Combination of $t\bar{t}$ cross section measurements and constraints on the mass of the top quark and its decays into charged Higgs bosons

V. M. Abazov
*Joint Institute for Nuclear Research, Dubna, Russia*

Kenneth A. Bloom
*University of Nebraska - Lincoln, kbloom2@unl.edu*

Gregory Snow
*University of Nebraska - Lincoln, gsnow1@unl.edu*

D0 Collaboration

Follow this and additional works at: [http://digitalcommons.unl.edu/physicsbloom](http://digitalcommons.unl.edu/physicsbloom)

Part of the [Physics Commons](http://digitalcommons.unl.edu/physicsbloom)

Abazov, V. M.; Bloom, Kenneth A.; Snow, Gregory; and Collaboration, D0, "Combination of $t\bar{t}$ cross section measurements and constraints on the mass of the top quark and its decays into charged Higgs bosons" (2009). *Kenneth Bloom Publications*. 287.

[http://digitalcommons.unl.edu/physicsbloom/287](http://digitalcommons.unl.edu/physicsbloom/287)
Combination of $t\bar{t}$ cross section measurements and constraints on the mass of the top quark and its decays into charged Higgs bosons


(The D0 Collaboration)

1 Universidad de Buenos Aires, Buenos Aires, Argentina
2 LAFEX, Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil
3 Universidade do Estado do Rio de Janeiro, Rio de Janeiro, Brazil
4 Universidade Federal do ABC, Santo André, Brazil
5 Instituto de Física Teórica, Universidade Estadual Paulista, São Paulo, Brazil
6 University of Alberta, Edmonton, Alberta, Canada; Simon Fraser University, Burnaby, British Columbia, Canada; York University, Toronto, Ontario, Canada; and McGill University, Montreal, Quebec, Canada
7 University of Science and Technology of China, Hefei, People’s Republic of China
8 Universidad de los Andes, Bogotá, Colombia
9 Center for Particle Physics, Charles University, Faculty of Sciences of the Czech Republic, Prague, Czech Republic
10 Czech Technical University in Prague, Prague, Czech Republic
11 Center for Particle Physics, Institute of Physics, Academy of Sciences of the Czech Republic, Prague, Czech Republic
12 Universidad San Francisco de Quito, Quito, Ecuador
13 LPC, Université Blaise Pascal, CNRS/IN2P3, Clermont, France
14 LPSC, , USAUniversité Joseph Fourier Grenoble I, CNRS/IN2P3, Institut National Polytechnique de Grenoble, Grenoble, France
15 CPPM, Aix-Marseille Université, CNRS/IN2P3, Marseille, France
16 LAL, , USAUniversité Paris-Sud, IN2P3/CNRS, Orsay, France
17 LPNHE, IN2P3/CNRS, , USAUniversités Paris VI et VII, Paris, France
18 CEA, Irfu, SPP, Saclay, France
19 IPHC, Université de Strasbourg, CNRS/IN2P3, Strasbourg, France
20 IPNL, Université Lyon 1, CNRS/IN2P3, Villeurbanne, France and Université de Lyon, Lyon, France
21 III. Physikalisches Institut A, RWTH Aachen University, Aachen, Germany
22 Physikalisches Institut, Universität Bonn, Bonn, Germany
23 Physikalisches Institut, Universität Freiburg, Freiburg, Germany
24 II. Physikalisches Institut, Georg-August-Universität Göttingen, Germany
25 Institut für Physik, Universität Mainz, Mainz, Germany
26 Ludwig-Maximilians-Universität München, München, Germany
27 Fachbereich Physik, University of Wuppertal, Wuppertal, Germany
28 Panjab University, Chandigarh, India
29 Delhi University, Delhi, India
30 Tata Institute of Fundamental Research, Mumbai, India
31 University College Dublin, Dublin, Ireland
32 Korea Detector Laboratory, Korea University, Seoul, Korea
33 SungKyunKwan University, Suwon, Korea
34 CINVESTAV, Mexico City, Mexico
35 FOM-Institute NIKHEF and University of Amsterdam/NIKHEF, Amsterdam, The Netherlands
36 Radboud University Nijmegen/NIKHEF, Nijmegen, The Netherlands
37 Joint Institute for Nuclear Research, Dubna, Russia
38 Institute for Theoretical and Experimental Physics, Moscow, Russia
We combine measurements of the top quark pair production cross section in $p\bar{p}$ collisions in the $\ell^+\ell^-$, $\ell\ell$, and $\tau\ell$ final states (where $\ell$ is an electron or muon) at a center of mass energy of $\sqrt{s} = 1.96$ TeV in 1 fb$^{-1}$ of data collected with the D0 detector. For a top quark mass of $170 \text{ GeV}/c^2$, we obtain $\sigma_{t\bar{t}} = 8.18^{+0.98}_{-0.87}$ pb in agreement with the theoretical prediction. Based on predictions from higher order quantum chromodynamics, we extract a mass for the top quark from the combined $t\bar{t}$ cross section, consistent with the world average of the top quark mass. In addition, the ratios of $t\bar{t}$ cross sections in different final states are used to set upper limits on the branching fractions $B(t \rightarrow H^+ b \rightarrow \tau^+ \nu b)$ and...
Precise measurements of the production and decay properties of the heaviest known fermion, the top quark, provide important tests of the standard model (SM) and offer a window for searches for new physics. In this paper we measure the top-antitop quark pair ($t\bar{t}$) production cross section and compare it with the SM prediction, extract the top quark pole mass from this measurement, and search for new physics in top quark decays analyzing ratios of the $t\bar{t}$ cross sections measured in different decay channels.

The inclusive $t\bar{t}$ production cross section ($\sigma_{t\bar{t}}$) is measured in different $t\bar{t}$ decay channels assuming SM branching fractions. The comparison of the results to predictions in next-to-leading order perturbative quantum chromodynamics (QCD), including higher order soft-gluon resummations [1–4], yields a direct test of the SM. Ratios of $\sigma_{t\bar{t}}$ measured in different final states are particularly sensitive to non-SM particles that may appear in top quark decays, especially if the boson in the decay is not a SM $W$ boson. An example is the decay into a charged Higgs boson ($t \rightarrow H^+ b$), which, as predicted in some models [5], can compete with the SM decay $t \rightarrow W^+ b$. Additionally, many experimental uncertainties cancel in the ratios. Furthermore, since $\sigma_{t\bar{t}}$ depends on the mass of the top quark ($m_t$), it can be used to extract $m_t$. Such a measurement is less accurate than direct mass measurements, but provides complementary information with different experimental and theoretical uncertainties.

Within the SM, each quark of the $t\bar{t}$ pair is expected to decay nearly 100% of the times into a $W$ boson and a $b$ quark [6]. $W$ bosons can decay hadronically into $q\bar{q}$ pairs or leptonically into $e\nu_e$, $\mu\nu_\mu$, and $\tau\nu_\tau$ with the $\tau$ in turn decaying into an electron, a muon, or hadrons, and associated neutrinos. If one of the $W$ bosons decays hadronically while the other one produces a direct electron or muon or a secondary electron or muon from $\tau$ decay, the final state is referred to as the $\ell +$ jets (or $\ell j$) channel. If both $W$ bosons decay leptonically, this leads to a dilepton final state containing a pair of electrons, a pair of muons, or an electron and a muon (the $\ell\ell$ channel), or a hadronically decaying tau accompanied either by an electron or a muon (the $\tau\ell$ channel).

Measurements of the individual $t\bar{t}$ cross sections in $\ell\ell$ and $\tau\ell$ channels using about 1 fb$^{-1}$ of $p\bar{p}$ data from the D0 detector at the Fermilab Tevatron collider at $\sqrt{s} = 1.96$ TeV are available in Ref. [7]. In the $\ell +$ jets channel, we use the same selection and background estimation as in Ref. [8], but a slightly larger data set and a unified treatment of systematic uncertainties with the $\ell\ell$ and $\tau\ell$ channels. We provide a brief summary of the event selection and analysis procedures below.

In each final state we select data samples enriched in $t\bar{t}$ events by requiring one or two isolated high transverse momentum ($p_T$) leptons for the $\ell +$ jets or $\ell\ell$ channel, respectively. At least two high $p_T$ jets are required for $\ell\ell$ and $\tau\ell$ events, and at least three for $\ell +$ jets events. Further, in all but the $e\mu$ channel, large transverse missing energy ($E_T$) is required to account for the large transverse momenta of neutrinos from $W$ boson or $\tau$ lepton decays. In the $e\mu$ final state, a requirement on the sum of the $p_T$ of the highest $p_T$ (leading) lepton and the two leading jets is imposed instead. In the $\mu\mu$ channel, the $E_T$ requirement is supplemented with a requirement on the significance of the $E_T$ measurement, estimated from the $p_T$ of muons and jets, and their expected resolutions. Additional criteria are applied on the invariant mass of the two opposite charge leptons of the same flavor in the $ee$ and $\mu\mu$ channels to reduce the dominant background from $Z/\gamma^* \rightarrow \ell^+ \ell^-$ events. In the $\ell +$ jets and $\tau\ell$ channels we require a minimum azimuthal angle separation between the $E_T$ vector and the lepton $p_T$, $\Delta\phi(\ell, E_T)$, to reduce background from multijet events, where jets are misidentified as electron, muon, or $\tau$. Details of lepton, jet, and $E_T$ identification are provided in Refs. [9,10]. The final selection in these channels demands at least one identified $b$ jet via a neural-network based algorithm [11]. In the $\ell +$ jets channel we separate events with one or $\geq 2b$-tagged jets due to their different signal over background ratio and systematic uncertainties.

To simplify the combination and extraction of cross section ratios, all channels are constructed to be mutually exclusive. In particular, events with two identified leptons are excluded from the $\ell +$ jets selection, and all $\tau\ell$ candidates are removed from the rest of the channels.

The compositions of the samples in the $\ell +$ jets, $\ell\ell$, and $\tau\ell$ channels are shown in Table I. $W +$ jets production dominates the background for the $\ell +$ jets events, while multijet production is the most important background in the $\tau\ell$ channel. Background in the $\ell\ell$ channels comes mainly from $Z +$ jets production. In the $\ell\ell$ channel, contributions from $W +$ jets production are part of the multijet background. The smaller contribution from diboson production is included in the category labeled “other background.” This category also includes the contribution from single top quark production in the $\ell +$ jets and $\tau\ell$ channels. The signal, $W +$ jets and $Z +$ jets backgrounds are simulated using ALPGEN [12] for the matrix element calculation and PYTHIA [13] for parton showering and hadronization. Diboson and single top backgrounds are simulated with the PYTHIA and SINGLETOP [14] generators, respectively. We estimate the multijet background from the control data samples. The difference in the ratio of $t\bar{t}$ and $W +$ jets events in the $e +$ jets and $\mu +$ jets final states is the result of the larger efficiency and misidentified lepton rate in the $e +$ jets channel compensating for the lower
lepton acceptance (|η| < 1.1) compared to the μ + jets channel (|η| < 2.0). In addition, the wider rapidity distribution of the W + jets events compared to tt events increases the W + jets background contribution in the μ + jets channel.

To calculate the combined cross section, we define a joint likelihood function as the product of Poisson probabilities for the 14 disjoint subsamples, as listed in Table I. Fourteen additional Poisson terms constrain the multijet background in the ℓ + jets and τ ℓ channels. In particular, for the ℓe and τμ channels, the multijet background is determined by counting events with an electron or muon and associated τ of the same electric charge, introducing a corresponding Poisson term per channel. In the ℓ + jets channel, we estimate the multijet background separately for each of the eight subchannels by using corresponding control data samples [15]. Four additional terms arise from applying this same method in evaluating the multijet background before b tagging.

Each systematic uncertainty is included in the likelihood function through one free “nuisance” parameter [15]. Each of these parameters is represented by a Gaussian probability density function with zero mean and a standard deviation of one; all are allowed to float in the maximization of the likelihood function, thereby changing the central value of the measured σ_{μ}. Correlations are taken into account by using the same nuisance parameter for a common source of systematic uncertainty in different channels scaled by the corresponding standard deviation (SD) of each individual channel. Thus, the likelihood function to be maximized is represented by the product

$$ \mathcal{L} = \prod_{i=1}^{14} \mathcal{P}(n_i, m_i) \times \prod_{j=1}^{14} \mathcal{P}(n_j, m_j) \times \prod_{k=1}^{K} \text{SD}_{ik} \times G(\nu_i; 0, 1), $$

where \( \mathcal{P}(n, m) \) is the Poisson probability to observe \( n \) events given the expectation of \( m \) events. The predicted number of events in each channel is the sum of the predicted background and expected tt events, which depends on σ_{μ}. In the product, \( i \) runs over the subsamples and \( j \) runs over the multijet background subsamples. The Gaussian distributions SD_{ik} \times G(\nu_i; 0, 1) describe the systematic uncertainties, \( K \) is the total number of independent sources of systematic uncertainty, \( \nu_i \) are the individual nuisance parameters, and SD_{ik} is 1 standard deviation for the source of uncertainty \( k \) in subsample \( i \).

Systematic uncertainties on the measured σ_{μ} are evaluated from sources that include electron and muon identification; τ and jet identification and energy calibration; b-jet identification; modeling of triggers; and background and integrated luminosity. All uncertainties are treated as fully correlated among channels and between signal and background. Systematic uncertainties arising from limited statistics of data or Monte Carlo samples used in estimating signal or backgrounds are considered to be uncorrelated. A detailed discussion on systematic uncertainties can be found in Refs. [7,8]. Table II shows a breakdown of uncertainties on the combined cross section. We evaluate the effect from each source by setting all uncertainties to zero except the one in question and redoing the likelihood maximization with respect to only the corresponding nuisance parameter. Since the method allows each uncertainty to change the central value, the total uncertainty on σ_{μ} differs slightly from the quadratic sum of the statistical and individual systematic uncertainties. The total systematic uncertainty on σ_{μ} exceeds the statistical contribution. The luminosity uncertainty of 6.1% which enters into the estimation of the majority of the backgrounds and the luminosity measurement of the selected samples is the dominant source of systematic uncertainty.
Table III summarizes the individual $\sigma_{\bar{t}t}$ measurements for the individual channels, as well as some of their combinations. Within uncertainties, all measurements are consistent with each other. The combined cross section for $\ell + \text{jets}$, $\ell\ell$, and $\tau\ell$ final states for a top quark mass of 170 GeV/$c^2$ is evaluated to be

$$\sigma_{\bar{t}t} = 8.18_{-0.87}^{+0.98} \text{ pb}, \quad (2)$$

in agreement with theoretical predictions [1–4]. The uncertainty is comparable to the one on the cross section combination from different methods in the $\ell + \text{jets}$ channel performed by D0 [8]. The observed number of events in the different channels is compared to the sum of the background and combined $\bar{t}t$ signal in Fig. 1(a).

We compute ratios $R_{\sigma}$ of measured cross sections, $R_{\sigma}^{\ell/\ell'} = \sigma_{\ell} / \sigma_{\ell'}$ and $R_{\sigma}^{\ell/\ell'^/}$, by generating pseudodata sets in the numerator and denominator in order to take into account the correlation between systematic uncertainties. $\sigma_{\bar{t}t}$ represent the measured cross sections in the corresponding channel. The pseudodata sets are created by varying the number of signal and background events around the expected number according to Poisson probabilities. All independent sources of systematic uncertainties are varied within a Gaussian distribution. Although the individual channels considered are exclusive, each channel can receive signal contributions from different $\bar{t}t$ decay modes. We calculate the contribution from dilepton events to the $\ell + \text{jets}$ final state as well as the contribution from dilepton and $\ell + \text{jets}$ events to the $\tau\ell$ final states using the corresponding observed cross sections in the individual channels when generating pseudodata sets. For each pseudodata set, we perform the maximization of Eq. (1) separately in the numerator and denominator, and divide the results. The central value is obtained from the mode of the distribution of $R_{\sigma}$, and the uncertainties are derived from the interval containing 68% of the pseudoeperiments. From these pseudoeperiments, we obtain $R_{\sigma}^{\ell/\ell'} = 0.86_{-0.17}^{+0.19}$ and $R_{\sigma}^{\ell/\ell'^/} = 0.97_{-0.29}^{+0.32}$, which is consistent with the SM expectation of $R_{\sigma} = 1$.

Extensions of the SM, based on supersymmetry or grand unification [5], require the existence of additional Higgs multiplets beyond the Higgs doublet of the SM. Some of these models, such as the two Higgs-doublet model or the minimal supersymmetric standard model, foresee the existence of physical degrees of freedom which can be associated with a charged scalar particle, the charged Higgs boson. If this charged Higgs boson is lighter than the top quark, it will appear in the top quark decays. We use the ratios to extract upper limits on the branching ratio $B \equiv B(t \to H^+ b)$. In particular, a charged Higgs boson decaying into a tau and a neutrino ($B(H^+ \to \tau\nu)$) results in more events in the $\tau\ell$ channel, while fewer events appear in the $\ell\ell$ and $\ell + \text{jets}$ final states compared to the SM prediction. In case of a leptonophobic ($B(H^+ \to e\bar{\nu}) = 1$) model, the number of dilepton events decreases faster than the number of $\ell + \text{jets}$ events for increasing $B(t \to H^+ b)$. We therefore use $R_{\sigma}^{\ell/\ell'^/}$ to set limits on the leptonophobic model, while $R_{\sigma}^{\ell/\ell'^/}$ is used to search for decays in which the charged Higgs bosons are assumed to decay exclusively to taus.

To extract the limits, we generate pseudodata sets assuming different branching fractions $B(t \to H^+ b)$. The signal for a charged Higgs boson is simulated using the PYTHIA Monte Carlo event generator [13], and includes decays of $\bar{t}t \to W^+ bH^-$ and its charge conjugate (WH) and $\bar{t}t \to H^+ bH^-$ (HH). For a given branching fraction $B$, we calculate the expected number of $\bar{t}t$ events per final state,

$$N_{\bar{t}t} = [(1 - B)^2 \cdot \epsilon_{WW} + 2B(1 - B) \cdot \epsilon_{WH} + B^2 \cdot \epsilon_{HH}] \sigma_{\bar{t}t} L,$$

(3)

where $\epsilon$ are the selection efficiencies for the different decays (WW refers to $\bar{t}t \to W^+ bW^-$ and L is the integrated luminosity. We add $N_{\bar{t}t}$ to the expected background and treat the sum as a new number of expected events in each channel. We then perform the likelihood maximization to extract $\sigma_{\bar{t}t}$ from these pseudodata as if they contained only SM $\bar{t}t$ production. This provides distributions for the ratios of cross sections for each generated $B$, which are compared to the observed ratio. We set limits on $B$ by

---

**TABLE II. Summary of uncertainties on the combined $\sigma_{\bar{t}t}$.**

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta \sigma_{\bar{t}t}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>+0.47</td>
</tr>
<tr>
<td>Lepton identification</td>
<td>+0.15</td>
</tr>
<tr>
<td>Tau identification</td>
<td>+0.02</td>
</tr>
<tr>
<td>Jet identification</td>
<td>+0.11</td>
</tr>
<tr>
<td>Jet energy scale</td>
<td>+0.19</td>
</tr>
<tr>
<td>Tau energy scale</td>
<td>+0.02</td>
</tr>
<tr>
<td>Trigger modeling</td>
<td>+0.11</td>
</tr>
<tr>
<td>$b$-jet identification</td>
<td>+0.34</td>
</tr>
<tr>
<td>Signal modeling</td>
<td>+0.17</td>
</tr>
<tr>
<td>Background estimation</td>
<td>+0.14</td>
</tr>
<tr>
<td>Multijet background</td>
<td>+0.12</td>
</tr>
<tr>
<td>Luminosity</td>
<td>+0.56</td>
</tr>
<tr>
<td>Other</td>
<td>+0.15</td>
</tr>
<tr>
<td>Total systematic uncertainty</td>
<td>+0.78</td>
</tr>
</tbody>
</table>
using the frequentist approach of Feldman and Cousins [16].

The observed and expected (i.e., for \( R_\sigma = 1 \)) limits for
the tauonic and the leptophobic charged Higgs boson models are shown in Figs. 1(b) and 1(c), respectively. In
the tauonic model the upper 95% C.L. limits on \( B \) range from 15% to 40% for 80 GeV/c^2 \( \leq M_{H^0} \leq 155 \) GeV/c^2,
improving the limits given in [17]. For the leptophobic
charged Higgs boson model, which is investigated here for
the first time, the upper limit on the \( B \) range is between
48% and 57% for the same mass range. Although indirect
bounds as those from the measured rate of \( b \to s \gamma \) [18]
appear stronger than the results from the direct search
presented here, they can be invalidated by the presence
of new physics contributions.

The interpretation of the direct measurement of the top
quark mass [6] has become a subject of intense discussion
in terms of its renormalization scheme [19]. The extraction
of this parameter from the measured cross section provides
complementary information, with different sensitivity to
theoretical and experimental uncertainties, relative to
direct methods that rely on kinematic details of the top quark
reconstruction. Simulated samples of \( t\bar{t} \) events generated at
different values of the top quark mass are used to estimate
the signal acceptance. The resulting measurements of \( \sigma_{t\bar{t}} \)
are fitted as a function of \( m_t \) [2]:

\[
\sigma_{t\bar{t}}(m_t) = \frac{1}{m_t^2} \left[ a + b(m_t - m_0) + c(m_t - m_0)^2 + d(m_t - m_0)^3 \right]
\]

where \( \sigma_{t\bar{t}} \) and \( m_t \) are in pb and GeV/c^2, respectively, and
\( m_0 = 170 \) GeV/c^2 [20]. The dependence on the top quark
mass is due to the mass dependence of the selection
efficiencies.

We compare this parametrization to a prediction in pure
next-to-leading-order (NLO) QCD [1], to a calculation
including NLO QCD and all higher-order soft-gluon re-
summations in next-to-leading logarithms (NLL) [2], to an
approximation to the next-to-next-to-leading-order
(NNLO) QCD cross section that includes all next-to-
next-to-leading logarithms relevant in NNLO QCD [3],
and to a calculation that employs full kinematics in the
double differential cross section beyond NLL using the soft
anomalous dimension matrix to calculate the soft-gluon
contributions at NNLO [4]. Figure 2 shows the experimental
and the theoretical [1–3] \( t\bar{t} \) cross sections as a function of
the top quark mass.

Following the method of Refs. [7,8], we extract the most
probable top quark mass values and the 68% C.L. band.
Since the theoretical predictions are performed in the pole
mass scheme, this defines the extracted parameter here.
The results are given in Table IV. All values are in good
agreement with the current world average of 171.2 \( \pm 2.1 \) GeV/c^2 [6].

In summary, we have combined the \( t\bar{t} \) cross section
measurements in \( \ell + \) jets, \( \ell\ell \), and \( \ell \tau \) channels to measure
\( \sigma_{t\bar{t}} = 8.18 \pm 0.98 \) pb for a top quark mass of 170 GeV/c^2.
For the first time, we have also calculated ratios of cross

![FIG. 1 (color online). (a) Expected and observed numbers of events versus channel used in measuring the combined \( \sigma_{t\bar{t}} \). The dashed band around the prediction indicates the total uncertainty. Upper limits on \( B(\ell \to H^+ b) \) for (b) tauonic and (c) leptophobic \( H^+ \) decays. The yellow band shows the ±1 standard deviation band around the expected limit.](https://example.com/fig1)

![FIG. 2 (color online). Experimental and theoretical [1–3] \( \sigma_{t\bar{t}} \) as a function of \( m_t \). The colored dashed lines represent the theoretical uncertainties due to the choice of the PDF and the renormalization and factorization scales. The point shows the measured combined \( \sigma_{t\bar{t}} \), the black dot-dashed line shows the fit with Eq. (4), and the gray band shows the corresponding total experimental uncertainty.](https://example.com/fig2)
sections and interpreted them in terms of limits on non-
standard model top quark decays into a charged Higgs
boson. All results are in good agreement with the SM
expectations. Finally, using different theoretical predic-
tions given in the pole mass scheme, we have extracted
the top quark mass from the combined $\sigma_{\tau\tau}$ and have found
the result to be consistent with the world average top quark

We thank the staffs at Fermilab and the collaborating
institutions, and acknowledge support from the DOE and
NSF (USA); CEA and CNRS/IN2P3 (France); FASI,
Rosatom, and RFBR (Russia); CNPq, FAPERJ, FAPESP,
and FUNDUNESP (Brazil); DAE and DST (India);
Colciencias (Colombia); CONACyT (Mexico); KRF and
KOSEF (Korea); CONICET and UBACyT (Argentina);
FOM (the Netherlands); STFC and the Royal Society
(United Kingdom); MSMT and GACR (Czech Republic);
CRC Program, CFI, NSERC, and WestGrid Project
(Canada); BMBF and DFG (Germany); SFI (Ireland); the
Swedish Research Council (Sweden); CAS and CNSF
(China); and the Alexander von Humboldt Foundation
(Germany).

### TABLE IV. Top quark mass with 68% C.L. region for different
theoretical predictions of $\sigma_{\tau\tau}$. Combined experimental and theo-
retical uncertainties are shown.

<table>
<thead>
<tr>
<th>Theoretical prediction</th>
<th>$m_t$ (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO [1]</td>
<td>165.5$^{+6.1}_{-5.9}$</td>
</tr>
<tr>
<td>NLO + NLL [2]</td>
<td>167.5$^{+3.8}_{-3.6}$</td>
</tr>
<tr>
<td>Approximate NNLO [3]</td>
<td>169.1$^{+5.9}_{-5.6}$</td>
</tr>
<tr>
<td>Approximate NNLO [4]</td>
<td>168.2$^{+5.9}_{-5.4}$</td>
</tr>
</tbody>
</table>

    Cacciari (private communications).
[20] We obtain $a = 6.82350 \times 10^9$, $b = 1.10480 \times 10^8$, $c = 8.80552 \times 10^5$, and $d = -1.767 \times 10^3$ for Eq. (4).