2010

Salinity dynamics of discharge lakes in dune environments: Conceptual model

Vitaly A. Zlotnik
University of Nebraska-Lincoln, vzlotnik1@unl.edu

Neville I. Robinson
Flinders University

Craig T. Simmons
Flinders University, craig.simmons@flinders.edu.au

Follow this and additional works at: http://digitalcommons.unl.edu/geosciencefacpub

Part of the Earth Sciences Commons

http://digitalcommons.unl.edu/geosciencefacpub/488

This Article is brought to you for free and open access by the Earth and Atmospheric Sciences, Department of at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Papers in the Earth and Atmospheric Sciences by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.
Salinity dynamics of discharge lakes in dune environments: Conceptual model

Vitaly A. Zlotnik, Neville I. Robinson, and Craig T. Simmons

Received 11 December 2009; revised 2 August 2010; accepted 27 August 2010; published 24 November 2010.

[1] Lakes of varying salinity are ubiquitous in dune environments with semiarid climates. Often these lakes become solute traps due to specific groundwater flow conditions. Over geological time after emergence of such lakes, the groundwater flow and mass transport processes lead to accumulation of solutes. This salinity increase can result in mineral deposition and free convection from lakes. However, high solute concentrations in these lakes are often not observed. We propose a conceptual model that estimates the lake salinity dynamics and yields concentration profiles beneath the lakes over centennial and millennial scales. Unlike previous approaches, a simple mathematical model is proposed based on coupling the lake solute mass balance with the advection-dispersion equation. We discuss limitations of the model and apply it to conditions of the Nebraska Sand Hills, USA. We show that the lake salinity concentration is consistent with the regional paleoclimate data. The proposed model can be used to constrain hydrologic paleoclimate reconstructions, which is a significant challenge for lakes in dune environments.


1. Introduction

[2] Shallow saline lakes in semiarid climates are common in semiarid and arid environments in North America, Africa, Asia, and Australia (see Gill [1996] and Yechezl and Wood [2002] for reviews). These lakes are closed to surface runoff and are supported by fresh groundwater seepage. Combination of hydrodynamic factors (regional groundwater flow, evaporation, precipitation, lake size, and hydrogeologic settings) may preclude seepage out of the lake even in the presence of ambient regional flow as shown by Winter [1976], Townley and Trevy [2000], and Zlotnik et al. [2009]. Such lakes represent groundwater sinks and are called discharge lakes in contrast to recharge lakes/playas that are the groundwater recharge areas [Wood, 2002] or flow-through lakes [Turner and Townley, 2006]. This is a commonly accepted classification [Born et al., 1979; Rosen, 1994]. All previous theoretical studies considered steady state flow and solute regimes because accurate determination of transient water-solute budgets of purely gaining lakes is very laborious and difficult experimental work [e.g., Turner and Townley, 2006]. Solutes from groundwater are captured by these lakes and become enriched over time.

[3] The importance of understanding lake solute dynamics is increasing with societal concerns: the open water surface of groundwater-fed lakes in deserts is perceived as a loss of water resources; sometimes, ideas are being promoted to drain such lakes [Chen et al., 2004], which may destroy fragile ecosystems and increase dust emissions.

[4] An example of discharge lakes can be found among lakes in the Nebraska Sand Hills, USA [Gosselin et al., 1994; Bleed and Ginsberg, 1998], the largest vegetated dune field in the Western Hemisphere. The underlying aquifer is the principal source of water in one of the major agricultural areas of the United States. High recharge rates in dunes and the thickness of sand and gravel beneath the Sand Hills provide 65% of the water stored in the aquifer. More than 1000 perennial or “near-perennial” lakes with areas >4 ha (Figure 1) exhibit salinity differences by 3 orders of magnitude due to evaporation in this semiarid climate [McCarraher, 1977; Gosselin et al., 1994; Bennett et al., 2007; Zlotnik et al., 2007]. Dunes emerged during extended mega droughts when the groundwater table was deep [Loepe et al., 1995; Miao et al., 2007]. Subsequent pluvial conditions led to groundwater recharge and water level rise on a geological time scale. Initially, fresh groundwater discharged steadily to the topographic depressions forming shallow lakes, but these lakes exhibited salinity increases over time, and now the solute concentration exceeds 100 g L\(^{-1}\) in some cases. Considering the discharge regime, a perennial lake of area 1 km\(^2\) with annual evaporation ~1000 mm and precipitation ~500 mm that is fed exclusively by fresh groundwater with solute concentration ~0.2 g L\(^{-1}\) receives 100 metric tons of solute annually. If the residence time of water in a shallow lake is 1 year, the evaporative enrichment of the lake solutes should result in a lake salinity of ~200 g L\(^{-1}\) within 1000 years. It is accepted that these lakes have existed for much longer than 1000 years [Loope et al., 1995], but observed values of salinity are ~100 g L\(^{-1}\) or below.
Why these lakes, which emerged as freshwater lakes at the onset of pluvial climate conditions in the Holocene, do not exhibit the expected salinity levels and lack significant mineral deposits in spite of the evaporation that greatly exceeds precipitation, are important questions to be answered.

Our analysis explicitly considers the discharge regime. Discrimination of discharge lakes among ubiquitous flow-through lakes would require extensive piezometer network and long-term observations [Meyboom, 1966; Wood and Sanford, 1995; Nimick, 1997]. An alternative is to use steady state lake solute balance as a tool suggested by Wood and Sanford [1990]. This balance relates lake concentration ($C_L$) to groundwater concentration ($C_0$), using inflow-to-outflow ratio as follows:

$$C_L = C_0 \left( \frac{Q_{in}}{Q_{out}} \right)$$

where $Q_{in}$ and $Q_{out}$ are volumetric inseepage and outseepage values for a lake, respectively. In a given example, $C_0 \sim 0.2 \text{ g L}^{-1}$ and $C_L \sim 100 \text{ g L}^{-1}$, the ratio $Q_{out}/Q_{in} = 0.2\%$. Thus, only a minuscule fraction of water escapes from such a lake, which makes direct detection of outseepage based on potentiometric data highly problematic. However, if the lake salinity might not yet be in equilibrium with climate, the ensuing steady state $C_L$ values are even higher, which leads to even smaller $Q_{out}/Q_{in}$.

Three hydraulic mechanisms are involved in the solute enrichment of discharge lakes or other surface water features. Advection is a major mechanism delivering solutes with fresh groundwater to the saline lake [Wood and Sanford, 1995; Yechieli and Wood, 2002]. Two other mechanisms that may retard the increase of salinity are mechanical dispersion and free convection. For example, Konikow and Arevalo [1993] used a 1-D advection-dispersion model to explore the salt flushing of marine connate water. Free convection was discussed in several studies of saline groundwater overlying fresh water [e.g., Wooding et al., 1997; van Duijn et al., 2002; Nield et al., 2008; Il'ichev et al., 2008]. Several studies addressed playas, or dry lakes, from hydraulic perspectives. Just recently, such an analysis was applied to the results of geophysical investigations of a sabkha system [Van Dam et al., 2009]. We will focus on these mechanisms assuming a discharge regime and apply our analysis to lakes in regions that have low $Q_{out}/Q_{in}$ or high $C_L/C_0$ ratios.

Nonhydraulic mechanisms that may affect lake salinity include salt dust emissions and atmospheric salt dust deposition have been first hypothesized in a classic work by Langbein [1961] and further explored by Wood and Sanford [1995]. These mechanisms are characteristic of dry lakes [Wood, 2002] but have been completely ignored for “wet” lakes.

To explain the phenomenon of constrained growth of lake salinity over long time scales, we use solute balance of a discharge lake in semiarid climate conditions, driven by surface evaporation of inseeping groundwater. A simple mathematical model couples the lake solute mass balance with the advection-dispersion equation. Our work is presented in the following sequence: (1) a coupled lake-aquifer model, (2) derivation of spatial and temporal changes of the aquifer solutes and lake salinity, (3) analysis of model results and limitations, and (4) assessment of this model in the light of available climate data for conditions of the Nebraska Sand Hills, USA.

2. Coupling Aquifer and Lake Fluxes at Geological Time Scale

Hydrodynamics of a discharge lake within ambient groundwater flow has been thoroughly investigated by Townley and Trefry [2000] and Zlotnik et al. [2009]. In Figure 2a, the cross section displays the lake capture zone and a curvilinear stream tube that delivers the groundwater...
solute balance equation, but necessary parameters are extremely rare, and quantification of deflation rates is in the stage of development [e.g., Reynolds et al., 2007].

3. Problem and Solution

3.1. Statement
[13] Our conceptual model includes solute transport from the aquifer to the lake, coupled with lake solute balance, which accounts for evaporation as the only water loss component. Consider a coordinate \( z \), positive upward, with origin at the lake-aquifer interface, a solute concentration in an aquifer \( C_A(z, t) \) with porosity \( n \), upward linear velocity \( u \), Darcy’s velocity \( q = nu \), and dispersion coefficient \( D \). We assume that the initial concentration of the groundwater and the lake at the onset of pluvial conditions is \( C_0 \). Our aim is to evaluate the growth of lake and aquifer concentration, the concentration gradient in the aquifer and flux \( q_s \) at the lake-aquifer interface. The transport equation for the aquifer solutes is as follows:

\[
\frac{\partial C_A}{\partial t} = D \frac{\partial^2 C_A}{\partial z^2} - u \frac{\partial C_A}{\partial z}, \quad -\infty < z < 0, \quad t > 0,
\]

with background concentration of fresh groundwater beneath the lake

\[
C_A(-\infty, t) = C_0, \quad t > 0
\]

and uniform initial condition

\[
C_A(z, 0) = C_0, \quad -\infty < z < 0.
\]

It is natural to assume equality of the lake and aquifer solute concentrations at the interface:

\[
C_A(0, t) = C_L(t).
\]

The closing additional condition for finding the lake concentration follows from the mass balance equation (1) using the definition \( q_s = n(-D \frac{\partial C_A}{\partial z} + u C_A)|_{z=0} \)

\[
H \frac{dC_L}{dt} = -n D \frac{\partial C_A}{\partial z} + q C_A, \quad z = 0, \quad t > 0.
\]

Condition (6) is the major difference with previous models. It governs increase of lake salinity that may be limited by mineral precipitation or free convection that will be discussed separately.

3.2. Solution
[14] The linear initial boundary value problem (2)–(6) lends itself to analytical treatment (see Appendix A). The closed form solution for vertical solute distribution in dimensionless parameters

\[
\bar{z} = \frac{z}{H}, \quad \bar{t} = \frac{D}{H^2 t}, \quad \bar{Pe} = \frac{u H}{D},
\]

where \( C_L(t) \) is the lake concentration changing with time \( t \) due to 1-D seepage from an adjacent underlying aquifer of infinite depth, and \( q_s \) is the conservative solute flux. Lake depth and concentration vary seasonally. Variations of the average value of depth under steady climatic and hydrologic conditions from year to year can be neglected [Donovan, 1994], and we use solute concentration as an average for 1 year or more.
where $Pe$ is a Peclet number as is follows

$$
C_A(\bar{y}, \bar{t}) = C_0 + \frac{nPe}{(n + Pe)} C_0 \exp(-\rho^2) \left\{ \sqrt{\frac{\bar{t}}{\pi}} + \frac{(a + b)}{4a(a + b)} \text{erfc}(\rho y_+) \right. \\
+ \left. \left[ \frac{(b - a)}{4a(a + b)} + a\bar{t} - \frac{x^2}{2} \right] \text{erfc}(\rho y-) + \frac{b}{(b^2 - a^2)} \text{erfc}(\rho y) \right\} 
$$

erfc $w$ = $e^w^2$ erf(c $w$), $a = Pe/2$, $b = a + n$, $y_+ = |\bar{y}|/\left(2\sqrt{\bar{t}}\right) + a\sqrt{\bar{t}}$ ; $y = |\bar{y}|/\left(2\sqrt{\bar{t}}\right) + b\sqrt{\bar{t}}$. 

where $\text{erfc}(\cdot)$ is the complementary error function. It is quite different from results of Wooding et al. [1997] and van Duijn et al. [2002]. Lack of steady state solution results from the solute balance condition instead of fixed solute concentration at the lakebed. The concentration in the lake is obtained by substituting $\bar{y} = 0$ into equation (8): $C_L(t) = C_A(0, \bar{t})$. Results are plotted in Figure 3 in dimensionless variables. The surface flux $q_S$ is calculated using equation (1)

$$
q_s(\bar{t}) = \frac{D}{H} \frac{\partial C(0, \bar{t})}{\partial \bar{t}} = \frac{Caq}{(n + Pe)} \left\{ a \left[ 1 + \text{erf}(a\sqrt{\bar{t}}) \right] + b \left[ \left( \rho^2 - a^2 \right) \text{erfc}(b\sqrt{\bar{t}}) \right] \right\} 
$$

with $\text{erf}(\cdot)$ as the error function.

3.3. Solutes in the Aquifer and the Lake at Large Times

At large times, the asymptotic expressions for $C_A$, $\partial C_A/\partial \bar{t}$, $\partial C_A/\partial \bar{y}$, and $q_s$ are as follows:

$$
C_A(\bar{y}, \bar{t})_{\bar{t} \rightarrow \infty} \sim C_0 + \frac{nPe^2}{(n + Pe)} C_0 \bar{t} \exp(\rho Pe) , \quad \bar{t} > \frac{\rho}{Pe} 
$$

$$
\frac{\partial C_A(\bar{y}, \bar{t})}{\partial \bar{t}}_{\bar{t} \rightarrow \infty} \sim \frac{nPe^2}{(n + Pe)} C_0 \exp(\rho Pe) , \quad \bar{t} > \frac{\rho}{Pe}. 
$$

The restriction $\bar{t} > |\bar{y}|/Pe$ for $C_A$ arises from the term $\text{erfc}(\bar{y})$ when $\bar{y}$ becomes negative. Of special interest is solute flux to the lake at large times $q_S(\bar{t})$:

$$
q_s(\bar{t})_{\bar{t} \rightarrow \infty} \sim q \frac{C_0 \bar{t}}{(n + Pe)}. 
$$

Note that the lake concentration is obtained by substitution of $\bar{y} = 0$ in equation (11) and is linear in time.

3.4. Model Limits

The scale of the boundary layer thickness beneath the lake is determined by the exponential function and is of the order of $H/Pe = D/u$.

At large times, the asymptotic expressions for $C_A$, $\partial C_A/\partial \bar{t}$, $\partial C_A/\partial \bar{y}$, and $q_s$ are as follows:

$$
C_A(\bar{y}, \bar{t})_{\bar{t} \rightarrow \infty} \sim C_0 + \frac{nPe^2}{(n + Pe)} C_0 \bar{t} \exp(\rho Pe) , \quad \bar{t} > \frac{\rho}{Pe} 
$$

$$
\frac{\partial C_A(\bar{y}, \bar{t})}{\partial \bar{t}}_{\bar{t} \rightarrow \infty} \sim \frac{nPe^2}{(n + Pe)} C_0 \exp(\rho Pe) , \quad \bar{t} > \frac{\rho}{Pe}. 
$$

This time of reaching $C_{max}$ is estimated by substituting the mineral precipitation concentration ($C_P$) or critical lake solute density ($C_C$), for $C_{max}$.

The concentration gradient beneath the lake increases linearly over time and reaches a maximum at time $T_{max}$; therefore, substitution $t = T_{max}$ into equation (15) yields this maximum gradient as follows

$$
\max \left| \frac{\partial C_A(0, \bar{t})}{\partial \bar{y}} \right| \sim Pe \frac{C_P}{H} = \frac{u C_P}{D}, 
$$

where dimensional variables are used.
[20] After reaching $C_{\text{max}}$, the lake salinity enters a steady state phase, and constant concentrations are then assigned at the lakebed surface:

$$C(0, t) = C_{\text{max}}, t > T_{\text{max}}. \quad (18)$$

The salinity profile developed by time $T_{\text{max}}$, or $C(z, T_{\text{max}})$, becomes an initial condition for this next phase. The eventual steady state profile $C(z, \infty)$ is obtained from equations (2) and (18):

$$C(\infty, z) = C_0 + (C_{\text{max}} - C_0) \exp(\frac{z}{D}), \quad (19)$$

which was discussed by Allison and Barnes [1985].

[21] Note that this approach simplifies a rich suite of geobiological processes that occur in actual systems and considers solute as a single conservative tracer. Mineral precipitation in evaporating lakes occurs in a certain sequence defined by both chemistry and water budget of the lake [see Wood and Sanford, 1990; Gosselin et al., 1994]. This process is subject to the striking seasonal and daily temperature variations and sometimes reversals [e.g., Smith et al., 1987], which easily move various minerals between liquid and solid phases. In addition, parameters of precipitation-dissolution may be affected by biological processes [Joeckel and Clement, 2005]. Further, transport of each ion should be simulated by a separate advection-dispersion equation and mineral thermodynamics [Wood and Sanford, 1990, Gosselin et al., 1994]. Accurate consideration of these processes is computationally feasible for a system of equations (one for each ion), plus at least one equation for temperature dynamics, but parameter demand and model calibration are hardly realistic with current knowledge. A compromise to obtain some constructive measure of mineral precipitation was introduced by Yechiel and Wood [2002]: they proposed to simulate these complex processes by using a single value of saturation concentration of the order $C_{\text{max}} = C_p \approx 200$ g L$^{-1}$ and assuming isothermal conditions. This value is consistent with observed total dissolved solids (TDS) at Dead Sea, Owens Lake, and others. We will use this simplified approach for our model analysis.

[22] Increase of solute density beneath the lakebed overlaying the freshwater may result in instability, leading to free convection, after which the 1-D solution of equation (8) is not applicable. Wooding et al. [1997, p. 1205] and van Duijn et al. [2002, equation 2.12, p. 156] provided stability analyses, based on the Rayleigh number $Ra$, which can be written in terms of field variables [Bauer et al., 2006], namely the lakebed hydraulic conductivity ($K$), the lake water density ($\rho_L$), and the background density of fresh water $\rho_0$ ($= 1000$ g L$^{-1}$):

$$Ra = K \frac{\rho_L - \rho_0}{\rho_0} \frac{g}{\rho_0}. \quad (20)$$

The critical value of $Ra$, or $Ra_{\text{C}}$, was estimated to be in the range $5.78 < Ra_{\text{C}} < 14.35$, below which there is definitely stability and above which there is definitely instability. Fundamentally, the ratio of lakebed hydraulic conductivity ($K$) to the Darcy’s seepage ($q$) controls the onset of free convection.

[23] The linear relationship between the solute density and concentration [e.g., Holzbecher, 1998] exists: $\rho \approx \rho_0 (1 + \beta C_L)$, where the dimensionless coefficient $\beta \approx 0.8 \times 10^{-3}$ when $C_L$ is given in g L$^{-1}$. The criterion can be used to find the lowest lake salinity when instability is feasible by using equation (20):

$$Ra = K \frac{\beta C_L}{\rho_0} \geq Ra_{\text{C}} = 5.78. \quad (21)$$

Then a simple criterion for critical lake concentration $C_C$, leading to the onset of free convection in field variables is

$$C_L \geq C_C = 8.3 \times 10^1 q \frac{K}{\rho_0}. \quad (22)$$

The time interval of validity of solutions (8), (10), (14), or $T_{\text{max}} = T_{\text{C}}$, can be calculated from (16) after substitution $C_{\text{max}} = C_C$. In general, the increase of lake salinity in discharge lakes will be limited by mineral precipitation or free convection, whichever is reached first.

4. Example: Lakes in the Nebraska Sand Hills, USA

[25] We apply our model to the lakes in the Nebraska Sand Hills, USA, over centennial and millennial scales. This application corroborates TDS dynamics with drought record inferred from Eolian sand and loess ages studies by Miao et al. [2007] and allows for a consistency check between hydrologic and sedimentological climate proxies, namely lake TDS and dune ages. Powerful, sustained Eolian activity (dune movement) ceased about 6500 years before present (YBP). Assuming a steady hydrologic regime over this period, we use our solute balance-based model and sensitivity analyses to assess time needed to reach mineral precipitation or develop free convection in these lakes. If this time is approximately 6500 years, we must observe mineral deposits. Absence of minerals will indicate other operating processes that interfere with steady hydrologic regime. These include erosion during droughts, seasonal Eolian process, lower seepage, and free convection. If free convection can be quantified or excluded, then just Eolian processes remain as important factors of lake solute balance as hypothesized by Langbein [1961] and shown qualitatively by Reynolds et al. [2007]. For example, evidence of significant droughts exists for 700–1000 YBP and 2300–4500 YBP [Miao et al., 2007].

[26] As a representative case, we use Alkali Lake (41°49′N, 102°36′W) with a seasonally variable concentration $C_L$ of the order ~100 g L$^{-1}$ [Olaguerra, 2007]. It has all the features of discharge lakes and exhibits frequent and massive but ephemeral, crust occurrences on lake beaches. To illustrate the salinity increase in pluvial, semiarid climates, we use typical lake and aquifer parameters: $H = 1$ m, $D = 10^{-9}$ m$^2$ s$^{-1}$, $\nu = 5 \times 10^{-8}$ m$^3$ s$^{-1}$, and $C_0 = 0.2$ g L$^{-1}$ [Gosselin et al., 1994, Zlotnik et al., 2007]. The rationale for parameter selection will be given below.

4.1. Paleohydrologic and Paleoclimatic Parameters ($q$, $n$, $H$)

[27] For a closed lake, inflow $q$ is the only water and solute source beside precipitation. If this lake is permanent or “wet,” the seepage must compensate excess of evaporation, $E$ (m yr$^{-1}$), over precipitation, $P$ (m yr$^{-1}$); therefore, $q = E - P$. For the present-day Sand Hills, NE, $q = E - P \sim \ldots
Effect of on a fraction of advective mass flux $D$ and $H$ [1997], $D_q m$ play limited roles in $= may become unimportant when lake salinity enters for shallow lakes are of the order $0.1 = in the range $0.1 [2008] adopted the between $10 ranging between $0.1$ and $100$, in the following, range from $1.0$ to $100$, $Pe \approx s$ of lacustrian sediments for several representative $D Pe 0.5/ m$ coefficient, or $Duijn et al$. [1992]. 

4.2. Lakebed Characteristics ($K$, $D$, $Pe$) 

[29] $K$ of lacustrian sediments for several representative lakes in the Sand Hills are in the range of $0.01$–$0.1$ m/d, or $1.3 \times 10^{-2}$–$1.3 \times 10^{-6}$ m s$^{-1}$ [Ong and Zlotnik, 2010] and is 2 or more orders of magnitude lower than $K$ of Eolian sediments in surrounding dunes [Zlotnik et al., 2007]. Flow orientation and anisotropy in $K$ play limited roles in analyses of a predominantly vertical flow system beneath the lakes.

[30] Data on dispersion in lacustrian sediments are rare. Konikow and Arevalo [1993], Wooding et al. [1997], van Duijn et al. [2002], and Nield et al. [2008] adopted the molecular diffusion coefficient as a proxy for the dispersion coefficient, or $D = D_m = 10^{-10}$–$10^{-9}$ m$^2$ s$^{-1}$. Friedman et al. [1997] provided actual field data for the Owens Lake cores in the same range. Consideration of microdispersion and macrodispersion at the scale of the boundary layer of thickness $L = 0.1$–$1.0$ m gives an estimate of the dispersion coefficient $D$ between $10^{-9}$ and $10^{-7}$ m$^2$ s$^{-1}$ [Gelhar et al., 1992].

[31] For our range of parameters $u$ and $H$, one can use $Pe = uH/D$ ranging between $0.1$ and $100$. In the following, we use $D = 10^{-9}$ m$^2$ s$^{-1}$ for analysis. Note that uncertainty in $Pe$ may become unimportant when lake salinity enters the linear range (see equations (13) and (14)).

4.3. Salinity Dynamics: Role of Advection and Dispersion 

[32] Equation (13) indicates that the solute flux into the lake based on advection mechanisms only, namely $qC_0$, differs from the actual flux $q_S$, which is especially apparent at large times and steady $q_S = q_S(\infty)$, when it becomes steady

$$\frac{q_S(\infty)}{qC_0} \sim \frac{Pe}{n + Pe}$$

(23)

[33] Note that the solute influx $q_S(t)$ was frequently assessed simply as $qC_0$ [Yechieli and Wood, 2002], without considering dispersion processes at the lake-aquifer interface. Figure 4 shows $q_S(t)/qC_0$ as a function of $Pe$ for various porosity values. For small $Pe$ in the range $0.1$–$1.0$, this error may be as high as $80\%$, but in our $Pe$ range from $1.0$ to $100$, this largest error is of the order $30\%$, and advection dominates solute influx.

[34] The concentration plots beneath the lake surface (Figure 5) at times $10^2$, $10^3$, $10^4$, and $10^5$ years indicate (1) solutes accumulate immediately under the interface in a boundary layer, exponentially decaying with depth at any given time; (2) the apparent scale of this layer extends downward with time; and (3) the gradient of salinity increases with time. These traits are consistent with the large time behavior in equations (9)–(11).

[35] Figure 6 demonstrates the effect of $Pe$ on mass accumulation in the boundary layer beneath the lakebed over $10^5$ years. It is apparent that the advancement of this “front” as a function of $Pe$ is consistent with the asymptotic expression for the concentration profile from equation (10). The boundary layer thickness beneath the lake is determined by the exponent in equation (10) and is of the order $H/Pe = Du$, which ranges from a fraction of a meter to several meters.

[36] In a paleohydrological context, advective accumulation of solutes is a dominant mechanism of lake salinization. Temporary reductions or reversal of velocity direction can lead to reversal of solute fluxes as in an example of playas of the Southern High Plains [Wood, 2002]. In this case lakes may lose sizable salt mass before the next pluvial period that

Figure 4. Effect of $Pe$ on a fraction of advective mass flux in total solute flux into the lake.

0.5 m yr$^{-1}$ $\approx 1.5 \times 10^{-8}$ m s$^{-1}$ [Wilhite and Hubbard, 1998]. With Eolian sand porosity $n = 0.3$, one obtains: $u = q/n \sim 0.5/0.3$ m yr$^{-1}$ $\approx 5 \times 10^{-8}$ m s$^{-1}$.

[28] Values of $H$ for shallow lakes are of the order $0.1$–$1.0$ m [Bleed and Ginsberg, 1998]. The seasonal fluctuations of $H$ are neglected when considering multiannual and centennial time scales.

Figure 5. Concentration profile beneath the lake at various times and various $Pe$. 

4.3. Salinity Dynamics: Role of Advection and Dispersion

[32] Equation (13) indicates that the solute flux into the lake based on advection mechanisms only, namely $qC_0$, differs from the actual flux $q_S$, which is especially apparent at large times and steady $q_S = q_S(\infty)$, when it becomes steady

$$\frac{q_S(\infty)}{qC_0} \sim \frac{Pe}{n + Pe}$$

(23)

[33] Note that the solute influx $q_S(t)$ was frequently assessed simply as $qC_0$ [Yechieli and Wood, 2002], without considering dispersion processes at the lake-aquifer interface. Figure 4 shows $q_S(t)/qC_0$ as a function of $Pe$ for various porosity values. For small $Pe$ in the range $0.1$–$1.0$, this error may be as high as $80\%$, but in our $Pe$ range from $1.0$ to $100$, this largest error is of the order $30\%$, and advection dominates solute influx.

[34] The concentration plots beneath the lake surface (Figure 5) at times $10^2$, $10^3$, $10^4$, and $10^5$ years indicate (1) solutes accumulate immediately under the interface in a boundary layer, exponentially decaying with depth at any given time; (2) the apparent scale of this layer extends downward with time; and (3) the gradient of salinity increases with time. These traits are consistent with the large time behavior in equations (9)–(11).

[35] Figure 6 demonstrates the effect of $Pe$ on mass accumulation in the boundary layer beneath the lakebed over $10^5$ years. It is apparent that the advancement of this “front” as a function of $Pe$ is consistent with the asymptotic expression for the concentration profile from equation (10). The boundary layer thickness beneath the lake is determined by the exponent in equation (10) and is of the order $H/Pe = Du$, which ranges from a fraction of a meter to several meters.

[36] In a paleohydrological context, advective accumulation of solutes is a dominant mechanism of lake salinization. Temporary reductions or reversal of velocity direction can lead to reversal of solute fluxes as in an example of playas of the Southern High Plains [Wood, 2002]. In this case lakes may lose sizable salt mass before the next pluvial period that
ZLOTNIK ET AL.: SALINITY DYNAMICS OF DISCHARGE LAKES

Figure 6. Effect of Pe on apparent thickness of the saline layer beneath the lake.

should be accounted for by the lake solute balance in equation (1). Required for our model is a certain stability of climate over the last 6500 years as indicated by Miao et al. [2007].

4.4. Model Sensitivity Analysis and Application to Paleodata

[37] According to the model, the uninterrupted pluvial episode will lead eventually to mineral precipitation at some time according to equation (16). This time depends on lake depth and seepage velocity (assuming that the aquifer characteristics, namely groundwater salinity, porosity, and diffusion coefficient do not change with climate). Consider depth values \( H = 0.1 \) and 1.0 m and velocity values \( u = 0.1 \times 10^{-8} \) and \( 0.5 \times 10^{-8} \) m s\(^{-1}\). Typically, the larger seepage corresponds to higher water table elevations near lakes and wetter climate conditions. Figure 7 provides insight into the process of solute accumulation in the lakes. Curve 1 indicates that the shallow lake (\( H = 0.1 \) m) with intensive seepage reaches saturation \( (C = 200 \text{ g L}^{-1}) \) in the shortest time of only \( T_p \approx 200 \) years. Contrary to that, a deeper lake \( (H = 1 \text{ m}) \) and smaller seepage \( (u = 0.1 \times 10^{-8} \text{ m s}^{-1}) \) extends this period to \( T_p \approx 10,900 \) years (curve 5).

[38] Smaller seepage for drier conditions \( (u = 2.5 \times 10^{-8} \text{ m s}^{-1}) \) and intermediate lake depth \( (H = 0.5 \text{ m}) \) are more consistent with a period of episodic Eolian activity [see Miao et al., 2007]. These parameters result in \( T_p \approx 2200 \) years for curve 4.

[39] “Mixed” characteristics (curve 2, \( H = 1 \text{ m} \) and \( u = 10^{-8} \text{ m s}^{-1} \), and curve 3, \( H = 0.5 \text{ m} \) and \( u = 0.5 \times 10^{-8} \text{ m s}^{-1} \)) result in \( T_p \approx 1400 \) and 2100 years, respectively. Considering the broad range of possible lake characteristics in the region due to the differences in geomorphology and geology following from the work of Townley and Trefry [2000] and Zlotnik et al. [2009], the \( T_p \approx 1400–2100 \) years appears to be a good estimate of time for developing salt deposits.

[40] This \( T_p \) estimate explains the lack of salt stored in and near these lakes using the data of Miao et al. [2007]. Accumulation of solutes started 6500 YBP and lasted till 4500 YBP, when the next drought resulted in “rapid” transition of the lakes to playa conditions, surface deflation, and loss of salts over the period 2300–4500 YBP. This phenomenon could be described by adding a deflation rate term in equation (1) over this period. Therefore, the starting moment of evaporating the lake water was reset to 2300 YBP, although some solutes could remain from the previous accumulation period. The following salt re-accumulation during the next pluvial period between 2300 and 1000 YBP could be interrupted by renewed Eolian activity 700–1000 YBP and repeated salt losses.

[41] An alternative mechanism to severe droughts is slow Eolian deflation of salt crust from the lake margins. It was originally hypothesized by Langbein [1961] and others [e.g., Bowler, 1986; Teller and Last, 1990; Nimick, 1997] and received new attention and quantification for sabkhas [Wood and Sanford, 1995] and playas [Gill, 1996; Wood, 2002]. Windborn salt “clouds” from such lakes are frequently observed [Langbein, 1961; Gosselin, 1997; Nimick, 1997]. Recently, technological advancements showed the importance of playas in generating salt dust on a world scale [e.g., Prospero et al., 2002]. Our observations show airborne dust from salt crust emerging annually at the saline lake margins in the Nebraska Sand Hills [Ong et al., 2009]. Continuous deflation dramatically affects the solute balance in conducive conditions.

[42] Observed low modern concentrations in lakes (\( \approx 100 \text{ g L}^{-1} \) and typically less) indicate that both processes can operate separately or simultaneously. Differentiation between rapid and slow Eolian pathways of salts losses and coupling of transport processes for salt in solute form (groundwater) and dust (from lakebed sediments) are a matter for future studies.

4.5. Role of Free Convection

[43] Criterion (22) to estimate the possibility of free convection as a mechanism of solute removal from the lake can be applied to the modern data for the Nebraska Sand Hills conditions.

Figure 7. Lake concentration dynamics for the regional Sand Hills conditions.
Hills: \( q = 1.3 \times 10^{-5} \text{ m s}^{-1} \) (or 0.5 m/yr) and \( K \) is in the range from \( 1.3 \times 10^{-7} \) to \( 1.3 \times 10^{-6} \text{ m s}^{-1} \) (from 0.01 to 0.1 m d\(^{-1}\)). \( C_t \) must reach or exceed a critical concentration \( C_{\text{max}} \) to have just a possibility (not certainty) of free convection:

\[
C_C = 8.3 \times 10^9 \frac{q}{K} \approx 8.3 \times 10^1 \frac{1.3 \times 10^{-8}}{1.3 \times 10^{-4}} = 83 \text{ g L}^{-1}.
\]

(24)

Here we applied the highest measured \( K \) values [Ong and Zlotnik, 2010] to obtain the lowest values of critical concentration that are most favorable for generating free convection.

[44] Free convection would be common only if the lakebed would have a much higher \( K \sim 1 \text{ m/d} \) \((\sim 10^{-3} \text{ m s}^{-1})\), which is comparable with \( K \) at surrounding dunes [see Zlotnik et al., 2007], equation (24) then yielding \( C_C = 8.3 \text{ g L}^{-1} \). However, such high \( K \) values are inconsistent with data for lacustrian sediments in the Nebraska Sand Hills but which may exist in sabkhas [Van Dam et al., 2009] and marine environments [Stevens et al., 2009].

[45] Lake concentrations in the modern lakes of the Nebraska Sand Hills rarely exceed 100 g L\(^{-1}\) and only over relatively short seasons and in small-volume drying pools, which are conduits for solute losses, and solute mass in these pools hardly can result in extensive spread of fingers. Therefore, free convection may play a marginal role in the lake salt balance, but its detection in the Sand Hills is extremely difficult.

5. Summary

[46] A new conceptual transient coupled lake-aquifer model yields salinity concentrations and gradients beneath a closed lake. This model is valid for time scales exceeding a decade. We neglect seasonal variations of lake salinity by considering processes that operate on much longer time scales comparable with climate changes. The advantages of this model include (1) presentation of lake dynamics on centennial and millennial scales in a quantitative fashion compared to just qualitative assessment of trends and (2) a simple analytical form of the solution. This also allows for determining time scales of reaching solute saturation or critical TDS for instability and free convection of lake water. However, this model considers just total solute mass in the lake and disregards geochemical evolution of lake water that may introduce some bias at high solute concentrations and is designed largely for lakes with consistent inseepage of groundwater. An important feature of this model is that lake salinity is a variable, which is calculated with consideration of climatic conditions in the process of model solution instead of being imposed on the lake-aquifer interface as was done previously [e.g., Wood and Sanford, 1995; Wooding et al., 1997; van Duijn et al., 2002].

[47] Advection of groundwater solutes and lake evaporation is the driving mechanism leading to lake salinity increase. It is shown that mechanical dispersion flux in groundwater, countering the advection flux with lake salinity increase, is too weak and insufficient to significantly slow down the lake salinization. Therefore, the linear salinity model is a good approximation for discharge lakes in the given climate conditions.

[48] The lake salinity model is limited by the mineral precipitation at solute saturation or free convection, which ever occurs first. An available stability criterion was reformulated in terms of field variables (lake concentration, lakebed hydraulic conductivity, and Darcy’s inseepage velocity) and was used for establishing a concentration for the onset of free convection.

[49] This model is illustrated by data from the Nebraska Sand Hills, USA, the largest vegetated dune field in the Western Hemisphere. Using existing lakebed hydraulic conductivity values, modern climate data, and salinity ranges, hydraulic estimates indicate that over the last 6500 years, lakes should collect more solutes than we observe in modern conditions.

[50] Assuming that undetected flow-through processes are absent, the model indicates several other processes acting sequentially or simultaneously that can result in an observed solute deficit:

[51] (1) Severe mega droughts that occurred at 700–1000 and 2300–4500 YBP could interrupt the steady hydrologic patterns that sustained lakes and solute accumulation.

[52] (2) Average inseepage was lower over the last 6500 years than in modern conditions.

[53] (3) Eolian deflation of the salt at the lake surface might occur due to seasonal fluctuations even in the modern, relatively wet climate (Langbein’s hypothesis).

[54] (4) Incidental free convection could be locally active over the spatially sparse and temporally nonsystematic conditions.

[55] Integration of these processes in a single model is a task for future studies of shallow lakes in dune environments in semiarid climate.

Appendix A: Laplace Transform Solutions

[56] We introduce an increase of aquifer concentration over the background: \( C = C_A - C_0 \). Then

\[
\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial z^2} - Pe \frac{\partial C}{\partial z} \quad \text{for} \quad -\infty < z < 0, \quad t > 0
\]

(A1)

\[
\frac{\partial C}{\partial t} = -n \frac{\partial C}{\partial z} + n Pe(C + C_0) \quad \text{for} \quad z = 0, \quad t > 0
\]

(A2)

\[
C(-\infty, t) = 0
\]

(A3)

\[
C(z, 0) = 0, \quad -\infty < z < 0.
\]

(A4)

With \( \mathcal{C}(\tau, p) \) being the Laplace transform of \( C(z, \tau) \), we obtain

\[
\mathcal{C}(\tau, p) = \frac{C_0}{p} + \frac{n Pe}{p - n a + n \sqrt{p + a^2}} \exp \left[ 2(a + \sqrt{p + a^2}) \right].
\]

(A5)

To obtain an explicit solution, we use the shift \( s = p + a^2 \) and present the expression in partial fractions

\[
\mathcal{C}(\tau, s) = \frac{C_0 n Pe}{n + Pe} \mathcal{E}_s \left( \frac{C_s}{s - a^2} \right) \left[ \frac{1}{\sqrt{s - a}} - \frac{1}{\sqrt{s + b}} \right],
\]

(A6)
where \( a = \frac{Pe}{2} \) and \( b = n + \frac{Pe}{2} \). With the aid of Laplace transform tables in the studies by Carslaw and Jaeger [1959, p. 495] and Van Genuchten and Alves [1982, p. 104], the inverse of \( \hat{C}(\tau, s) \) multiplied by \( e^{-as^2} \) due to the s-shift produces the expression for \( \hat{C}(\tau, T) \) and then \( C_A(\tau, T) = C(\tau, T) + C_0 \) as given above. Results for \( \partial C_A(\tau, T)/\partial \tau \) and \( \partial C_A(\tau, T)/\partial T \) follow from a similar technique.

[57] The asymptotic small time behavior of various characteristics at the aquifer surface is obtained by inversion of the \( \hat{C}(0, p) \) expansion for large \( p \) that leads to expressions

\[
C_A(0, T) = C_L(\tau) + C_0 + n Pe C_0 \tau + O(\tau^{3/2}) \tag{A7}
\]

\[
\frac{\partial C_A(0, T)}{\partial \tau} = \left[ 2 \frac{1}{\sqrt{\pi}} - n \left( \frac{1}{\frac{Pe}{2}} \right) \right] n Pe C_0 + O(\tau^{3/2}) \tag{A8}
\]

\[
\frac{\partial C_A(0, T)}{\partial T} = \left[ 1 - 2n \frac{1}{\sqrt{\pi}} + n \left( \frac{1}{\frac{Pe}{2}} \right) \right] n Pe C_0 + O(\tau^{3/2}) \tag{A9}
\]

Expressions (A6)–(A8) substituted in the mass balance equation (1) satisfy it asymptotically with accuracy \( O(\tau^{7/2}) \). In addition,

\[
q_3(\tau) = q_C + O(\tau^{1/2}). \tag{A10}
\]

[58] The asymptotic large time behavior of various characteristics at the aquifer surface is obtained by inversion of \( \hat{C}(0, p) \) for a small \( p \) expansion.

[59] Acknowledgments. This material is based upon work supported by the National Science Foundation under grant EAR-0609982 and a fellowship from Center for Coastal and Catchment Environments Flinders University, South Australia, to VAZ and CTS in 2008. We thank D. Loope (UNL); L. Konikow, S. Ward (both USGS); and W. Wood (Michigan State University, formerly USGS) for fruitful discussions at different stages of this study and review and J.B. Ong (UNL) and T. Halihan (OSU) for technical assistance. Reviews from two additional anonymous reviewers helped to improve quality of the article. M. Koepsel and M. French of Crescent Lake National Wildlife Refuge, provided assistance with logistics of data collection for this study.

References


Born, S. M., S. A. Smith, and D. A. Stephenson (1979), Hydrogeology of glacial terrain lakes, with management and planning applications, J. Hydrol., 43, 7–43.


Miao, X., J. A. Mason, J. B. Swinehart, D. B. Loope, P. R. Hanson, R. J. Gaelle, and X. Liu (2007), A 10,000 year record of dune activity, dust storms, and severe drought in the central Great Plains, Geology, 35(2), 119–122.


N. I. Robinson and C. T. Simmons, School of the Environment and National Centre for Groundwater Research and Training, Flinders University, GPO Box 2100, Adelaide, SA 5001, Australia. V. A. Zlotnik, Department of Earth and Atmospheric Sciences, University of Nebraska–Lincoln, Lincoln, NE 68588-0340, USA. (vzlotnik1@unl.edu)