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ABSTRACT

Evaluations for milk, fat yield, and fat test for Holstein sires in artificial insemination were calculated from first lactation records of Holstein daughters by best linear unbiased prediction procedures. Indirect evaluations for fat test also were calculated by adding best linear unbiased predictions for milk and fat yield to breed averages for milk and fat yield to calculate test and then subtracting breed average fat test. The indirect evaluation for test had a simple correlation of .99 with fat test from best linear unbiased prediction. Dollar value indexes were calculated from evaluations for milk yield and from evaluations for fat test by best linear unbiased prediction or by the indirect method with a price of $17.64 per 100 kg of milk and a fat test differential of $.1764 per .1%. The correlation between the dollar values was .99. Dollar value from best linear unbiased prediction of fat test was correlated .96, .93, and .23 with the best linear unbiased predictions for milk yield, fat yield, and fat test.

INTRODUCTION

Dollar value often is calculated to consider milk and fat test in a single index of economic merit. Dollar value sometimes is calculated in the same manner as milk value is calculated in many states:

$$\hat{V} = (\hat{M} + \overline{M})\ [B + D(\hat{T} + \overline{T} - T_{\underline{b}})] \qquad [1]$$

where $\hat{V}$ is sire evaluation for dollar value ($/lactation), $\hat{M}$ is sire evaluation for milk (kg), $\overline{M}$ is breed average milk yield (kg), B is the base milk price ($/kg), D is the price differential for fat test ($/kg per percent), $\hat{T}$ is the sire evaluation for fat test (percent), $\overline{T}$ is the breed average fat test (percent), and $T_{\underline{b}}$ is the base test for the milk differential (percent).

The $\hat{V}$ as calculated is a prediction of the gross value of mature production of milk in a 305-day lactation of a sire's daughter in breed average conditions. It usually is taken as a deviation from dollar value for average milk and fat test of the breed.

The index of equation [1] is a special case of a quadratic economic function of milk and fat evaluations. Wilton et al. (6) showed that if selection indexes are calculated for milk from milk and fat test and for fat test from both variables, then $\hat{V}$ has properties of the selection index including minimum prediction error variance. Wilton and Van Vleck (5) showed that evaluation for milk calculated from only milk data and fat test evaluation calculated from only fat test would result in 99.8% of the progress expected from using both variables to index each other.

Use of dollar value for selection rather than milk or fat yield is supported by several studies. Spahr (4) used herdmate evaluations to calculate dollar value by equation [1]. He found that the correlation of Holstein milk yield with dollar value was .88. The correlations for other breeds were higher, ranging from .90 for Guernseys to .96 for Ayrshires. Fat yields were more highly correlated than milk yield with dollar value (.96 for Holsteins and .96 to .99 for other breeds). On the other hand, Wilton and Van Vleck (5) found theoretically using selection index principles that milk yield would give 95% of the progress expected from selecting on dollar value while selection on fat yield would give only 92%. Thus, the dollar value index could result in 4 to 12% more genetic progress for gross income than selection on daughter yields of milk or fat.

Some caution should be applied in extending these results to mixed model cases. The proof that the quadratic index has minimum variance

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of prediction error applies only for the selection index where all effects are random and true means are known. Henderson (1, 2) has shown that linear functions of solutions to mixed model equations which are best linear unbiased predictors (BLUP) of random variables provide predictors of the same linear functions of fixed and random effects (if they are predictable) which have minimum variance of prediction error. It has not been shown that quadratic functions of these solutions are predictors with minimum error variance of the corresponding quadratic functions of the effects. Quadratic functions of BLUP solutions, however, probably provide good predictors of quadratic merit functions such as dollar value for milk and fat test. Some encouragement comes from the fact that under normality, functions of BLUP solutions provide maximum likelihood estimates of the fixed effects and of the conditional means of the random effects (1). By the invariance property of maximum likelihood, quadratic functions as well as linear functions would be maximum likelihood predictors.

There are computational reasons for evaluating fat yield rather than fat test. Milk and fat yield have the same heritability. When sire evaluations are calculated by BLUP (1), it is possible to use the same coefficient matrix with several sets of right-hand sides if the same heritabilities and repeatabilities apply. The majority of the work of setting up the BLUP equations is in calculating the coefficient matrix. There is little extra work to calculate right-hand sides for fat yield at the same time right-hand sides are calculated for milk. However, if fat test is to be evaluated, its higher heritability means that the coefficient matrix will be different, which can add considerably to the computing costs depending on the model and computing procedure. Two separate runs are required when random effects are absorbed, as when sire evaluation involves all lactations, natural service sires, or sire-by-herd interaction.

Two approximate alternatives are to calculate an appropriate fat test from BLUP equations for milk and fat yield or to use BLUP equations for milk and fat yield directly in the linear index of USDA. The two alternatives are equivalent as shown now.

The appropriate fat test evaluation is calculated from evaluations for fat and milk yield as:

$$\tilde{T} = [100 \times (\hat{F} + \overline{F})/(\hat{M} + \overline{M})] - \overline{T} \qquad [2]$$

where $\tilde{T}$ is an approximate sire evaluation for fat test (%), $\hat{F}$ and $\hat{M}$ are sire evaluations for fat yield and milk yield, and $\overline{M}$, $\overline{F}$, and $\overline{T}$ are the breed averages for milk, fat, and test (%).

This approximate evaluation for fat test can be used in the quadratic index [1]. However, a linear index for dollar value based on milk and fat yield was suggested as an alternative to the quadratic index by Norman and Dickinson (3) and currently is used in the USDA sire evaluation. It is

$$I = (\hat{M} + \overline{M})(B - D \times T_{\underline{b}}) + (\hat{F} + \overline{F})(100 \times D) \qquad [3]$$

where I is calculated dollar value and other symbols are as defined for equation [1].

The linear USDA index [3] is identical to the quadratic index [1] when fat test is calculated from milk and fat yield [2] as shown by substituting [2] into [1]:

$$\begin{aligned} V &= (\hat{M} + \overline{M})(B + D) \left[\frac{100 \times (\hat{F} + \overline{F})}{(\hat{M} + \overline{M})} - \overline{T} + \overline{T} - T_{\underline{b}}\right] \\ &= (\hat{M} + \overline{M})(B - D \times T_{\underline{b}}) + D \times 100 \times (\hat{F} + \overline{F}). \end{aligned}$$

The index in [3] ranks bulls the same regardless of breed average milk and fat (as would equation [1] with calculated fat test from [2]). This implies a breeder need not be concerned with his herd's production when determining the emphasis to place on evaluations for milk and fat.

However, the conclusion is different if fat test is evaluated directly for equation [1]. Sires rated by equation [1] will differ in rank when the breed average (or herd) is changed (5).

The practical difference between direct use of [1] and the use of [2] to calculate fat test to use in [1] can be resolved only by recognizing that it is inconsistent to expect all three traits – milk yield, fat yield, and fat test – to be transmitted additively and independent of herd. Which two traits best fit the assumption of additivity and no interaction with herd production is not clear.

TABLE 1. Sire evaluations for fat test and dollar value by BLUP vs. indirect calculation.

	BLUP		Test evaluation			Dollar evaluation		
Sire	Milk evaluation	Fat evaluation	BLUP	Calculated	Difference	BLUP	Calculated	Difference
	(kg)		(%)			($)		
1	313	12	.04	.04	-.00	61	61	-0
2	197	4	-.03	-.03	-.00	31	32	-1
3	480	12	-.05	-.05	.00	79	79	0
4	-166	-16	-.17	-.14	-.04	-50	-46	-4
5	-294	-16	-.07	-.07	.00	-60	-60	0
6	97	12	.15	.15	.01	36	35	1
7	229	-2	-.13	-.13	-.00	25	25	-0
8	355	13	.03	.03	-.00	67	67	-0
9	221	-8	.02	.02	-.00	-37	-37	-0
10	464	11	-.05	-.05	-.00	77	77	-0
11	152	20	.27	.24	.04	61	56	4
12	403	19	.09	.09	.00	83	83	0
13	0	-6	-.11	-.08	-.04	-13	-9	-4
14	-49	-9	-.09	-.08	-.01	-20	-18	-2
15	470	34	.30	.27	.03	122	118	4
16	293	5	-.06	-.05	-.01	45	46	-2
17	508	10	-.13	-.10	-.03	74	78	-4
18	-68	-2	.08	.03	.05	-3	9	6
19	592	17	-.06	-.03	-.03	98	102	-3
20	-15	4	.10	.10	.00	9	9	0

The questions addressed in this study are how similar is the calculated test evaluation [2] to the BLUP evaluation and how does dollar value computed from the calculated test compare with dollar value computed from BLUP evaluations.

METHODS AND DATA

Holstein sire evaluations for milk and fat yield from the January, 1974, Northeast AI Sire Comparison (NEAISC) were used to calculate fat test as in [2] and dollar value (deviated from breed average) as in equation [1]. The same first lactation records used to evaluate milk and fat yield also were used to evaluate fat test. Fat test was calculated for each first lactation record as 305-day age-season adjusted fat yield divided by 305-day age-season adjusted milk yield. Then BLUP sire evaluations were calculated in the same manner as milk and fat yield in the NEAISC with fixed sire groups, random sire within group effects, and fixed herd-year-season effects (1).

A heritability of .50 was assumed for fat test resulting in a ratio of $\sigma_e^2/\sigma_s^2 = 7$. The NEAISC uses heritabilities of milk and fat yield of .25 or a ratio of $\sigma_e^2/\sigma_s^2 = 15$. The milk price used was \$17.64 per 100 kg and \$.1764 per .1% fat differential from 3.5%. Breed averages were milk 6804 kg and test 3.6%.

RESULTS AND DISCUSSION

The sire evaluations were calculated for all sires with daughters freshening since 1956. There were 76 sires with 20 or more daughters. The evaluations of a sample of every fourth one of these based on registration number is in Table 1. The BLUP evaluations for fat test are compared with those calculated from milk and fat yield by equation [2]. Dollar values from the BLUP fat test are compared with dollar values from the calculated fat test. The largest absolute difference in evaluations for fat test for any sire is .05. The largest absolute difference in dollar value evaluations is \$6.

The differences between BLUP and calculated fat test values are small relative to variation in the evaluations. The variances of the evaluations and differences between evaluations were calculated for the 76 sires with 20 or more daughters and are shown as standard deviations in Table 2. The variance of differences between

TABLE 2. Correlations and standard deviations among sire evaluations.

Evaluation	BLUP test	Calculated test	Test difference	BLUP dollar	Calculated dollar	Dollar difference	BLUP milk	BLUP fat
BLUP test	1.00	.99	.71	.23	.20	.71	-.06	.57
Calculated test	.99	1.00	.62	.23	.21	.62	-.06	.57
BLUP test – calculated test	.71	.62	1.00	.17	.12	.99	-.05	.35
BLUP dollar	.23	.23	.17	1.00	.99	.16	.96	.93
Calculated dollar	.20	.21	.12	.99	1.00	.12	.96	.92
BLUP dollar – calculated dollar	.71	.62	.99	.16	.12	1.00	-.05	.34
BLUP milk	-.06	-.06	-.05	.96	.96	-.05	1.00	.78
BLUP fat	.57	.57	.35	.93	.92	.34	.78	1.00
Standard deviation	.1066	.0954	.0168	44.48	44.20	2.077	243.3	10.66

evaluations for fat test by BLUP and those by equation [2] is 2.5% of the total variation in the BLUP fat tests. The variance in dollar value differences is only .22% of the variance in dollar value calculated from BLUP equations for fat test.

The simple correlations among the various evaluations and differences are in Table 2. The correlation between BLUP and calculated fat test evaluations is .992. The dollar value procedures have an even larger correlation, r = .999.

Theoretical predictions of relative progress due to selection on milk or fat yield (5) are supported by this study. The correlation of .96 between BLUP evaluations for milk yield and dollar value implies that selection for milk would give about .96 the progress in dollar value as selection on that trait which compares to .95 predicted by Wilton and Van Vleck (5). Table 2 shows the correlation between fat yield and dollar value from BLUP test of .93 as compared to the predicted value of .92.

Although the differences are small, they are associated systematically with fat test. The test difference (BLUP-calculated) is associated positively with BLUP test (r = .71) but appears essentially independent of milk yield (r = −.05). Similarly, the dollar differences (BLUP-calculated) also are associated with BLUP test and not with milk (r = .71 and −.05).

In conclusion, the indirect calculation of fat test described in equation [2] is adequate and can save computer costs. Likewise, dollar values are essentially the same whether they are based on evaluations for milk and fat yield or on evaluations for milk and fat test.

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