# Theory of spin loss at metallic interfaces

## SUPPLEMENTAL MATERIAL

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### I. CIRCUIT THEORY IN THE PRESENCE OF SPIN-FLIP SCATTERING

Consider two metallic nodes separated by a scattering region. The current in each node depends on the potential drop and on the spin accumulation drop between the nodes. The current evaluated for node 2 is [1]

$$\hat{I}_2 = G_0 \sum_{nm} \left[ \hat{t}'_{mn} \hat{f}_1 (\hat{t}'_{nm})^\dagger - \Big( M_2 \hat{f}_2 - \hat{r}_{mn} \hat{f}_2 (\hat{r}_{nm})^\dagger \Big) \right], \tag{1}$$

where $G_0 = e^2/h$, $\hat{r}_{mn}$ is the spin-dependent reflection amplitude for electrons reflected from channel $n$ into channel $m$ in node 2, and $\hat{t}'_{mn}$ is the spin-dependent transmission amplitude for electrons transmitted from channel $n$ in node 1 into channel $m$ in node 2. Note that the ensuing results can be easily rewritten for the current $\hat{I}_1$ in node 1. Spin-flip scattering at the interface makes the matrices $\hat{r}_{mn}$ and $\hat{t}'_{mn}$ non-diagonal in spin space.

Let us introduce a matrix:

$$\check{S}_{mn} = \begin{pmatrix} \hat{r}_{mn} & \hat{t}'_{mn} \\ \hat{t}_{mn} & \hat{r}'_{mn} \end{pmatrix}, \tag{2}$$

where $\hat{r}'$ and $\hat{t}$ are the amplitudes of reflection and transmission into node 1. Charge conservation requires $\check{S}\check{S}^\dagger = \check{1}$, and, therefore,

$$\sum_{mn} \check{S}_{mn} \check{S}^\dagger_{mn} = \check{M} = \hat{\sigma}^0 \otimes \hat{M}, \tag{3}$$

where $\hat{\sigma}^0$ is a unit matrix in spin space, the symbol $\otimes$ denotes the Kronecker product, and $\hat{M}$ is a diagonal matrix with elements $M_{ii} = M_i$ representing the number of channels in electrode $i$. We extract only part of Eq. (3) that contains $\hat{r}_{mn}$ and $\hat{t}'_{mn}$ coefficients:

$$\sum_{mn} \hat{r}_{mn} \hat{r}^\dagger_{mn} + \hat{t}'_{mn} (\hat{t}'_{mn})^\dagger = M_2 \hat{\sigma}^0 \tag{4}$$

leading to three independent constraints on the elements of the $S$ matrix. If the system has time reversal symmetry, the total $S$ matrix also satisfies $S = S^T$.

The spin-dependent distribution functions in nodes 1 and 2, as well as the current matrix, can be expressed via the Pauli matrices $\hat{\sigma}^1$, $\hat{\sigma}^2$, $\hat{\sigma}^3$ and the unit matrix $\hat{\sigma}^0$: $\hat{f}_1 = \hat{\sigma}^0 f_1^0 + \hat{\sigma} \bar{f}_1^s$, $\hat{f}_2 = \hat{\sigma}^0 f_2^0 + \hat{\sigma} \bar{f}_2^s$, $\hat{I} = (\hat{\sigma}^0 I^0 + \hat{\sigma} \bar{I}^s)/2$. We express the scattering amplitudes with the help of notations proposed in Ref. [2]. Denoting the unit matrix as $\hat{\sigma}^0$, we define $\mathcal{R}^{\mu\nu}_{mn} = \mathrm{Tr}[(\hat{r}_{mn} \otimes \hat{r}^*_{mn}) \cdot (\hat{\sigma}^\mu \otimes \hat{\sigma}^\nu)]/4$ and $\mathcal{T}^{\mu\nu}_{mn} = \mathrm{Tr}[(\hat{t}'_{mn} \otimes \hat{t}'^*_{mn}) \cdot (\hat{\sigma}^\mu \otimes \hat{\sigma}^\nu)]/4$.

The circuit theory expression (1) can now be rewritten in the form of Eqs. (4)-(5) of the main text, with the following definitions of the conductances:

$$G = 2G_0 \sum_{mn} \mathcal{T}^{\nu\nu}_{mn}, \quad G^s_i = 2G_0 \sum_{mn} (\mathcal{T}^{i0}_{mn} + \mathcal{T}^{0i}_{mn} + i\varepsilon_{ijk} \mathcal{T}^{jk}_{mn}), \tag{5}$$

$$G^t_i = 4G_0 \sum_{mn} i\varepsilon_{ijk} \mathcal{T}^{jk}_{mn}, \quad G^r_i = 4G_0 \sum_{mn} i\varepsilon_{ijk} \mathcal{R}^{jk}_{mn}, \tag{6}$$

$$\mathcal{G}^t_{ij} = 2G_0 \delta^{kl}_{ij} \sum_{mn} (\mathcal{T}^{kl}_{mn} + \mathcal{T}^{lk}_{mn} + i\varepsilon_{klv} [\mathcal{T}^{0v}_{mn} - \mathcal{T}^{v0}_{mn}]), \tag{7}$$

$$\mathcal{G}^r_{ij} = 2G_0 \delta^{kl}_{ij} \sum_{mn} (\mathcal{R}^{kl}_{mn} + \mathcal{R}^{lk}_{mn} + i\varepsilon_{klv} [\mathcal{R}^{0v}_{mn} - \mathcal{R}^{v0}_{mn}]), \tag{8}$$

where $\delta_{ij}^{kl} = \delta_{ik}\delta_{jl} - \delta_{ij}\delta_{kl}$, and summation over the repeated indices is assumed everywhere.

In the case of a non-magnetic (disordered) interface with axial symmetry, $\bar{G}^s = \bar{G}^t = \bar{G}^r = 0$, while the tensors $\hat{\mathcal{G}}^t$ and $\hat{\mathcal{G}}^r$ are diagonal in the reference frame aligned with the symmetry axis. These simplifications lead to the following expressions for the currents in the nodes:

$$I_0 = G\Delta f_0, \tag{9}$$

$$\bar{I}_2^s = (G - \mathcal{G}^t)\Delta \bar{f}_s - \mathcal{G}_2^{sl} \bar{f}_2^s, \tag{10}$$

$$\bar{I}_1^s = (G - \mathcal{G}^t)\Delta \bar{f}_s + \mathcal{G}_1^{sl} \bar{f}_2^s, \tag{11}$$

where we introduced the spin-loss conductance $\mathcal{G}_{1(2)}^{sl} = \mathcal{G}_{1(2)}^r + \mathcal{G}^t$ calculated along one of the symmetry axes in the nodes.

## II. RENORMALIZATIONS FOR OHMIC CONTACTS

It is well known that interface resistances in transparent Ohmic contacts are renormalized by the Sharvin resistance [3, 4]. The circuit theory can be generalized to account for the drift contributions in the nodes by renormalizing the conductances $G$, $\mathcal{G}^t$, and $\mathcal{G}_{1(2)}^{sl}$. This can be done by connecting nodes 1 and 2 to proper reservoirs with spin-dependent distribution functions $\hat{f}_L$ and $\hat{f}_R$ via transparent contacts. The currents in the nodes then become $\hat{I}_1 = 2G_0M_1(\hat{f}_L - \hat{f}_1)$ and $\hat{I}_2 = 2G_0M_2(\hat{f}_2 - \hat{f}_R)$, where $M_{1(2)}$ describe the number of channels in the nodes. Substituting these currents in Eqs. (9), (10), and (11), we arrive at the amended circuit theory:

$$I_0 = G(\Delta f_0 + \frac{I_0}{4G_0M_1} + \frac{I_0}{4G_0M_2}), \tag{12}$$

$$I_1^s = (G - \mathcal{G}^t)(\Delta f_s + \frac{I_1^s}{4G_0M_1} + \frac{I_2^s}{4G_0M_2}) + \mathcal{G}_1^{sl}(f_s^1 + \frac{I_1^s}{4G_0M_1}), \tag{13}$$

$$I_2^s = (G - \mathcal{G}^t)(\Delta f_s + \frac{I_1^s}{4G_0M_1} + \frac{I_2^s}{4G_0M_2}) - \mathcal{G}_2^{sl}(f_s^2 - \frac{I_2^s}{4G_0M_2}). \tag{14}$$

These equations are equivalent to Eqs. (9)-(11) after the substitution $G \to \tilde{G}$, $\mathcal{G}^t \to \tilde{\mathcal{G}}^t$, and $\mathcal{G}_{1(2)}^{sl} \to \tilde{\mathcal{G}}_{1(2)}^{sl}$, where

$$\frac{2}{\tilde{G}} = \frac{2}{G} - \frac{1}{2G_0M_1} - \frac{1}{2G_0M_2}, \tag{15}$$

$$\frac{2}{\tilde{G} - \tilde{\mathcal{G}}^t + \frac{\tilde{\mathcal{G}}_1^{sl}\tilde{\mathcal{G}}_2^{sl}}{\tilde{\mathcal{G}}_1^{sl} + \tilde{\mathcal{G}}_2^{sl}}} = \frac{2}{G - \mathcal{G}^t + \frac{\mathcal{G}_1^{sl}\mathcal{G}_2^{sl}}{\mathcal{G}_1^{sl} + \mathcal{G}_2^{sl}}} - \frac{1}{2G_0M_1} - \frac{1}{2G_0M_2}, \tag{16}$$

$$\frac{1}{\tilde{\mathcal{G}}_1^{sl}} = \frac{1}{\mathcal{G}_1^{sl}} - \frac{1}{2G_0M_1} - \frac{\mathcal{G}_2^{sl}/\mathcal{G}_1^{sl} - M_2/M_1}{\mathcal{G}_1^{sl} + \mathcal{G}_2^{sl} + 2\mathcal{G}_1^{sl}\frac{\mathcal{G}_2^{sl} - 2G_0M_2}{G - \mathcal{G}^t}}, \tag{17}$$

$$\frac{1}{\tilde{\mathcal{G}}_2^{sl}} = \frac{1}{\mathcal{G}_2^{sl}} - \frac{1}{2G_0M_2} - \frac{\mathcal{G}_1^{sl}/\mathcal{G}_2^{sl} - M_1/M_2}{\mathcal{G}_1^{sl} + \mathcal{G}_2^{sl} + 2\mathcal{G}_2^{sl}\frac{\mathcal{G}_1^{sl} - 2G_0M_1}{G - \mathcal{G}^t}}. \tag{18}$$

Note that these equations can be further simplified in the symmetric case, $\mathcal{G}_1^{sl} = \mathcal{G}_2^{sl}$ and $M_1 = M_2$.

## III. TRANSPORT IN $N_1|N_2$ SUPERLATTICE

We now assume that we have a superlattice constructed out of repeated interfaces between two normal metals $N_1$ and $N_2$. We take nodes in both $N_1$ and $N_2$ layers, and the conductances $\tilde{G}$, $\tilde{\mathcal{G}}^t$, $\tilde{\mathcal{G}}_1^{sl}$, and $\tilde{\mathcal{G}}_2^{sl}$ describe the two nodes. We arrive at the following equations for the spin current in node $i$:

$$I_i^s = (\tilde{G} - \tilde{\mathcal{G}}^t)(f_{i-1}^s - f_i^s) - \tilde{\mathcal{G}}_1^{sl} f_i^s, \tag{19}$$

$$I_i^s = (\tilde{G} - \tilde{\mathcal{G}}^t)(f_i^s - f_{i+1}^s) + \tilde{\mathcal{G}}_2^{sl} f_i^s, \tag{20}$$

which leads to the recursive formula:

$$\frac{2\tilde{\mathcal{G}}^{sl}}{\tilde{G}-\tilde{\mathcal{G}}^{t}} f_i^s = f_{i-1}^s - 2f_i^s + f_{i+1}^s, \tag{21}$$

where $\tilde{\mathcal{G}}^{sl} = (\tilde{\mathcal{G}}_1^{sl} + \tilde{\mathcal{G}}_2^{sl})/2$. This recursive equation has the following solution:

$$f_s^i = C_1 e^{\delta i} + C_2 e^{-\delta i}, \tag{22}$$

where

$$\delta = \ln\left[1 + \frac{\tilde{\mathcal{G}}^{sl}}{\tilde{G}-\tilde{\mathcal{G}}^{t}}\left(1+\sqrt{1+\frac{2(\tilde{G}-\tilde{\mathcal{G}}^{t})}{\tilde{\mathcal{G}}^{sl}}}\right)\right], \tag{23}$$

and the constants $C_1$ and $C_2$ depend on the boundary conditions.

## IV. ACCOUNTING FOR THE BULK CONTRIBUTION

Within the circuit theory, spin transport across a non-magnetic interface that is axially symmetric (either microscopically or after averaging over crystallite orientations) is fully characterized by four conductances: $\tilde{G}$, $\tilde{\mathcal{G}}^t$, $\tilde{\mathcal{G}}_1^{sl}$, and $\tilde{\mathcal{G}}_2^{sl}$. We will also refer to the quantities $\tilde{\mathcal{G}}^s = \tilde{G} - \tilde{\mathcal{G}}^t$, which appear in Eqs. (19)-(20), as spin conductances. In the main text of the paper we have neglected the resistivities of the bulk metallic layers and assumed that spin relaxation occurs only at the interfaces, in order to simplify the resulting expressions. These features can be restored by placing the circuit nodes in the middle of the bulk layers. A contact between two nodes is then defined to include both the physical interface and the adjacent bulk regions extending up to these nodes, as shown in Fig. S1.

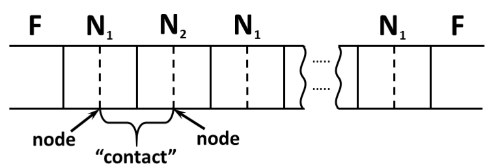


FIG. S1. Partitioning of the multilayer in nodes and contacts.

Spin-transport in a bulk diffusive region $i$ is assumed to obey the Valet-Fert model, which yields $\tilde{\mathcal{G}}_{bi}^s = \tilde{G}_{bi}\delta_i/\sinh\delta_i$ and $\tilde{\mathcal{G}}_{bi}^{sl} = \tilde{G}_{bi}\delta_i\tanh(\delta_i/2)$, where $\delta_i = t_i/l_{sf}^i$ is defined similar to the spin-memory loss parameter for an interface. We have added a subscript $b$ to distinguish bulk and interface conductances in the following. There is only one $\tilde{\mathcal{G}}_{bi}^{sl}$ parameter, because a bulk region is left-right symmetric. Thus, two parameters $\tilde{G}_{bi}$ and $\delta_i$ completely describe a diffusive bulk layer. (A general interface can not be fully described in this way, because four independent conductances can not be reduced to two parameters $\tilde{G}$ and $\delta$.)

Introducing the conductances $\tilde{G}_a$, $\tilde{\mathcal{G}}_a^s$, $\tilde{\mathcal{G}}_{a1}^{sl}$, and $\tilde{\mathcal{G}}_{a2}^{sl}$ for the composite three-layer "contact," we can apply Eq. (9) from the main text to obtain

$$\mathcal{D}^2 = \frac{\tilde{\mathcal{G}}_{a1}^{sl} + \tilde{\mathcal{G}}_{a2}^{sl}}{\tilde{\mathcal{G}}_a^s}, \tag{24}$$

which now fully takes into account the bulk contributions. The composite conductances can be obtained by concatenating the interface with the adjacent bulk regions using the circuit theory:

$$\tilde{\mathcal{G}}_a^s = \frac{\tilde{\mathcal{G}}_{b1}^s\tilde{\mathcal{G}}_{b2}^s\tilde{\mathcal{G}}^s}{(\tilde{\mathcal{G}}_{b1}^s+\tilde{\mathcal{G}}_{c1}^{sl})(\tilde{\mathcal{G}}_{b2}^s+\tilde{\mathcal{G}}_{c2}^{sl}) + (\tilde{\mathcal{G}}_{b1}^s+\tilde{\mathcal{G}}_{b2}^s+\tilde{\mathcal{G}}_{c1}^{sl}+\tilde{\mathcal{G}}_{c2}^{sl})\tilde{\mathcal{G}}^s}, \tag{25}$$

$$\tilde{\mathcal{G}}_{a1}^{sl} = \frac{(\tilde{\mathcal{G}}_{b1}^s+\tilde{\mathcal{G}}_{c1}^{sl})[\tilde{\mathcal{G}}_{b2}^s(\tilde{\mathcal{G}}_{c2}^{sl}+\tilde{\mathcal{G}}_{b2}^{sl}) + \tilde{\mathcal{G}}_{b2}^{sl}\tilde{\mathcal{G}}_{c2}^{sl}] + [(\tilde{\mathcal{G}}_{b1}^s+\tilde{\mathcal{G}}_{c1}^{sl}+\tilde{\mathcal{G}}_{c2}^{sl})\tilde{\mathcal{G}}_{b2}^{sl} + \tilde{\mathcal{G}}_{b2}^s(\tilde{\mathcal{G}}_{c1}^{sl}+\tilde{\mathcal{G}}_{c2}^{sl}+\tilde{\mathcal{G}}_{b2}^{sl})]\tilde{\mathcal{G}}^s}{(\tilde{\mathcal{G}}_{b1}^s+\tilde{\mathcal{G}}_{c1}^{sl})(\tilde{\mathcal{G}}_{b2}^s+\tilde{\mathcal{G}}_{c2}^{sl}) + (\tilde{\mathcal{G}}_{b1}^s+\tilde{\mathcal{G}}_{b2}^s+\tilde{\mathcal{G}}_{c1}^{sl}+\tilde{\mathcal{G}}_{c2}^{sl})\tilde{\mathcal{G}}^s}, \tag{26}$$

where $\tilde{\mathcal{G}}_{ci}^{sl} = \tilde{\mathcal{G}}_{bi}^{sl} + \tilde{\mathcal{G}}_i^{sl}$. The expression for $\tilde{\mathcal{G}}_{a2}^{sl}$ is obtained from $\tilde{\mathcal{G}}_{a1}^{sl}$ by interchanging the indices 1 and 2. We also have $\tilde{G}_a^{-1} = \tilde{G}^{-1} + \tilde{G}_{b1}^{-1} + \tilde{G}_{b2}^{-1}$.

Expanding of Eq. (24) to first order in spin-flip scattering results in

$$\mathcal{D}^2 \approx \frac{\mathcal{G}_1^{sl} + \mathcal{G}_2^{sl} + 2\mathcal{G}_{b1}^{sl} + 2\mathcal{G}_{b2}^{sl}}{\tilde{G}_a}, \tag{27}$$

Equation (27) shows that to lowest order in spin-flip scattering there are only two relevant parameters for the interface in a periodic $N_1/N_2$ multilayer with diffusive layers: its renormalized conductance $\tilde{G}$ and the symmetric spin-loss conductance $\mathcal{G}^{sl} = (\mathcal{G}_1^{sl} + \mathcal{G}_2^{sl})/2$. Under these conditions, the treatment based on the Valet-Fert model, with $\delta$ given by Eq. (8) of the main text, gives the same result as the full circuit theory. This justifies our treatment in the main text, where the correspondence with the Valet-Fert model was established for a multilayer with vanishing bulk resistance and spin relaxation.

Higher-order correction to $\mathcal{D}$ is always positive, which means that we have slightly overestimated $\delta$. However, this correction is very small for the Cu/Pd interface; for $\delta = 0.4$ and typical parameters for bulk Pd [5] the correction to $\delta^2$ is less than 0.01. The correction may, however, be significant for interface with strong spin-flip scattering, such as Cu/Pt with $\delta \sim 1$ [5].

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