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Frequency Demodulation-Aided Condition Monitoring for Drivetrain Gearboxes

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Abstract—Condition monitoring and fault diagnosis (CMFD) of drivetrain gearboxes has become a prominent challenge in assorted industries. Current-based diagnostic techniques have significant advantages over traditional vibration-based techniques in terms of accessibility, cost, implementation and reliability. This paper proposes a current-based, frequency demodulation-aided CMFD method for drivetrain gearboxes. A mathematical model is developed for a drivetrain consisting of a two-stage gearbox and a permanent magnet synchronous generator (PMSG), from which the characteristic frequencies of gearbox faults in the PMSG stator current are derived. An adaptive signal resampling method is proposed to convert the variable fault characteristic frequencies to constant values for the drivetrain running at variable speed conditions. A demodulation method, combining the Hilbert transform, a finite impulse response (FIR) differentiator, and a phase unwrapping algorithm, is developed to extract the instantaneous frequency (IF) patterns that are related to the fault-induced gearbox vibration. A fault detector is proposed for diagnosis of gearbox faults using statistical analysis on the extracted fault signatures. Experimental studies are carried out to validate the effectiveness of the proposed method.

Keywords—Adaptive resampling, condition monitoring, current, drivetrain, fault diagnosis, frequency demodulation, gearbox, Hilbert transform, permanent magnet synchronous generator (PMSG)

I. INTRODUCTION

Gearboxes are widely used in the drivetrains of modern wind turbine systems, hybrid electric vehicles, aircrafts, and electric trains. CMFD of gearboxes has become a prominent challenge in modern industries [1]-[4]. Since gearboxes are considered the most troublesome part in drivetrains, detecting gearbox faults at an early stage and repairing the faulted gearbox in a timely manner is highly desired to reduce the downtime and prevent catastrophic damage of the system.

Gearbox faults have been successfully detected and evaluated by using electric machine stator current signals [5], [6]. Similar techniques have also been utilized to detect bearing faults [7], [8]. In [5], [6], the sidebands across the harmonic components of the current signals are used as the features for fault diagnosis. These sidebands are products of signal modulation, where the stator line currents are modulated by the torsional vibrations from the gear at specific frequencies. These diagnostic approaches are considered indirect method, since the characteristic frequencies of the gearbox vibration are indirectly observed from the sidebands, i.e., the modulated features. Therefore, an innovative technique that can demodulate the current signals to find the signatures directly related to gearbox faults and subsequently achieve the direct observation of the gearbox fault characteristic frequencies is of great interest for current-based CMFD for driven gearboxes.

This paper proposes a frequency demodulation-aided CMFD method for drivetrain gearboxes. A mathematical model is derived for an emulated drivetrain system consisting of a two-stage gearbox and a PMSG to identify the characteristic frequencies of gearbox faults, e.g., gear tooth breaks, in the PMSG stator current signals. Since the fault characteristic frequencies vary with the shaft rotating frequency of a drivetrain, which is usually variable, it is difficulty to employ conventional frequency analysis methods to detect the faults. An adaptive signal resampling algorithm is thus proposed to convert the variable fault characteristic frequencies to constant values to facilitate the following diagnosis process. A frequency demodulation algorithm is proposed, which uses the Hilbert transform to extract the instantaneous phase angle (IPA) of the resampled stator current. An FIR differentiator is proposed to estimate the IF from the IPA with the aid of a phase unwrapping algorithm. A fault detector is then developed based on the statistical analysis of the IF magnitudes to effectively detect the gearbox faults. Experimental studies are carried out to validate the proposed method.

II. CHARACTERISTIC FREQUENCIES OF DRIVETRAIN GEARBOXES

This paper considers a drivetrain consisting of a two-stage gearbox connected with a PMSG, as shown in Fig. 1, where \( z_1 \), \( z_2 \), \( z_3 \) and \( z_4 \) are the tooth numbers of the four gears in the gearbox. The characteristics frequencies of gearbox vibration include the input shaft frequency \( f_1 \), pinion shaft frequency \( f_2 \), output shaft frequency \( f_3 \), and two gear meshing frequencies \( f_{m1} \) and \( f_{m2} \). Due to the torsional vibrations induced by transmission errors in the input, pinion, and output wheels and the stiffness variation of the gear teeth contact, the gearbox adds rotational and meshing frequency components into the torque signature of the output shaft [9]. The gearbox characteristic frequencies then modulate the stator currents of the PMSG and yield sidebands across the dominant components in the currents. The stator phase A current can be modeled as in (1) [5], [6], with a phase modulation term in (2), where only the fundamental frequency component and its sidebands of the stator current are considered:
Fig. 1. Schematic of a drivetrain consisting of a two-stage gearbox connected to a PMSG with characteristic vibration frequencies.

\[ I_m = I_o \cos(2\pi f_1 t) \]
\[ + \frac{1}{2} \{ A_{r1} \cos[2\pi(f - f_1) t + \chi(t) + \phi_1] \]
\[ + A'_{r1} \cos[2\pi(f + f_1) t + \chi(t) + \phi_1] \}
\[ + \frac{1}{2} \{ A_{r2} \cos[2\pi(f - f_2) t + \chi(t) + \phi_2] \]
\[ + A'_{r2} \cos[2\pi(f + f_2) t + \chi(t) + \phi_2] \}
\[ + \frac{1}{2} \{ A_{r3} \cos[2\pi(f - f_3) t + \chi(t) + \phi_3] \]
\[ + A'_{r3} \cos[2\pi(f + f_3) t + \chi(t) + \phi_3] \]  

where \( I_o \) is a constant term, \( f_1 \) is the fundamental frequency of the stator current, \( \phi_{1i} (i = 1, 2, 3) \) represents the phase angle, \( A_{s1i} (i = 1, 2, 3) \) or \( A'_{s1i} (i = 1, 2, 3) \) represents the magnitude of each current component, and the phase modulation term can be expressed as follows.

\[ \chi(t) = a_1 \cos(2\pi f_1 t + \theta_1) \]
\[ + a_2 \cos(2\pi f_2 t + \theta_2) + a_3 \cos(2\pi f_3 t + \theta_3) \]

where \( a_i (i = 1, 2, 3) \) is the magnitude of each term.

III. FREQUENCY DEMODULATION FOR CALCULATION OF INSTANTANEOUS FREQUENCIES

Appropriate demodulation methods can separate the useful information related to gearbox faults from dominant components in the current signals to facilitate the extraction of gearbox fault signatures from the current signals.

A. Hilbert Transform

This paper proposes to use the Hilbert transform to obtain the IPA of the current signals to facilitate the fault diagnosis. The Hilbert transform of a signal \( x(t) \) is defined by an integral transform [10]:

\[ H[x(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau \]  

A complex signal, whose imaginary part is the Hilbert transform of the real part, is called an analytic signal:

\[ X(t) = x(t) + jH[x(t)] \]  

Therefore, the IPA, which is denoted as \( \varphi(t) \), can be assumed for the signal \( x(t) \) using a traditional representation of the analytical signal in its trigonometric form as in (5).

\[ \varphi(t) = \tan^{-1} \left( \frac{H[x(t)]}{x(t)} \right) \]  

The IPA of one phase stator current can be demodulated from the stator current signal by using (5) and the result is (6).

\[ \varphi_{st}(t) = A_{f1}2\pi f_1 t + A_{f2}2\pi(f_1 \pm f_2) t \]
\[ + A_{f3}2\pi(f_1 \pm f_3) t + B_{f1}\sin(2\pi f_1 t) \]
\[ + B_{f2}\sin(2\pi f_2 t) \]

where \( A_{fi} \) and \( B_{fi} \) (\( i = 1, 2, 3 \)) are the amplitudes of the frequency components.

The resultant IPA signal can then be used to extract the fault signatures described in the following Section.

B. FIR Differentiator

The IF of a signal (in Hz) is defined as the time derivative of the IPA of the signal

\[ IF(t) = \frac{1}{2\pi} \frac{d\varphi(t)}{dt} \]  

Applying (7) to (6), the IF of the stator current signal is calculated as

\[ IF_{st}(t) = A_{f1}'f_1 + A_{f2}'(f_1 \pm f_2) + A_{f3}'(f_1 \pm f_3) \]
\[ + B_{f1}'\sin(2\pi f_1 t) + B_{f2}'\sin(2\pi f_2 t) \]

where \( A_{f1}' \), \( A_{f2}' \), \( A_{f3}' \), \( B_{f1}' \), \( B_{f2}' \) and \( B_{f3}' \) are the amplitudes of the frequency components.

In the discrete-time system, the time derivative of the IPA in (7) can be approximated by differentiation of two subsequent samples as:

\[ IF[n] = \frac{1}{2\pi} \frac{\varphi[n+1] - \varphi[n]}{T_s} \]  

where \( T_s \) is the sampling period.

However, (9) introduces much noise in real-system applications. A better alternative is to use an FIR differentiator as in (10) to approximate the time derivative of the IPA with a smaller estimation error. This FIR differentiator implements a numerical differentiation formula from the Lagrange polynomial approximation [11], [12]. A fourth-order FIR differentiator is illustrated in Fig. 2. In the designed FIR differentiator, the delayed signal at each path is weighted by a corresponding coefficient and then summed at the output node.

\[ IF[n] = \frac{T_s}{2\pi} \left\{ -\frac{1}{12} \varphi[n] + \frac{2}{3} \varphi[n-1] - \frac{2}{3} \varphi[n-2] + \frac{1}{12} \varphi[n-4] \right\} \]  

The equation for the analytical signal, which is expressed as a series of 

\[ \varphi(t) = \tan^{-1} \left( \frac{H[x(t)]}{x(t)} \right) \]  

where \( H[x(t)] \) is the Hilbert transform of the signal \( x(t) \), can be separated into real and imaginary parts.

\[ x(t) + jH[x(t)] \]  

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where \( A_{f1}' \), \( A_{f2}' \), \( A_{f3}' \), \( B_{f1}' \), \( B_{f2}' \) and \( B_{f3}' \) are the amplitudes of the frequency components.

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The equation for the analytical signal, which is expressed as a series of 

\[ \varphi(t) = \tan^{-1} \left( \frac{H[x(t)]}{x(t)} \right) \]  

where \( H[x(t)] \) is the Hilbert transform of the signal \( x(t) \), can be separated into real and imaginary parts.
where $f_{SF} = 1/T_{SF}$ is the sampling rate.

C. Phase Unwrapping Algorithm

Mathematically, the IPA values obtained directly by using the Hilbert transform in (6) lie between $-\pi$ and $\pi$. To avoid arithmetic overflow, a phase unwrapping algorithm is proposed to correct the phase angle to produce a smoother phase curve by adding multiples of $2\pi$, when absolute jumps between two consecutive samples of the IPA are greater than a default jump tolerance, which is defined as $\pi$ in this paper. The flowchart of the algorithm is illustrated in Fig. 3 and its operation is shown in Fig. 4. With the proposed phase unwrapping algorithm, the direct result of the Hilbert transform can be successfully unwrapped to form a smooth line. The unwrapped phase $\varphi'[n]$ in Fig. 3 is then used for the calculation of the IF in (10). Equation (8) indicates clearly that the torsional vibrations with specific frequencies will induce characteristic frequency components in the IF signature of the PMSG stator currents. Therefore, the changes of these characteristic frequencies can be viewed as the features for fault diagnosis.

IV. ADAPTIVE SIGNAL FEATURE EXTRACTION

The characteristic frequencies of a gearbox fault in the frequency domain of a stator current signal are related to the gearbox shaft rotating frequencies, and become nonstationary when the shaft rotating frequencies vary with time. Conventional spectrum analysis methods are not able to provide physically meaningful results on the nonstationary signals [5]. To overcome the limitation of the conventional methods, an adaptive signal resampling algorithm similar to that in [5] is developed to convert the variable characteristic frequencies of a gearbox fault to constant values. After that, the proposed frequency demodulation-based method can be employed to detect the fault for the drivetrain gearbox running in variable-speed conditions.
extracted from the PSD to evaluate the condition of the gearbox.

V. FAULT DETECTOR

After extracting the fault signatures from the stator current signal using the proposed adaptive feature extraction algorithm, a fault detector is necessary to evaluate the fault signatures for fault diagnosis. This section proposes a fault detector based on the statistical analysis on the fault signatures extracted in Section IV. The power of the three vibrating frequency components \( M_i \) (\( i = 1 \cdots 3 \)), i.e., the sinusoidal components of the three shaft rotating frequency of the gearbox, in (8) is first normalized using (11).

\[
M'_i = \frac{M_i}{\sum M_i / 3}
\]  

(11)

An evaluation metric, called the normalized power difference \( NPD_i \) (\( i = 1, 2, 3 \)), of the vibrating (i.e., fault characteristic) frequency components between the healthy case and faulty case is defined in (12) and will be utilized for diagnosis of gearbox faults.

\[
NPD_i = M'_{i, \text{fault}} - M'_{i, \text{health}}
\]

(12)

VI. EXPERIMENTAL RESULTS

A. Experimental System

Experimental studies are carried out to validate the proposed model and fault diagnosis method. Fig. 6 shows the experimental system. It consists of a 300-W PMSG driven by a variable-speed induction motor through two back-to-back connected SIEMENS gearboxes, which are two-stage helical gearboxes with a total gear ratio of 10.57. One gearbox (i.e., the speed reducer) reduces the shaft speed of the induction motor driving by an adjustable-speed AC drive. The second gearbox (i.e., the test gearbox) is used to emulate the drivetrain in real-system applications. The test gear is mounted at the input shaft of the test gearbox and pretreated by removing one gear tooth as shown in Fig. 7. The system is first tested in constant-speed conditions and then variable-speed conditions. One phase stator current of the PMSG and vibration of the test gearbox are acquired via a Fluke current clamp and an accelerometer, respectively, and recorded by a National Instrument (NI) data acquisition system with a sampling rate of 10 kHz.

![Fig. 7. The test gear with one tooth removed.](image)

B. Results for System in Stationary Conditions

The PSD spectrum of the IF of the PMSG stator current obtained by the proposed method is shown in Fig. 8 for the gearbox with healthy gears, when the PMSG is running at 297 RPM. The characteristic frequencies of the gearbox vibration are clearly depicted on the PSD spectrum. For comparison, the PSD spectrum of the gearbox vibration signal in the same operating condition is provided in Fig. 9, which is generated directly by using the Fast Fourier Transform method. It can be found that the characteristic frequencies of gearbox vibration are not observable in the PSD of the vibration signal, because these frequency components are covered by high-level noise. On the other side, since the vibration of the gearbox modulates the current signal of the PMSG, the characteristic frequencies of the gearbox fault can be observed indirectly in the current signal. Moreover, since the PMSG functions as a low-pass filter such that the noise of the gearbox vibration does not modulate the current signal, the characteristic frequencies of the gearbox vibration clearly appear in the PSD spectrum of the current signal without the need of a low-pass filter, which is otherwise required when processing the vibration signals. The comparison between Fig. 8 and Fig. 9 reveals the advantages of the proposed method.
The PSD spectrum of the IF of the PMSG stator current for the gearbox with a gear tooth break fault is also generated by using the proposed method, as shown in Fig. 10, while other operating condition is the same for the system. The characteristic frequencies of gearbox vibration in the IF of the PMSG stator current are also clearly found in the PSD spectrum. Comparing the PSD spectra in Fig. 8 and Fig. 10 indicates that the magnitudes of the frequency components at \( f_1 \), \( f_2 \) and \( f_3 \) in the healthy condition are different from those in the faulty case. This difference is caused by the gearbox fault and will be quantitatively analyzed by the proposed fault detector later.

### C. Results for System in Nonstationary Conditions

The proposed adaptive signal resampling method is then applied to the stator current samples to facilitate the diagnosis of the tooth break fault for the gearbox operating in variable-speed conditions. Fig. 11 shows the estimated speed by a phase-locked-loop (PLL) method [13], and the estimated PMSG load by the Hilbert transform [14].

Subsequently, the original and objective sample indexes are calculated based on the estimated shaft rotating frequency for interpolation, as described in Fig. 5. Totally 16 iterations of interpolation are executed. A base frequency of 291 RPM is chosen for the purpose of comparing with the baseline case, whose speed is fixed at 291 RPM. After adaptive signal resampling, frequency demodulation-based feature extraction is performed to generate the PSD of the IF of the resampled normalized PMSG stator current, as shown in Fig. 12.

The characteristic frequencies of the gearbox vibration, i.e., the three shaft rotating frequencies predicated by the
mathematical model in Section II, can be observed in Fig. 12. Although the frequency at $f_2$ is interfered by higher-level noise compared to Fig. 8, the fault characteristic frequencies can still be clearly identified from the PSD spectrum of the resampled normalized current signal generated by the proposed method, which converts the variable-speed condition to a constant-speed condition. The power of these characteristic frequency components is then used as the signatures by the fault detector for gearbox fault diagnosis.

D. Results of Fault Detector

It has been concluded previously that the gearbox fault can alter the distribution of the characteristic frequency components in the PSD of the current signal. The proposed fault detector is then applied for quantitative analysis for both the constant-speed and variable-speed cases, and the results are shown in Fig. 13. The resultant NPDs are quite similar for both the constant-speed and variable-speed cases, i.e., revealing a “V-shape”. This “V-shape” NPD distribution indicates that the gear tooth break fault has relatively decreased the gearbox vibration at the second shaft rotating frequency $f_2$ while relatively increasing the vibration at the third shaft rotating frequency $f_3$. Meanwhile, little variation at the first shaft rotating frequency $f_1$ is observed, which indicates that the gear tooth break has little impact on it.

Moreover, the scaled PSD of the IF obtained from the normalized stator current in variable-speed conditions. Therefore, the performance of the fault detector is not affected by load or rotating speed variations.

![Fig. 13. NPD distributions for constant-speed and variable-speed conditions.](image)

VII. CONCLUSIONS

A frequency demodulation aided CMFD technique has been proposed for a drivetrain gearbox. A mathematical model has been developed to derive the characteristic frequencies of the gearbox vibration in generator stator current measurements. A frequency demodulation-based IF calculation algorithm combining the Hilbert transform, an FIR differentiator, and phase unwrapping has been designed to extract the characteristic frequencies of gearbox vibration from the PMSG stator current. An adaptive signal resampling algorithm has been proposed to process the stator current signals to convert the nonstationary fault characteristic frequencies to stationary values for the gearbox operating with variable rotating speeds. The adaptive signal resampling and IF calculation constitute an adaptive feature extraction algorithm to extract the signatures of gearbox faults, which are the power of the characteristic frequency components in the PSD spectra of the IF of the processed stator current signal. A fault detector based on the statistical analysis on the extracted fault signatures has been developed to quantitatively detect the gearbox faults. Experimental studies have shown that the characteristic frequencies of gearbox vibration can be clearly observed in the PSD spectra of the IF of the resampled PMSG stator current signals, which have then been used by the fault detector to effectively identify the gear tooth break fault. The effectiveness of the proposed method has also been validated by the experiments for variable load and rotating speed conditions.

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REFERENCES