tuning (EWFT) for reasons that are evident. We emphasize that the sharp distinction between these terms exists only in models such as mSUGRA that are assumed to be a valid description to very high scales and is absent in low-scale models such as the phenomenological MSSM [16].

In this paper, we quantify the severity of fine-tuning in the mSUGRA model, keeping separate the contributions from the two different terms. We are motivated to do so for the following two different reasons:

(i) First, as emphasized, $C_2$-type terms appear only if the theory is applicable out to scale $\Lambda \gg m_{\text{low}}$, while the $C_3$-type terms are always present. In this sense, the fine-tuning from the $C_3$-type terms is ubiquitous to all models, whereas the fine-tuning associated with the (potentially larger) $C_2$-type terms may be absent, depending on the model.

(ii) Second, as we will explain below, there are two very different attitudes that one can adopt for the fine-tuning from $C_2$-type terms. Keeping the contributions from $C_2$ and $C_3$ separate will allow the reader the choice as to how to interpret our results and facilitate connection with previous studies.

The remainder of this paper is organized as follows. In Sec. II we introduce our measures of fine-tuning. As usual, we adopt the degree to which various contributions from the minimization of the one-loop effective potential in the MSSM Higgs boson sector must cancel to reproduce the observed value of $M_Z^2$ as our measure of fine-tuning. We use these considerations to introduce two different measures. The first of these is the least stringent one and relies only on the weak-scale Lagrangian that arises from mSUGRA, with total disregard for its high-scale origin, and is referred to as electroweak fine-tuning (EWFT). The other measure that we introduce incorporates the high-scale origin of mSUGRA parameters and is therefore referred to as high-scale fine-tuning (HSFT). In Sec. III, we present contours for both HSFT and EWFT in several $m_{\text{SUSY}}$ planes along with excluded regions from LHC7 sparticle searches and LHCb limits from $B_\ell \rightarrow \mu^+ \mu^-$ searches.2 We find that while LHC7 sparticle mass limits typically require EWFT at $\sim 1\%$ level, the requirement that $m_\mu \sim 125$ GeV leads to much more severe EWFT in the $0.1\%$ range in the bulk of parameter space. As anticipated, HSFT is even more severe. We also find that the hyperbolic branch/focus point region (HB/FP) [19]—while enjoying lower EWFT than the bulk of mSUGRA parameter space—still requires fine-tuning at about the percent level. The fine-tuning situation is exacerbated by the requirement of large $|A_0|/m_0|$ for which the

\[ m_\phi^2 = m_{\phi 0}^2 + C_1 \frac{g^2}{16\pi^2} \Lambda^2 + C_2 \frac{g^2}{16\pi^2} m_{\text{low}} \log \left( \frac{\Lambda^2}{m_{\text{low}}^2} \right) + C_3 \frac{g^2}{16\pi^2} m_{\text{low}}^2. \]
II. FINE-TUNING

We begin by first writing the Higgs potential whose minimization determines the electroweak gauge boson masses as

\[ V_{\text{Higgs}} = (m_{H_u}^2 + \mu^2)|h_u^0|^2 + (m_{H_d}^2 + \mu^2)|h_d^0|^2 \]

\[ -2B\mu(h_u^0 h_d^0 + H.c.) + \frac{1}{8}(g^2 + g'^2)(|h_u^0|^2 - |h_d^0|^2)^2 \]

\[ + \Delta V, \]

where the radiative corrections (in the one-loop effective potential approximation) are given in the DR scheme by

\[ \Delta V = \sum_i \frac{(-1)^{j_i}}{64\pi^2} \text{Tr} \left[ (\mathcal{M}_i^\dagger \mathcal{M}_i)^2 \left| \log \frac{M_i^2}{M_i^2} - \frac{3}{2} \right| \right]. \]  

Here, the sum over \( i \) runs over all fields that couple to Higgs fields, \( \mathcal{M}_i^\dagger \mathcal{M}_i \) is the Higgs field dependent mass squared matrix (defined as the second derivative of the tree level potential), and the trace is over the internal as well as any spin indices. One may compute the gauge boson masses in terms of the Higgs field vacuum expectation values \( v_u \) and \( v_d \) by minimizing the scalar potential in the \( h_u^0 \) and \( h_d^0 \) directions. This leads to the well-known condition

\[ M_Z^2 = \frac{(m_{H_u}^2 + \Sigma_u^d) - (m_{H_d}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} \]  

(2.3)

Here the \( \Sigma_u^u \) and \( \Sigma_u^d \) terms arise from first derivatives of \( \Delta V \) evaluated at the potential minimum and \( \tan \beta = v_u/v_d \). At the one-loop level, \( \Sigma_u^u \) contains the contributions \( \Sigma_u^u(\tilde{t}_{1,2}), \Sigma_u^u(\tilde{b}_{1,2}), \Sigma_u^u(\tilde{t}_{1,2}), \Sigma_u^u(\tilde{W}_{1,2}), \Sigma_u^u(h, H), \Sigma_u^u(H^\pm), \Sigma_u^u(W'^\pm), \Sigma_u^u(Z), \) and \( \Sigma_u^u(t) \). \( \Sigma_d^d \) contains similar terms along with \( \Sigma_d^d(b) \) and \( \Sigma_d^d(t) \) while \( \Sigma_d^d(t) = 0 \) \[14\].

Although we have highlighted third-generation matter fermion contributions here because these frequently dominate on account of their large Yukawa couplings, we note that there are also first/second generation contributions \( \Sigma_u^u(\tilde{q}, \tilde{e}) \) and \( \Sigma_d^d(\tilde{q}, \tilde{e}) \) that arise from the quartic \( D \)-term interactions between the Higgs sector and matter scalar sector even when the corresponding Yukawa couplings are negligibly small. These contributions are proportional to \( (T_3 - Q_i \sin^2 \theta_w) \times F(m_i^2) \), where \( T_3 \) is the hypercharge, \( Q_i \) is the electric charge and \( F(m_i^2) = m_i^2(\log \frac{m_i^2}{\Lambda^2} - 1) \) of the \( i \)th matter scalar. Although the scale of these is set by the electroweak gauge couplings rather than the top Yukawa coupling, these can nevertheless be sizeable if the squarks of the first two generations are significantly heavier than third generation squarks. However, in models such as mSUGRA—where all squarks of the first two generations (and separately, the corresponding sleptons) are nearly mass degenerate—these contributions largely cancel. Indeed, the near cancellation (which would be perfect cancellation in the case of exact degeneracy) occurs within each generation, and separately for squarks and for sleptons. These terms, summed over each of the first two generations, are always smaller than the other terms in the \( C_i \) and \( B_i \) arrays used to define our fine-tuning criterion below and so do not alter our fine-tuning measure defined below.

The reader may wonder that we are treating the first two generations differently from the third generation in that for the latter we consider the contributions from each squark separately (i.e., not allow for cancellations of the contributions to, say, \( \Sigma_u^u \) from different third generation squarks), while we sum the contributions from the entire first/second generation to obtain a tiny contribution. The reason for this is that the mSUGRA framework predicts degenerate first/second generation squarks (and sleptons) while the top squark masses (remember that top squarks frequently make the largest contribution to \( \Sigma_u^u \)) are essentially independent. In an unconstrained framework such as the mSUSY [16] we would not combine the contributions from the first/second generation scalars; if these are very heavy and have large intrageneration splitting, their contribution to \( \Delta_{\text{EW}} \) can be significant.

A. Electroweak scale fine-tuning

One measure of fine-tuning, introduced previously in Refs. [12,14], is to posit that there are no large cancellations in Eq. (2.3). This implies that all terms on the right-hand side are comparable to \( M_Z^2/2 \), i.e., that each of the three tree-level terms \( C_{H_u} = |m_{H_u}^2/(\tan^2 \beta - 1)|, C_{H_d} = |m_{H_d}^2/(\tan^2 \beta - 1)|, C_{\mu} = | - \mu^2| \) and each \( C_{\Sigma_{u,d}} \) is less than some characteristic value \( \Lambda \), where \( \Lambda \sim M_Z^2 \). (Here, \( i \) labels SM and supersymmetric particles that contribute to the one-loop Higgs potential and includes the sum over matter sfermions from the first two generations.) This leads to a fine-tuning measure.

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\[ \Delta_{\text{EW}} \equiv \max (C_i)/(M_Z^2/2). \tag{2.4} \]

A feature of defining the fine-tuning parameter solely in terms of weak-scale parameters is that it is independent of whether the SUSY particle spectrum is generated using some high-scale theory or generated at or near the weak scale, as in the pMSSM or possibly in gauge-mediation [20]; if the spectra and weak-scale couplings from two different high-scale theories are identical, the corresponding fine-tuning measures are the same. However, as we will see in Subsection II B, in theories such as mSUGRA, \( \Delta_{\text{EW}} \) does not capture the entire fine-tuning because Eq. (2.3) does not include information about the underlying origin of the weak scale mass parameters.

It is worthwhile to note that over most parameter space, the dominant contribution to \( \Delta_{\text{EW}} \) comes from the weak-scale values of \( m_{H_u}^2 \) and \( \mu^2 \). To see this, we note that unless \( \tan \beta \) is very small, aside from radiative corrections, we would have simply that \( M_Z^2/2 \approx -m_{H_u}^2 - \mu^2 \). As is customary, the value of \( \mu^2 \) is selected so that the correct value of \( M_Z^2 \) is generated. In this case, over much of parameter space \( \Delta_{\text{EW}} \sim |\mu|^4/(M_Z^2/2) \). Only when \( |\mu| \) becomes small do the radiative corrections become important—providing the largest contribution to Eq. (2.3). Thus, contours of fixed \( \Delta_{\text{EW}} \) typically track the contours of \( |\mu| \) except when \( |\mu| \) is small; in this latter case, \( \Delta_{\text{EW}} \) is determined by the \( |\Sigma_u^2| \) whose value is loop suppressed. In Fig. 1 we show the surface of \( |\mu| \) values in the \( m_0 \) vs \( m_{1/2} \) plane of mSUGRA/CMSSM for \( A_0 = 0 \) and \( \tan \beta = 10 \). Here, \( \mu \) is small either at low \( m_0 \) and \( m_{1/2} \) (the bulk region [21]), or in the HB/FP region [19] at large values of \( m_0 \).

\[ \begin{align*}
M_Z^2/2 & = \left( m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 + \Sigma_u^2 \right) \tan^2 \beta - 1 \\
& = \left( m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 + \Sigma_u^2 \right) \tan^2 \beta - 1 - (\mu^2(\Lambda) + \delta \mu^2). \tag{2.5} \end{align*} \]

Following the same spirit that we had used in our earlier analyses [14], we can now define a fine-tuning measure that encodes the information about the high-scale origin of the parameters by requiring that each of the terms on the right-hand side of Eq. (2.5) be smaller than a preassigned \( \Delta_{\text{HS}} \) times \( M_Z^2/2 \). The high-scale fine-tuning measure \( \Delta_{\text{HS}} \) is thus defined to be

\[ \Delta_{\text{HS}} \equiv \max (B_i)/(M_Z^2/2), \tag{2.6} \]

with

\[ \begin{align*}
B_{H_u} & = |m_{H_u}^2(\Lambda)/(\tan^2 \beta - 1)|, \\
B_{\delta H_u} & = |\delta m_{H_u}^2/(\tan^2 \beta - 1)|, \\
B_{H_\nu} & = |m_{H_\nu}^2(\Lambda)\tan^2 \beta/(\tan^2 \beta - 1)|, \\
B_{\delta H_\nu} & = |\delta m_{H_\nu}^2 \tan^2 \beta/(\tan^2 \beta - 1)|, \text{ etc.} 
\end{align*} \]

defined analogously to the set \( C_i \) in Sec. II A. As discussed above, in models such as mSUGRA whose domain of validity extends to very high scales, because of the large logarithms one would expect that (barring seemingly accidental cancellations) the \( B_{\delta H} \) contributions to \( \Delta_{\text{HS}} \) would be much larger than any contributions to \( \Delta_{\text{EW}} \) because the \( m_{H_u}^2 \) evolves from \( m_0^2 \) to negative values.

As we have noted, \( \Delta_{\text{EW}} \) indeed provides a measure of EWFT that is determined only by the sparticle spectrum; by construction, it has no information about any tuning that may be necessary in order to generate a given weak scale SUSY mass spectrum. Thus, while for a given SUSY spectrum \( \Delta_{\text{EW}} \) includes information about the minimal amount of fine-tuning that is present in the model, \( \Delta_{\text{HS}} \) better represents the fine-tuning that is present in high-scale models.

The reader may have noticed that—unlike in our definition of \( \Delta_{\text{EW}} \) in Eq. (2.4) where we have separated out the
contributions from various sources and required each of these to not exceed some preassigned value—we have neglected to separate out the various contributions to $\Delta m_{\text{Higgs}}^2$ that determine $\Delta_{\text{HS}}$. We have done so mainly for convenience,3 but this will also help us to connect up with what has been done in the literature.

Before closing this section, we remark that our definition of $\Delta_{\text{HS}}$ differs in spirit from that used by some groups [15]. These authors write the $m_{\text{Higgs}}^2$ as a quadratic function of the high-scale parameters $\xi_i = \{m_0, m_{1/2}, A_0\}$ for mSUGRA, i.e.,

$$m_{\text{Higgs}}^2 = \sum_{i,j} a_{ij} \xi_i \xi_j,$$

(2.7)

and substitute this (along with the corresponding form for $m_{\text{Higgs}}^2$) in Eq. (2.3) to examine the sensitivity of $M_Z^2$ to changes in the high-scale parameters.4 In the resulting expression, the coefficient of $m_0^2$ in Eq. (2.7) is often very small because of cancellations with the large logarithms, suggesting that the region of mSUGRA with rather large $m_0$ (but small $m_{1/2}$ and $A_0$) is not fine-tuned: we feel that this is misleading and so have separated the contributions from the large logarithms in our definition of $\Delta_{\text{HS}}$. Combining all $m_{\text{Higgs}}^2$ contributions into a single term effectively combines $m_i^2(\Lambda) + \delta m_i^2$ into a single quantity which (aside from the one-loop terms $\Sigma_{\mu i}$ and $\Sigma_{\mu j}$) evidently is the weak scale value of $m_i^2$ in our definition of $\Delta_{\text{HS}}$. Except for these one-loop correction terms, $\Delta_{\text{HS}}$ then reduces to $\Delta_{\text{EW}}$.1

In defining $\Delta_{\text{HS}}$ as above, we have taken the view that the high-scale parameters as well as the scale at which we assume the effective theory to be valid are independent. In the absence of an underlying theory of the origin of these parameters, we regard cancellations between terms in Eq. (2.7) that occur for ad hoc relations5 between model parameters and lead one to conclude that $M_Z$ is not fine-tuned as fortuitous and do not incorporate it into our definition of high-scale fine-tuning. We emphasize that we would view the fine-tuning question very differently if indeed the high-scale parameters were all related from an underlying

3Unlike for $\Delta_{\text{EW}}$ where we have separated the contributions by particles (and treated these as independent) for the electroweak scale theory, in a constrained high-scale model, these would not be independent. Instead, we could separate out contributions that have independent origins in the high-scale model. For instance, for the mSUGRA model we should separately require contributions from gauginos, scalars and $A$ parameters to $\delta m_{\text{Higgs}}^2$ to be small. We have not done so here mainly for expediency. In this sense if accidental cancellations reduce $\Delta_{\text{HS}}$ to very small values, this should be interpreted with care.

4Typically these authors use $\Delta = 93.15 \text{ GeV}/m_i$ (where $a_i$ labels the input parameters) as a measure of the sensitivity to parameters [11]. This prescription agrees with our $\Delta$ at tree level, but differs when loop corrections are included.

5It may be argued that such an analysis is helpful as a guide to model builders attempting to construct models of natural SUSY.

meta-theory.6 In that case, though, as we just mentioned, $\Delta_{\text{EW}}$ would be an adequate measure of fine-tuning.

### III. RESULTS IN $m_0$ VS $m_{1/2}$ PLANE

We present our first results as contours of $\Delta_{\text{HS}}$ and $\Delta_{\text{EW}}$ in the $m_0$ vs $m_{1/2}$ plane of the mSUGRA/CMSSM model. For all plots, we take $m_t = 173.2$ GeV and we generate SUSY particle mass spectra from Isasugra v7.83 [23]. In Fig. 2, we show contours of $\Delta_{\text{HS}}$ in frame a) and for $\Delta_{\text{EW}}$ in frame b). For both frames, we take $A_0 = 0$, $\tan \beta = 10$ and $\mu > 0$. The gray-shaded regions running from the extreme left of the plot, across the bottom and on to the right are excluded by either a $\tilde{t}_1$ as LSP (left side), LEP1 constraints (bottom) or lack of appropriate EWSB (right side). The region marked LEP2 is excluded by LEP2 chargino searches ($m_{\tilde{\chi}} > 103.5$ GeV) [24]. The region below the contour labeled LHC7 is excluded by lack of a SUSY signal from SUSY searches at LHC7 with 5 fb$^{-1}$ of data [8,9]. The dashed portion of the contour is our extrapolation of LHC7 results to higher values of $m_0$ than are shown by the Atlas/CMS collaborations. We also denote regions where the calculated [25] branching fraction $B_s \to \mu^+ \mu^-$ falls outside its newly measured range from LHCb observations [26], which now require

$$2 \times 10^{-9} < BF(B_s \to \mu^+ \mu^-) < 4.7 \times 10^{-9} \ (95\% \ CL).$$

(3.1)

However, for the low value of $\tan \beta$ in this figure (and also in subsequent figures with $\tan \beta = 10$), the LHCb does not lead to any constraint because the SUSY contribution, which grows rapidly with $\tan \beta$, is rather small. The green-shaded region is where the thermally-generated relic density of neutralinos (computed using ISARED [27]) satisfies $\Omega^N_\chi h^2 < 0.1194$, the $2\sigma$ upper limit on the density of cold dark matter obtained by the WMAP collaboration [28]. This region encompasses the stau-coannihilation strip [29] (extreme left), the bulk region [21] (bottom left corner) and the well-known focus point/hyperbolic branch region [19] of the model. The shaded region labeled $a_\mu$ is where the measured muon magnetic moment [30] satisfies $4.7 \times 10^{-10} \leq a_\mu \leq 52.7 \times 10^{-10}$, within 3$\sigma$ of its theoretical value [31]. For $A_0 = 0$ adopted in this figure, $m_t < 123$ GeV over the entire parameter plane, so that mSUGRA is excluded for $A_0 \sim 0$ (as noted in Ref. [6]) unless one has very high values of $m_0$ and $m_{1/2}$ [32].

As might be anticipated, $\Delta_{\text{HS}}$ grows with increasing values of $m_0$ or $m_{1/2}$, so that we expect contours of fixed $\Delta_{\text{HS}}$ to be oval shaped in the $m_0$ vs $m_{1/2}$ plane. This is readily seen in frame a) of Fig. 2, except that because the oval is extremely elongated since the scales on the two axes

6This situation seems to occur in the so-called mixed-modulus-anomaly mediated SUSY breaking models for some ranges of the mixing parameter $\alpha$ as emphasized in Ref. [22].
are very different, we only see a small part of this contour (which appears as nearly vertical lines) for very large values of $\Delta_{\text{HS}}$. We have checked that $\Delta_{\text{HS}} < 150$ is already excluded by LHC searches, so high-scale fine-tuning of less than a percent is now mandatory for $A_0 = 0$. If we take the high-scale origin of the mSUGRA model seriously, we see that without a theory that posits special relations between the parameters that could lead to automatic cancellation of the large logarithms that enter $\Delta_{\text{HS}}$, we are forced to conclude that LHC data imply that the theory is fine-tuned to a fraction of a percent. For the portion of the plane compatible with LHC constraints on sparticles, the smallest values of $\Delta_{\text{HS}}$ occur where $m_0$ and $m_{1/2}$ are simultaneously small. As $m_0$ moves to the multi-TeV scale, $\Delta_{\text{HS}}$ exceeds 1000, and fine-tuning of more than part per mille is required.

In frame (b) of the figure, we show contours of constant $\Delta_{\text{EW}}$. Over most of the plane, these contours tend to track contours of constant $\mu^2$ since $M_2^2/2 \sim -m_{H_u}^2 - \mu^2$ so that when $|m_{H_u}^2| \gg M_2^2/2$, then $-m_{H_u}^2 \sim \mu^2$. Thus, along the contours of $\Delta_{\text{EW}}$, the value of $m_{H_u}^2$ is independent of $m_0$ at least until the contours turn around at large values of $m_0$ and $m_{1/2}$. This is just the focus point behavior discussed in the second paper of Ref. [19].\footnote{More precisely, the discussion in this paper was for a fixed value of $m_{1/2}$ so that the range of $m_0$ was limited because we hit the theoretically excluded region. We see though that the same value of $m_{H_u}$ can be obtained if we simultaneously increase $m_0$ and $m_{1/2}$ so that we remain in the theoretically allowed region.} The $\Delta_{\text{EW}}$ contours, for large values of $m_0$ bend over and track excluded region on the right where $\mu^2$ becomes negative. This is the celebrated hyperbolic branch [19] of small $|\mu|$. The contours of $\Delta_{\text{EW}}$ then bend around for very large values of $m_0$ because $\Sigma^\mu_\nu$ contributions, especially from $\tilde{g}$ loops, increase with $m_0$ and begin to exceed $-m_{H_u}^2 \approx \mu^2$. Indeed, Fig. 2(b) shows that there is a region close to (but somewhat removed from) the “no EWSB” region on the right where $\Delta_{\text{EW}}$ becomes anomalously small even for large values of $m_0$ and $m_{1/2}$. It is instructive to see that while this low EWFT region is close to the relic-density consistent region with small $\mu$ [19], it is still separated from it.\footnote{Much of the literature treats these regions as one. While this is fine for some purposes, it seems necessary to be clear on the difference when discussing either dark matter or EWFT. Note that $\Delta_{\text{HS}}$ is large in both regions.} While $\Delta_{\text{EW}} \sim 100$ is excluded at low $m_0$, this 1% EWFT contour, even with the resolution of our scan, extends out to very large $m_0 \sim 6$ TeV values for $m_{1/2}$ as high as 1 TeV! While these plots show that relatively low EWFT ($\Delta_{\text{EW}}$ of a few tens) is still allowed by LHC7 constraints on sparticles, it is important to realize that these planes are now excluded since they cannot accommodate $m_h \sim 125$ GeV.

Before moving on to other planes, we remark that for the smallest values of $m_0$ in the LHC-allowed regions of the figure, $\Delta_{\text{HS}} \sim \Delta_{\text{EW}}$. As we have explained, $\Delta_{\text{HS}}$ is determined by the value of $|\delta m_{H_u}^2|$ [see Eq. (2.5)], which for $m_0 \sim 0$ is just $|m_{H_u}^2|$ that determined $\Delta_{\text{EW}}$ when $m_0$ is very
We thus see that the two measures are roughly comparable for small values of $m_0$ but deviate from one another as $m_0$ is increased. We see that $\Delta_{\mathrm{HS}}$ typically exceeds $\Delta_{\mathrm{EW}}$ by an order of magnitude, because of the large logarithm of the ratio of the GUT and weak scales, except in the HB/FP region where $\Delta_{\mathrm{EW}}$ is exceptionally small.

In Fig. 3 we show the $m_0$ vs $m_{1/2}$ plane for $\tan\beta = 50$ and $A_0 = 0$. The contours in both frames are qualitatively very similar those for the $\tan\beta = 10$ case. As expected, regions of low $\Delta_{\mathrm{EW}}$ extend to very large $m_0$ and $m_{1/2}$ in the HB region. One difference from the $\tan\beta = 10$ case discussed above is that this time the HB region largely overlaps with the relic-density-consistent green-shaded region. Note also that for this large value of $\tan\beta$ there is a considerable region (left of the LHCb contour) that is now excluded due to too large a value of $\text{BF}(B_s \to \mu^+ \mu^-)$. Again, the entire region of plane shown is excluded by the LHC Higgs discovery at 125 GeV.

In Fig. 4, we show contours of $\Delta_{\mathrm{HS}}$ and $\Delta_{\mathrm{EW}}$ for $\tan\beta = 10$ and $A_0 = -m_0$. The first thing to notice is that the HB/FP region does not appear. The region at extremely large $m_0$ is still theoretically excluded, but...
more typically because $m_A^2$ turns negative (or there are tachyons) not because $\mu^2$ turns negative.\(^9\) In addition, the very large $m_0 \approx 7\text{–}9 \text{ TeV}$ region yields a value of $m_h > 123 \text{ GeV}$; thus, the bulk of this plane is still excluded. The contours of $\Delta_{\text{HS}}$ are qualitatively similar to the $A_0 = 0$ cases, and LHC7 still excludes $\Delta_{\text{HS}} < 100$, so again a HSFT of more than 1% is required. In the region with $m_h > 123 \text{ GeV}$, $\Delta_{\text{HS}} \approx 1.5 \times 10^4$, and extreme HSFT is required. Moving to frame (b), we note that though the contours of fixed $\Delta_{\text{EW}}$ now run from top left to lower right, these still follow the lines of fixed values of $\mu^2$. Moreover, values of $\Delta_{\text{EW}}$ below 100 are excluded by just the LHC7 sparticle mass constraints. If one also imposes $m_0 > 123 \text{ GeV}$, then $\Delta_{\text{EW}} \approx 2000$ is required over the entire plane shown.

In Fig. 5, we show the $m_0$ vs $m_{1/2}$ plane for $A_0 = -m_0$ but with $\tan \beta = 50$. We see this is qualitatively very similar to the previous figure aside from the sizeable LHCb excluded region on the low $m_0$ portion of the plane. Again the theoretically excluded region occurs at values of $m_0$ far beyond the range shown. Note though that the contour of $m_h = 123 \text{ GeV}$ has moved to slightly lower $m_0$ values. Still, requiring $m_h > 123 \text{ GeV}$ requires $\Delta_{\text{HS}} > 5 \times 10^3$, and $\Delta_{\text{EW}} \approx 700$.

\(^9\)For $m_{1/2} = 500 \text{ GeV}$, this happens for $m_0 \approx 22 \text{ TeV}$. We mention that this breakdown of parameter space could be an artifact of the ISAJET algorithm for computing the sparticle mass spectrum in mSUGRA. An approximate tree-level spectrum is first required in order to evaluate the radiative corrections that can potentially yield a valid solution using an iterative procedure. But in the absence of a nontachyonic, tree-level spectrum with the correct EWSB pattern, the program is unable to compute the radiatively corrected mass spectrum.

According to Ref. [6], large mixing in the top squark sector and consequently the largest values of $m_h$ occur in mSUGRA for $A_0 \sim -2m_0$. In Fig. 6, we show contours of $\Delta_{\text{HS}}$ and $\Delta_{\text{EW}}$ for $\tan \beta = 10$ and $A_0 = -2m_0$. We note again that the HB/FP region does not appear in this plane. Notice also that the contours of $m_h = 123 \text{ GeV}$ have moved all the way down to $m_0 \sim 2 \text{ TeV}$; thus, now much of the mSUGRA plane shown is allowed by the LHC Higgs-like resonance discovery. In fact, the portion of the plane with $m_0 \approx 6\text{–}8 \text{ TeV}$ gives too large a value of $m_h > 127 \text{ GeV}$. The portion of the $m_0$ vs $m_{1/2}$ plane allowed by both LHC7 sparticle searches and by having $m_h \sim 123\text{–}127 \text{ GeV}$ requires $\Delta_{\text{HS}} \sim 10^3\text{–}10^4$, or $0.1$ -- 0.01% HSFT. The EWFT required is $\Delta_{\text{EW}} \approx 10^3$, also large. The lesson learned here is that the remaining mSUGRA regions with $m_h \sim 123\text{–}127 \text{ GeV}$, and which obey sparticle mass constraints, are highly fine-tuned, even with the less restrictive EWFT measure.

Figure 7 shows the mSUGRA plane for $A_0 = -2m_0$ but with $\tan \beta = 50$. In this case, large theoretically excluded parameter regions appear and these only grow larger until the entire parameter space collapses for even higher $\tan \beta \sim 55\text{–}60$ [33]. The region on the right is forbidden because $m_h^2$ turns negative, not because $|\mu|$ becomes small: this is why there is no DM-allowed region for large values of $m_0$. The low $m_{1/2}$ and low $m_0$ portions of the plane marked LHCb are excluded due to too large a $B_i \rightarrow \mu^+ \mu^-$ branching fraction. The $m_h = 123 \text{ GeV}$ contour nearly coincides with $\Delta_{\text{HS}} = 10^3$ and $\Delta_{\text{EW}} = 500$. In this case, values of $m_0 \approx 6 \text{ TeV}$ are excluded as giving rise to too heavy a value of $m_h$. Thus, again the regions with $m_h \sim 123\text{–}127 \text{ GeV}$ and obeying LHC7 sparticle search constraints, are highly fine-tuned.
Before closing this section, we digress to compare our results for the EWFT measure with some results in the recent literature [15] for the fine-tuning within the mSUGRA/CMSSM model calculated using the procedure described at the end of Sec. II B. We have already argued at the end of that section that the fine-tuning measure that results from substituting $\mu^2$ in Eq. (2.7) and the analogous expression for $\mu^2$ into Eq. (2.3) should match our EWFT measure. To check this, we have compared our results in Fig. 2(b) to those in Fig. 1 of the first paper of Ref. [15]. There, these authors show the minimum value of their fine-tuning parameter $\Delta$ in the $m_0 - m_{1/2}$ plane, marginalizing over a range of $A_0$ and $\tan\beta$. We see that the shapes of their $\Delta$ contours are qualitatively similar (except in the large $m_0$ region where the contours turn around because radiative correction effects are important) to those of the contours in frame (b) of Figs. 2 and 3. We use our $A_0 = 0$ figures for this comparison because of all the figures these have the smallest value of $\Delta_{EW}$. We have also checked that for any chosen value of $m_0$ and $m_{1/2}$, $\Delta$ of Antusch et al. has a magnitude similar to (but never larger than) the corresponding lowest $\Delta_{EW}$ that we obtain for any choice of $A_0$ and $\tan\beta$.

FIG. 6 (color online). Contours of (a) $\Delta_{HS}$ and (b) $\Delta_{EW}$ in the mSUGRA model with $A_0 = -2m_0$, $\tan\beta = 10$ and $\mu > 0$. The value of $m_t$ as well as the various shaded/colored regions are as in Fig. 2.

FIG. 7 (color online). Contours of (a) $\Delta_{HS}$ and (b) $\Delta_{EW}$ in the mSUGRA model with $A_0 = -2m_0$, $\tan\beta = 50$ and $\mu > 0$. The value of $m_t$ as well as the various shaded/colored regions are as in Fig. 3.
IV. SCAN OVER MSUGRA PARAMETER SPACE

While the results of the previous section provide an overview of both the EWFT and the HSFT measures in light of LHC7 and LHC8 constraints on sparticle and Higgs boson masses, we only presented results for particular choices of $A_0$ and $\tan \beta$, and for $\mu > 0$. In this section, we present results from a scan over the complete mSUGRA parameter space with the following range of model parameters:

\[
\begin{align*}
m_0: & \quad 0\text{--}15 \text{ TeV}, \\
m_{1/2}: & \quad 0\text{--}2 \text{ TeV}, \\
-2.5 < A_0/m_0 < 2.5, & \\
\tan \beta: & \quad 3\text{--}60.0.
\end{align*}
\]

We will show results for both $\mu > 0$ and $\mu < 0$. For each solution generated, we require that

1. Electroweak symmetry be radiatively broken (REWSB),
2. The neutralino $\tilde{\chi}_1$ is the lightest MSSM particle,
3. The light chargino mass obeys the LEP2 limit that $m_{\tilde{\chi}_1} > 103.5$ GeV [24],
4. $m_h = 125 \pm 2$ GeV, in accord with the recent Higgs-like resonance discovery at LHC [1,2],
5. The calculated value of $\text{BF}(B_s \to \mu^+ \mu^-)$ lie within $(2\text{--}4.7) \times 10^{-9}$ in accord with recent LHCb measurements [26] and
6. The mass spectra obey LHC7 sparticle mass constraints for the mSUGRA model [8,9].

Our first results are shown in Fig. 8 for (a) $\Delta_{\text{HS}}$ and (b) $\Delta_{\text{EW}}$ vs $m_0$. Solutions with $\mu < 0$ are shown as blue circles while solutions with $\mu > 0$ are shown in red crosses. Note that here, and in subsequent figures, there are many points for $\mu < 0$ (blue circles) that are not visible as these are covered by the red crosses for $\mu > 0$. In frame (a), we see that $\Delta_{\text{HS}}$ values occupy a rather narrow band which increases monotonically with $m_0$. Values of $m_0 \lesssim 1$ TeV are excluded by the requirement $m_h > 123$ GeV. The $\mu > 0$ and $\mu < 0$ solutions occupy essentially the same band. This is not surprising because the large logarithms are essentially independent of the sign of $\mu$. The minimum allowed value of $\Delta_{\text{HS}}$ is $\sim 1000$, so that at least 0.1% fine-tuning is required of all remaining mSUGRA solutions. The minimum for $\Delta_{\text{HS}}$ occurs at $m_0 \sim 1500$ GeV. This minimal $\Delta_{\text{HS}}$ solution is shown as a benchmark point in Sec. V. For $m_0$ as high as 15 TeV, $\Delta_{\text{HS}}$ increases to nearly $10^5$. In frame (b), we show $\Delta_{\text{EW}}$ vs $m_0$. Here, the shape of the allowed region is very different from the $\Delta_{\text{HS}}$ case in frame (a). Low values of $m_0$ can give $m_h > 123$ GeV only if $|A_0/m_0|$ is sizeable and, as we have already seen, yield $\Delta_{\text{EW}}$ of at least several hundred. Smaller values of $\Delta_{\text{EW}}$ are obtained only in the HB/FP region where $m_0$ is large. In other words, in the “hole region” in frame (b), we have $m_h < 123$ GeV. The point with the minimum value of $\Delta_{\text{EW}} \sim 100$ occurs at $m_0 \sim 7900$ GeV, and is shown as the electroweak benchmark point in the next section. Over the remaining mSUGRA parameter space, at best 1% EWFT is required.

In Fig. 9, we show the distributions of $\Delta_{\text{HS}}$ and $\Delta_{\text{EW}}$ vs $m_{1/2}$. The sharp edge on the left is a reflection of the lower limit on $m_{1/2}$ from LHC7 searches. In frame (a) for $\Delta_{\text{HS}}$, we see that the minimal $\Delta_{\text{HS}}$ is spread across a wide spectrum of $m_{1/2}$ values. This is consistent with the behavior of $\Delta_{\text{HS}}$ shown in Fig. 2(a) where the HSFT contours are nearly vertical, indicating little dependence on $m_{1/2}$. In frame (b), the minimal values of $\Delta_{\text{EW}}$ are also spread across the $m_{1/2}$ range. For both $\Delta_{\text{HS}}$ and $\Delta_{\text{EW}}$, there may be a slight preference for lower $m_{1/2}$ values.

FIG. 8 (color online). Fine-tuning measures $\Delta_{\text{HS}}$ and $\Delta_{\text{EW}}$ vs $m_0$ from a scan over mSUGRA/CMSSM parameter space for $\mu > 0$ (red crosses) and $\mu < 0$ (blue circles). We take $m_t = 173.2$ GeV.
In Fig. 10 we show how $\Delta_{HS}$ and $\Delta_{EW}$ are distributed vs $A_0 = m_0$. In frame (a), we see that minimal $\Delta_{HS} \sim 1000$ is obtained for $A_0/m_0 \sim -2$, which is also the vicinity of where $m_h$ is maximal for given $m_0$ and $m_{1/2}$ values. There is also a minimum at $A_0/m_0 \sim 2.5$, with $\Delta_{HS}$ reaching only to $\sim 3000$.\(^{10}\) In frame (b), the value of $\Delta_{EW}$ is even more correlated with $A_0/m_0$. For $|A_0/m_0| \leq 1$, $\Delta_{EW}$ tends to be smaller than for larger values of $|A_0/m_0|$. The solutions with the least EWFT occur at $A_0/m_0 \sim \pm 0.6$, with the minimal $\Delta_{EW} \sim 100$. Once again, this occurs in the HB/FP region mentioned above. For larger magnitudes of $A_0/m_0$, the HB/FP region is absent, and $\Delta_{EW}$ is much larger. The gap in the plots around $A_0/m_0 \sim 0$ occur because it is nearly impossible to generate $m_h$ as heavy as 123–127 GeV for such low values of trilinear couplings [6].

In Fig. 11, we plot $\Delta_{HS}$ and $\Delta_{EW}$ vs $\tan \beta$. The minimal $\Delta_{HS}$ and $\Delta_{EW}$ solutions are spread uniformly across a range of $\tan \beta$ values. At very low $\tan \beta \approx 10$ values, it is difficult to generate solutions with $m_h \approx 123$ GeV unless mSUGRA parameters are extremely large, leading to high fine-tuning.

\(^{10}\)The asymmetry of the minimum of $\Delta_{HS}$ with respect to the sign of $A_0$ may only be a reflection of the fact that it is more difficult to generate large values of $m_h$ for positive values of $A_0$.
V. LOWEST FINE-TUNING MSUGRA BENCHMARKS

What is apparent from our results so far is that, after imposing LHC7 sparticle mass constraints and requiring that \(m_{h} = 125 \pm 2\) GeV on the mSUGRA/CMSSM model, the viable solutions are fine-tuned to at least 1% even with the less stringent EWFT measure. With a fine-tuning measure that knows about the high-scale origin of mSUGRA parameters, the required fine-tuning is increased by an order of magnitude. Nonetheless, our understanding of how SUSY breaking parameters arise is extremely limited and it remains possible that nature may appear fine-tuned to a certain degree. With this in mind, we exhibit and qualitatively examine the features of the lowest \(\Delta_{\text{HS}}\) and the lowest \(\Delta_{\text{EW}}\) solutions in the mSUGRA/CMSSM framework. These are listed in Table 1 as solutions HS1 and EW1.

Solution HS1 has \(\Delta_{\text{HS}} = 1100\) and so requires \(\sim 0.1\%\) fine-tuning. The EWFT parameter \(\Delta_{\text{EW}} \approx 600\), requiring \(\sim 0.2\%\) fine-tuning. HS1 has \(m_{0} \approx 1500\) GeV, lying at the lower edge of the band of solutions shown in Fig. 8. With \(m_{\tilde{g}} \approx 1660\) GeV, and \(m_{\tilde{q}} \approx 2000\) GeV, this solution lies beyond the reach of LHC8 searches with up to 30 fb\(^{-1}\) [34], but should be accessible to LHC14 searches with \(\sim 10\)–20 fb\(^{-1}\) [35]. The relatively light top squarks allow for \(\tilde{g} \to t\tilde{t}_{1}\) decay at \(\sim 100\%\), followed by \(\tilde{t}_{1} \to t\tilde{Z}_{1}\). Thus, gluino pair production will give rise to \(t\tilde{t}\) + \(E_{\text{T}}^\text{miss}\) events at LHC and may be searchable even in the multijet plus \(E_{T}^\text{miss}\) channel[36]. First-generation squark pair production and corresponding \(\tilde{q}\tilde{g}\) production will augment this rate since typically \(\tilde{q} \to q\tilde{g}\) for first and second generation squarks. Production of second and third generation squarks will be suppressed by parton distribution functions. The HS1 solution has \(\Omega_{\tilde{Z}_{1}}^\text{th} h^{2} \approx 12\), so would produce too many neutralinos in the early universe under the standard cosmology. Late time entropy production [37] or neutralino decay to a lighter state, e.g., \(\gamma + \text{axino}\) in extended models [38], can bring such a model into accord with the measured relic abundance. The \(b \to s\gamma\) branching fraction is somewhat below measured values, although additional flavor-violating Lagrangian soft terms could bring this value into accord with measurements without affecting LHC phenomenology.

The solution EW1 has \(\Delta_{\text{HS}} \approx 1.5 \times 10^{4}\), but \(\Delta_{\text{EW}} \approx 100\) so that the latter requires EWFT at the 1% level. The reader may wonder whether it makes sense to talk about low values of \(\Delta_{\text{EW}}\) when \(\Delta_{\text{HS}}\) is so much larger. In this connection, it may be worth allowing for the possibility that the mSUGRA framework may itself one day be derived from an underlying theory along with specific relations between seemingly unrelated mSUGRA parameters that lead to cancellations of the terms containing the large logarithms, as discussed at the end of Sec. II B.

Returning to the EW1 point in the Table with \(m_{\tilde{g}} \approx 1600\) GeV and \(m_{\tilde{q}} \approx 6\)–8 TeV, we see that this model is only accessible to LHC14 searches with \(\sim 50\)–100 fb\(^{-1}\) of integrated luminosity [35]. In this case, gluino pair production would be followed by gluino three-body decays to multijet plus multilepton plus \(E_{T}^\text{miss}\) final states. The final states would be rich in \(W\) and \(Z\) bosons, leading to distinctive signatures [39]. The thermally produced neutralino abundance \(\Omega_{\tilde{\chi}_{1}}^\text{th} h^{2} \approx 10\), so again a nonstandard cosmology as well as an extension of the spectrum is needed to bring this solution in accord with the measured dark matter density.

Both HS1 and EW1 points will need yet other new physics to bring them in accord with the E821 measurement [30] of the muon magnetic moment if this discrepancy continues to hold up.
TABLE I. Input parameters and masses in GeV units for the two mSUGRA/CMSSM benchmark points with the lowest values of $\Delta_{\text{HS}}$ and $\Delta_{\text{EW}}$ after imposing $m_0 = 125 \pm 2$ GeV and also the LHC7 sparticle mass bounds. We take $m_t = 173.2$ GeV.

<table>
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<th>Parameter</th>
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<th>EW1</th>
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<td>$m_{1/2}$</td>
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<td>6682.5</td>
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| $\Omega_{\tilde{Z}_1} h^2$ | 12.3 | 9.4 |
| $\text{BF}(b \to s \gamma) \times 10^4$ | 2.7 | 3.1 |
| $\text{BF}(B_d \to \mu^+ \mu^-) \times 10^9$ | 4.4 | 3.8 |
| $\sigma^{\text{SI}}(Z_p) \,(\text{pb})$ | $1.4 \times 10^{-11}$ | $1.6 \times 10^{-10}$ |
| $\Delta_{\text{HS}}$ | 1105 | $1.5 \times 10^4$ |
| $\Delta_{\text{EW}}$ | 582.9 | 92.4 |

VI. CONCLUDING REMARKS

The recent discovery of a 125 GeV Higgs-like resonance at LHC has set a strong new constraint on supersymmetric models. In addition, the lack of evidence for a SUSY signal at LHC now requires masses of strongly interacting sparticles in models such as mSUGRA/CMSSM to be above the 1 TeV scale. If LHC searches for sparticles continue without a new physics signal, then the little hierarchy problem—how to reconcile the Z and Higgs boson mass scale with the scale of SUSY breaking—will become increasingly acute in models such as mSUGRA.11

11We do not note that the little hierarchy problem may be solved within the context of the MSSM if we go to nonuniversal SUGRA models: see e.g., Refs. [14,40]. Alternatively, invoking extra singlets [41] or extra vectorlike matter [42] may provide additional contributions to $m_0$ while maintaining light top squarks which seem to be required for low electronegative fine-tuning.

In this paper, we have reported on results from the calculation of two measures of fine-tuning in the mSUGRA/CMSSM model. The first—$\Delta_{\text{HS}}$—which includes information about the high-scale origin of mSUGRA parameters—is the more stringent one. The second, $\Delta_{\text{EW}}$, depends only on the physical spectrum and couplings, and so is universal to all models that yield the same weak scale Lagrangian. Our results incorporate the latest constraints from LHC7 sparticle searches along with a light Higgs scalar with $m_h \sim 123–127$ GeV. We find $\Delta_{\text{HS}} \geq 10^3$, or at best 0.1% fine-tuning. The more model-independent EWFT gives a $\Delta_{\text{EW}} \geq 10^2$, or at best 1% fine-tuning. The minimum value of $\Delta_{\text{EW}}$ tends to occur near the FP region which extends to large values of $m_0$ and $m_{1/2}$ but which does not always overlap with the neutralino relic density allowed HB region. We will leave it to the reader to assess how much fine-tuning is too much, and also how much credence one should give to $\Delta_{\text{HS}}$ in light of our ignorance of physics at or around the GUT scale.12

From a scan over the entire mSUGRA/CMSSM parameter space including LHC sparticle and Higgs mass constraints, we do find viable regions where EWFT is at the 1% level, even for gluino and squark masses well beyond LHC reach. These regions are characterized by $m_0 \sim 8$ TeV and $A_0 \sim 0.6 m_0$. Since these points are spread across a wide range of $m_{1/2}$ values ranging up to and perhaps beyond 2 TeV, it appears that regions of parameter space with EWFT at the 0.5–1% levels (but with very large values of $\Delta_{\text{HS}}$) will persist even after the most ambitious LHC SUSY searches are completed.

To conclude, we remind the reader it was the realization that SUSY can solve the big hierarchy problem which provided the rationale for low scale SUSY. This remains unaltered by LHC and Higgs mass constraints. The underlying hope was that with sparticles close to the weak scale, there would be no hierarchy problem. The data seem to indicate that, at least in the mSUGRA framework, EWFT at the percent level is mandatory. It is difficult to say whether these considerations point to the failure of the mSUGRA model, or whether the little hierarchy is the result of an incomplete understanding of how soft supersymmetry breaking parameters arise. While we continue to regard models with low EWFT as especially interesting, it appears difficult to unilaterally discard SUSY models that are fine-tuned at a fraction of a percent or a part per mille, given that these provide the solution of the much more pressing big hierarchy problem. Our results provide a quantitative measure for ascertaining whether or not the remaining mSUGRA/CMSSM model parameter space is excessively fine-tuned, and so could provide impetus for considering alternative SUSY models.

12Of course, if we take the mSUGRA model to be the final high-scale theory, we would no doubt take $\Delta_{\text{HS}}$ to be our fine-tuning measure, but the judgement to be made is whether one should treat mSUGRA in this manner.
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