2013

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Energy Conversion Congress and Exposition (ECCE), 2013 IEEE
Year: 2013
Pages: 3934 - 3940, DOI: 10.1109/ECCE.2013.6647222

Adaptive Feature Extraction and SVM Classification for Real-Time Fault Diagnosis of Drivetrain Gearboxes

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***Abstract*—Drivetrain gearboxes play an important role in many modern industrial applications. This paper presents a novel method consisting of adaptive feature extraction and support vector machine (SVM)-based classification for condition monitoring and fault diagnosis of drivetrain gearboxes operating in variable-speed conditions. An adaptive signal resampling algorithm, a frequency tracker, and a feature generation algorithm are integrated in the proposed method for effective extraction of the features of gearbox faults from the stator current signal of the AC electric machine connected to the gearbox. A radial basis function kernel-SVM classifier is designed to identify the fault in the gearbox according to the fault features extracted. Experimental studies are performed for a drivetrain gearbox with a gear crack fault connected with a permanent magnet synchronous machine. Results show that the fault can be effectively identified by the proposed method.**

***Keywords*—Adaptive resampling, classification, condition monitoring, fault diagnosis, drivetrain gearbox, permanent magnet synchronous generator (PMSG), support vector machine (SVM)**

I. INTRODUCTION

Gearboxes are widely used in the drivetrains of many mechanical and electromechanical systems, such as wind energy systems, (hybrid) electric vehicles, aircrafts, etc. Condition monitoring and fault diagnosis (CMFD) of drivetrain gearboxes is of great importance to ensure safe operation of these systems [1]-[4]. Compared to conventional vibration-based diagnostic techniques, current-based (i.e., mechanical sensorless) diagnostic techniques have gained increasing attention in recent years because of their advantages in terms of cost, implementation, reliability, accessibility, and robustness [5], [6].

Prior work has demonstrated the use of current-based methods for effective detection of gearbox faults [5], [7], [8]. However, the condition of an operating system is usually unknown. The previous work assumed that the gearbox had been in a faulty condition and did not address the diagnostic task when the condition of the gearbox is unknown. To solve this problem, two issues need to be investigated. First, an appropriate signature of the fault needs to be extracted from the current signals for fault diagnosis. Second, an appropriate method is needed to effectively identify the condition of the gearbox.

Pattern recognition methods offer an effective means to solve the problem of fault identification. Among various pattern recognition methods, the support vector machines (SVMs), which are designed by using the statistical learning theory, are found to be remarkably effective in real-world applications [9], [10]. They can generate a satisfactory signal generalization capability using a small set of data points. In addition, the SVMs possess some useful properties for the problems of classification in terms of the complexity and efficiency of computation, uniqueness of solution, and simplicity of implementation [11]. Therefore, this paper proposes to utilize the SVM to design a classifier for fault identification of drivetrain gearboxes in a real-time CMFD system.

This paper proposes a novel real-time CMFD method for drivetrain gearboxes operating in variable-speed conditions. A three-step feature extraction scheme consisting of adaptive signal resampling, frequency tracking, and feature generation is proposed to extract the frequency-domain features of gearbox faults from the generator stator current measurements. An SVM classifier is designed to classify the condition of a gearbox to be healthy or faulty according to the fault features extracted. Experimental studies are carried out to validate the proposed method.

II. CHARACTERISTIC FREQUENCIES OF DRIVETRAIN GEARBOXES

In a drivetrain gearbox, the shaft rotation and gear meshing will introduce torsional vibrations on the shaft. The

This work was supported in part by the U.S. National Science Foundation under grant ECCS-1308045 and the Nebraska Center for Energy Sciences Research.

shaft torsional vibrations will subsequently cause magnetic field anomaly of an AC machine connected with the gearbox via the shaft. As a result, the mutual and self-inductances of the machine will change, generating sidebands across current frequencies [12]. Mathematical expression has been derived to show how the torsional vibrations affect the current signals of the AC machine and demonstrate the characteristic frequencies of gearbox faults in the current frequency spectrum of the AC machine [5], [13]. Those studies provide a theoretical basis for current-based CMFD for drivetrain gearboxes, which is valid for both permanent magnet synchronous machines (PMSGs) and induction machines [13].

This paper considers a drivetrain consisting of a two-stage gearbox connected with a PMSG, as shown in Fig. 1, where z_1 - z_4 denote the tooth numbers of the four gears in the gearbox. The characteristic frequencies of the gearbox vibration include the input shaft frequency f_1, pinion shaft frequency f_2, output shaft frequency f_3, and two gear meshing frequencies f_{m1} and f_{m2}. These vibrations are induced by transmission errors in the input, pinion, and output wheels and the stiffness variation of gear tooth contact. The rotational and meshing frequency components appear in the torque signature of the gearbox output shaft and are called torsional vibrations [7]. The characteristic frequencies of the gearbox vibration then modulate the stator currents of the PMSG and generate sidebands across the dominant components of the currents [5]. Accordingly, the frequencies of these sidebands depend on the input, pinion and output shaft frequencies as well as the fundamental and harmonics of the currents, and can be expressed in the following form:

$$f_{sideband} = k f_s \pm l f_1 \pm m f_2 \pm n f_3 \qquad (1)$$

where f_s is the fundamental frequency of the stator current, and $k, l, m, n = 1, 2, 3, \cdots$.

A gearbox fault will change the amplitudes of these sidebands, which therefore can be used as the feature for fault diagnosis.

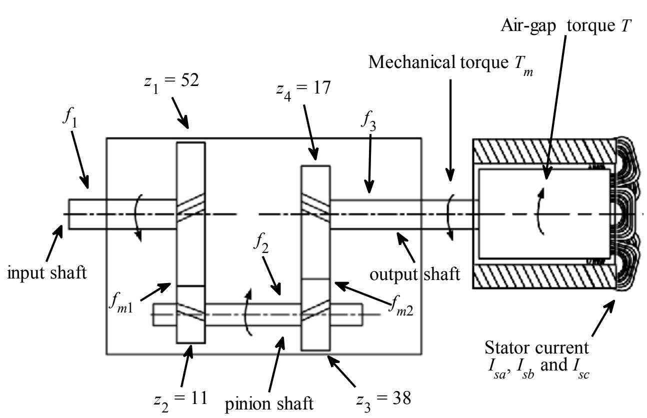

Fig. 1. Schematic of a drivetrain consisting of a two-stage gearbox connected to a PMSG with characteristic vibration frequencies.

III. ADAPTIVE FEATURE EXTRACTION SCHEME

Feature extraction is of vital importance in the implementation of classification. Proper feature extraction can help simplify the design of the SVM. On the contrary, improper feature extraction will deteriorate the performance or even lead to failure of the designed SVM.

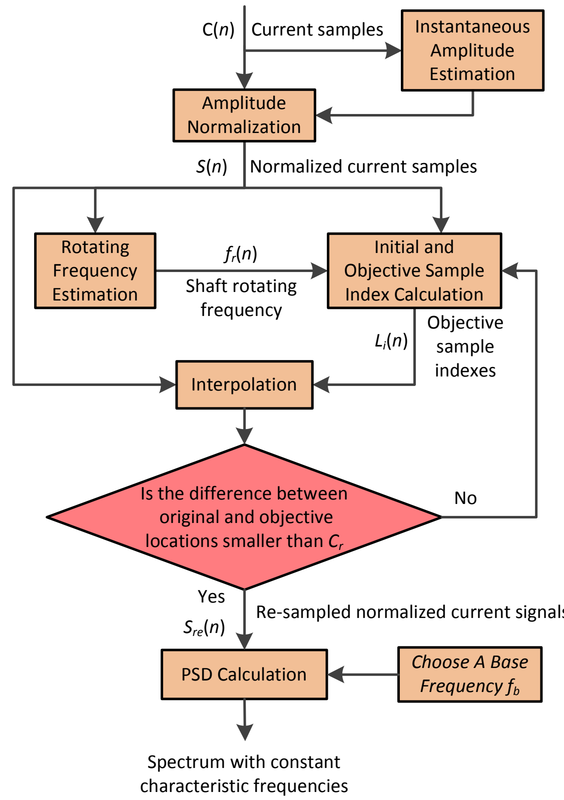

Fig. 2. Schematic of the adaptive signal resampling algorithm.

A. Adaptive Signal Resampling

The characteristic frequencies of gearbox faults in the stator current are related to gearbox shaft speeds, and become nonstationary when the shaft speeds vary with time. It is difficult to extract the fault signatures from the nonstationary stator current of the PMSG using conventional spectrum analysis methods. This paper uses an adaptive signal resampling algorithm to convert the current samples from a fixed sampling rate to an adaptive sampling rate to make the values of the objective frequencies, such as the fault characteristic frequencies, constant, as illustrated in Fig. 2. The details of the algorithm are described in [5]. By using the adaptive resampling algorithm, the time-varying characteristic frequencies of the gearbox in the stator current are converted to constant values in the PSD spectrum of the resampled signal. The magnitudes of certain frequency components in the resultant PSD spectrum can be then used to generate signatures to evaluate the condition of the gearbox.

B. *Frequency Tracker*

The characteristic frequencies of the gearbox in Fig. 1 are related to each other because of their mechanical connection.

$$f_2 = \frac{z_4}{z_3} f_3 \qquad (2)$$

$$f_1 = \frac{z_2}{z_1} f_2 = \frac{z_2}{z_1}\frac{z_4}{z_3} f_3 \qquad (3)$$

The rotating frequency of the gearbox output shaft, f_3, is identical to that of the PMSG shaft, which is proportional to the fundamental frequency of the PMSM stator current via the following relationship.

$$f_3 = \frac{2}{P} f_s \qquad (4)$$

where P is the number of poles in the PMSG.

The objective features of faults are different among different data records, as the operating points, e.g., the speed and load, may vary over time. To facilitate the fault feature extraction in this work, a frequency tracker (Fig. 3) is developed to solve this problem by utilizing the mechanical relationship among the gearbox shafts. In the proposed frequency tracker, a frequency detector is designed to detect the fundamental frequency of the resampled stator current signal. Then, according to (2)-(4), the characteristic frequencies, f_1, f_2 and f_3, of the gearbox can be calculated. After that, an objective frequency extractor is designed to extract the objective frequency components with their magnitudes according to (1). In this works, the objective frequency components are the sidebands around the fundamental frequency of the resampled stator current. The frequency tracker does not require complex signal processing. Another advantage is that the speed and load variations among different data records will not affect the performance of the proposed frequency tracker.

C. *Feature Generation*

Acquiring the most relevant features is of vital importance for constructing a reliable CMFD system. This paper uses the magnitudes of the first four pairs of the sidebands around the fundamental frequency extracted from the frequency tracker to construct the fault features. These sidebands are caused by the gearbox input shaft rotating frequency in the current PSD spectrum, i.e., $l = 1, \cdots, 4$ in (1), with $k = 1$ and $m = n = 0$. The sidebands are first normalized with respect to the magnitude of the fundament frequency f_s. Two features are then generated from the normalized sideband magnitudes: the standard deviation defined in (5) and the summation defined in (6).

$$\sigma = \left[\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2 \right]^{\frac{1}{2}} \qquad (5)$$

$$M = \sum_{i=1}^{N} x_i \qquad (6)$$

where x_i $(i = 1, 2, \ldots, n)$ is the normalized magnitude of each sideband, $\bar{x}$ is the mean value, and N is number of sideband pairs being processed.

The adaptive signal resampling algorithm, the frequency tracker, and the feature generation algorithm constitute the proposed adaptive feature extraction scheme, as shown in Fig. 3. In summary, the adaptive signal resampling algorithm does the PSD analysis for the current signal; the resultant PSD spectrum is used by the frequency tracker to find the magnitudes of the objective frequency components; and the obtained magnitudes are then used by the feature generation algorithm to generate the features for diagnosis of the gearbox faults.

IV. SVM-Based Classification

A. *Principles of SVM*

The basic idea of the SVM-based classification is to construct a hyper-plane as the decision plane, which separates the positive (+1) and negative (-1) classes with the largest margin (Fig. 4). The margin is the sum of the distances from the hyper-plane to the two boundaries constructed by closest data points of each of the two classes. These closest data points are defined as Support Vectors (SVs).

Suppose that there is a given training data set $G = \{(x_i, y_i), i = 1 \ldots P\}$. Each sample $x_i \epsilon R^D$ belongs to a class $y_i \in \{+1, -1\}$. The hyper-plane of the SVM can be expressed as follows:

$$\boldsymbol{\omega} \cdot x + \mathbf{b} = 0 \qquad (7)$$

where $\boldsymbol{\omega}$ is a weight vector and $\boldsymbol{b}$ is a bias vector.

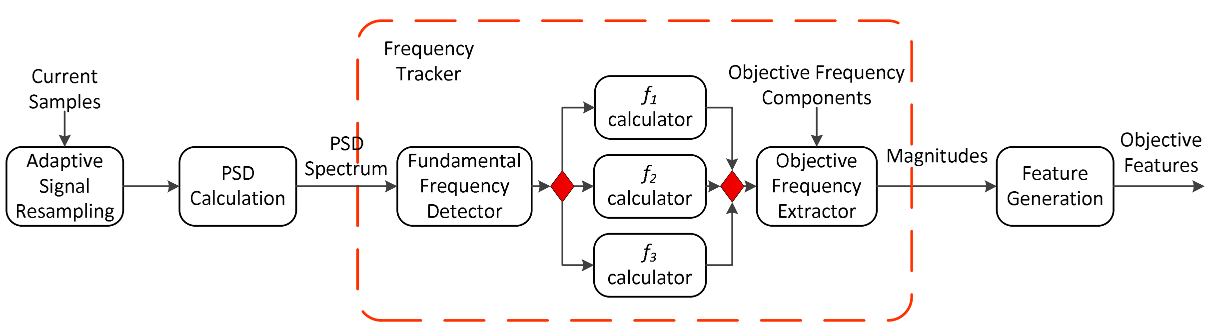

Fig. 3. Schematic diagram of the proposed feature extraction scheme.

Thus, the following decision function can be used to classify any data point x in either class " $+$ 1" or "-1":

$$f(x) = \mathrm{sgn}(\boldsymbol{\omega} \cdot x + \mathbf{b}) \tag{8}$$

where $\mathrm{sgn}(\cdot)$ is the operation to find the sign of a value.

According to (8), a SVM-based classifier can be written as follows:

$$f(x) = \mathrm{sgn}\left[\sum_{i=1}^{P} a_i y_i K(x, x_i) + \mathbf{b}\right] \tag{9}$$

which is subject to

$$\sum_{i=1}^{l} \alpha_i y_i = 0 \tag{10}$$

where $\alpha_i \geq 0$ are Lagrange multipliers, $K(x, y)$ is the kernel function that maps the data from the input space to a feature space.

The SVM possesses some useful properties for solving the classification problem [11]:

- The solution is unique for the optimization problem of constructing a SVM.
- The learning process in constructing a SVM is computationally efficient.
- A set of support vectors are obtained simultaneously with constructing the decision rule.
- .Designing a new SVM can be accomplished by changing only the kernel function.

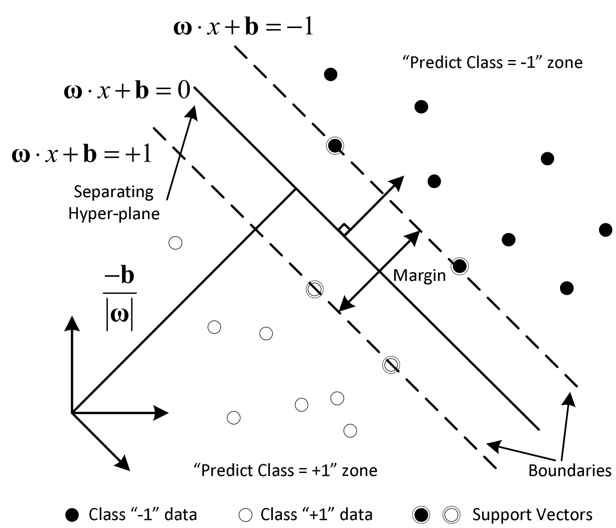

Fig. 4. SVM classification.

B. Kernel Function

In the problem of classification, the selection of the kernel function $K(x, y)$ is a key issue in designing the SVM. The kernel function maps the input vectors into a higher-dimensional feature space through some nonlinear separating hyper-plane and, thus, makes the data linearly separable in the feature space although the original input vectors are nonlinearly separable in the input space, as shown in Fig. 5. Hence, the kernel substitution provides a route for obtaining a nonlinear algorithm from the algorithms previously restricted to handling linear separable datasets [10]. The kernel function is the key affecting the learning and generalization abilities of an SVM. It determines the transformation and characteristic space of the designed SVM. Four types of kernel functions are currently commonly used, which are linear function, polynomial function, radial basis function (RBF), and sigmoidal function. Among them the RBF kernel shown in (11) is believed the best choice for practical applications owing to its excellent learning ability and high efficiency for classification.

$$K(x, y) = \exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right) \tag{11}$$

where σ is the width of the RBF kernel.

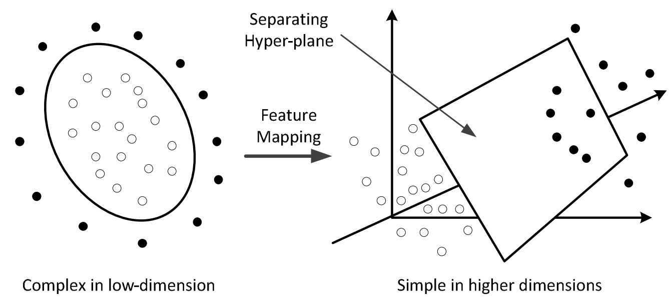

Fig. 5. Feature mapping by using the kernel function.

Various SVMs with different kernel functions are constructed to investigate their performance. . The selection of the parameters of the SVMs is performed by trial and error, which is the most commonly used method in SVM design. These SVMs are compared in terms of the number of SVs and classification accuracy, as shown in Table I. Usually the computational complexity of a SVM is proportional to the number of SVs. Fewer SVs means faster classification of the test samples. Although Table I indicates that the polynomial-SVM has the least number of SVs, its accuracy is a little worse than that of the RBF-SVM. Considering the learning and extension abilities [9], the RBF-SVM is considered to have the most accurate, reliable, and efficient performance.

TABLE I
COMPARISON OF SVMS WITH DIFFERENT KERNEL FUNCTIONS

	Linear	Polynomial	RBF	Sigmoidal
SV #	46	20	28	80
Accuracy	88%	95%	96%	89%

C. Generalization Ability Analysis

In the research on machine learning and pattern recognition, the dataset is usually divided into training and test subsets. The training subset is used to train the SVM model, while the test subset is used to evaluate the prediction accuracy (i.e., generalization ability [10]) of the model for unknown samples.

How to divide a complete dataset into training and test subsets is also an important issue. Two general rules are commonly followed [14]:

- The number of samples in the training subset must be sufficiently large, which is at least more than 50% of the total samples;
- The samples in the training and test subsets must be selected uniformly from the complete dataset.

The second rule is especially important. The purpose of uniform sample selection is to reduce the bias between the training/test subsets and the original complete dataset. A general practice is to do random selections. When there are sufficient samples, random selections of data samples will lead to uniform selections of the data samples.

This paper adopts a random sample selection method [14] to generate the training and test subsets. The original dataset is divided into training and test subsets according to a pre-determined control factor, p $(0.5 \leq p < 1)$. Suppose that there are totally $(M + N)$ samples in the original complete dataset, where M samples are Class "-1" and N samples are Class "+1". Then a number of pN samples will be chosen randomly into training subset, and the rest $(1 - p)(M + N)$ samples will constitute the test subset. Through the random selection process, the generalization performance of the designed SVM classifier for an independent dataset can be assessed. An example dataset division is presented in Fig. 6.

Original Dataset

Sample 1	Sample 2	- - -	Sample M	Sample (M+1)	Sample (M+2)	- - -	Sample (M+N)
Class "-1"	Class "-1"	- - -	Class "-1"	Class "+1"	Class "+1"	- - -	Class "+1"

Random Sampling

Training Subset

Sample 2	Class "-1"
Sample (M+2)	Class "+1"
Sample (M+4)	Class "+1"
Sample 1	Class "-1"
Sample 6	Class "-1"
Sample (M+2)	Class "+1"
Sample (M+8)	Class "+1"
- - -	- - -

≥50% (M+N)

Test Subset

Sample 4	Class "-1"
Sample 11	Class "-1"
Sample (M+3)	Class "+1"
Sample 7	Class "-1"
Sample 9	Class "-1"
Sample (M+9)	Class "+1"
Sample 14	Class "-1"
- - -	- - -

≤50% (M+N)

Fig. 6. Example of dataset division.

V. EXPERIMENTAL STUDIES

A. *Experiemetnal System*

The proposed method is applied for diagnosis of a gear crack fault in a test drivetrain gearbox. Fig. 7 shows the experimental system setup, which consists of a 300-W PMSG driven by a variable-speed induction motor through two back-to-back connected SIEMENS gearboxes. They are two-stage helical gearboxes with a total gear ratio of 10.57. One gearbox (i.e., the speed reducer) reduces the shaft speed of the induction motor. The second gearbox (i.e., the test gearbox) is used to emulate a drivetrain gearbox in real-system applications. The test gear is mounted at the input shaft of the test gearbox and pretreated by artificially generating a gear crack from the corner of the key way to the tooth root (Fig. 8). One phase stator current of the PMSG is recorded via a Fluke current clamp and National Instrument (NI) data acquisition system with a sampling rate of 10 kHz. Each data record lasts 210 seconds, during which the rotating speed of the PMSG is varied randomly in the range of 297 to 891 RPM. A total number of 161 data records are collected, including 80 for the healthy case and 81 for the gear crack case.

Fig. 8. The test gear with a crack.

B. *System in Nonstationary Conditions*

The proposed technique is testified when the system is operated in variable-speed conditions, where the rotating speed of the PMSG input shaft (i.e., the output shaft of the test gearbox) randomly varies in a range of 297 to 891 RPM. Each speed lasts for 8 second.

As reported in the previous work [5], the classical FFT analysis cannot be used directly to extract effective fault features from the nonstationary stator current signal caused by the variable shaft frequencies. As shown in Fig. 9, no specific characteristic frequencies of the gearbox are observable in the PSD spectrum of the stator current signal obtained directly from the classical FFT analysis. Therefore,

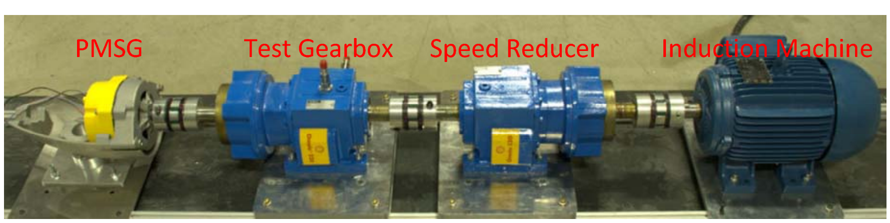

Fig. 7. The experimental system.

no fault signatures can be extracted from the nonstationary stator current signal caused by the variable shaft rotating frequencies using the classical FFT analysis.

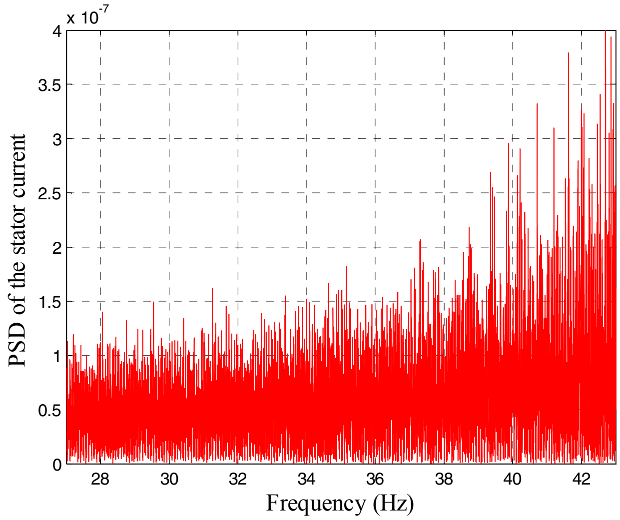

Fig. 9. PSD spectrum of the stator current signal obtained directly from classical FFT analysis.

The proposed adaptive feature extraction scheme is utilized to facilitate the diagnosis of the gear crack fault in variable-speed conditions. As shown in Fig. 10, the shaft rotating speed of the PMSG is estimated by a phase-locked loop (PLL) method [15]. Meanwhile, the Hilbert transform [16] is used to calculate the instantaneous load of the PMSG, which ranges from 30% to 70% of the rated power of the PMSG The load connected to the PMSG is purely resistive and, therefore, is proportional to the rotating speed of the PMSG.

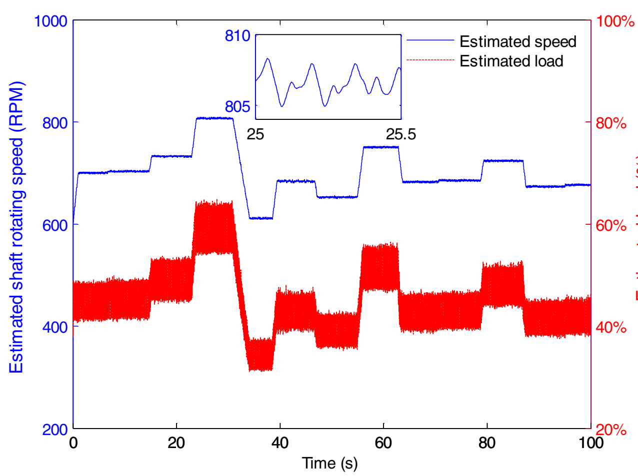

Fig. 10. PMSG input shaft rotating frequency estimated by the PLL method [15] and load estimated by the Hilbert transform [16] .

Figs. 11 and 12 exhibit the PSD spectra of the stator current obtained from the proposed method for the system with and without the gear crack in the test gearbox, respectively. As expected, the characteristic frequencies predicted in Section II are observed. Moreover, the gear fault altered the distribution of the sidebands around the fundamental frequency of the stator current, which is related to the gearbox vibration. The main change in the current PSD spectrum caused by the gear crack appears in the input shaft-related sidebands, i.e., $k = 1$, m = 0, $n = 0$ and $l = 1, 2, 3, \ldots$ in (1).

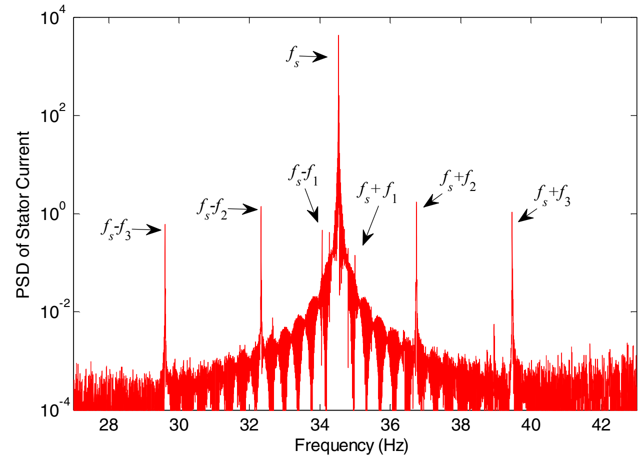

Fig. 11: PSD spectrum of stator current for healthy gearbox.

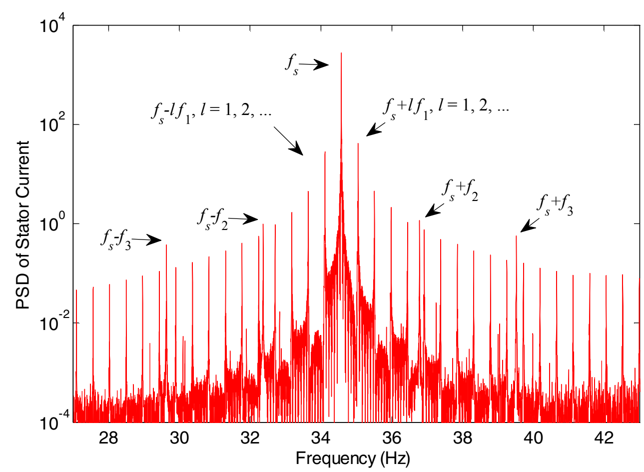

Fig. 12: PSD spectrum of stator current for gearbox with gear crack.

The proposed feature extraction scheme is then applied to the PSD spectrum of the PMSG stator current data. Each data record produces one set of features. The complete dataset have 161 sets of features, which are then divided into the training subset and test subset using the aforementioned random selection method. Based on the premise of generalization ability, the critical case where $p = 0.5$ is considered. In other words, the training subset has 81 data records and the test subset has 80 data records.

According to the comparison in Section III, a SVM classifier with a RBF kernel is designed to evaluate the condition of the test gearbox. The SVM has two inputs, which are the two features generated by the adaptive feature extraction scheme described in Section III. The output of the SVM is binary coded, where Class “-1” indicates the gearbox is in healthy condition and Class “+1” indicates the gearbox is in faulty condition. Different widths are tested for the RBF kernel function and an optimal value of $\sigma = 2.673$ is chosen for the designed SVM, which provides the best classification accuracy.

Monte Carlo studies have been performed to testify the effectiveness of the proposed method, where the 161 feature sets generated by the proposed method are used for fault diagnosis. Different training/test subsets are generated for classification in different Monte Carlo cases, where the use of different training/test sets leads to different cases of classification using the SVM classifier [17]. Results show that the average classification accuracy of the proposed method is 91.5%, where the highest accuracy is 96.25% and the lowest accuracy is 87.5%. These results clearly indicate that the gear crack fault can be properly detected by the proposed method consistently.

VI. CONCLUSIONS

A novel method consisting of effective adaptive feature extraction and SVM-based classification has been presented for CMFD of drivetrain gearboxes operating in variable-speed conditions. The proposed feature extraction method consists of an adaptive signal resampling algorithm, a frequency tracker, and a feature generation algorithm for effective extraction of the features of gearbox faults in the frequency domain of the PMSG stator current signal. A RBF kernel-SVM classifier has been designed to identify the faults in the gearbox according to the fault features extracted. A random selection method has been adopted to generate the training and test subsets form the original complete dataset, which ensures the generalization ability of the proposed method. Experimental studies have been performed for a PMSG-connected drivetrain gearbox with a gear crack fault; and the experimental results have shown that the fault can be effectively identified by the proposed method. Monte Carlo studies have shown that the SVM classifier has consistently satisfactory classification accuracy.

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