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SUBJECTIVE IMPRESSION OF DISCOMFORT GLARE FROM SOURCES OF NON-UNIFORM LUMINANCE

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SUBJECTIVE IMPRESSION OF DISCOMFORT GLARE FROM SOURCES OF NON-UNIFORM LUMINANCE

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The intent of this study was to further investigate the effects of spatial frequency and position on discomfort glare. Most of the discomfort research in the past has used sources of uniform luminance, so not much is known about how non-uniformity affects the perception of glare.

An apparatus was designed and built specifically for this study, but it was also designed to have significant flexibility for future work. Two different experiments were performed with this apparatus: a paired comparison experiment; and, a rating scale experiment. For both experiments, 6 levels of spatial frequency and 4 levels of position were studied.

The results show that both spatial frequency and position are significant predictors of discomfort glare, as is the interaction between the two. As spatial frequency increases, discomfort increases. As position increases, discomfort decreases. Spatial frequency affects discomfort more at positions close to the line of sight than at positions far from the line of sight.

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Chapter 1 - Introduction

Glare is defined by the Illuminating Engineering Society of North America (IESNA) as "the sensation produced by luminance within the visual field that is sufficiently greater than the luminance to which the eyes are adapted to cause annoyance, discomfort or loss in visual performance and visibility" (Rea 2000). This overall sensation of glare is then subdivided into two types: disability glare and discomfort glare. Disability glare is "the effect of stray light in the eye whereby visibility and visual performance are reduced" (Rea 2000). Discomfort glare is "glare that produces discomfort. It does not necessarily interfere with visual performance or visibility" (Rea 2000).

Within the lighting community, there has been a great amount of research done in the field of discomfort glare. The psychophysical reason that we experience discomfort glare is not very well understood, although many have attempted to determine the fundamental cause. We do, however, know that the major factors that affect our perception of discomfort glare are as follows: the luminance of the light source, the luminance of the background (with or without the glare source), the visual size of the light source, and the relative position of the light source (in relation to the observer's point of fixation) (Poulton 1991). As the luminance of the glare source increases, the sensation of discomfort increases. As the background luminance increases, the sensation of discomfort glare decreases. As the size of the glare source increases, the discomfort glare sensation increases until the source itself significantly influences the background luminance, which reduces discomfort glare. As the angle between the point of fixation

(on-axis) and the luminaire increases, the sensation of discomfort glare decreases. Even though these factors are widely accepted as factors affecting discomfort glare, there are still several different glare evaluation systems. These systems differ in the coefficients and exponents that are applied to the individual factors (e.g. size, luminance, and position). In addition, the different models vary in the methods used to address multiple glare sources, and whether or not the glare source itself is considered to be part of the background luminance.

The impetus behind the development of the traditional discomfort glare models was the development of large area sources, specifically the fluorescent lamp. This lamp is much more energy efficient than the incandescent lamp that preceded it. These lamps were first used in luminaires without lenses (with a direct view of the lamp), then in luminaires with diffusing or prismatic lenses. The lighted environment today uses a wide variety of fluorescent as well as other sources in direct and indirect luminaires with all types of shielding media.

It is imperative that the lighting community has a method of quantifying the level of discomfort felt by observers, so that new lighting systems can be evaluated to ensure discomfort is minimized. If such a method does not exist, the lighting community will not have a good metric for comparing different lighting installations, and no means of intelligently discussing discomfort glare with one another. If the method is not used, there is no opportunity to improve our lit environments.

Currently, the IESNA Lighting Handbook 9th Edition includes direct glare as a design issue and does give a rule of thumb indicating that luminaire luminances should be

less than 10,000 cd/m^2 or should be not more than 100 times the luminance of the immediate background. While this is certainly good to have, it is simplifying the issue of discomfort glare down to only two factors, namely luminaire luminance and background luminance.

The handbook also proposes the use of Visual Comfort Probability (VCP) as the discomfort glare evaluation system of choice, although very few practitioners use it, as it has been tested and validated for lensed fluorescent troffers only (Rea 2000), which are luminaires with a relatively uniform luminance gradient, and which are not commonly used today. Many of the luminaires used today have a non-uniform luminance gradient; therefore, the VCP system has not been proven to be an effective metric for commonly-used luminaires.

Another popular glare metric, the CIE's Unified Glare Rating (UGR), also does not address sources of non-uniform luminance, although the CIE (2002) published extensions to the UGR, including a complex extension, which attempts to address this issue of non-uniformity. However, the document was not well referenced and was not supported by much research.

Mr. L. Bedocs of the UK urged the CIE to develop a metric, stating, "The CIE Guide must have glare limits. Therefore we must have one system upon which we agree. We cannot have recommendations without an agreed system" (Lofberg 1987). In response to a question about what the IESNA could do to promote glare evaluation, Boyce et al (2003) noted, ". . . what the IESNA should do is concentrate on developing a system that can be used to predict when a proposed lighting installation is likely to be

considered uncomfortable because of glare. The first step would be to identify a suitable metric for predicting discomfort glare. The most promising metric is the UGR formula of the CIE which is used in many countries. However, before introducing the UGR formula into practice it would be necessary to carry out a field validation trial to better establish the relationship between the UGR value of an installation and the percentage of people in North America finding it uncomfortable."

Therefore, the principle investigator (PI) believes research should be undertaken to validate the UGR with its extensions through laboratory-controlled experiments, then move to a field study to see if UGR could be accepted as the United States' discomfort glare metric. The purpose of the proposed direction is to determine if the UGR with the extensions accurately reflects human response to glare sensation. This study is the first step in that direction, which investigates the complex extension. This extension seems to be the most applicable to light fixtures currently used and would therefore have the biggest impact on the design community. The intent of this research is to better understand how humans respond to a complex source.

The PI believes that a complex source differs from a uniform source primarily in its luminance gradient. The frequency with which the gradient varies from luminaire to luminaire is the spatial frequency. The effect of spatial frequency on discomfort glare has been studied previously by Waters et al (1995). In that paper, four independent variables were investigated: the spatial frequency of the periodic gratings (2 levels – 1 cycle per degree and 4 cycles per degree); the modulation of the gratings (2 levels – 0.42 and 0.83); the gradient wave shape (2 levels – sine and square wave patterns); and, the position of the stimuli within the field of view (3 levels – 0, 10, and 20 degrees above the

line of sight). The experimenter simultaneously showed each subject a gradient stimulus and a uniform stimulus side by side. The subject was asked to identify which caused more discomfort, the left or the right stimulus, or if they were the same. The uniform stimulus was then changed, based on the subject's answer, to try to get the discomfort of the two stimuli to be equal, i.e. if the uniform source was considered more discomforting, a darker one (lower luminance) was chosen for the next trial. The investigator found that the effect of position on discomfort glare was significant, as was the interaction of position by spatial frequency. It was concluded that a non-uniform glare source is considered less discomforting in the periphery than a uniform source. It was also concluded that, in the periphery, low frequency gratings were considered less discomforting than higher frequency gratings. This previous research serves as the basis for the current research, which will look at many more levels of the variables of spatial frequency and position to attempt to expand on the results found by Waters et al.

The author understands that this direction of research, while very practical, will most likely not lead to a better understanding of the physical or psychological causes of discomfort glare. The research required to truly understand those causes is on a very different path from that suggested above. Both paths, however different, are certainly worthwhile. The former will be more quickly accomplished, and will therefore make a more immediate impact on the lighting community. The latter is a much more involved process of which intermediate steps will most likely not have much practical impact on the lighting community but will be of great importance to the lighting community when completed. Both types of research really must be undertaken to achieve the complete picture needed to truly understand discomfort glare, i.e. in the short term, a worldwide

system which predicts human response reasonably well; and, in the long term, an understanding of the human mechanisms that are at work in the perception of the glare sensation.

Some might argue that if a glare prediction system cannot perfectly predict discomfort glare, then why bother. The PI feels that it is better to have a system that describes glare reasonably well for most modern lighting systems than not to have a system at all. Having a good system in place that lighting designers can use as a tool will surely improve the lit environment – a worthy venture.

Dissertation Outline

This dissertation consists of four additional chapters: Chapter 2 is the Review of Literature; Chapter 3 is the Methodology section; Chapter 4 is the Results section; and, Chapter 5 is the Conclusion section.

Chapter 2 gives a detailed discussion of the research that has been performed involving discomfort glare. Primarily, that research has led to the development of glare metrics, which are outlined.

Chapter 3 outlines the methodology used for the experiments performed in this study. The apparatus that was designed and built for this experiment is described, along with the independent and dependent variables, a description of the stimuli used, and the subjects used for the experiments.

Chapter 4 discusses the data that was collected and how each set of data was analyzed.

Chapter 5 discusses the findings from the experiments and how they relate to the real world. There is also some discussion of future topics worthy of investigation.

The appendices include additional information vital to the document including an extensive bibliography.

Chapter 2 Review of Literature

The significant research on discomfort glare will be discussed in terms of the glare metric which was developed from that research. Four major glare metrics are discussed below, including the Visual Comfort Probability, the Unified Glare System, The British Glare Index, and the Glare Limiting System.

Visual Comfort Probability

Luckiesh and Holladay (1925) were the first to apply psychological appraisal to glare. They developed a scale of comfort-discomfort, or degrees of sensation, from a scarcely noticeable sensation to intolerable and painful sensations. Their study provided the background for the original comprehensive study that began the development of the VCP system, which was conducted by Luckiesh and Guth (1949). This study (Luckiesh and Guth 1949), fueled by the development of the fluorescent lamp, explored the effects of glare source luminance (adjusted by the subject, ranged from 315 to 1600 footlamberts), glare source size (maintained at 0.0011 steradian), background or field luminance (maintained at 10 footlamberts), and the position of the glare source within the visual field (maintained at on-axis viewing), on the metric "Borderline Between Comfort and Discomfort" (BCD).

The experimental environment in this study consisted of an extended visual field of uniform brightness created by two-thirds of an 80-inch photometric sphere with a lamp located near the center to provide uniform field luminance. Light sources mounted

behind circular openings in the sphere surface provided glare sources. Subjects were placed in the center of the sphere.

The experimental technique consisted of evaluating the sensation of the glare source when the source was momentarily exposed to view in the uniform luminance background. Short exposure to the glare source maintained an adaptation luminance as close to the background luminance as possible. The exposure cycle involved three one-second "on" periods, separated by one-second "off" periods, followed by a five-second "off" period. Subjects were permitted as many cycles as necessary to make an appraisal. To estimate BCD, fifty subjects adjusted the source luminance to their own criterion of BCD.

In subsequent experiments, the effects of field luminance (varied among 1, 10, and 100 footlamberts), source size (ranged from 0.0001 to 0.126 steradian), and source position (varied between 0 and 100 degrees from the line of sight, vertically, horizontally, and diagonally) were individually studied. These experiments tested ten subjects deemed representative of the fifty subjects used for the previous study. Multiple sources and linear sources were studied with only tentative conclusions, and additional experimental data was deemed necessary in these areas (Luckiesh and Guth 1949).

This study (Luckiesh and Guth 1949) generated a large amount of discussion. Questions were raised regarding the experience of the ten observers and why these observers were chosen from the original fifty. Discussion of continuous versus momentary exposure was deemed in need of further study. Also, the difference in human

response between laboratory and field measurements was also considered an important variable that warranted attention.

In a continuing series of investigations, Guth (1951) presented studies exploring the effects of a task, and continuous versus momentary exposure of the glare source on BCD. In the first series of experiments, the experimental environment was the same as for the previous studies (Luckiesh and Guth 1949), using the 80-inch diameter sphere, except for the addition of a test object that required resolution of a series of horizontal and parallel lines. A second series of experiments used a simulated visual environment. This simulated environment consisted of the details of a room projected upon a vertical plane five feet in front of the observer. Slots cut in the ceiling area, with lamps mounted behind, provided the glare sources.

No significant difference in BCD was found between continuous and momentary exposure when the glare source was above the line of vision. When a task was included, BCD was found at lower luminance levels (indicating more discomfort) within the region of zero degree to twenty degree above the line of vision and at higher luminance levels (indicating less discomfort) above twenty degree.

Guth and McNelis (1959) developed a discomfort glare evaluator for field evaluations to substantiate or modify conclusions derived from laboratory data. This evaluator permitted rating of complex combinations of sources in actual environments not possible in the laboratory. Using the momentary exposure methods of earlier studies, the data obtained with the evaluator showed a consistent relationship with laboratory BCD data.

Guth, again with McNelis (1961), presented further data on discomfort glare using a simulated environment of an earlier study (Guth 1951) and the BCD metric. They investigated visual discomfort with various combinations of luminous elements common to lighting systems of that day. The paper was considered a report of progress on the subject of discomfort glare and not a preferred method of evaluation.

Guth (1963) ultimately proposed a method for the evaluation of discomfort glare using data from his and others' past studies, performed with simulated rooms, scale model rooms, and actual rooms. His proposal of a formula for multiple sources or discretized larger sources was shown to relate directly to experimental subjective data. Other previously proposed summations of multiple glare sources (Einhorn 1961) were simpler but did not show the degree of discomfort involved. Guth presented a chart for converting the discomfort glare rating to a percentage of observers expected to judge a lighting condition at, or more comfortable than, BCD. He termed this metric discomfort glare estimate (DGE). Later, the term Visual Comfort Probability (VCP) was used for this metric (Bradley and Logan 1964).

Using the Guth studies above, the work of Bradley and Logan (1964), and additional experimental data by Allphin (1961), the IESNA RQQ committee prepared an "Outline of a standard procedure for computing Visual Comfort Ratings for Interior Lighting" (IESNA 1966). After additional experimental data by Allphin (1966, 1968) and work on computational procedures and their application (Guth 1966, McGowan & Guth 1969), a modified standard procedure for computing visual comfort ratings for interior lighting was put forth by the RQQ committee of the IESNA (1973). With the addition of calculation procedures (Levin 1973, 1975, DiLaura 1976) and methods for

simplifying the calculations (Fry 1976, Goodbar 1976), a recommended procedure including a section on simplified calculations was published in the IES Lighting Handbook 1984 Reference Volume (Kaufman 1984). The simplified procedure has not been included in subsequent editions of the handbook. The modified standard procedure (IESNA 1973) was reprinted as LM-42 with the addition of a preamble approved by the board of directors of IESNA in 1991.

Even though the IESNA officially recommends VCP, the handbook clearly highlights its limitations. “This system was tested and validated using lensed direct fluorescent systems only. VCP should not be applied to very small sources such as incandescent and high-intensity discharge luminaires, to very large sources such as ceiling and indirect systems, or to nonuniform sources such as parabolic reflectors” and “The procedure has never been proven to accurately model the discomfort caused by parabolic fluorescent luminaires, although many lighting professionals continue to apply it in such situations.” (Rea 2000). Given these restrictions, it could be argued that the IESNA does not in fact have a discomfort glare evaluation system that is at all applicable, or at best it has a system that is valid for only a very small percentage of new designs (those using lensed direct fluorescent systems only).

The basic VCP equation for a single glare source is:

$$Glare\ Sensation\ (M) = \frac{0.5\ L_s\ Q}{P\ F_v^{0.44}} \quad \textbf{(2.1)}$$

Where: L_S = Luminance of the glare source (cd/m^2)

$Q = 20.4\ \Omega_S + 1.52\ \Omega_S^{0.2} - 0.075$

Ω_S = Solid angle subtended at the eye by the glare source (steradians)

F_v = Average luminance of the entire field of view including the glare source (cd/m^2)

P = An index of the position of the glare source with respect to the line of sight calculated for any interior luminaire within the field of view, limited to 53º above a horizontal line of sight (IESNA 1973 preamble), as derived by Luckiesh and Guth, and given by the following:

$$P = exp\left[\left(35.2 - 0.31899\alpha - 1.22e^{-2\alpha/9}\right)10^{-3}\beta + (21 + 0.26667\alpha - 0.002963\alpha^2)10^{-5}\beta^2\right] \quad \textbf{(2.2)}$$

Where: α = Angle from vertical of the plane containing the source and the line of sight (degrees)

β = Angle between the line of sight and the line from the observer to the source (degrees)

The effect of multiple glare sources is considered by first calculating Discomfort Glare Rating (DGR), then converting to VCP (as described below). DGR is calculated using the following summation equation:

$$Discomfort\ Glare\ Rating\ (DGR) = \left(\sum_{i=1}^{n} M_i\right)^a \quad \textbf{(2.3)}$$

Where: M = Glare Sensation from the single source equation above

a = n$^{-0.0914}$

n = The number of glare sources in the field of view

Higher values of DGR represent higher glare. Once the DGR has been determined for an installation, it is converted to VCP either by a conversion chart or by the following relationship:

$$VCP = \frac{100}{\sqrt{2\pi}}\int_{-\infty}^{6374-1.3227\ln DGR} e^{-t^2/2}dt \quad \textbf{(2.4)}$$

The higher the value of VCP, the greater the percentage of people who would find the glare from the installation acceptable. The IESNA recommends a VCP of 70 or greater for office lighting installations and a VCP of 80 or greater for office lighting installations where computer terminals will be present, which is expected for most offices today (Rea 2000).

British Glare Index

Overlapping the research of Guth in the United States, Hopkinson of the United Kingdom started major work on discomfort glare in the late 1940's. This work (Petherbridge & Hopkinson 1950) is the basis for the British Glare Index. To define the magnitude of the discomfort sensation, Hopkinson used a four point semantic scale: just intolerable; just uncomfortable; just acceptable; and, just imperceptible. His apparatus consisted of a model made from black and white photographs of a classroom. The glare sources were created by holes in the photograph with lights behind. Adaptation luminance was provided by front illumination. For a number of different source luminances, subjects were asked to adjust the adaptation luminance so that the source appeared at one of the points on the semantic scale.

Einhorn (1979) observed that Hopkinson's basic research found that adaptation luminance outweighs the effect of source size in the determination of discomfort glare. This finding conflicted with the research of Sollner (1965) and Collins and Plant (1971), questioning the validity of Hopkinson's formulation.

Application and recommendations of the British Glare Index system were published in 1967 (IES-London) and revised in 1985 (CIBSE). These recommendations were part of the 1984 CIBSE Code on Interior Lighting (CIBSE 1984), and the 1997 version of the same document (CIBSE 1997). However, the 2002 Code now recommends the UGR (CIBSE 2002).

The basic British Glare Index formula for a single glare source is:

$$Glare\ Sensation\ (g) = \frac{0.9\ L_S^{1.6} \omega_S^{0.8}}{L_b\ P^{1.6}} \tag{2.5}$$

Where: L_S = Luminance of the glare source (cd/m^2)

ω_S = Solid angle subtended by the glare source at the eye (steradians)

L_b = Average luminance of the field of view excluding the glare source (cd/m^2)

P = An index of the position of the glare source with respect to the line of sight, as derived by Luckiesh and Guth (determined from a table of values based on the geometry of the situation, which includes sources up to 62º above the line of sight) (CIBSE 1985)

As with the VCP system, multiple glare sources are combined with a summation formula:

$$Glare\ Index\ (GI) = 10\ \log_{10}\left[0.5\ \left(\sum g\right)\right] \tag{2.6}$$

Where: g = the glare sensation of the individual glare sources found in the previous equation.

Different glare limits or ranges are specified for different working environments. The polarity of the scale in the British Glare Index system is opposite to that of the VCP system. Larger glare indices indicate more glare sensation.

Glare Limiting System

DeBoer (1958), of Germany, believing the summations of individual glare sources used in the VCP and British Glare Index systems to be inaccurate, proposed a different summation formula. Arndt, Bodmann and Muck (1959) studied the various summation formulas and found that none of them agreed with observations of multiple sources. DeBoer was convinced that a reliable system should be based on subjective appraisals of entire lighting installations, not summations of individual observations.

The investigations (Bodmann, Sollner & Senger 1966, Sollner 1965, Bodmann & Sollner 1965) used one-third scale models. Appraisals were made by ten to fifteen observers of approximately 850 glare situations using a seven point semantic scale: no glare; glare between nonexistent and slight; glare slight; glare between slight and severe; glare severe; glare between severe and intolerable; and, glare intolerable. From that research, the luminance curve method was proposed. Fischer (1972) approximated Sollner's system with a mathematical frame and transformed the luminance curve method to a glare limiting method. This glare limiting method specifies luminance limits for different quality classes of lighting situations that are part of the German lighting standard (Fischer 1991).

The glare limiting system is different from the VCP and British glare index systems in that there is no equation that defines the sensation of glare and the parameters influencing the glare sensation, but rather simply sets luminance limits. Consequently, it is restricted in use to finding whether the glare produced by a specific installation will fall above or below the glare threshold defined by the limiting curve.

Unified Glare Rating

The International Commission on Illumination (CIE) committee TC 3-4 developed the CIE Glare Index (CGI) in an attempt to combine the best points of the major discomfort glare evaluation systems including VCP, the British Glare Index, and the Glare Limiting System. This compromise, developed by Einhorn of South Africa (Einhorn 1979), is published in CIE Publication No. 55 (CIE 1983). The basic formula is as follows:

$$CIE\ Glare\ Index\ Formula\ (CGI) = 10\ \log_{10}\left[0.1\ \frac{1+E_d/500}{E_d+E_i}\sum\frac{L^2\omega}{p^2}\right] \quad \textbf{(2.7)}$$

Where: E_d = direct vertical illuminance at the eye from all sources (lux)

E_i = indirect illuminance at the eye (lux)

L = luminance of the luminous parts of each luminaire in the direction of the observer's eye (cd/m^2)

ω = solid angle of the luminous parts of each luminaire at the observer's eye (steradians)

p = Guth position index for each luminaire (displacement from the line of sight)

The indirect portion of illuminance at the eye, E_i represents the background luminance of the room surfaces, against which the glare sources are viewed. The direct portion, E_d, is the illuminance from the source(s). This direct portion from the CGI formula has since been excluded from the UGR formula, as the CIE could not find a way to include it in the simplified glare calculation procedures. "For practical purposes this has little effect when the formula is applied to rooms having illuminances within the usual range recommended for working interiors" (CIE 1995).

The quantity, $(1+E_d/500)/E_d+E_i$, allows for co-variance in the numerator and adaptation in the denominator. Co-variance means that glare will vary directly with direct illuminance at the eye (versus contra-variance which means glare will decrease as the illuminance at the eye increases (Einhorn 1979)). Adaptation allows for realism in very dark rooms where E_i would be very low, such that the UGR value does not become infinite (for example, glare is not infinite in a cave lit by a candle, where E_i would be almost zero (Poulton 1979)). In a 1989 CIE TC 3-13 session, this quantity was replaced with simply $1/L_b$ in the final UGR formula for the following reasons: "The computation of E_d requires tremendous computation time and effort. As omission of E_d does not result in significant loss of accuracy, it was decided to omit the same." And, "Covariance and adaptation are factors that need further investigation before they can be included into general usage." (Pai & Gulati 1995). By replacing the factor $(1+E_d/500)/(E_d+E_i)$ with simply $1/L_b$ (where L_b is the background luminance excluding glare sources), UGR does not explicitly allow for co-variance nor for the direct component of adaptation.

The two constants (the "10" and the "0.1") from the CGI formula, have since been changed in the UGR formula to be the best representation of the original formula proposed by Hopkinson. Incorporating these modifications into the CGI formula, the CIE has defined the UGR formula to be as follows:

$$CIE\ Unified\ Glare\ Rating\ (UGR) = \ 8\ \log_{10}\left[\frac{0.25}{L_b}\sum\frac{L^2\omega}{p^2}\right] \qquad \textbf{(2.8)}$$

Where: L_b = background luminance (cd/m^2)

L = luminance of the luminous parts of each luminaire in the direction of the observer's eye (cd/m^2)

ω = solid angle of the luminous parts of each luminaire at the observer's eye (steradians)

p = Guth position index for each luminaire (displacement from the line of sight)

The background luminance, L_b, is calculated here without the glare sources themselves. It is therefore defined as "that uniform luminance of the whole surroundings which produces the same illuminance on a vertical plane at the observer's eye as the visual field under consideration excluding the glare sources." It is given by the following formula, which considers a full hemisphere, not only what the eye sees:

$$L_b = \frac{E_i}{\pi} \tag{2.9}$$

The CIE (1995) points out that "The UGR is relatively insensitive to errors in L_b; for example, an error +33% in L_b will result in an error of the UGR of 1 unit."

The luminaire luminance, L, is given by the following formula:

$$L = \frac{I}{A_p} \tag{2.10}$$

Where: I = the luminous intensity of the luminaire in the direction of the observer (candela)

A_p = projected area of the luminaire (m^2)

The solid angle, ω, is given by the following formula:

$$\omega = \frac{A_p}{r^2} \tag{2.11}$$

Where: A_p = projected area of the luminous parts of the luminaire (m^2)

r = distance from the observer to the center of the luminous parts of the luminaire (m)

The position index is determined by interpolating from a table of values as given in CIE 1995. The table's two parameters are H/R and T/R, where R, T, and H are defined as the distance projected onto the line of sight, the horizontal offset from the line of sight, and the height above the eye, respectively. The CIE procedure stipulates that luminaires with values of T/R outside the range of 0 to 3 should be ignored, which means that luminaires to the left or right beyond 71º off the line of sight should not be included in the calculation. Similarly, luminaires with high H/R values (1.7 and above, approximately 60º above the point of fixation) should be ignored, as this indicates locations that would be hidden from view by eyebrows and foreheads. Recently, however, research has shown that luminaires in this area can be potential glare sources (Ngai & Boyce 2000). Incorporating this issue of overhead glare would be a minor modification to the UGR – that of simply increasing the allowable values of H/R and T/R to include positions above 60º. "From the results collected in this study, it is concluded that the approximate level of discomfort produced by a luminaire seen between 55 degrees from the line of sight and the edge of the visual field can be predicted using the Unified Glare Rating system" (Boyce et al 2003). Boyce et al used 75º above the line of sight to be the limit of the visual field in this study. They also found that sources above 75º can still create discomfort, if the luminances are high enough. They suggest that ". . . at these angles the luminaire is outside the field of view and any discomfort is caused by different mechanisms than those that operate when the luminaire is in the field of view, so the UGR formula does not apply." Additional research is required to understand what is causing the discomfort from these sources outside the field of view. When that is better understood, the glare formula may have to be reevaluated.

The UGR formula results in values between 10 and 30, with higher values suggesting higher glare (Sorensen 1991). A value of 20 or below would be considered reasonable for a typical office environment. UGR is intended to be an interval scale, where the differences between the values represent equally perceptible differences in psychological values. The UGR formula's constants, 8 and 0.25, were chosen to give values similar to the British Glare Index scale, where the smallest perceptible difference is equivalent to a change of one unit, and a scale of recommended values should have 3-unit steps (Lowson 1981).

The UGR formula, "combines features of the Einhorn and the Hopkinson formulae and incorporates the Guth position index. It may be regarded as being composed of the best parts of the major formulae. . ." (CIE 1995). It produces a glare rating which is intended to show subjective discomfort response to a visual environment.

Recent research in a full-scale simulated office and an actual office has shown that, for installations with a single glare source, the correlation of UGR values with subjective ratings was excellent at 0.96. With multiple glare sources, the correlation was still very high at 0.95, but the calculated values are consistently higher than the subjective ratings, suggesting that the UGR formula overestimates the glare sensation. Akashi et al (1996) suggest that a modification be made to the UGR formula to include the multiplication of the quantity $n^{-0.006}$ to account for that overestimate, where n is the number of glare sources. This is very similar to the VCP formula.

Additional research has confirmed this high correlation of UGR with subjective ratings. Boyce et al found a correlation of 0.96 when looking specifically at a single

luminaire at 55, 65, and 75 degrees above the line of sight, with varying luminaire luminance (Boyce et al 2003).

The CIE reported in 1995 that the UGR system was developed with sources that subtend a maximum of 0.1 steradian at the eye. They also caution that the UGR should not be used for small sources (those which subtend a solid angle of less than 0.0003 steradian) (CIE 1995). This represents, for example, an MR-16 lamp (2" diameter face) seen from 7 feet at 45º off the line of sight. The CIE then published extensions to the UGR formula in 2002, in which they use projected area rather than solid angle to define the boundary between "small", "normal", and "large" sources. They state in a summary that the basic UGR formula is valid for sources with a projected area between $0.005m^2$ ($7.75in^2$) and $1.5m^2$ ($16.15ft^2$) but is not valid for sources smaller than $0.005m^2$ or larger than $1.5m^2$, as it tends to overestimate perceived glare for small sources and underestimate perceived glare for larger sources. Similarly, the UGR formula may not be accurate for complex sources, such as specular luminaires. The CIE has since published supplemental recommendations to handle large, small, and complex sources (CIE 2002), which are discussed below.

Unified Glare Rating - Extensions

A need to extend the UGR to currently used lighting systems prompted the development of the extensions of the UGR including small, large (luminous ceiling, and uniform indirect), large (transition from normal to luminous ceiling), and complex lighting systems (CIE 2002).

UGR Extensions - Small Sources.

The CIE recommends the following modification to the basic UGR formula for small sources more than 5º off the line of sight:

$$UGR = 8\log\left[\frac{0.25}{L_b}\sum 200\ \frac{I^2}{r^2p^2}\right] \tag{2.12}$$

This was developed from two research efforts – that of Claus Benz (Benz 1966) and of Brendon Paul (Paul 1997, Paul & Einhorn 1999). The CIE (2002) suggests that Benz's research showed that intensity, rather than luminance, determines glare for very small sources off the line of sight (Benz 1966). In this research, subjects were asked to increase the luminance of a glare source until a certain glare sensation was achieved. This research looked at the effect of changing the size of the glare source (ranged from 10^{-4} sr to 10^{-6} sr), the position within the field of view (ranged from 0º to 29º in both the horizontal and vertical directions), and the ambient luminance (was either 2.2 apostilb or 6.3 apostilb (0.7 cd/m^2 to 2 cd/m^2)). This research shows that as the solid angle of the source increases, the acceptable source luminance decreases for a given glare level, and this inverse relationship between size and luminance is more pronounced off the line of sight than on-axis. Unfortunately, the PI does not understand how this research supports the CIE's statement that intensity, rather than luminance, determines glare, because that statement would suggest that the projected area is constant, so that luminance is directly

proportional to intensity rather than intensity divided by the projected area. The author does not believe Benz's research supports the CIE's claim.

The CIE also does not address whether or not Benz's findings, measured at mesopic levels, are applicable at typical office lighting levels.

Paul's research shows that it is actually the projected area, not the solid angle, which is the determining factor of glare for interior lighting for small sources (Paul 1997, Paul & Einhorn 1999). Similar to the Benz research, subjects were asked in a laboratory environment to adjust the source luminance until a certain glare sensation was achieved. The effect of changing the distance from the subject to the glare source was studied (ranging from 1.37m to 3.94m). Paul theorized that the definition of a small source had to be either that it had a constant effective area or a constant effective solid angle, so he analyzed the relationship between the distance from the subject to the glare source and the ratio of E_d^2/E_i. This relationship allowed Paul to determine if, for small sources, the effective solid angle is constant (the projected area varies with distance) or the effective projected area is constant (the solid angle varies with distance), as shown below.

Since

$$CIE\ Unified\ Glare\ Rating\ (UGR) = \ 8\ \log_{10}\left[\frac{0.25}{L_b}\sum\frac{L^2\omega}{p^2}\right] \quad \textbf{(2.13)}$$

Then, for simplicity,

$$Glare = \ 8\log g \quad \textbf{(2.14)}$$

Where:

$$g = \ (0.785/E_i)\sum\frac{L^2\,\omega}{p^2} \quad \textbf{(2.15)}$$

because

$$L_b = \frac{E_i}{\pi} \tag{2.16}$$

Then

$$g = \frac{c\, L^2\, \omega}{E_i} \tag{2.17}$$

Where

$$c = \frac{0.785}{p^2} = constant \tag{2.18}$$

Then

$$g = c\left(\frac{E_d^2}{\omega^2}\right)\frac{\omega}{E_i} \tag{2.19}$$

because

$$L = \frac{E_d}{\omega} \tag{2.20}$$

So

$$g = c\left(\frac{E_d^2}{E_i}\right)\frac{r^2}{A} \tag{2.21}$$

because

$$\omega = \frac{A}{r^2} \tag{2.22}$$

Assuming glare remains unchanged, i.e. "just uncomfortable", g would be constant as subjects adjusted the source luminance at different distances from the source. From (2.21), if g is constant and if the effective solid angle is constant, the ratio of E_d^2/E_i would also be constant. If, however, g is constant and if the effective projected area is constant, the ratio of E_d^2/E_i would vary inversely with r^2. Paul (1997) found that the latter is true, suggesting that for a small source off the line of sight, the effective

projected area is constant. This research seems to be all that is required to allow the CIE to modify the original UGR formula for small sources, as given in (2.12) above.

He then determined that the value of the constant effective projected area for a small source should be 0.005m^2. He did this by comparing the subjective evaluations for the small source to subjective evaluations for a "normal" (0.2m^2) source. He showed that, by asking subjects to adjust source luminance for both a small and a large source to the same glare sensation, he can then equate those findings, as follows:

If $g_{normal} = g_{small}$, then

$$\left(\frac{E_d^2}{E_i}\right)_{normal} \frac{r^2}{A_{normal}} = \left(\frac{E_d^2}{E_i}\right)_{small} \frac{r^2}{A_{small}} \qquad \textbf{(2.23)}$$

When the indirect illuminances and the distances are the same, the area of the small source can be determined as follows, since the area of the normal source is known:

$$A_{small} = \left(\frac{E_{d\ small}^2}{E_{d\ normal}^2}\right) A_{normal} \qquad \textbf{(2.24)}$$

Paul determined, from averaging across subjects, that the 95% confidence limits for the area of the small source are 0.0043m^2 (lower limit) and 0.0061m^2 (upper limit). He therefore recommends 0.005m^2 as the cutoff for a small source.

The CIE therefore recommends the following for small sources, positioned at least 5º off the line of sight:

$$L = \frac{I}{A_p} = 200\ I \qquad \textbf{(2.25)}$$

Where: L = luminance (cd/m^2)

I = intensity toward the eye (candela)

A_p = projected area of the luminous parts of the luminaire (m^2) = $0.005m^2$

This suggests that for any source with a projected area less than $0.005m^2$ ($7.75in^2$), the “effective” projected area should be taken as $0.005m^2$ ($7.75in^2$). When this expression for luminance is combined with the original UGR formula, and substituting ($0.005/r^2$) for ω, the modified UGR formula for small sources becomes the equation previously given in (2.12). Additional research is needed to validate this modification, as proposed by the CIE.

The following is an example of the impact of this extension, as outlined by the CIE (CIE 2002): a 15W bare incandescent lamp is located 2m (6.6ft) above the point of fixation, 4m (13ft) away from the observer (see Figure 2-1). The background luminance is $30cd/m^2$, the intensity in the direction of the observer is 160cd, and the filament luminance is $4x10^6cd/m^2$. The projected area of the filament is $4x10^{-5}m^2$. The position factor is 2.9. With the original UGR formula (without the small source extension), the calculated UGR is 36, suggesting completely intolerable glare. With the small source extension, the calculated UGR is 19, suggesting unacceptable glare, but not uncomfortable and certainly not intolerable. An informal evaluation of this situation would indicate that the small source extension provides a more realistic representation of the glare sensation.

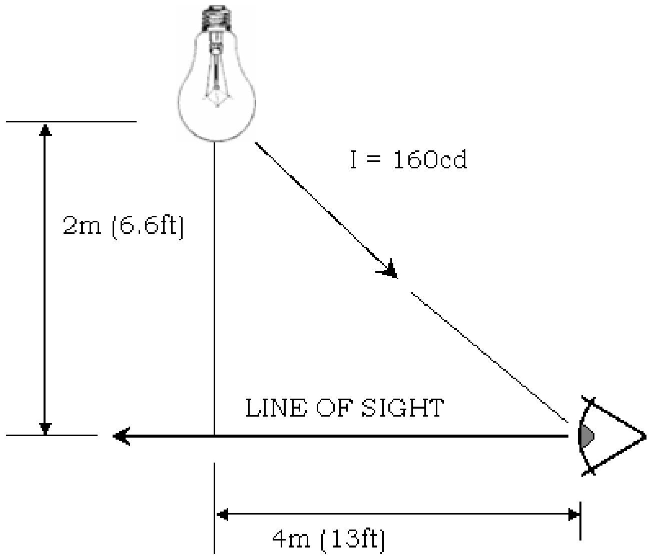


Figure 2-1. Geometric relationship for small source example

UGR Extensions - Large Sources: Luminous Ceiling. Indirect Lighting.

For large sources, including luminous ceilings and uniform indirect lighting, the CIE determined that, "an extension of the UGR formula would be too tolerant and permit unacceptable glare" (CIE 2002). Based on previous research (Hopkinson & Collins 1963) a single formula would not accurately express the glare sensation from a diffusing luminous ceiling, and instead a simple rule is to be used, which states that to achieve a specific UGR rating, an average illuminance be maintained below the following:

300 lux	for a UGR = 13
600 lux	for a UGR = 16
1000 lux	for a UGR = 19
1600 lux	for a UGR = 22

The existing CIE documentation (CIE 2002) is not clear on the location of the illuminance, but it appears from a review of Hopkinson's work (Hopkinson & Collins 1963) that the maximum average illuminance they refer to is at the workplane.

Hopkinson asked subjects to increase the luminance of a luminous ceiling in a scale model until they reached a specified glare criterion (i.e. just perceptible, just acceptable, just uncomfortable, and just intolerable) for several different combinations of wall and floor reflectances. Based on this ceiling luminance, the geometry of the room, and the room reflectances for each trial, average workplane illuminance can be calculated. It appears that the above illuminance limits were calculated specifically from the trial in Hopkinson's research when he used wall and floor reflectances of 0.48 and 0.20, respectively, because trials with different reflectances give very different illuminance values than those recommended above.

UGR Extensions - Large Sources: Transition Region.

For sources that are between what would be considered normal (luminaire projected area between 0.005m^2 and 1.5m^2) and the large, luminous ceiling, the CIE recommends the following modification to the UGR formula:

$$GGR = \left(\frac{0.18}{CC} - 0.18\right) 8\log\left(\frac{0.785}{E_i}\frac{L^2\omega}{p^2}\right) + \left(1.18 - \frac{0.18}{CC}\right) 8\log\left[\frac{2\left(1+\frac{E_d}{220}\right)}{(E_i+E_d)\left(\frac{L^2\omega}{p^2}\right)}\right] \quad \textbf{(2.26)}$$

Where: GGR = Large room (‘Great room’) Glare Rating

CC = ceiling coverage, or the ratio of the projected area of source toward nadir (typically the luminous area) to the area lit by one source (area of room divided by number of sources)

E_i = indirect illuminance at the eye (lux)

L = luminance of the source toward the eye (cd/m^2)

ω = solid angle size of source as seen (steradians)

p = position index

E_d = direct illuminance at the eye due to the source(s) (lux)

This portion of the CIE document did not reference any research, so it is unclear exactly how this expression was derived. It is coherent at both ends, meaning it gives the same result as the basic UGR formula for normal sources, and it gives the same result as the luminous ceiling rule for large sources. It appears to have been crafted solely to meet this coherency criterion. The author feels that research is needed to verify this recommendation.

UGR Extensions - Non-Uniform Indirect Lighting.

For non-uniform indirect lighting, the CIE recommends a formula for an illuminance limit as follows:

$$E_{av} = 1500 - \left(2.1 - \frac{1.5}{RI} - 1.4R_w\right) L_s \qquad \textbf{(2.27)}$$

Where: E_{av} = Average room illuminance (lux)

RI = Room Index, given by the following formula:

$$RI = \frac{(room\ length\ x\ room\ width)}{(room\ length + width)\ mounting\ height\ above\ workplace}$$

R_w = Reflectance of wall

L_s = average luminance of bright spots on the ceiling from indirect lighting or ”the average of the brighter half of an uplighter spot, about 0.75 … 0.95 of the peak luminance value, depending on ceiling luminance distribution” (cd/m^2) (CIE 2002)

The existing CIE documentation (CIE 2002) is not clear on the location of the illuminance, but it appears that the average room illuminance they refer to is at the workplane.

This formula gives a value of E_{av}, for which the average illuminance should not exceed. Notice that as the luminance of the source increases, the allowable average illuminance decreases. This calculated E_{av} is such that UGR = 19. If different UGR values are desired, the E_{av} must be multiplied by the following constants:

0.3 for UGR = 13

0.6 for UGR = 16

1.6 for UGR = 22

This portion of the CIE document did not reference any research, so it is unclear exactly how this expression was derived. In addition, the CIE does not clearly define what non-uniform indirect lighting actually is, nor does it adequately define L_s. The document in fact admits that this is an approximation that “does not claim great accuracy” (CIE 2002). Research and additional definition of source luminance are needed to verify this recommendation.

UGR Extensions - Complex Sources.

To discuss complex sources, such as parabolic louvered luminaires, the CIE first suggests that there are two basic types of luminaires. First, they define the diffusing

source as a luminaire that has constant luminance over the area of the luminaire. This luminance changes, however, with the viewing angle. Conversely, they define the specular source as a luminaire that has constant luminance regardless of the viewing angle, because the intensity and projected area both vary.

Since $L^2\omega = \frac{LI}{r^2}$, then for a diffusing source,

$$L = \frac{I}{A_o \cos\gamma} \quad and \quad \omega = \frac{A_o \cos\gamma}{r^2} \quad then \quad L^2\omega = \frac{I^2}{r^2 A_o \cos\gamma} \tag{2.28}$$

Where: γ = angle from nadir toward eye (degrees)

I = luminous intensity of source toward eye (candela)

For the specular source, the CIE suggests that an "effective" luminance must be defined, which would be higher than the luminance for a diffusing source because a diffusing source would average the intensity over the entire projected area of the source. The CIE suggests that the projected area of a specular source would be smaller because the dark patches in the source should not be included in the projected area. And, since the projected area is less than that of a diffusing source, the luminance would be higher. They recommend that the effective luminance can be defined at the point where intensity is at its maximum, I_{max} at γ_{max}. Therefore, for the specular source,

$$L = \frac{I_{max}}{A_o \cos\gamma_{max}} \quad then \quad L^2\omega = \frac{I\, I_{max}}{r^2 A_o \cos\gamma_{max}} \tag{2.29}$$

Where: γ_{max} = angle from nadir of maximum intensity (degrees)

I_{max} = maximum intensity of source (candela)

This expression for $L^2\omega$ (whether for the diffusing or for the specular source) is then used in the basic UGR formula to calculate a glare rating. The quantities in this

expression should be readily available from luminous intensity data for the fixture being specified, so this would be a fairly simple formula to apply in the design process. The CIE suggests that semi-specular luminaires would be somewhere in between the diffusing source and the specular source, and suggests that the mean of the two might give the best solution. This portion of the CIE document did not reference any research, so the author feels research is needed to verify this definition of specular and diffusing luminaires. For example, a specular parabolic luminaire at high angles has a low luminance (in fact, it often looks like it is turned off), and at low angles has a high luminance. Therefore, the CIE definition of constant luminance seems unreasonable. In addition, there is a lack of definition as to when to use the uniform UGR formula versus the complex UGR formula.

The following is an example of the impact of the complex source extension, as outlined by the CIE (CIE 2002): a single low brightness batwing fixture with luminous aperture of $0.4m^2$ ($4.3ft^2$) is located 1.8m (5.9ft) above the point of fixation, 2.14m (7ft) away from the observer (see Figure 2-2). The background luminance is $30cd/m^2$, and the intensity in the direction of the observer is 1200cd at an angle of 50º above nadir. The maximum intensity is 3600cd at an angle of 30º above nadir. The position factor is 5.3. Using the original UGR formula without the complex extension, the UGR would be 18.6; however, with the complex extension, the calculated UGR is 21.4. The former gives an indication of unacceptable glare; the latter suggests just uncomfortable glare (Akashi et al 1996).

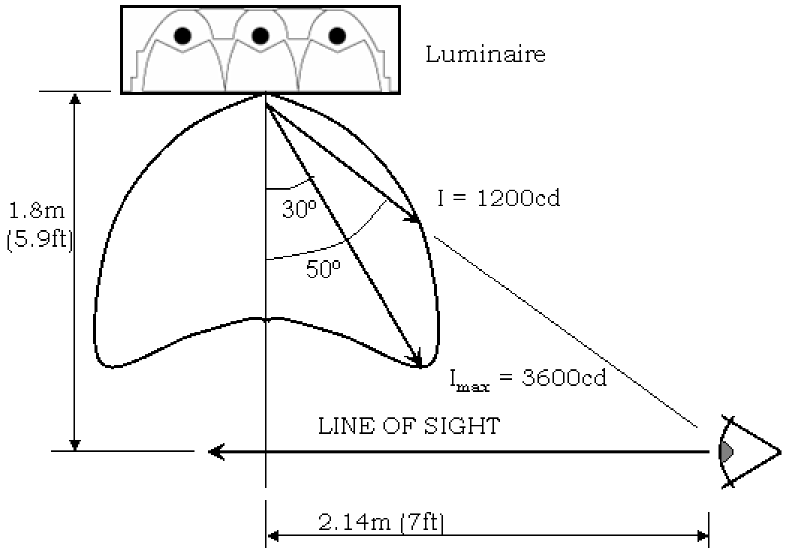


Figure 2-2. Geometric relationship for complex extension example

Comparison of VCP with UGR

So how do the two primary glare evaluation systems (VCP and UGR) compare? Mistrick and Choi (1999) conducted a comparison of VCP and UGR systems in terms of their formulation and performance when applied to conventional lighting systems. These authors compared many different fluorescent luminaires, including parabolic louvered and prismatic lensed luminaires, in a general lighting layout. The correlation between VCP and UGR was 0.82 for the data analyzed; meaning 68 percent of the variance in UGR values can be explained by VCP in a simple regression. While this correlation is fairly high, Mistrick and Choi wanted to determine the cause of the discrepancies. They compared two simple cases. First, a prismatic lensed luminaire was evaluated with both VCP and UGR. VCP predicted that the glare rating would change based on where in the room the observer was, while UGR predicted little change. Second, a parabolic louvered luminaire was evaluated. VCP predicted little change based on where the observer was,

while UGR predicted that the glare rating would change. The authors stated that they felt the UGR was more realistic, in that an observer probably would not sense much of a change in glare rating based on location with a prismatic lensed luminaire, but would with a parabolic luminaire (Mistrick and Choi 1999). This work did not use the CIE's extension for complex sources.

Discomfort Glare Experimental Problems

There is very little known of the basic mechanisms that lead to the perception of discomfort glare. Fry et al (Fugate & Fry 1956, Fry & King 1971, 1975) studied the relation of pupil size and fluctuation and the role of the iris in the sensation of discomfort glare. They found that fluctuation in pupil size generates discomfort. Berman et al (1991, 1994) examined electrical activity associated with facial muscles and its relation to discomfort glare. His conclusion from this research is that, although there is a significant correlation between facial muscle movement and subjective discomfort glare ratings, the facial movement is probably not the cause of the discomfort, but rather is caused by the discomfort, and therefore the cause of discomfort glare is still not understood. This does suggest, however, that an objective measure of discomfort glare might be possible, rather than having to rely on subjective measures.

The subjective studies of discomfort glare have shown a large variance in subject response. Past studies have shown that the correlation between predicted values and subject response is low (Manabe 1976, Stone & Harker 1973, Boyce et al 1979). The Manabe (1976) study, in particular, was very extensive. Sixty-three observers evaluated 42 installations. Correlations were computed between each evaluation system and the

subject appraisals. The correlation between the VCP system and subject appraisal was 0.63, leaving a significant amount of variance unexplained. However, more recent studies, where UGR was used as the glare prediction system, have shown the correlation to be higher (Akashi et al 1996, Boyce et al 2003), suggesting that UGR is a better prediction system than others.

Factors contributing to this low correlation could be many. There are problems associated with luminance measurement of modern luminaires, there are procedural factors and psychological factors that could have an effect on the low correlation, and demographic variables of subjects could also have an effect. Luminance values are calculated from the photometry measurements taken of luminaires. Illuminance is the measured quantity, and then intensity (and luminance) is back-calculated from the illuminance measurements. This gives an intensity distribution from the luminaire which is really based on an average illuminance measurement. For a uniform luminance fixture, this is probably an acceptable solution, but for a non-uniform luminaire, this does not give very detailed information.

There are many procedural factors that could have an effect on the low correlation. If instruction is given to the subject, it may affect the outcome, as was demonstrated by Bennett (1972b). Different instructions provided highly significant differences in discomfort glare responses with large changes in variance. The meaning of glare to subjects may also vary widely (Clarke et al 1991), contributing to the large variance. Lulla and Bennett (1981) reported that a subject's response depended upon the range of stimuli presented (the range effect). The duration of exposure (intermittent or constant) was studied by Guth (1951) with no significant difference found. This subject

has been debated but no comprehensive study has been conducted. The effect of the presence of a visual task on discomfort glare perception has been studied (Guth 1951, Ostberg et al 1975, Sivak and Flannagan 1991) showing that discomfort glare is task dependent.

There are psychological factors that could also have an effect on the low correlation. Difficulty of the task has been shown to have influence (Ostberg et al 1975, Sivak and Flannagan 1991), and the mood or level of anxiety and stress could also have an effect. Long duration (hours) of exposure to a glare source may be perceived differently than short duration.

Bennett conducted several studies (1972a, 1974, 1976, 1977) on the effects of demographic variables on discomfort glare. He found that age was negatively correlated with BCD, where older observers had lower luminance thresholds. He also found small correlations between the discomfort glare sensation and eye color, and whether the subject's occupation occurred primarily indoor or outdoor.

With so much variability in subject responses, should the lighting community just throw up its hands in dismay, because there will never be a perfect system? The PI believes that, while this variability will never be eliminated, there is still much to be gained from a valid, widely-used glare prediction system that effectively predicts discomfort from modern luminaires.

Chapter 3 - Methodology

Most of the research involving discomfort glare has used one of two primary data collection techniques. The first is accomplished by asking the subject to adjust the brightness of a test source to match a reference source (for example Luckiesh & Guth 1949, Guth 1951, Putnam & Faucett 1951, Putnam & Bower 1958, Allphin 1961). This technique has been very successful, but it has two substantial drawbacks. First, it takes the subject a fair amount of time to complete the task, which means that only a minimal number of trials can be performed by each subject. Second, dimming an incandescent lamp changes the color slightly, which then must be addressed.

The second technique used is semantic differential scaling, where the subject is shown one stimulus and must rate it on a particular scale (for example Reid & Toenjes 1952, Bodmann et al 1966, Sollner 1965, Bodmann & Sollner 1965, Boyce et al 1979, Boyce et al 2003, Akashi et al 1996). This technique has also been very successful, but has the drawback that it is sometimes difficult for the subject to remember how much discomfort constitutes a rating of "4" (as opposed to "5" or "3"), which would seem to make the data more variable.

Jacobs et al (1992) studied three different methods for assessing discomfort glare. In the first method, they asked the subjects to adjust the brightness of a test source until it was at the border of "disturbing" discomfort (which is similar to the first technique mentioned above). For the second method, they asked the subject whether or not the glare exceeded the border for "disturbing" discomfort. And the third method was a semantic differential scaling, where the subject was asked to mark the degree of discomfort on a categorical scale (which is similar to the second technique mentioned

above). They found that the semantic differential scaling method was the most reliable and showed the least variation across subjects.

It was determined that the current investigation of the effect of subjective evaluation of discomfort glare from sources of non-uniform luminance would utilize one of these historical techniques and one additional technique. Study #1 was a paired comparison experiment in which the subject saw two separate stimuli and chose which caused more discomfort, the left or the right. The paired comparison design has not been used much (if ever) in discomfort glare studies. The idea for using this technique came from other Architectural Engineering graduate students who were working in the acoustics area. They often used the paired comparison technique to determine which of two stimuli was considered louder or more reverberant. Parizet et al (2005) compared several different techniques in studying in-car ventilation noise and determined that "the discrimination power was greater for the paired comparison test than for the evaluation ones." In addition, the principal investigator felt that it would be a relatively simple task for a subject to simply determine which of two stimuli (when presented simultaneously) caused him more discomfort, which should therefore give data with little variability.

After making the decision to utilize the paired comparison technique, two additional decisions needed to be made. First, there was the question of how many pairs each subject needed to see. Does every subject need to see every pair? According to Bock and Jones (1968), "Thus, if n stimuli are to be compared, n(n-1)/2 pairs of stimuli must be presented if all possible distinct pairs are to be judged." Therefore, the decision was made to show every subject every pair. Second, the issue of whether or not to allow

ties needed to be determined. Torgerson (1958) states "The subject must designate one of the pair as greater. No equality judgments are allowed." Thus, no ties were allowed.

Study #2 was a semantic differential rating scale experiment where the subject saw only one stimulus, and was asked to rate the level of discomfort on a scale of 1 through 7. A seven point rating scale was used by both Akashi et al (1996) and by Boyce et al (2003). Akashi's rating scale was used verbatim for this study.

Independent Variables

Two independent variables were used in this study: spatial frequency of the periodic grating; and, position of the stimulus within the visual field. These two independent variables were chosen specifically because of the findings of Waters et al (1995), in which he found that the position and the position by spatial frequency interaction both significantly affected the subjective impression of discomfort glare. Spatial frequency seems to be the ideal mechanism to characterize a "complex" source, because any image can be broken down into a sum of sinusoids by Fourier Analysis. This suggests that for a non-uniform source such as a parabolic troffer, it is possible to take an image of that luminaire with a digital camera, and determine the different sinusoidal frequencies required to generate that image.

The investigator also believes that the spatial frequency will have a different effect on discomfort as the position changes (a significant interaction), due to the fact that the size of receptive fields increases from the fovea to the periphery of the retina.

Spatial Frequency

Six levels of spatial frequency were chosen to include in the study (see Table 3-1). Spatial frequency is simply the number of sine waves imaged within a given distance on the retina, measured in cycles per degree of visual angle. This gives an idea of the size of the luminance pattern. The levels chosen were based on research performed by Hilz and Cavonius (1974) who were looking at contrast sensitivity based on spatial frequency at different eccentricities. They found that, within the fovea, the sensitivity is greatest at 8 cycles per degree. At higher eccentricities, they found that the sensitivity peaks at lower spatial frequencies (see Figure 3-1). The investigator, therefore, chose frequencies that would be equally spaced on a log scale, starting at 8 cycles per degree. This led to the choice of 8, 4, 2, 1, and 0.5 cycles per degree. A uniform stimulus, equating to infinite cycles per degree, was also chosen as a benchmark. It was not possible to study frequencies lower than 0.5 cycles per degree because of the size of the stimulus that was used. A frequency of 0.25 cycles per degree, for instance, would have required a larger stimulus than what was being used to be able to show the subject at least one full cycle of the sine wave. The size of the stimulus could have been enlarged to allow for lower frequencies but was set based on prior research (see discussion in Stimuli Size). Frequencies higher than 8.0 cycles per degree were not used because of a precision limitation in Adobe Illustrator (the graphics program used to generate the stimuli). The spatial frequency was varied using paper targets.

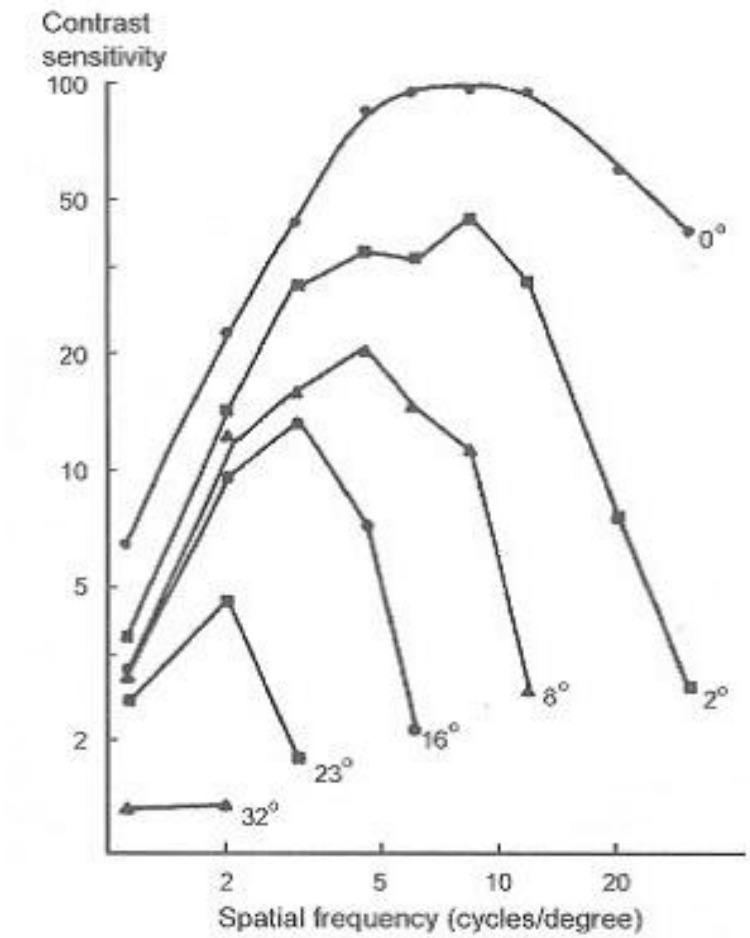


Figure 3-1. Contrast sensitivity vs. spatial frequency for different eccentricities (from Hilz and Cavonius, 1974)

Position

Four levels of position were chosen to include in the study (see Table 3-1). The Guth position index is only valid up to 60 degrees above the line of sight (Luckiesh & Guth 1949). In early dealings with the apparatus, it was determined that subjects would have a difficult time seeing anything above 40 degrees due to the chin/forehead rest. Therefore, 40 degrees was set as the highest position, and three other levels were chosen to be equally spaced, including 30, 20, and 10 degrees above the line of sight. These four

positions correspond with the positions used by Guth (1951) in his seminal BCD experiments. The positions are nominal, as the stimuli are offset horizontally from the line of sight by approximately 4" due to the nature of the paired comparison experiment and the design of the apparatus.

It was decided to only use vertical changes in position, as horizontal changes might encourage dominant eye biases, and would reduce the binocular effect. The position was varied by rotating the light fixture and target about the subject's eyes, with the subject fixated on the fixation point at all times (see Apparatus section for more details).

Table 3-1. Experimental Parameters

Position (4 levels)	10, 20, 30, 40 degrees above the line of sight
Spatial Frequency (6 levels)	Uniform, 0.5, 1.0, 2.0, 4.0, and 8.0 cycles per degree
Average Luminance of Stimuli	15250 cd/m^2
Range of Luminance of Sinusoidal Stimuli	5000 cd/m^2 to 33500 cd/m^2
Modulation of Sinusoidal Stimuli	0.75
Stimuli Visual Size	0.00714 steradians
Average Background Luminace	77cd/m^2

Control Variables

The stimulus size, background luminance, number of glare sources within the field of view, and average luminance were control variables for both studies, while the stimulus position within the field of view and the spatial frequency were varied. Other variables relating to the subject such as age, eye color, gender, and work environment were not controlled, although the information was collected for each subject. Other variables relating to the subject such as mood, amount of caffeine intake, and amount of sleep were not controlled, nor was that type of information collected.

Exposure time also was not a controlled variable. Guth (1951) studied the effect of exposure time on subjects' determination of the borderline between comfort and discomfort (BCD). In the first of his two studies, the subjects were shown the stimulus for three one second "on" periods, each separated by a one second "off" period. In the second of his two studies, the subjects were shown the stimulus continually, for as long as they required. He found no significant difference between the two presentation methods. Therefore, for the current study, the principal investigator decided to allow each subject as long as necessary to make his judgment for each trial. Typically, this was very short, on the order of 5 seconds. The time between trials was also not controlled but was fairly consistent, as it was the time required to change the positions and frequencies of the stimuli. Typically, this was approximately 15 seconds.

Apparatus

The apparatus was custom-designed and custom-built specifically for these experiments (see Figure 3-2 through Figure 3-7), but also built for flexibility for future experiments. It was constructed of 80/20 Inc.'s extruded aluminum modular framing system. It consisted of two independently operable arms that could be rotated about the subject's eyes, which allowed for the position of the stimuli to be changed. Each arm rotated via a gear in 2.5 degree increments from 90 degrees above horizontal to 30 degrees below horizontal (see Figure 3-8). Each arm held a 2000W Robert Juliat 710SX incandescent theatrical lighting fixture on one end and a target on the other end, onto which the stimulus was placed. The fact that the fixture and the target rotated together and were always the same distance from the subject means that the stimulus always

remained a constant size (projected area) and constant luminance, relative to the subject, as it stayed normal to the subject throughout the rotation (see Figure 3-9). The length of the arm could be adjusted from approximately 1.2 meters to approximately 3 meters. The apparatus was designed to have significant flexibility so that it could be easily reconfigured for future work.

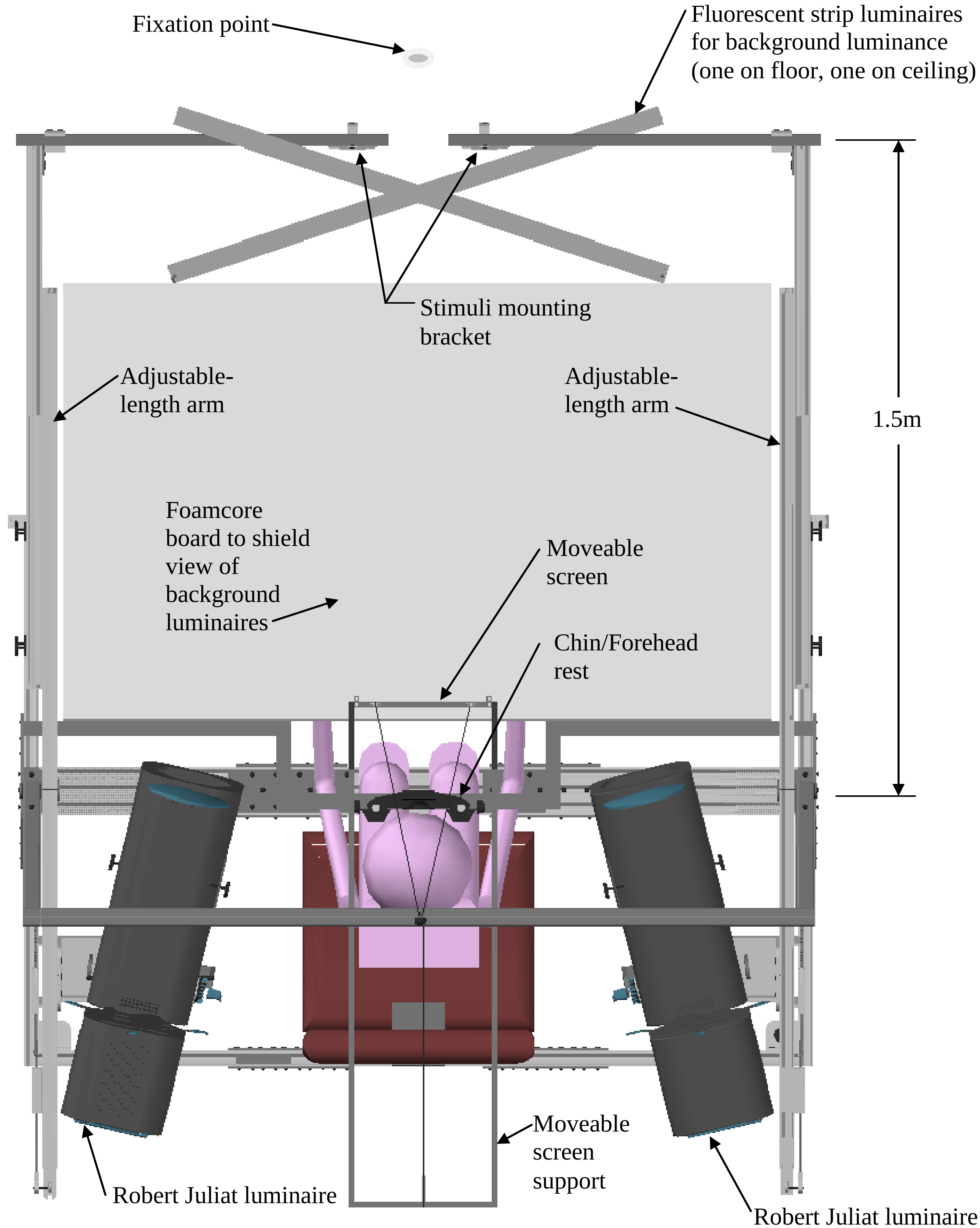


Figure 3-2. Plan view of apparatus (not to scale)

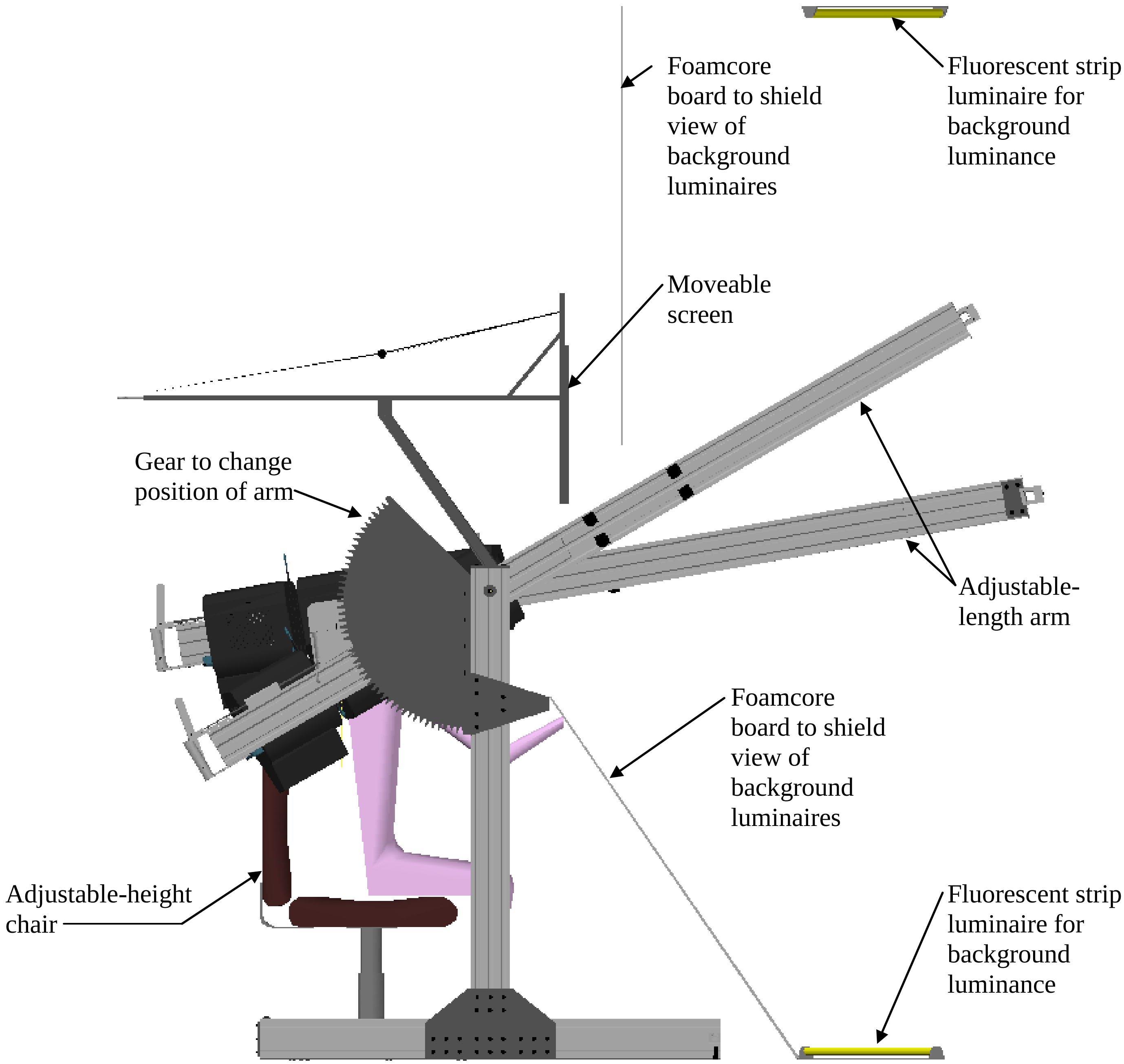


Figure 3-3. Elevation view of apparatus (not to scale)

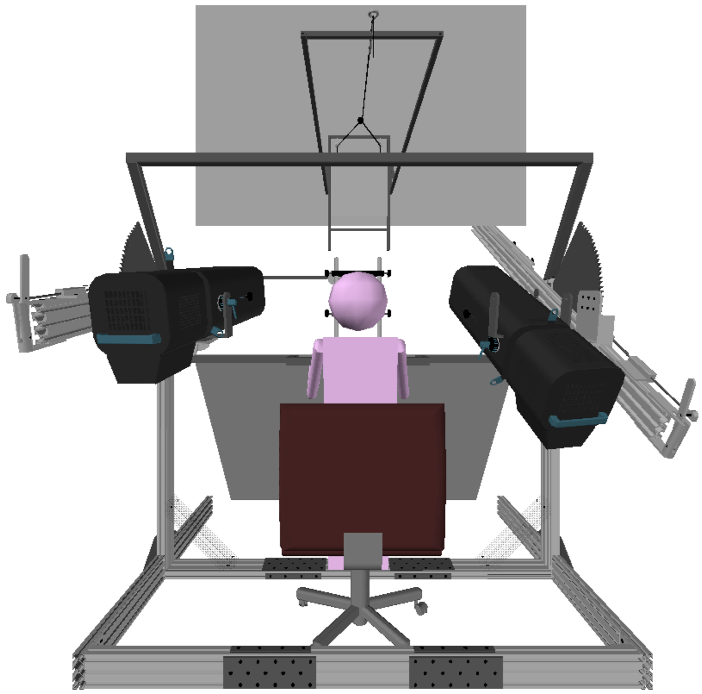


Figure 3-4. Perspective view of apparatus from behind subject (not to scale)

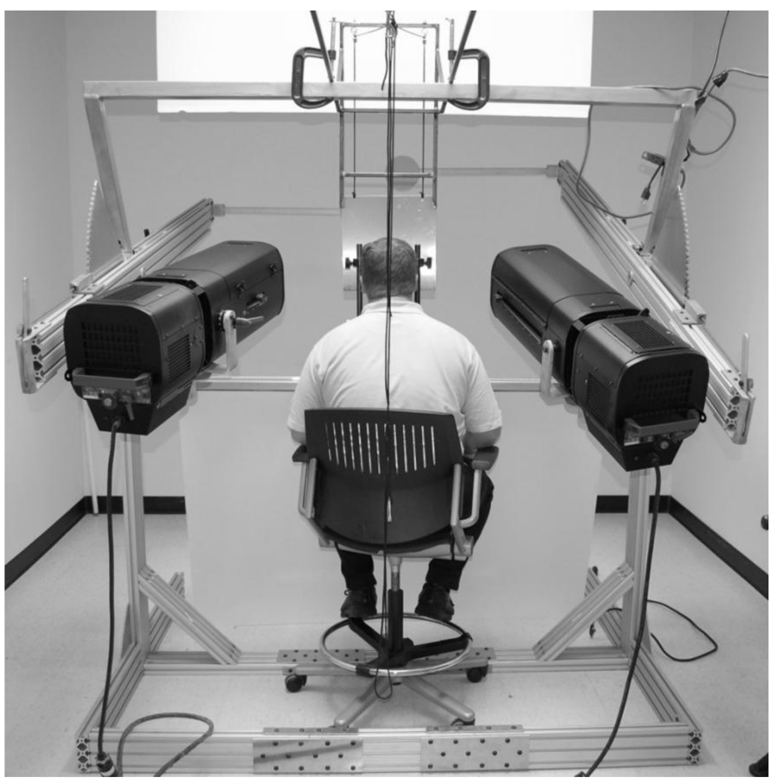


Figure 3-5. Photograph of apparatus from behind subject

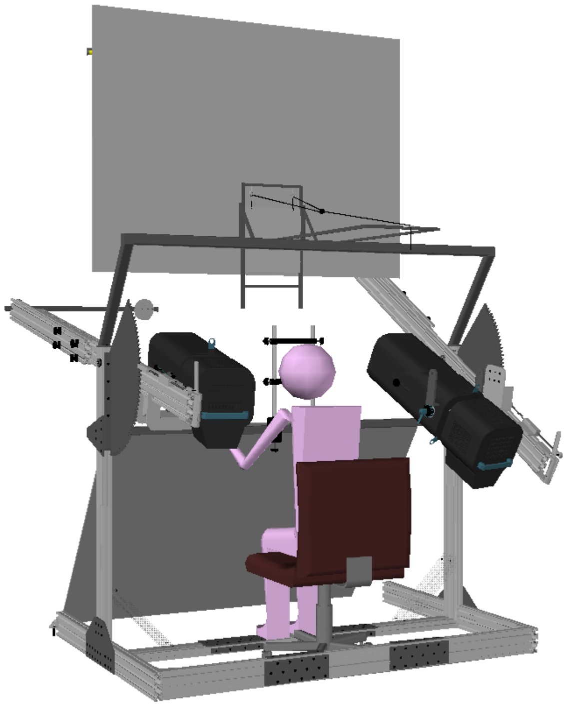


Figure 3-6. Perspective view of apparatus from behind and left of subject (not to scale)

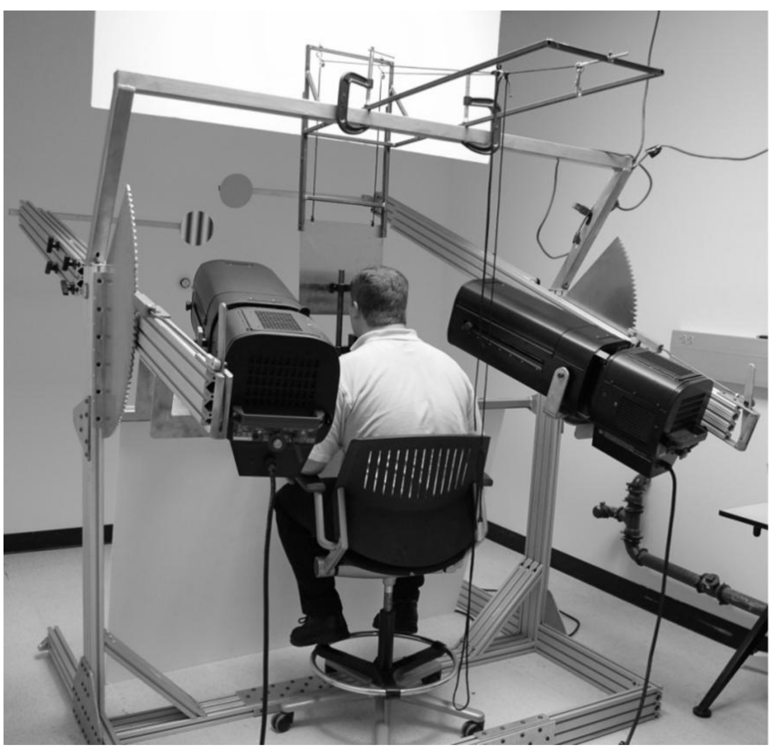


Figure 3-7. Photograph of apparatus from behind and left of subject

Figure 3-8. Gear used to change the position of the arm

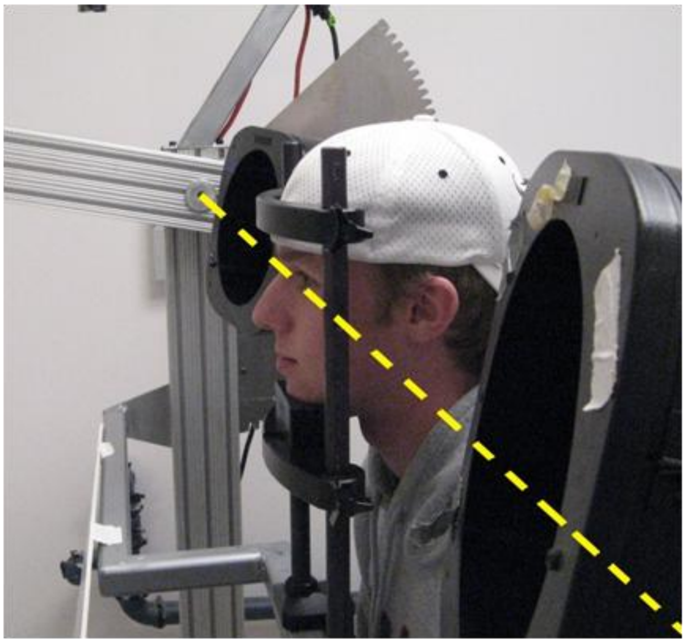
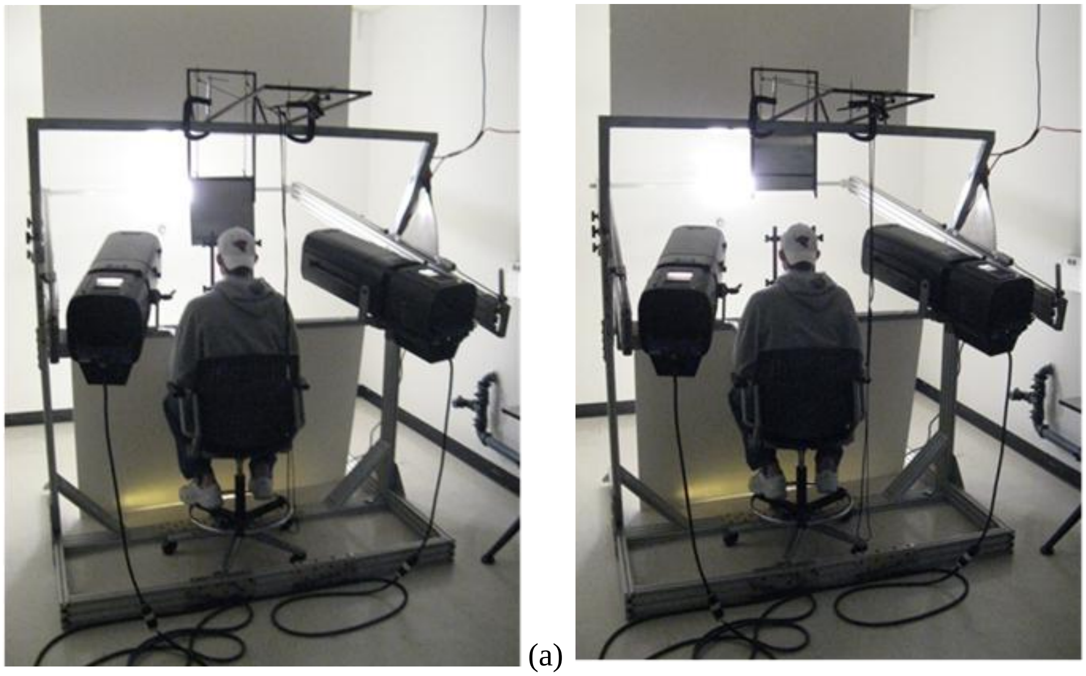


Figure 3-9. Image of a subject sitting in the apparatus showing the subject's eyes in line with the point of rotation for the two arms.

The subject was positioned with a chin/forehead rest 1.5 meters from the centerline of the two targets. The two lighting fixtures were wired into a 30A, 120V lighting contactor, so that both fixtures could be operated simultaneously with a toggle switch.

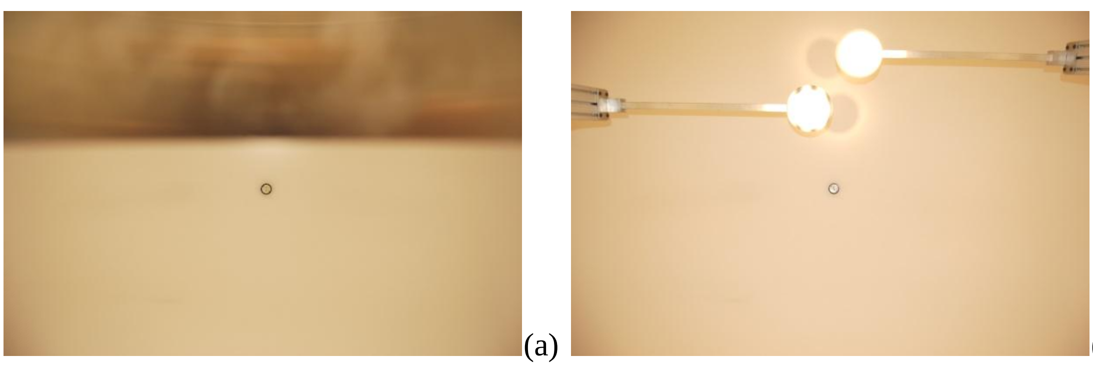
Mounted on the wall beyond the apparatus, directly in line with the subject's eyes, was a yellow-orange LED, which was used as a fixation point. In order to vary the independent variable of position, the arms of the apparatus were rotated. But that only changed the position of the stimuli if the subject's line of sight was held constant. The fixation point was used for that purpose. It blinked on and off at irregular intervals in an attempt to keep the subject's interest. During the experiment, the subject was instructed many times to "remember to look only at the fixation point at all times." Additionally, one of the assistants running the experiment also checked to make sure the subject was indeed looking at the fixation point as he made his determination by watching the subject's eyes from the side of the apparatus. If the subject was not looking at the fixation point, the trial was repeated.

A moveable screen shielded the subject's view of the stimuli while the stimuli were being changed between trials (see Figure 3-10). It was in its down position while the stimuli were being changed and was raised so that it was completely out of the field of view when the subject made his determination. When in its down position, the stimuli were blocked, but the subject still had a full view of the fixation point, so that the subject never lost sight of the fixation point. The luminance of the screen when the background luminaires were on was 8 cd/m^2. Figure 3-11 shows the moveable screen from the perspective of the subject.



(a) (b)

Figure 3-10. Image of a subject sitting in the apparatus with (a) the stimuli blocked from his field of view with the moveable screen down (between trials) and (b) the stimuli visible with the screen up



(a) (b)

Figure 3-11. Image of what a subject would see sitting in the apparatus with (a) the stimuli blocked from his field of view with the moveable screen down (between trials) and (b) the stimuli visible with the screen up

One design issue with the apparatus that required careful consideration was the fact that the light fixtures were not normal to the targets. Because there needed to be space for a subject to sit, the fixtures had to be moved out from where the targets were (left fixture had to be moved farther left of the left target, and right fixture had to be

moved farther right of the right target). The location of the targets could have been moved farther out to be in line with the fixtures, but that would mean there would have been a substantial gap between the targets. Then, determining which target was more discomforting might not have been a binocular task. It was therefore decided to move the fixtures out and leave the targets closer in, but that meant that there would be an eccentricity to the projection from the fixtures. This caused several problems. First, the luminance of the target was not uniform, as the fixture was physically closer to one side of the target than the other. This problem was solved by the “uniformity” gobo (see discussion below). And second, projecting the stimuli (as was originally intended) from the fixtures would mean that a circular stimulus would project as an ellipse. This problem was solved by not projecting the stimuli (see the Stimuli discussion below).

Background luminance was provided by two, single lamp fluorescent strip fixtures, one mounted to the ceiling above the apparatus and the other mounted to the floor. The main purpose of the fixtures was to provide a uniform background for the stimuli over the extents of the rotation. It was most important that the area directly behind the stimuli from 10 degrees to 40 degrees above the line of sight was uniform (see Figure 3-12). The fixtures were rotated slightly to minimize shadows on the wall from the apparatus arms. White foam core was mounted between the fixtures and the subject so that the subject did not have a direct view of the fixtures.

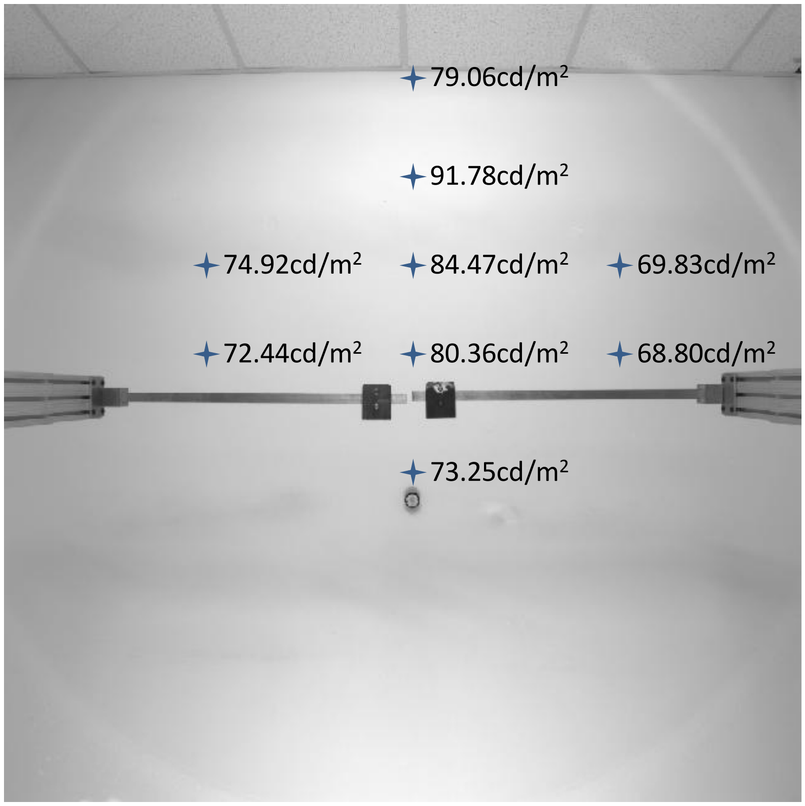


Figure 3-12. Digital Imaging Photometer image of background with luminance values superimposed

The background luminance was fairly constant over the duration of the experiments (see Table 3-2).

Table 3-2. Luminances at different positions of the background at different times during the duration of the experiment

Position	Background Luminance after subject #10 (cd/m^2)	Background Luminance after subject #19 (cd/m2)	Background Luminance after subject #28 (cd/m2)	Background Luminance after subject #34 (cd/m2)
1	79.06	78.30	74.85	76.30
2	91.78	90.05	86.11	87.76
3	74.92	69.27	68.46	70.13
4	84.47	82.73	80.03	81.01
5	69.83	70.56	67.56	67.89
6	72.44	70.39	67.26	68.08
7	80.36	79.06	77.07	77.35
8	68.80	68.36	65.68	65.72
9	73.25	73.10	71.62	81.51
Average	77.21	75.76	73.18	75.08
Std Dev	7.48	7.33	6.89	7.56

The output from the Robert Juliat fixtures projected a circle of light on the targets that was too large. Therefore, an iris was inserted in each fixture to reduce the diameter of the projected circle of light (see Figure 3-13).

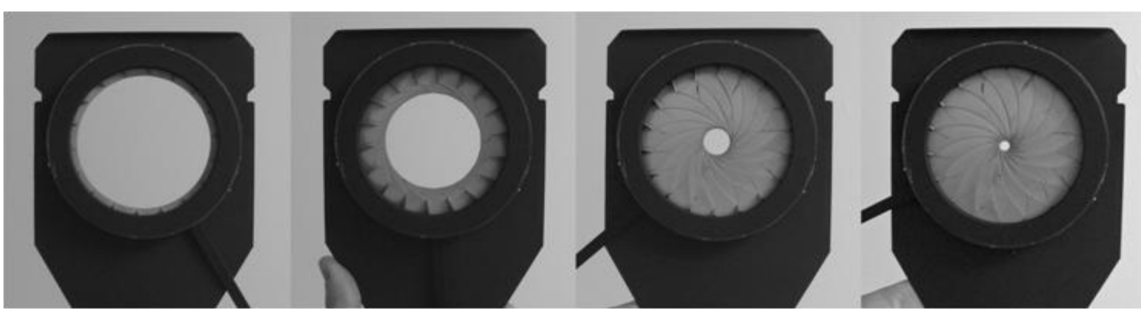


Figure 3-13. Adjustable internal iris used in the Robert Juliat fixtures to reduce the diameter of the circle of light projected.

Measurement Equipment

There were two pieces of equipment used for the measurements in this study: a luminance meter; and, a digital imaging photometer.

The luminance meter was a Minolta LS-110, 1/3º (Serial Number 79923018), and was mounted on a tripod. The meter was calibrated by Minolta in July 2006 (calibration certificate no. KMSA-6580).

The digital imaging photometer was a Radiant Imaging PM-1611-0, and was mounted on a tripod (see Figure 3-14). It contains a black and white charge-coupled device (CCD) with a 1024 by 1024 pixel resolution. Through the Prometric software (Version 8.5), the photometer is capable of taking an image of a scene, just like a typical digital camera, but then determining a luminance value at each of the 1024 by 1024 matrix of pixels based on the calibration of the lens being used. This matrix of luminance values can then be exported to Excel or Matlab, where the data can be analyzed.

Figure 3-14. Digital Imaging Photometer mounted on a tripod with a zoom lens

The digital imaging photometer is not a stand-alone device – it requires a computer to control it and to receive the data when it takes an image (as the photometer has only temporary memory storage). The photometer was designed to be controlled from a desktop computer, so the PCI card that comes with the camera is typically installed in a desktop computer. For this experiment, it was necessary to have the photometer be more mobile, so control from a laptop computer was preferred for convenience. Unfortunately, the PCI card did not physically fit in the laptop computer, so an external PCMCIA data converter was utilized (see Figure 3-15). From the external PCMCIA converter, a data cable was connected to the back of the digital imaging photometer (see Figure 3-16).

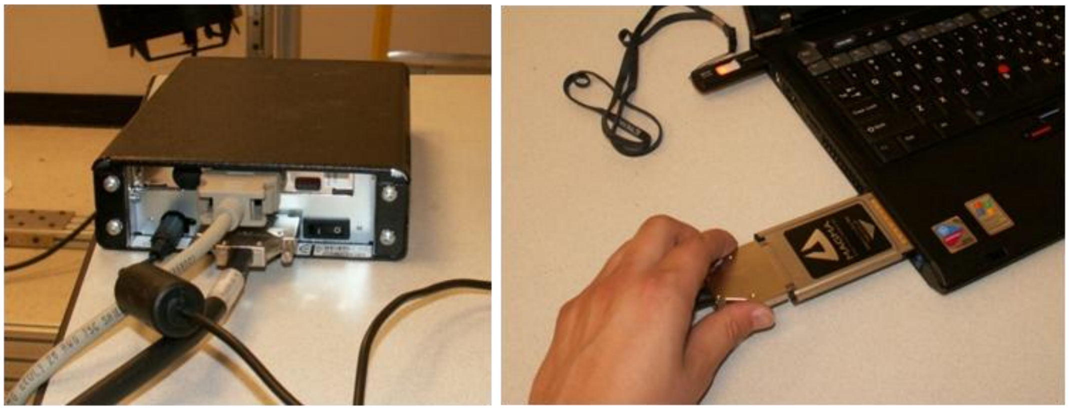


Figure 3-15. PCMCIA data converter (left) and PCMCIA card being inserted into laptop (right)

Figure 3-16. View of the back of the digital imaging photometer showing the data connection port (top right) and the power connection port (bottom)

Three different lenses were used with the digital imaging photometer (see Figure 3-17). The first was a Promaster zoom lens, the second was a Sigma 50mm lens, and the third was a Tokina 17mm wide angle lens. The Tokina lens was used primarily for background luminance measurements, while the Promaster zoom lens was used to measure the stimuli, as it has a very narrow field of view. The Sigma lens was primarily

used for testing purposes. Each of the lenses used has manually adjustable f-stops. Changing the f-stop changes the aperture diameter, which changes the amount of light admitted into the camera (see Figure 3-18), which could change the luminance values given by the software. Therefore, the appropriate f-stop was determined for each lens, depending on what was being measured, and that f-stop was used throughout the experiment.

Figure 3-17. Three lenses available for the Digital Imaging Photometer. Lenses are in order from largest to smallest focal length (narrow to wide field of view) from left to right. The focal lengths are 70-210mm, 50mm, and 17mm.

Figure 3-18. Images of Promaster zoom lens showing changes in aperture based on different f-stops. Left is f/22 and right is f/3.5.

Three neutral density filters were used in the measurement of the stimuli (see Figure 3-19). Different neutral density filters have different transmittance values. ND1 has a transmittance of 0.0869, while ND2 and ND8 have transmittances of 0.0094 and 0.4375, respectively. Depending on the luminance of the scene to be measured, neutral density filters were used to reduce the amount of light that reached the photometer. For the measurement of the experiment’s stimuli, a combination of two neutral density filters were required to reduce the light enough so that valid measurements could be made (ND2 and ND8). All of the neutral density filters can be used on each of the lenses, even though the lenses have different diameter barrels. This is accomplished with the mounting rings shown in Figure 3-20.

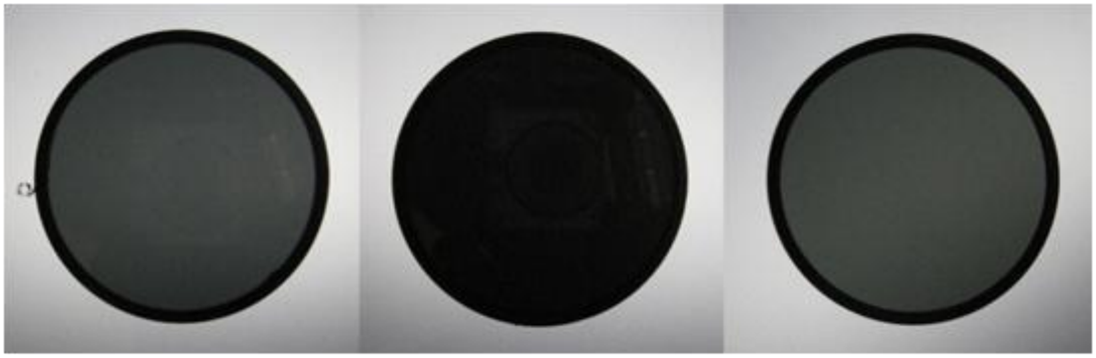


Figure 3-19. Neutral density filters used for the digital imaging photometer lenses. From left to right: ND1, ND2, and ND8.

Figure 3-20. Three mounting rings used to attach the neutral density filters to the lenses (one for each lens).

All three lenses were calibrated with and without the neutral density filters using a 12" diameter uniform luminance sphere (see Figure 3-21), which allowed a large range of luminances to be measured. To calibrate a specific lens/neutral density/f-stop combination, an image was taken of a flat field (from the uniform luminance sphere) with the photometer. That image should theoretically show every pixel at the same luminance (if the uniform luminance sphere truly gives a flat field); however, the image most likely will not show the same luminance at each pixel, due to imperfections in the lens and/or filter. In the calibration process an image was taken and a calibration record was generated, which basically corrected each individual pixel so that the same luminance

was given for each pixel. That calibration record was then used for each subsequent image taken with that lens/neutral density/f-stop combination.

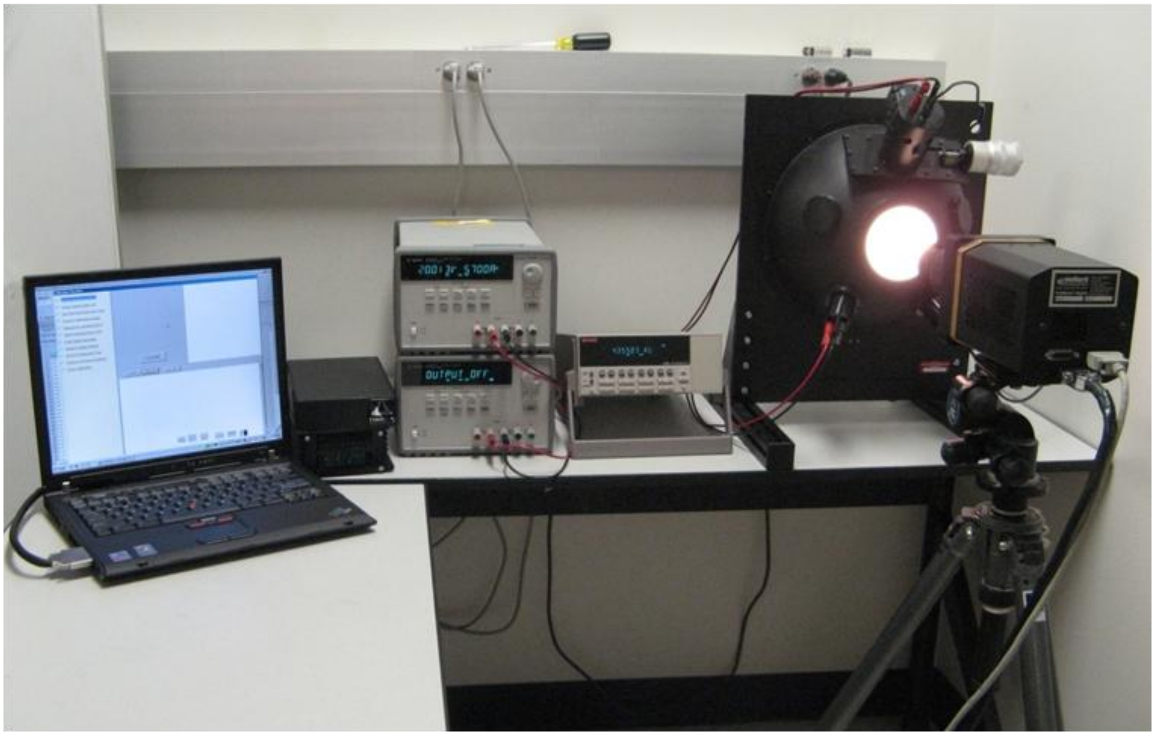


Figure 3-21. Components required for calibrating the digital imaging photometer. From left: laptop, PCMCIA data transfer terminal, photometer power supply, sphere lamp power supplies, picoammeter, 12" uniform luminance sphere, and photometer.

The sphere used in this study is unique in that two incandescent lamps were used to obtain the luminances within the sphere that were required. Typically, only one lamp is necessary. But due to the extremely high luminances of the stimuli in this experiment, two lamps were required (see Figure 3-22). The primary lamp was completely external to the sphere, and was mounted to the sphere with a micrometer-driven aperture (see Figure 3-23). The aperture can be opened and closed to change the amount of light getting into the sphere. The primary lamp was shielded by a baffle so the flux from the

lamp had to bounce before hitting the interior walls of the sphere (see Figure 3-24), which helped to create a uniform field.

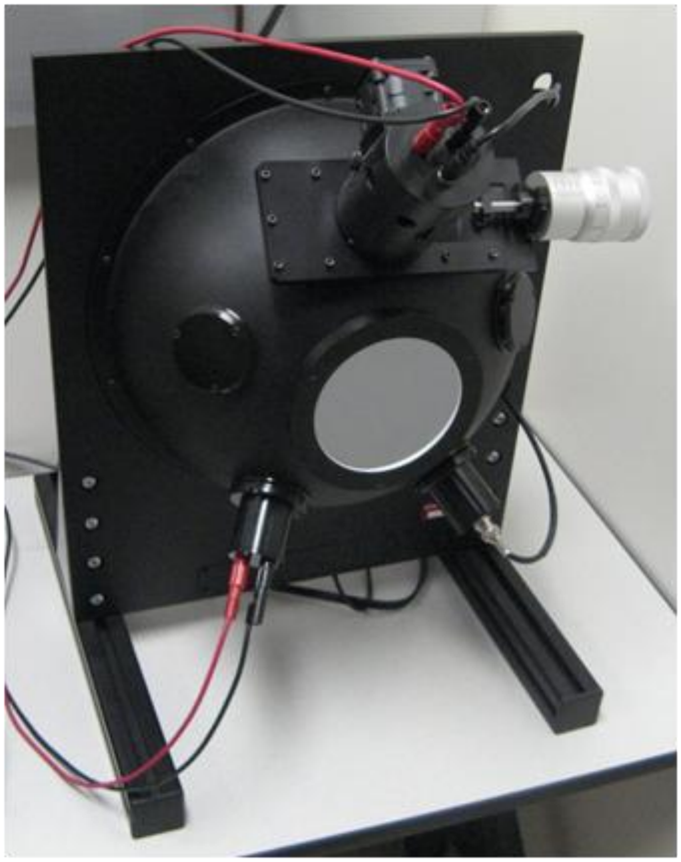


Figure 3-22. Close-up view of the sphere, showing connections to the primary lamp (top), secondary lamp (bottom left), and luminance detector (bottom right).

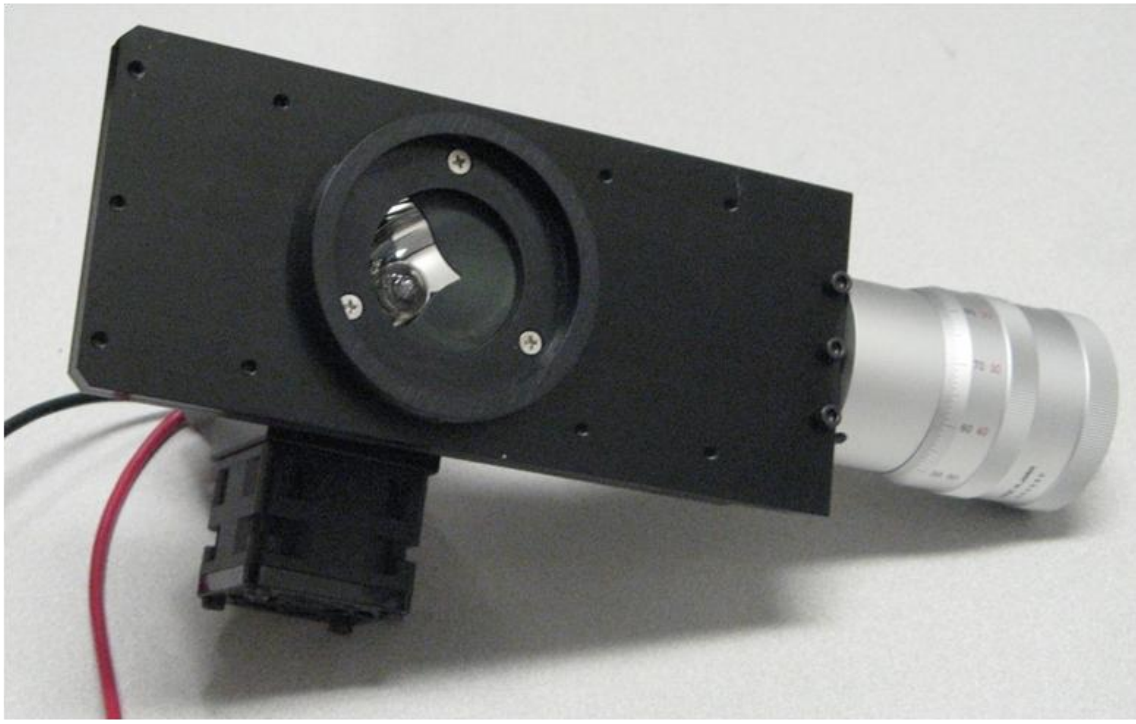


Figure 3-23. External housing for the primary lamp used in the sphere.

Figure 3-24. View of the opening in the sphere for the primary lamp, showing the internal baffle used to create a more uniform field.

It was necessary to add the second lamp in order to generate the luminances inside the sphere that were required (see Figure 3-25). The secondary lamp was mounted completely internal to the sphere, and did not have a baffle. This did cause the interior of the sphere to not be completely uniform, and was even visible when the wide angle lens was used (see Figure 3-26). However, for this particular study, it was not a problem. The two primary lenses used in the study were a zoom lens and a wide angle lens. The zoom lens was used to measure the stimuli, and had a very narrow field of view. The wide angle lens was used to measure the background. To calibrate the zoom lens, both lamps in the sphere needed to be on to generate higher luminances, which did create non-uniformity in the sphere. However, because the field of view of the zoom lens was small, it was possible to image only a small portion of the sphere which was very uniform. To calibrate the wide angle lens, only one lamp needed to be on, as it was used to take background luminance measurements where the luminances were much lower. With only one lamp on, the uniformity of the sphere was very good.

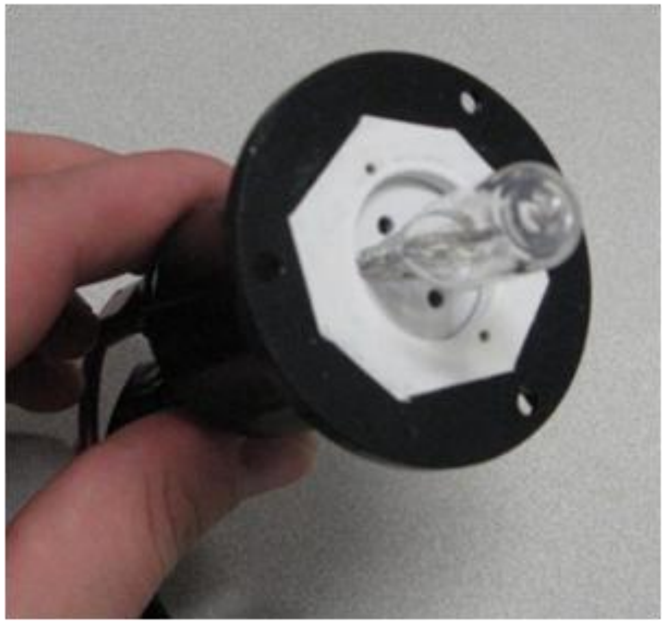


Figure 3-25. Secondary lamp, used to generate higher luminances within the sphere

Figure 3-26. View of the inside of the sphere, showing how the secondary lamp could be visible

Stimuli

Artwork Generation

The stimuli used for this study were originally intended to be projected images from the Robert Juliat 710SX fixtures with the use of gobos. Gobos are typically glass or metal and have a pattern etched into the glass or cut out of the metal. A gobo is inserted inside a theatrical fixture to affect the pattern of light that projects from the fixture. "A" size, 100mm diameter gobos, manufactured by Apollo Design Technology, Inc., were intended to be used (see Figure 3-27) in these studies. The gobos were made from 1.1mm thick borosiliciate glass, with custom designed black and white artwork, generated in Adobe Illustrator CS2, version 12.0.1. Apollo's precision was 20,000 dpi.

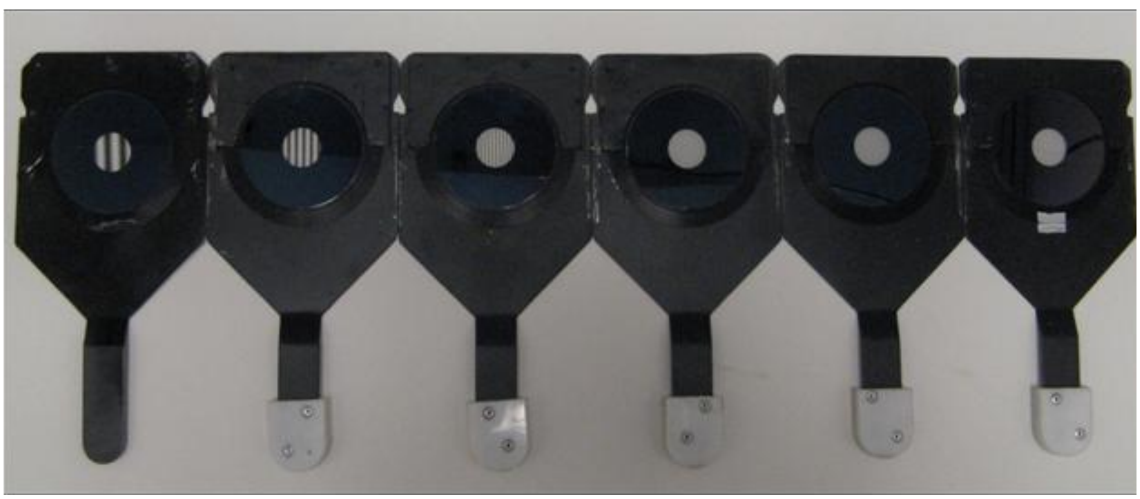


Figure 3-27. Gobos (mounted in gobo holders) with different frequencies : from left, 0.5cpd; 1.0cpd; 2.0cpd; 4.0cpd; 8.0cpd; and, Uniform.

The artwork for the gobos was oval, measuring 0.02402m on the horizontal diameter, and 0.02477m on the vertical diameter. The artwork was intentionally made to be oval, as opposed to circular, to account for the eccentricity in the apparatus, so that the projected image would be circular.

There were two types of stimuli – uniform and non-uniform. The single, uniform stimulus was made from a uniform percent grey (see Figure 3-28). The percent grey used for the uniform stimulus was chosen to match the average luminance of the non-uniform stimuli.

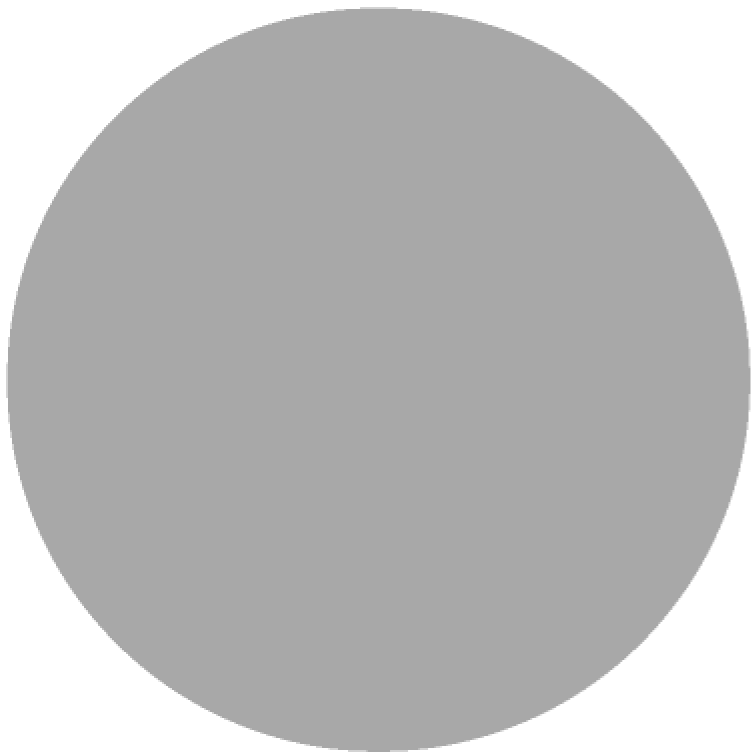


Figure 3-28. Adobe Illustrator image of Uniform stimulus

The five non-uniform stimuli were a series of sine wave gradients which varied in their spatial frequency (see Figure 3-29 to Figure 3-33). All five of the stimuli were created to have the same average luminance. They were created by discretizing each cycle of the sine wave into 24 equally-sized rectangles and shading each rectangle with Adobe Illustrator’s linear gradient tool. This tool allowed the PI to define the grey scale value at either edge of the rectangle and then filled the area with a linear transition from

the left to the right edge. For each of the sine wave gradients, the same percent black values were used at the edges of the rectangles to ensure that the modulation was consistent across the stimuli. The width of each rectangle varied based on the spatial frequency being created, but was always 1/24th of a cycle. The width of one cycle was determined based on the spatial frequency being created and the viewing distance of 1.5 meters.

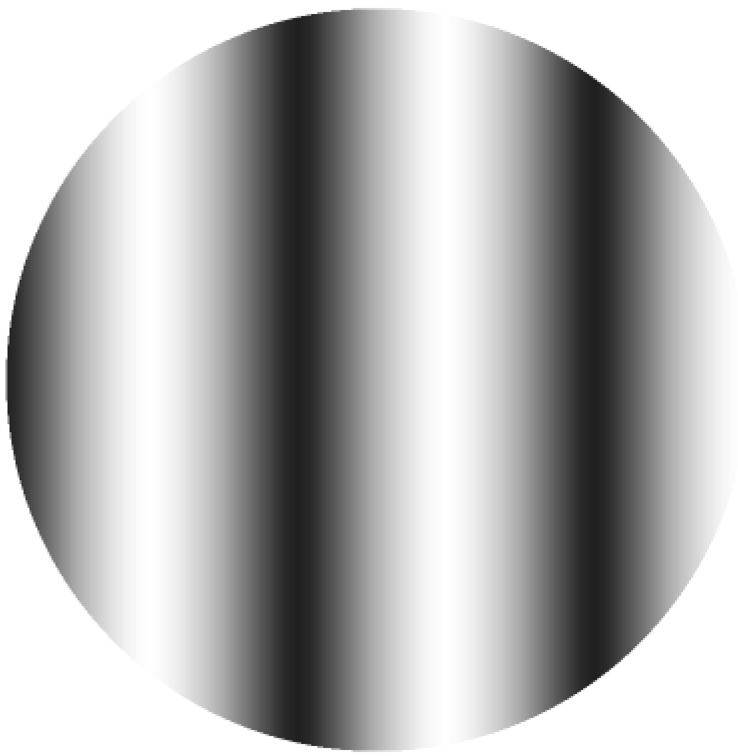


Figure 3-29. Adobe Illustrator image of 0.5 cycles per degree

Figure 3-30. Adobe Illustrator image of 1.0 cycles per degree

Figure 3-31. Adobe Illustrator image of 2.0 cycles per degree

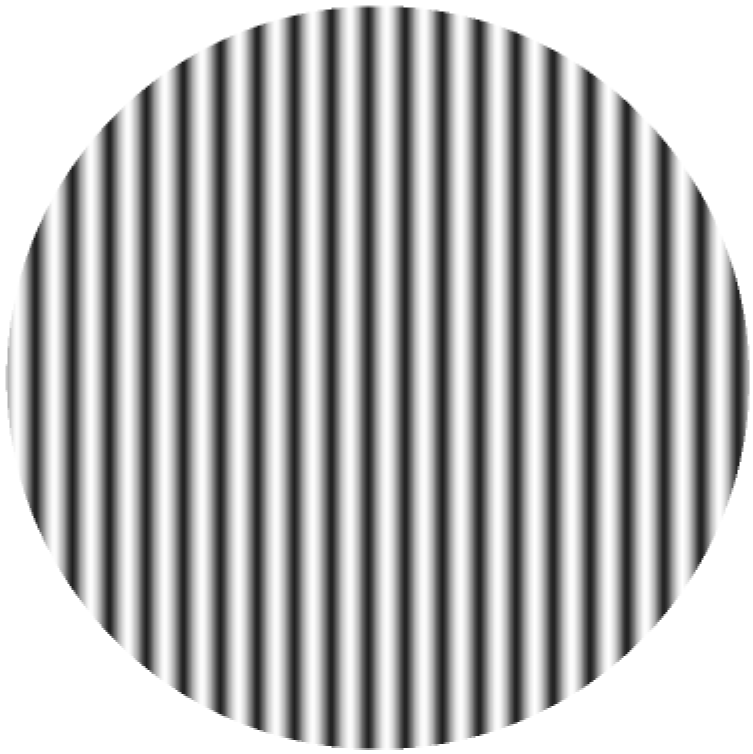


Figure 3-32. Adobe Illustrator image of 4.0 cycles per degree

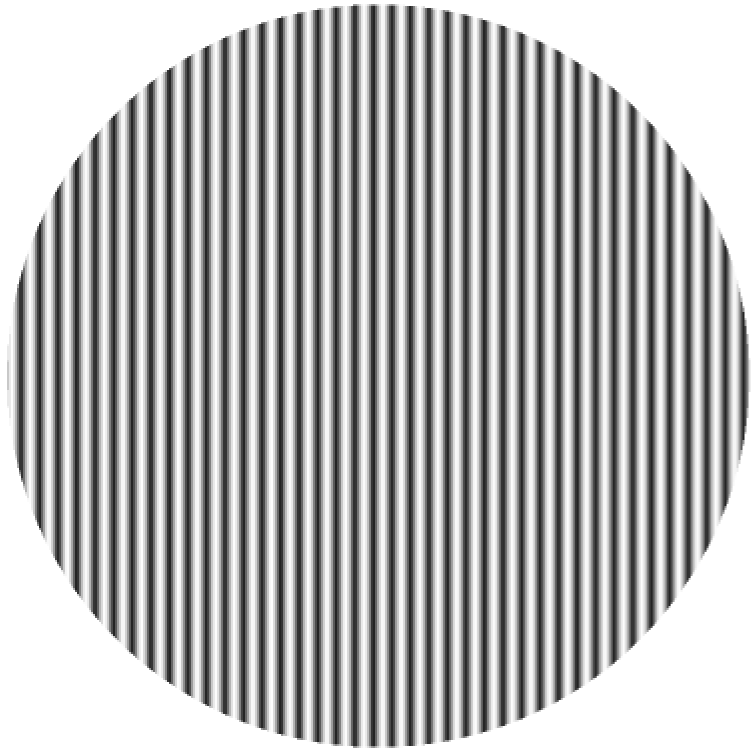


Figure 3-33. Adobe Illustrator image of 8.0 cycles per degree

To generate the gobos, the first step was to determine how percent black and transmittance were related. To do that, a figure was created in Adobe Illustrator that had 21 squares each of a uniform percent black, ranging from 0 to 100 percent black in 5 percent increments (see Figure 3-34). This figure was made into a gobo, which was inserted into the Robert Juliat fixture. The transmittance of each percent black was calculated from the projected image. This was done by measuring the luminance of each square as projected by the fixture with the gobo inserted, then measuring the luminance of the exact same position as projected by the fixture without the gobo. The transmittance is then simply the ratio of the luminance with the gobo to the luminance without the gobo.

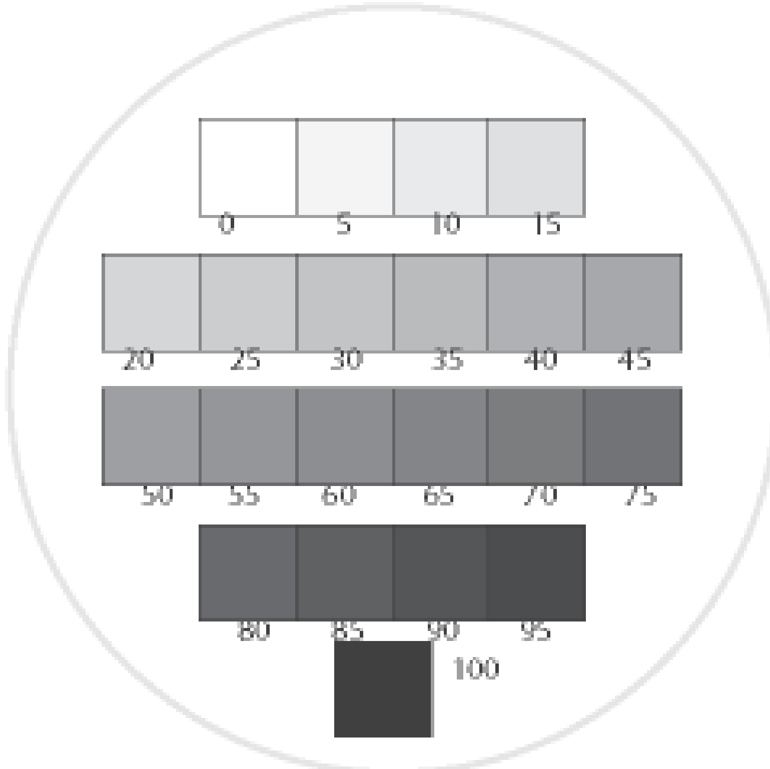


Figure 3-34. Adobe Illustrator image of different percent greys

That data was plotted (see Figure 3-35), and a best fit curve was fit to the raw data with an r value of 0.9957. The formula of the best fit curve was used to generate transmittance values for percent black values ranging from 0 to 100 in 1 percent increments. A transmittance value was also calculated for angles ranging from 0 to 360 degrees in 15 degree increments to determine what the transmittance would have to be at each angle to simulate a sine wave (see Figure 3-36). The formula used to calculate that transmittance value is shown here.

$$\tau_\theta = \sin\theta \quad \left(\frac{\tau_{max} - \tau_{min}}{2} + \frac{\tau_{max} + \tau_{min}}{2}\right) \tag{3.1}$$

Where: τ_θ = transmittance to be determined at a certain angle

Θ = angle in degrees

τ_{max} = maximum transmittance measured from gobo percent blacks (at 0% black)

τ_{min} = minimum transmittance measured from gobo percent blacks (at 100% black)

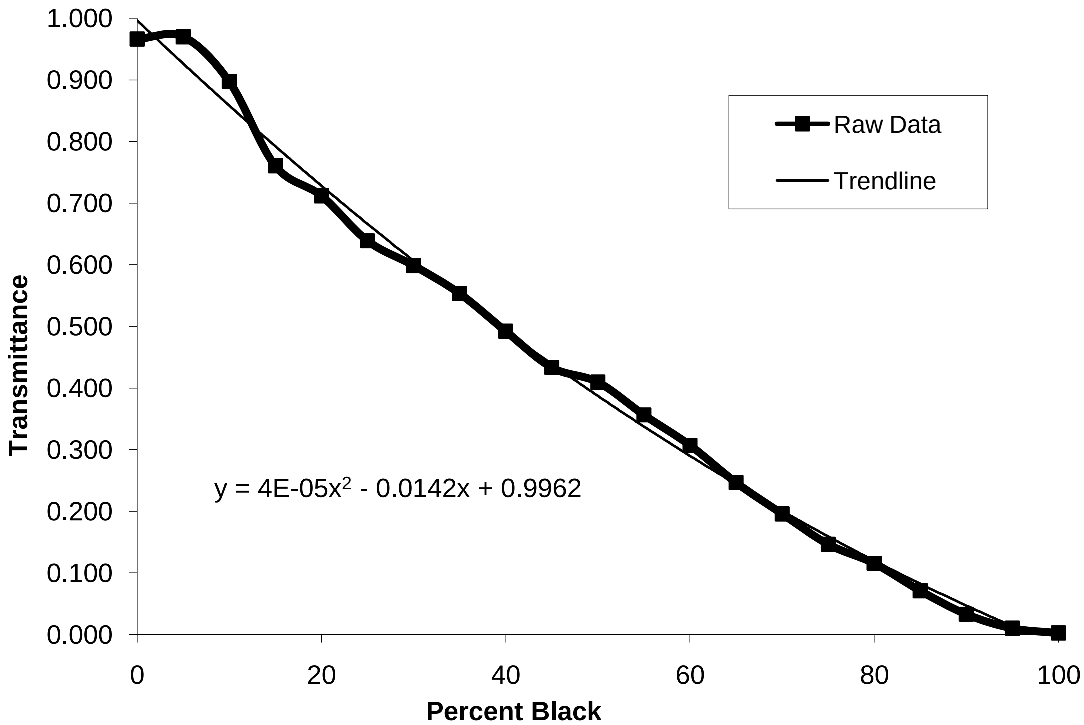


Figure 3-35. Transmittances of percent black values

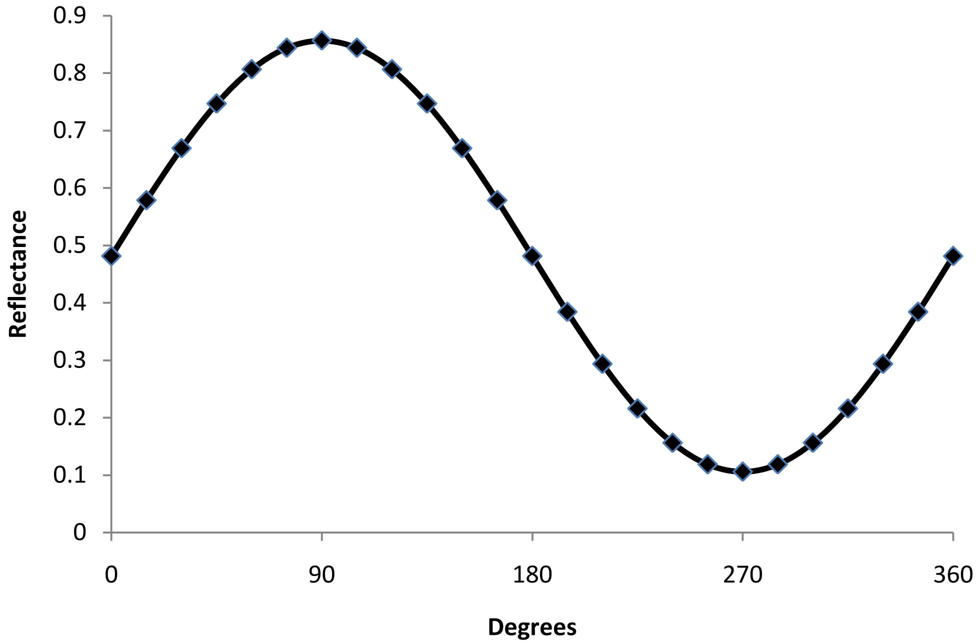


Figure 3-36. Calculated transmittance values required to simulate a sine wave

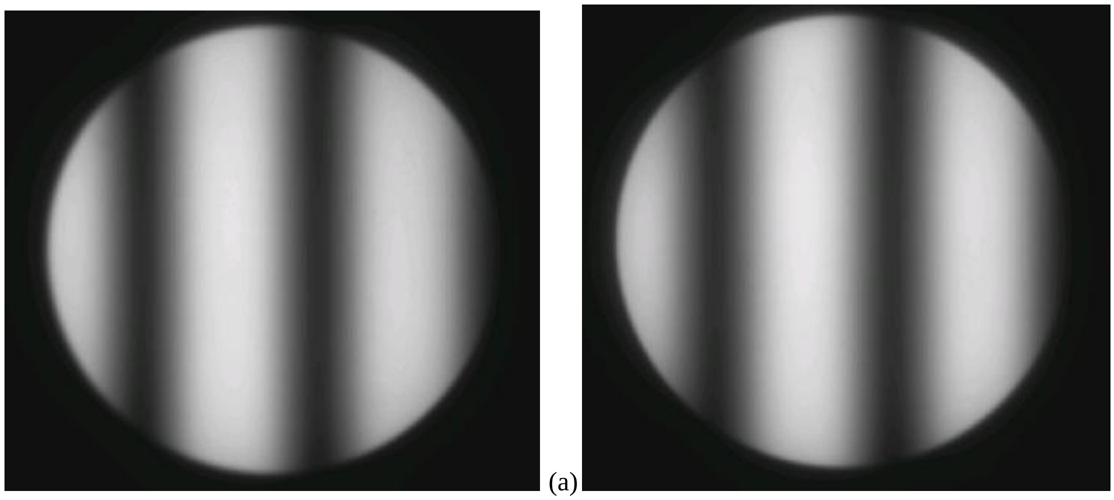
These transmittances were then converted back to percent black values based on the values calculated from the best fit line. It was these percent black values that were used in the Adobe Illustrator files for each of the 24 rectangles per cycle of sine wave to create the sine wave gradients (see Table 3-3). Apollo used these files to generate the gobos.

Table 3-3. Percent blacks for each of the 24 rectangles in each cycle of the sine wave gradients

Rectangle Number	Extents of Percent Black in that Rectangle
1	42-31
2	31-21
3	21-13
4	13-7
5	7-4
6	4-3
7	3-4
8	4-7
9	7-13
10	13-21
11	21-31
12	31-42
13	42-54
14	54-67
15	67-79
16	79-89
17	89-97
18	97-99
19	99-97
20	97-89
21	89-79
22	79-67
23	67-54
24	54-42

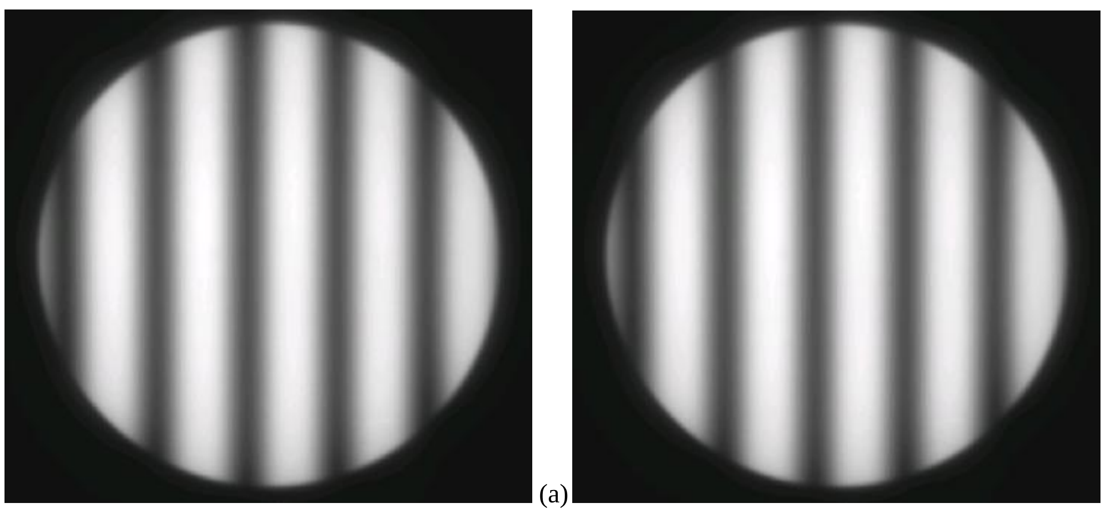
Gobo Problems

The actual stimuli, as projected by the Robert Juliat fixture with the gobo in the fixture, are shown in Figure 3-37 to Figure 3-42. These images are taken with the Radiant Imaging Digital Imaging Photometer.



(a) (b)

Figure 3-37. Digital Imaging Photometer images of the projected 0.5 cycles per degree gobo from the (a) left Robert Juliat fixture and (b) right Robert Juliat fixture



(a) (b)

Figure 3-38. Digital Imaging Photometer images of the projected 1.0 cycles per degree gobo from the (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

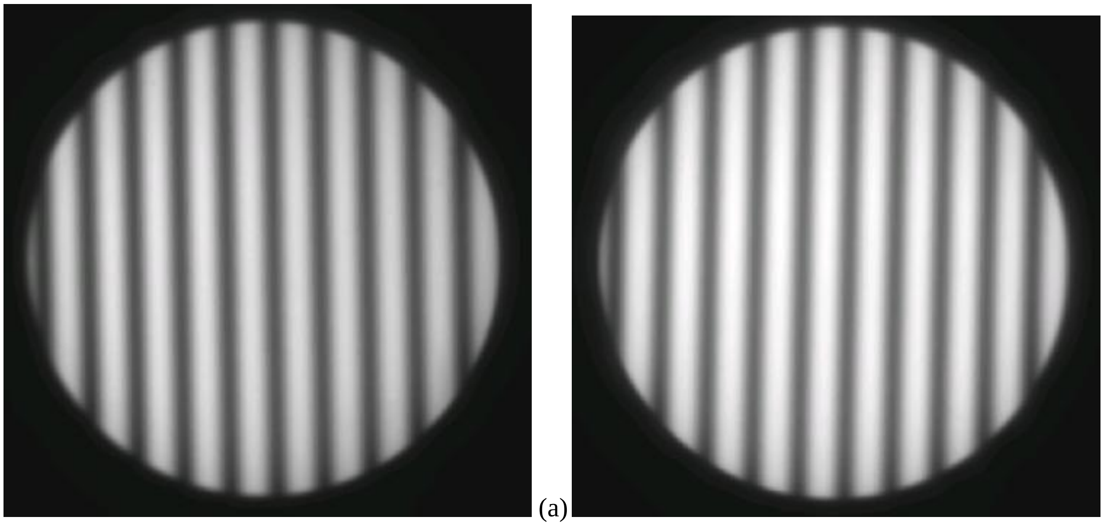

(a) (b)

Figure 3-39. Digital Imaging Photometer images of the projected 2.0 cycles per degree gobo from the (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

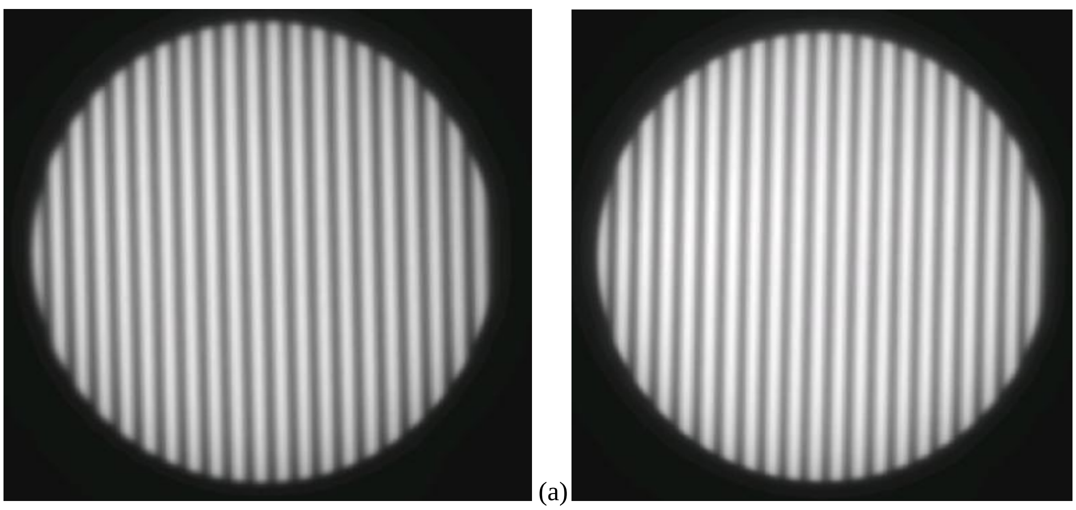
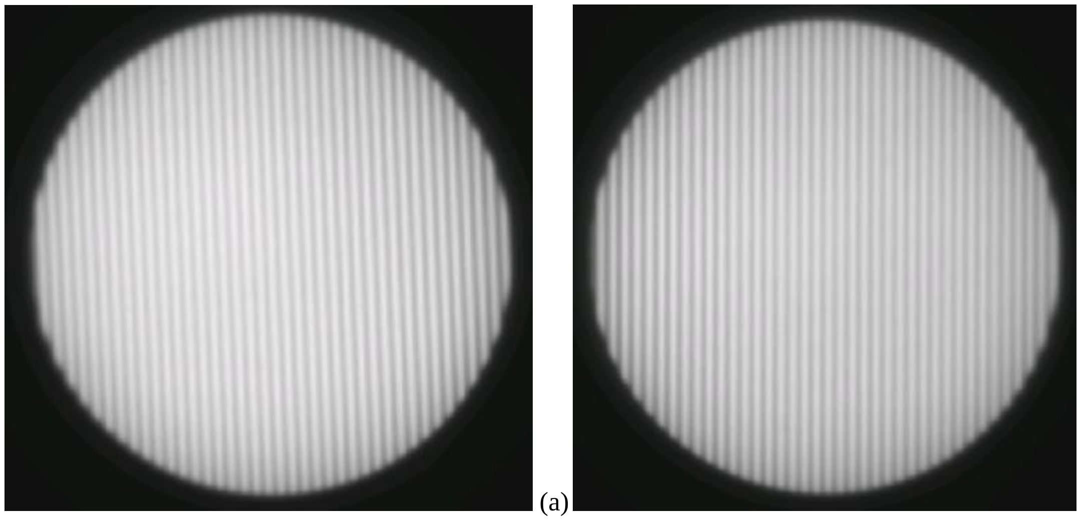
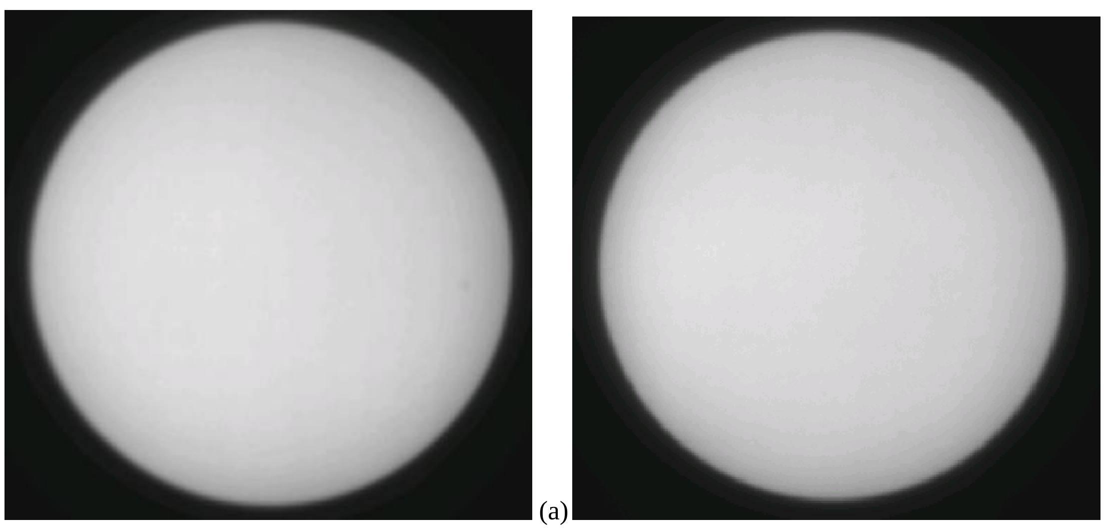

(a) (b)

Figure 3-40. Digital Imaging Photometer images of the projected 4.0 cycles per degree gobo from the (a) left Robert Juliat fixture and (b) right Robert Juliat fixture



(a) (b)

Figure 3-41. Digital Imaging Photometer images of the projected 8.0 cycles per degree gobo from the (a) left Robert Juliat fixture and (b) right Robert Juliat fixture



(a) (b)

Figure 3-42. Digital Imaging Photometer images of the projected Uniform gobo from the (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

Coordinating with Apollo in making the gobos was a year-long process. They are experts in making gobos for theatrical applications. In most theatrical applications, the precision of the gobo is not important, but in this application, precision was of the utmost importance. Their process in making gobos is a chemical process, whereby the glass (which is initially completely black) is dipped into an acid bath which eats away at the

black ink to leave only the pattern that is intended to remain. How light or dark this pattern gets is determined by how long the glass is submerged in the acid bath and whether the acid is relatively new or old. Apollo is capable of making only one gobo of this size at a time, and it is a manual process. Therefore, their process is not repeatable. This made for several challenges in working with Apollo.

The first major problem found was that the average luminance of the five different sine wave gratings was different (see Table 3-4). As average luminance was not intended to be an independent variable in this study, it was critical that it did not change from one stimulus to the next. All of the gobos were made with the same 24 rectangles per cycle, all having the same percent black values, so the average luminance should not have varied from one frequency gobo to another. It was determined that this error was most likely due to the fact that Apollo's process is manual, so if one gobo was submerged for 10 seconds, and the next was submerged for 12 seconds, there could certainly be a difference in average luminance. The gobos were remade, taking care that the submerge time was consistent. The average luminance for these new gobos was much more similar (see Table 3-5).

Table 3-4. Average luminance of stimuli as projected through gobos with left Robert Juliat fixture

Frequency (cycles per degree)	Average Luminance (cd/m^2)
0.5	11020
1.0	13053
2.0	13760
4.0	15857
8.0	13450

Table 3-5. Average luminance of stimuli as projected through remade gobos with left Robert Juliat fixture

Frequency (cycles per degree)	Average Luminance (cd/m^2)
0.5	21836
1.0	22098
2.0	22175
4.0	22242
8.0	22053

Similarly, the average luminance of the uniform (0 or infinite cycles per degree) gobo was not what it was expected to be. The zero crossing of the sine wave was at 41% black. Therefore, a uniform patch of 41% black should have given the same average luminance as the sine wave frequencies. When it did not, a series of gobos was ordered, to try to determine what percent black would be needed in a uniform patch to match the average luminance of the sine waves. Eleven gobos were ordered ranging from 25% to 45% black, in 2% increments. The average luminance of each of those gobos was measured and was expected to follow some linear-like trend with the percent black values. This was not the case (see Table 3-6). The 43% uniform gobo was determined to be the closest match to the sine wave gobos.

Table 3-6. Average luminance of different percent black gobos as projected with left Robert Juliat fixture.

% black	Average Luminance (cd/m^2)
25	25776
27	24780
29	26536
31	24779
33	23920
35	25764
37	23110
39	23268
41	19178
43	21853
45	19346

At that point, it seemed clear that Apollo was simply not going to be capable of delivering the precise, repeatable quality that was required for this experiment. A different type of manufacturer was contacted, who could offer better precision: a semi-conductor manufacturer. The semi-conductor process is significantly more precise, so that manufacturer certainly could have provided what was required, but it would have been at a much higher cost than Apollo. In addition, Apollo is knowledgeable about the specifics of a gobo – that it needs to withstand very high heat and the physical dimensions of how it fits into a gobo holder and into the fixture, which the semi-conductor manufacturer did not know. It seemed that the semi-conductor manufacturer could provide the precision, but would have trouble providing the product needed.

Apollo offered to create the gobos using a new process they were still developing, a digital process. This seemed to be the perfect solution to the problems. They remade the sine wave gobos using the digital process, and the average luminances were very

consistent. But it was then that a new problem was discovered. It seemed that as the spatial frequency increased, the modulation (contrast) decreased (see Figure 3-43).

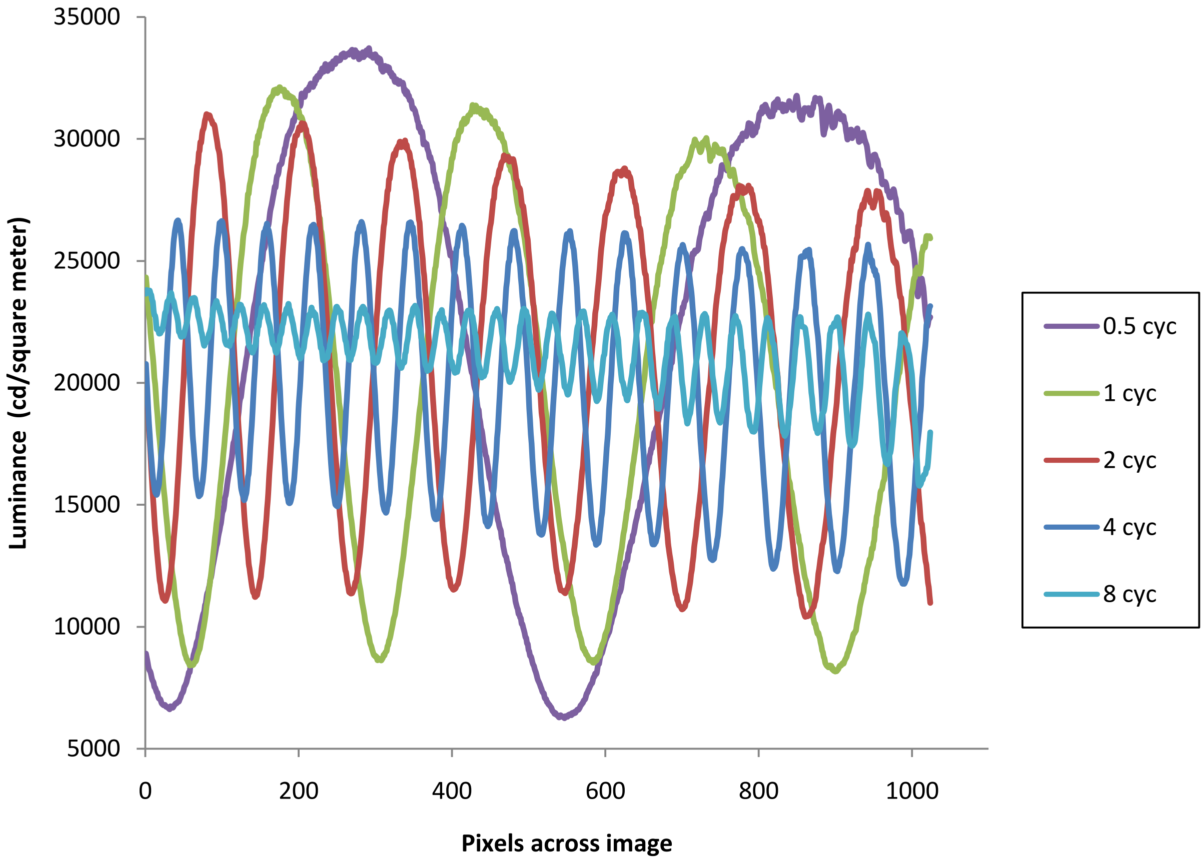


Figure 3-43. Luminance values of different frequency gobos as projected with left Robert Juliat fixture

It was originally speculated that the problem was simply a measurement error. The images were taken with the digital imaging photometer at the position of the subject, which was 1.5m from the target. From that distance, the target took up only a small amount of the image area of the photometer. The output from the photometer was a 1024 x 1024 matrix of luminance values, which means the smaller the target was within the image, the fewer pixels would be used to represent that target (see Figure 3-44). With

this geometry, there was a significant difference in the number of pixels per rectangle of the five different sine waves (because there are 24 rectangles per sine wave in each of the different spatial frequency gobos). For the 0.5 cycle per degree projection, there were 8.35 pixels per rectangle. For the 8.0 cycle per degree projected image, there were only 0.539 pixels per rectangle. This means that for the 8.0 cycle per degree image, each pixel would have to average more of an area than for the 0.5 cycle per degree image. This would explain the apparent change in modulation shown in Figure 3-43.

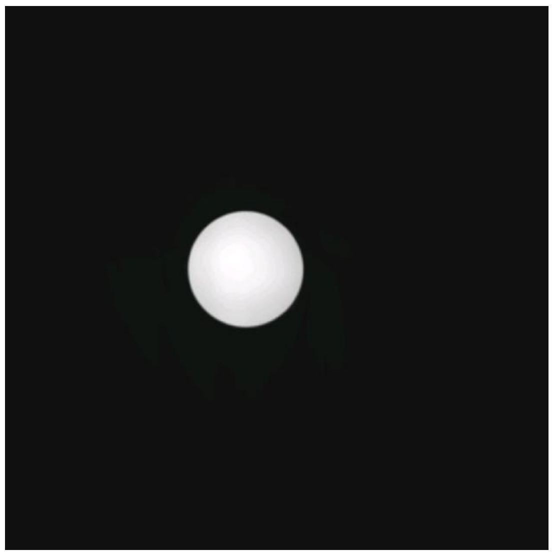


Figure 3-44. Image taken with digital imaging photometer of the target from the perspective of the subject, showing how little of the field of view is taken up by the stimulus

To test the theory that the problem was in the measurement technique, the digital imaging photometer was moved much closer to the target, approximately 0.5m away, so that there would be more pixels per rectangle for all of the different frequencies. That was as close as possible without blocking the projection from the Robert Juliat fixture. However, the same results were found – the modulation decreased as the spatial frequency increased. That suggested that the measurement method was not the culprit.

Assuming this was yet another gobo problem, the gobos themselves (rather than the projected images from the gobos) were measured. The gobo was mounted in front of the opening of the uniform luminance sphere, which served to backlight the gobo. The digital imaging photometer was used to take images of the gobos themselves. Figure 3-45 shows a single row of luminance values (from the 1024 x 1024 matrix output from the digital imaging photometer) plotted for each one of the different gobos. This shows that the gobos themselves have approximately the same modulation.

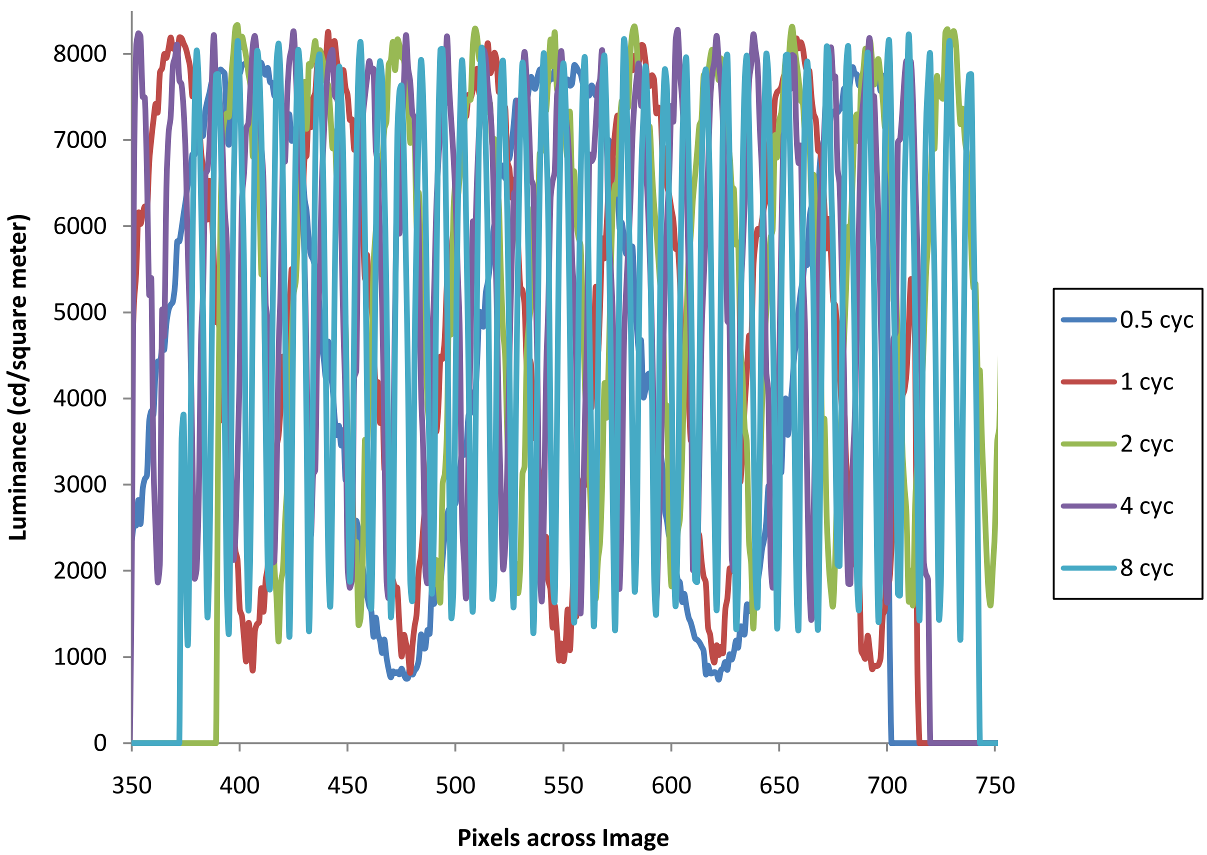


Figure 3-45. Graph of luminance vs. position for different gobos, measured in front of a uniform luminance sphere

Since the problem of decreasing contrast with increasing spatial frequency seemed to only happen with the projected images, and not with the gobo themselves, it appeared that the light fixture was to blame. After long discussions with the Robert Juliat staff, it was determined that the problem was due to the modulation transfer function of the fixture. A modulation transfer function is a property of any optical system (i.e. the Robert Juliat fixtures used in this study), and is defined as the modulation of the projected image divided by the modulation of the original image at all spatial frequencies (Hecht & Zajac 1987, Driscoll 1978). Because of diffraction, even a "perfect" lens (which does not exist in reality), has a modulation transfer function that is not flat across spatial frequency. Therefore, for any single optical system, the modulation transfer function dictates that as the spatial frequency increases, the contrast decreases.

For this experiment, the modulation transfer function had significant implications. It meant that there was a confounded variable in the experiment. Modulation was not intended to be an independent variable, but it changed as the spatial frequency changed. This was not acceptable. There were a few possible solutions to this problem. First, it would have been possible to create the same modulation across gobos if the focus point of the Robert Juliat fixture was changed for each gobo or if the aperture was changed. This was not thoroughly researched, but it seemed possible. It was not seriously considered for this experiment, because it just would not have been practical, as there were 300 trials per subject, and refocusing or changing the aperture for each trial seemed extreme. Second, it would have been possible to change the modulation of the gobos themselves, so that when they were projected, they all would have had the same modulation. This would be done by changing the Adobe Illustrator file that was used to

manufacture the gobos, so that all of the gobos would no longer have the same 24 rectangles per cycle. Instead, the percent blacks used for the edges of each rectangle (and the fountain fill within the rectangle) would have to be different for each of the gobos. This would imply that the highest frequency gobo (8 cycles per degree) would stay as it was, but for all of the others, the modulation would have been decreased to the level of the 8 cycles per degree gobo. From Figure 3-43, the modulation of the 8 cycles per degree gobo was calculated to be approximately 0.069, and the modulation of the 0.5 cycles per degree gobo was approximately 0.62 (and the other frequencies fell between those two). The modulation of a typical 3 lamp, 18-cell T8 parabolic troffer is approximately 0.8. To decrease the modulation of the four lowest frequency gobos to match the modulation of the 8 cycles per degree gobo would be moving too far from reality of the types of fixtures used in practice today. A modulation of 0.069 is almost uniform (a perfectly uniform field would have a modulation of 0), so this solution did not seem feasible. It was therefore decided not to project the sinusoidal images, but rather to print them on paper instead, as this phenomenon of modulation transfer function only affects projected images.

Printed Stimuli

When it was determined that the gobos would not work, the decision was made to use printed stimuli instead. The benefit to this was that there should not be a difference in modulation between the stimuli. In addition, it would be beneficial to have more control over the manufacturing process than there had been with the gobo process. The downside was that it would take more time to change the stimuli between trials, as

someone would physically have to walk to the end of the apparatus and change the stimuli (rather than simply change out the gobos from behind the apparatus which could happen quicker). The high luminance needed to create a glare situation would be achieved by illuminating the printed targets with the Robert Juliat luminaires.

The process of determining the relationship between percent black and reflectance (rather than transmittance) had to be repeated for printed media. Figure 3-34 was printed onto high reflectance paper, and the reflectance of each percent grey was calculated by measuring the luminance of the percent grey and a 99% reflectance standard under the same lighting conditions, where the reflectance of each percent grey was the product of the luminance of the percent grey and the reflectance of the standard, divided by the luminance of the standard. That data was plotted (see Figure 3-46), and a best fit curve was fitted to the raw data with an R^2 value of 0.998. A similar process was followed as outlined above to calculate the percent black values required for each rectangle (see Table 3-7).

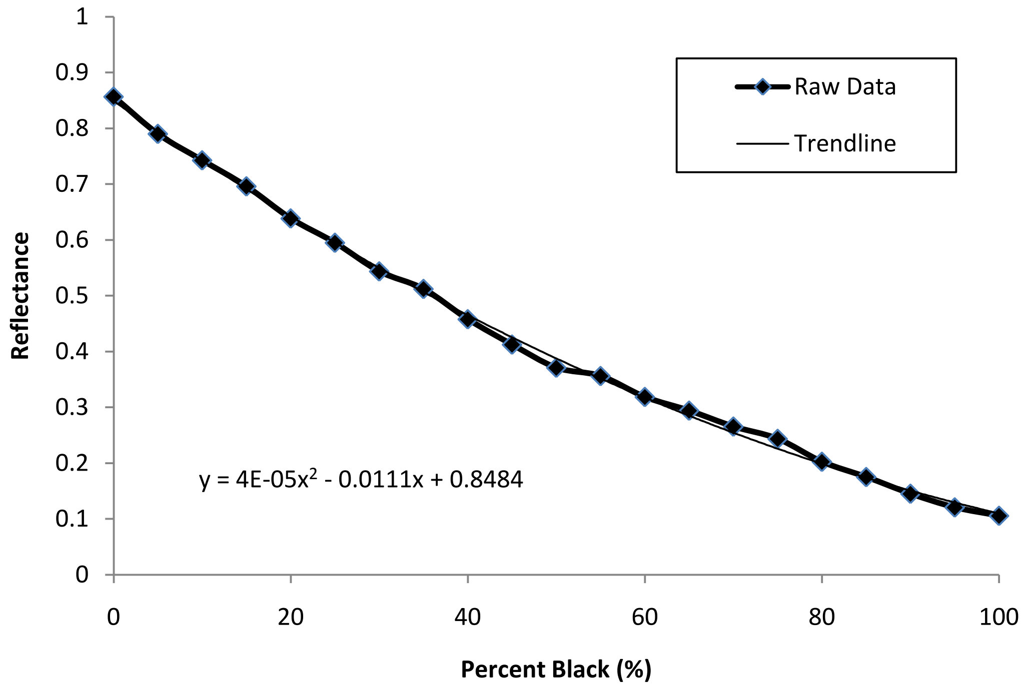


Figure 3-46. Reflectances of percent black values

Table 3-7. Percent blacks for each of the 24 rectangles in each cycle of the sine wave gradients

Rectangle Number	Extents of Percent Black in that Rectangle
1	39 - 29
2	29 - 20
3	20 - 12
4	12 - 6
5	6 - 2
6	2 - 0
7	0 - 2
8	2 - 6
9	6 - 12
10	12 - 20
11	20 - 29
12	29 - 39
13	39 - 53
14	53 - 67
15	67 - 80
16	80 - 90
17	90 - 97
18	97 - 100
19	100 - 97
20	97 - 90
21	90 - 80
22	80 - 67
23	67 - 53
24	53 - 39

The sine wave gradients were then printed onto Environment Text's Ultra Bright White paper using a color printer, then heat mounted to foam core (see Figure 3-47). The foam core targets were then attached to a metal L-shaped bracket with duct tape. The metal bracket was used as the handle and also used to mount the target to the apparatus. To change the stimuli, the L-shaped bracket was simply slipped down over a metal plate

on the apparatus (see Figure 3-48). This allowed the stimuli to always be mounted at the same location, relative to the subject’s eyes. It also made for quick changing of the stimuli between trials.

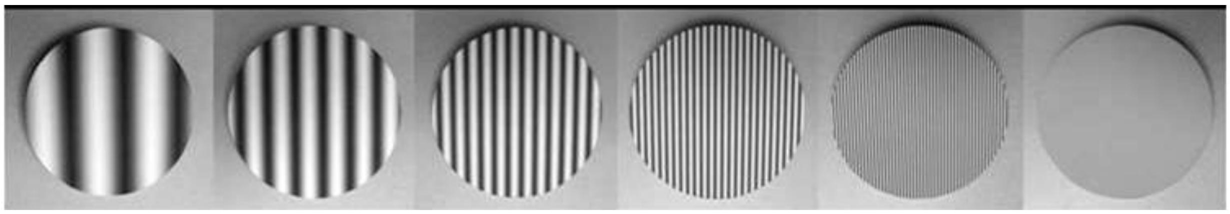


Figure 3-47. Printed stimuli. From left: 0.5, 1, 2, 4, and 8 cycles per degree, and Uniform

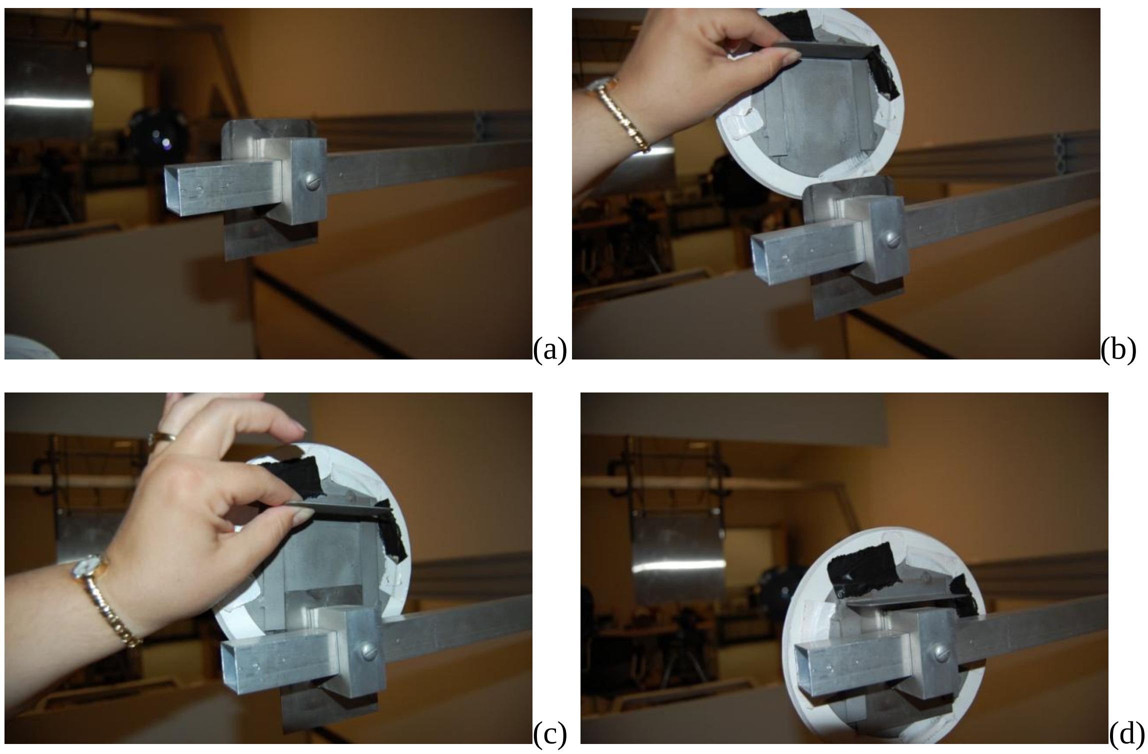


Figure 3-48. Printed stimulus on foam core, mounted to L-shaped bracket, being mounted to metal plate on apparatus

The paper targets, illuminated by the Robert Juliat fixtures, are shown in Figure 3-49 to Figure 3-53. These images are taken with the Radiant Imaging Digital Imaging Photometer.

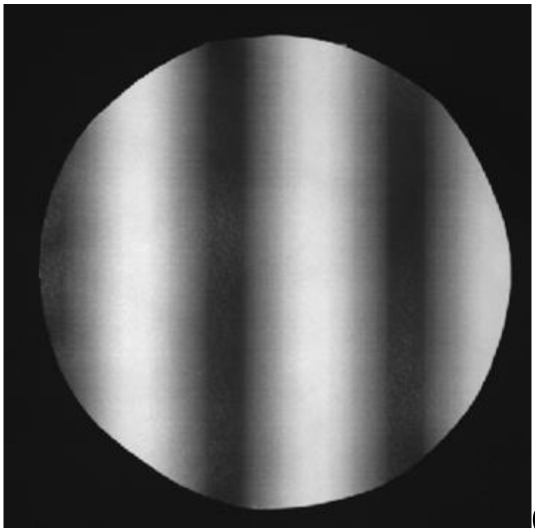
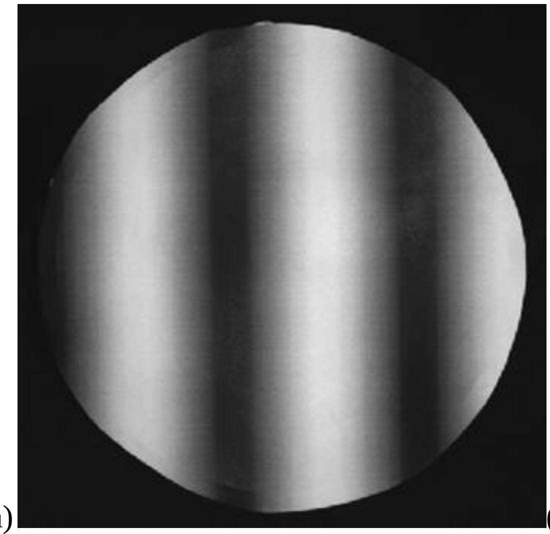

(a)

(b)

Figure 3-49. Digital Imaging Photometer images of 0.5 cycles per degree paper targets illuminated from (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

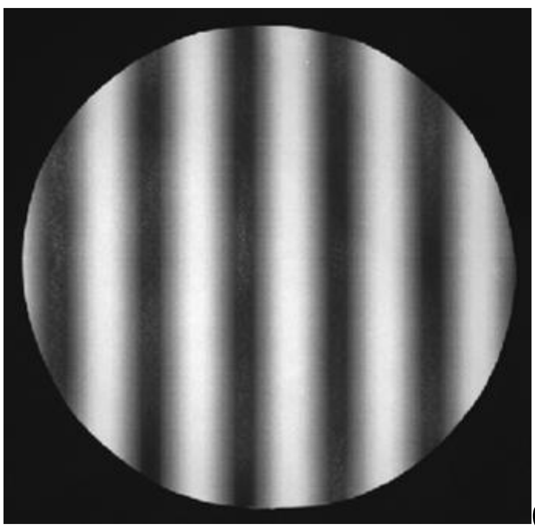
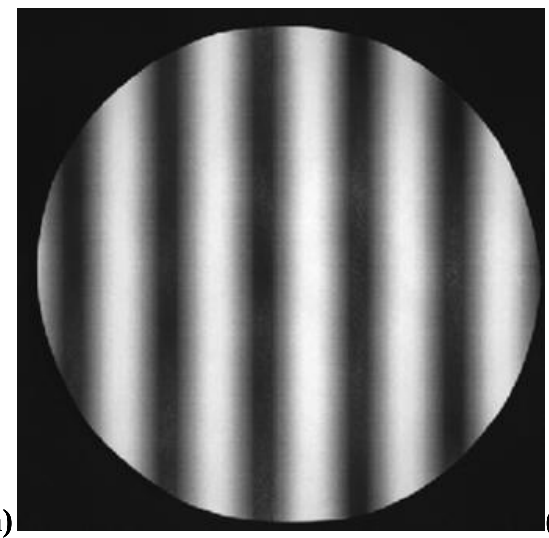

(a)

(b)

Figure 3-50. Digital Imaging Photometer images of 1.0 cycles per degree paper targets illuminated from (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

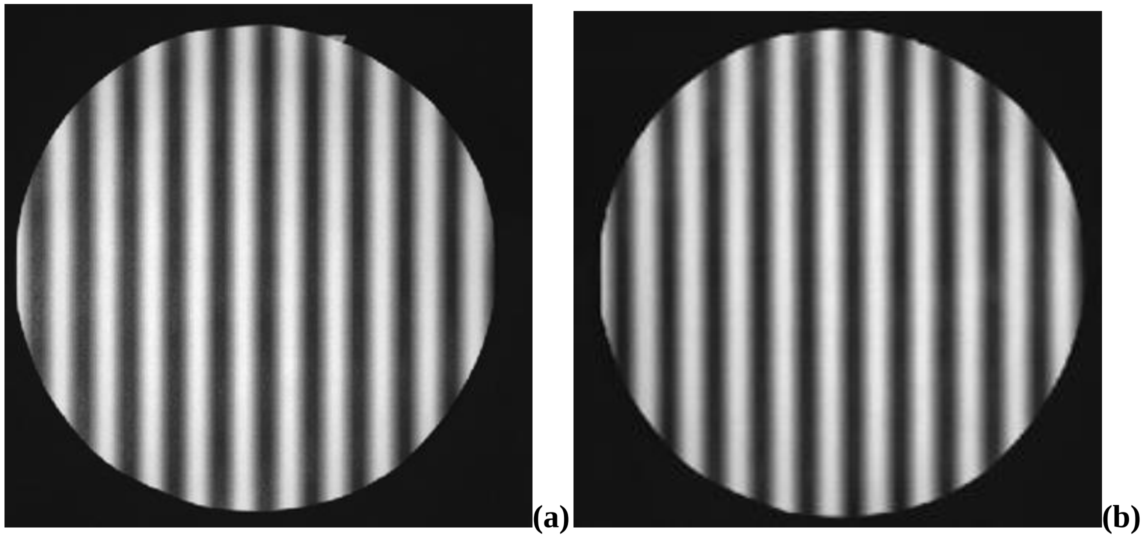


Figure 3-51. Digital Imaging Photometer images of 2.0 cycles per degree paper targets illuminated from (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

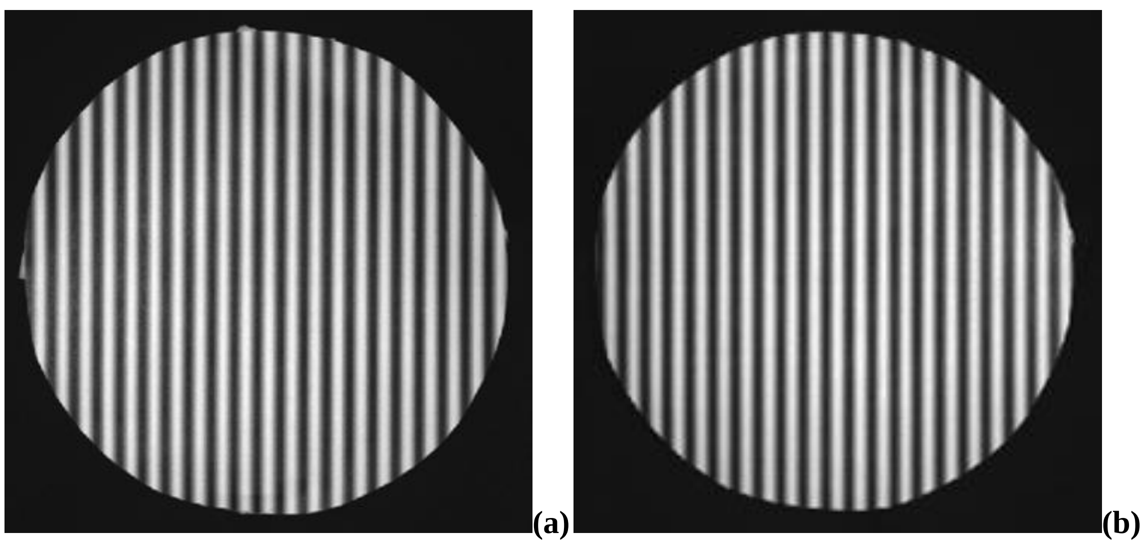
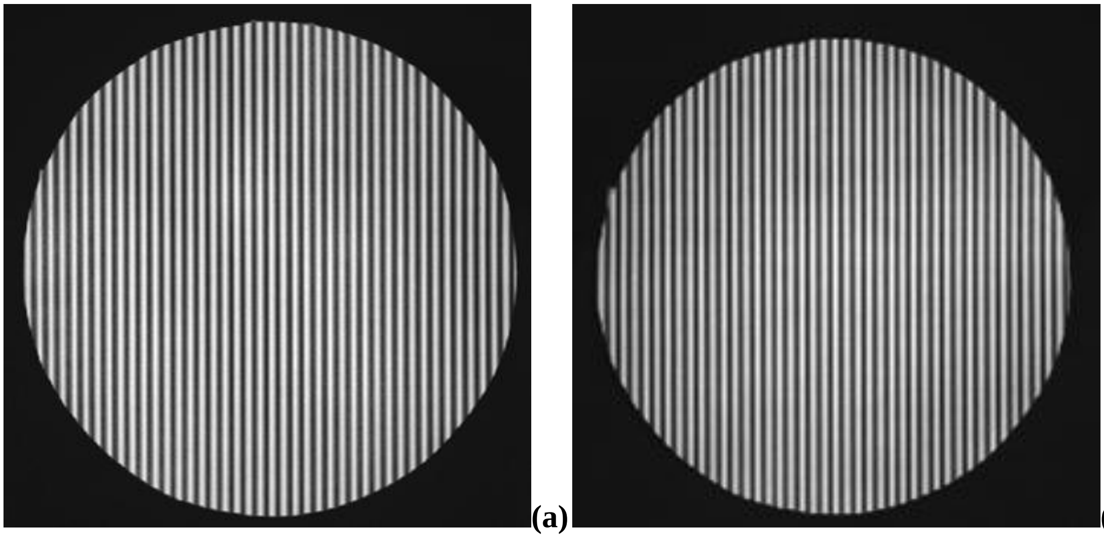


Figure 3-52. Digital Imaging Photometer images of 4.0 cycles per degree paper targets illuminated from (a) left Robert Juliat fixture and (b) right Robert Juliat fixture



(a) (b)

Figure 3-53. Digital Imaging Photometer images of 8.0 cycles per degree paper targets illuminated from (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

The Digital Imaging Photometer software was also used to export each of these measurements as a 1024 by 1024 matrix of luminance values. It was important that the stimuli were illuminated uniformly, so the PI could be certain that what the subjects were responding to was either position or frequency. If the stimuli were illuminated uniformly, then for any of the different spatial frequencies, when any row of the matrix was plotted, it would look like a perfect sine wave, with each successive cycle having the same amplitude. The following graphs show the plotted luminance across the images of the stimuli (from left to right), where the luminance value at each horizontal position is actually the average of ten rows of data – rows 512 through 522 (see Figure 3-54 through Figure 3-63). Those rows were chosen because they are near to the center of the 1024 row matrix of values.

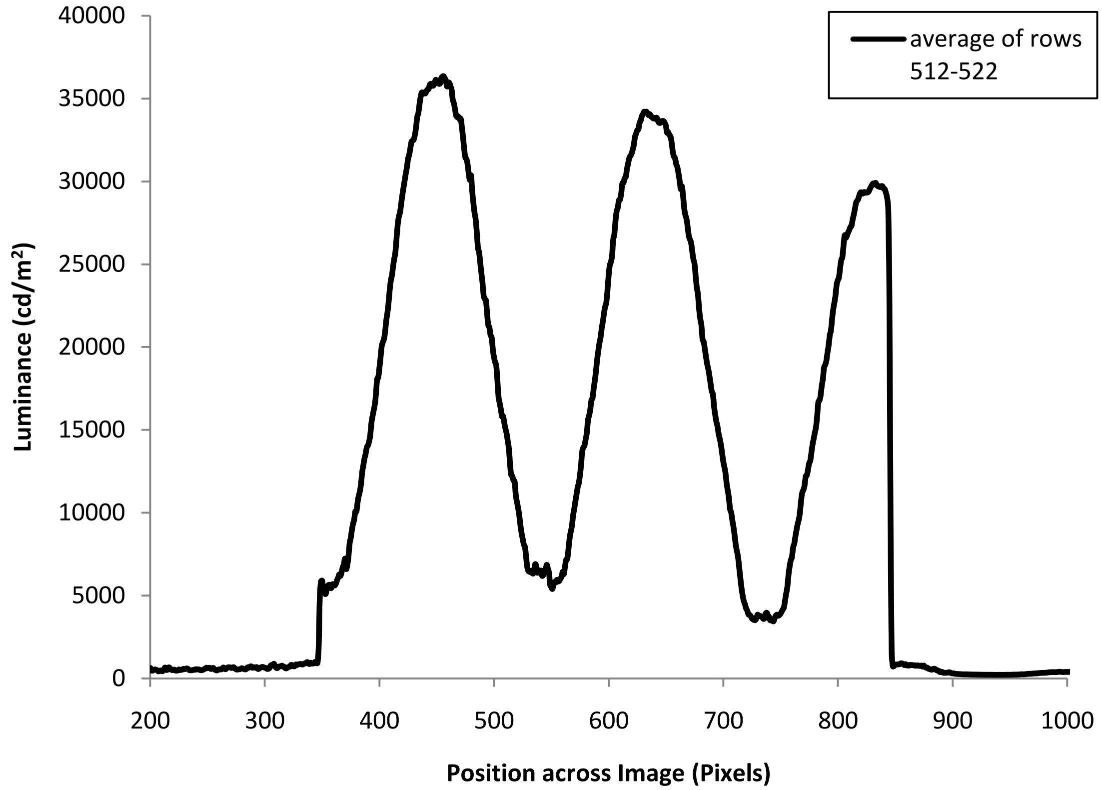


Figure 3-54. Graph of luminance vs. position for 0.5 cycles per degree paper target illuminated by left Robert Juliat fixture

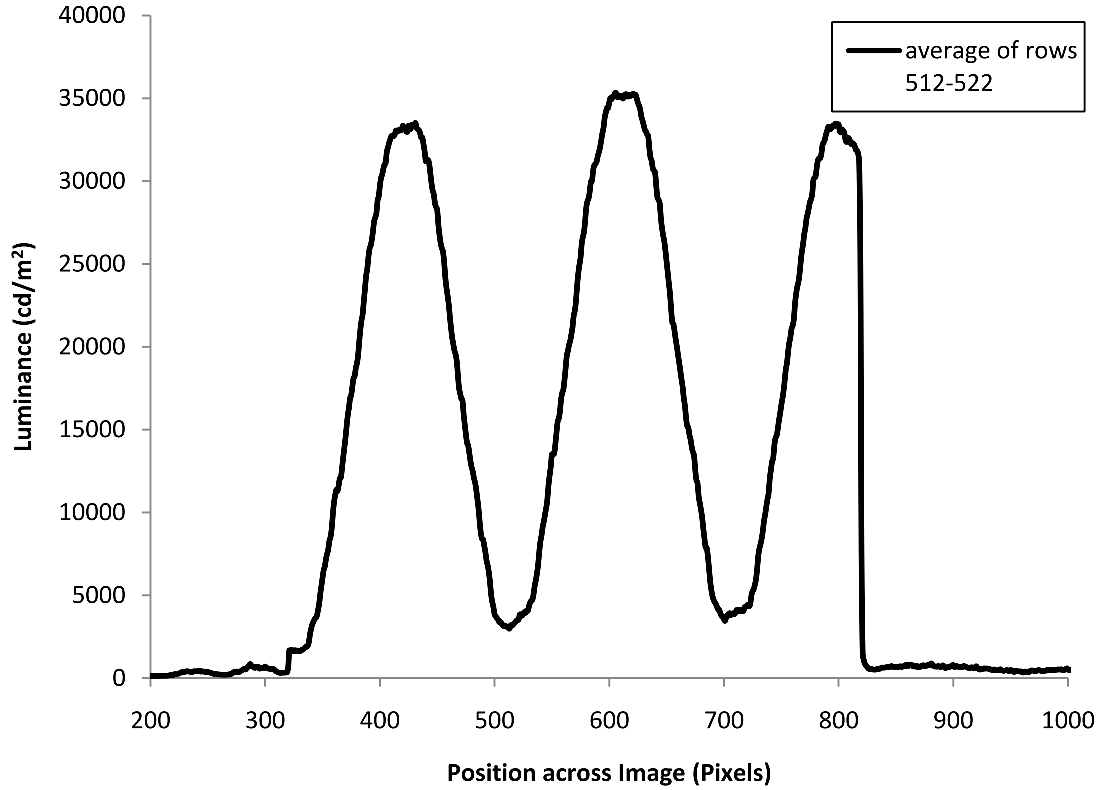


Figure 3-55. Graph of luminance vs. position for 0.5 cycles per degree paper target illuminated by right Robert Juliat fixture

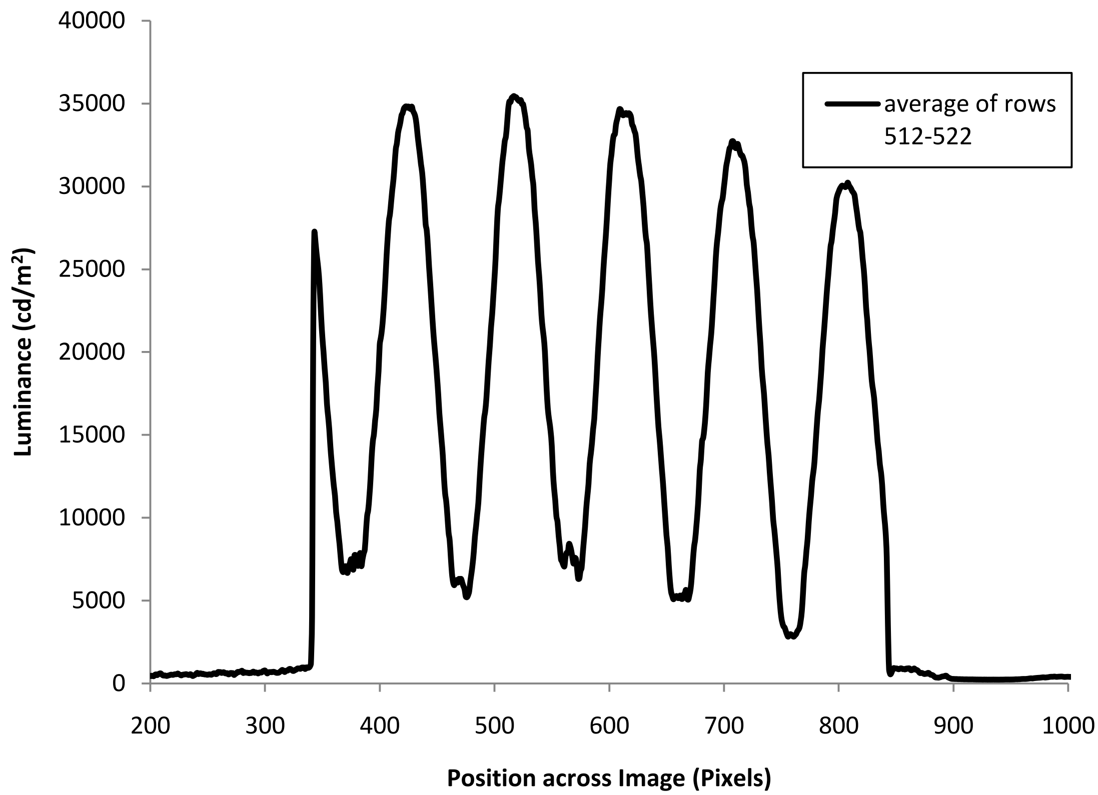


Figure 3-56. Graph of luminance vs. position for 1.0 cycles per degree paper target illuminated by left Robert Juliat fixture

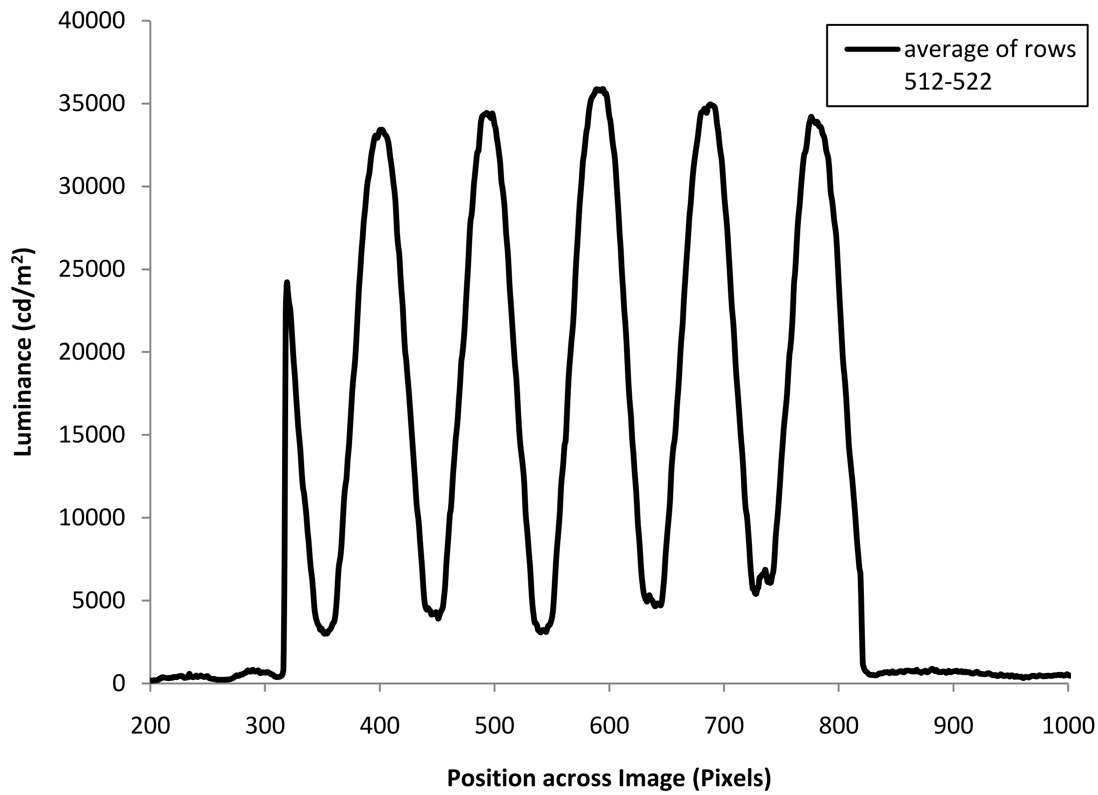


Figure 3-57. Graph of luminance vs. position for 1.0 cycles per degree paper target illuminated by right Robert Juliat fixture

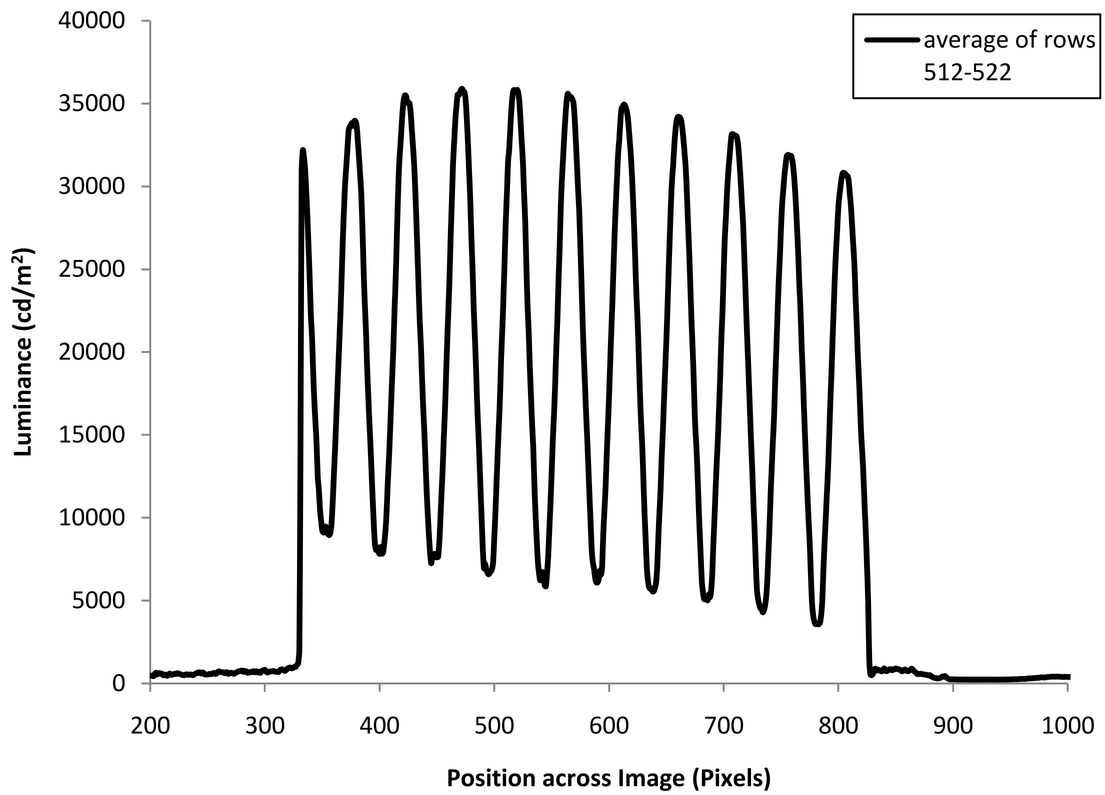


Figure 3-58. Graph of luminance vs. position for 2.0 cycles per degree paper target illuminated by left Robert Juliat fixture

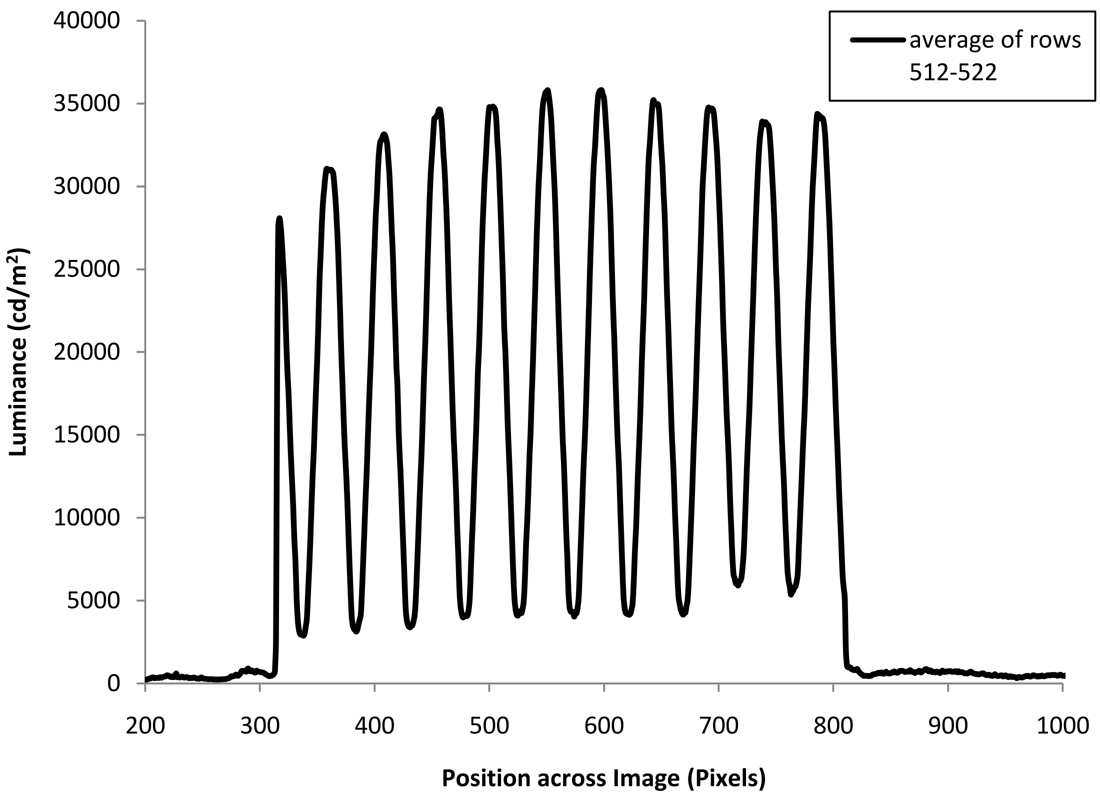


Figure 3-59. Graph of luminance vs. position for 2.0 cycles per degree paper target illuminated by right Robert Juliat fixture

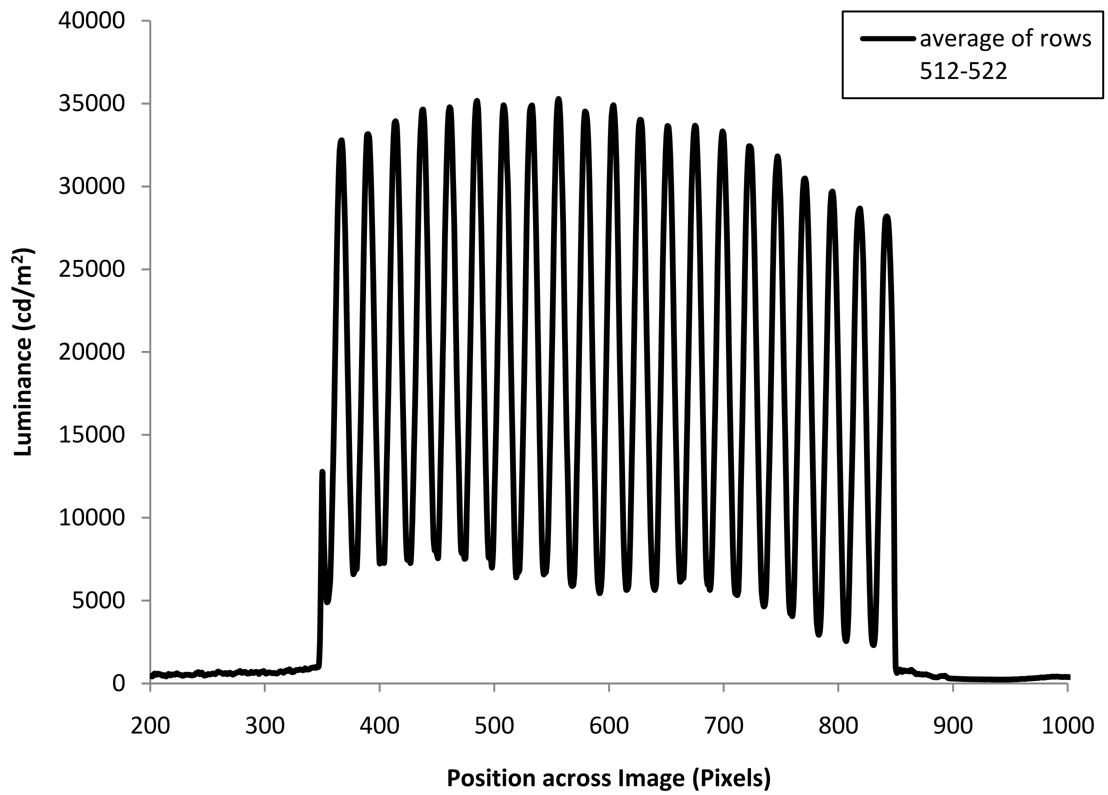


Figure 3-60. Graph of luminance vs. position for 4.0 cycles per degree paper target illuminated by left Robert Juliat fixture

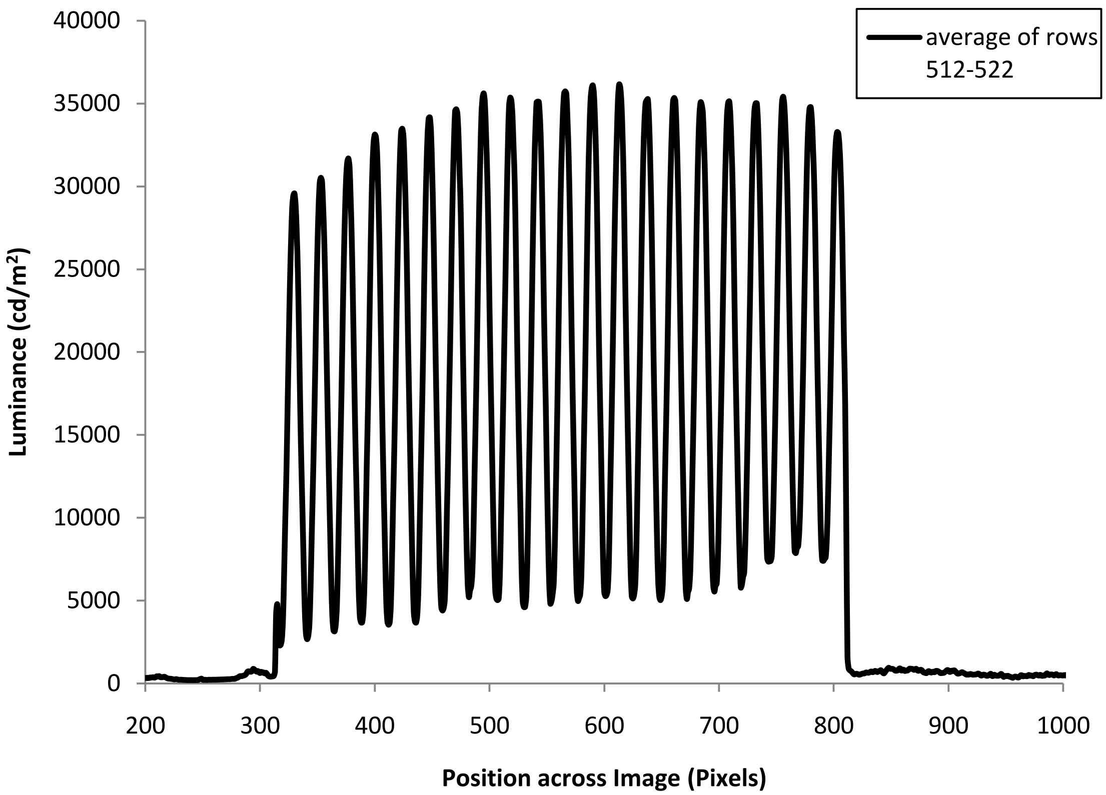


Figure 3-61. Graph of luminance vs. position for 4.0 cycles per degree paper target illuminated by right Robert Juliat fixture

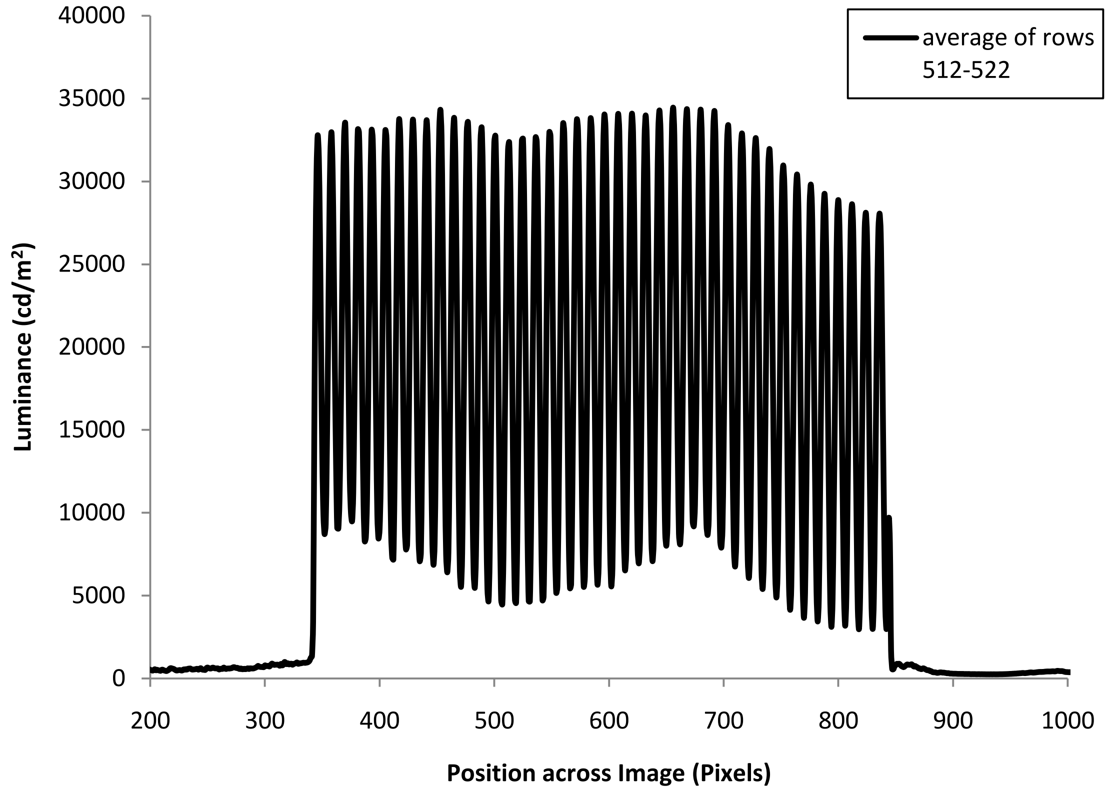


Figure 3-62. Graph of luminance vs. position for 8.0 cycles per degree paper target illuminated by left Robert Juliat fixture

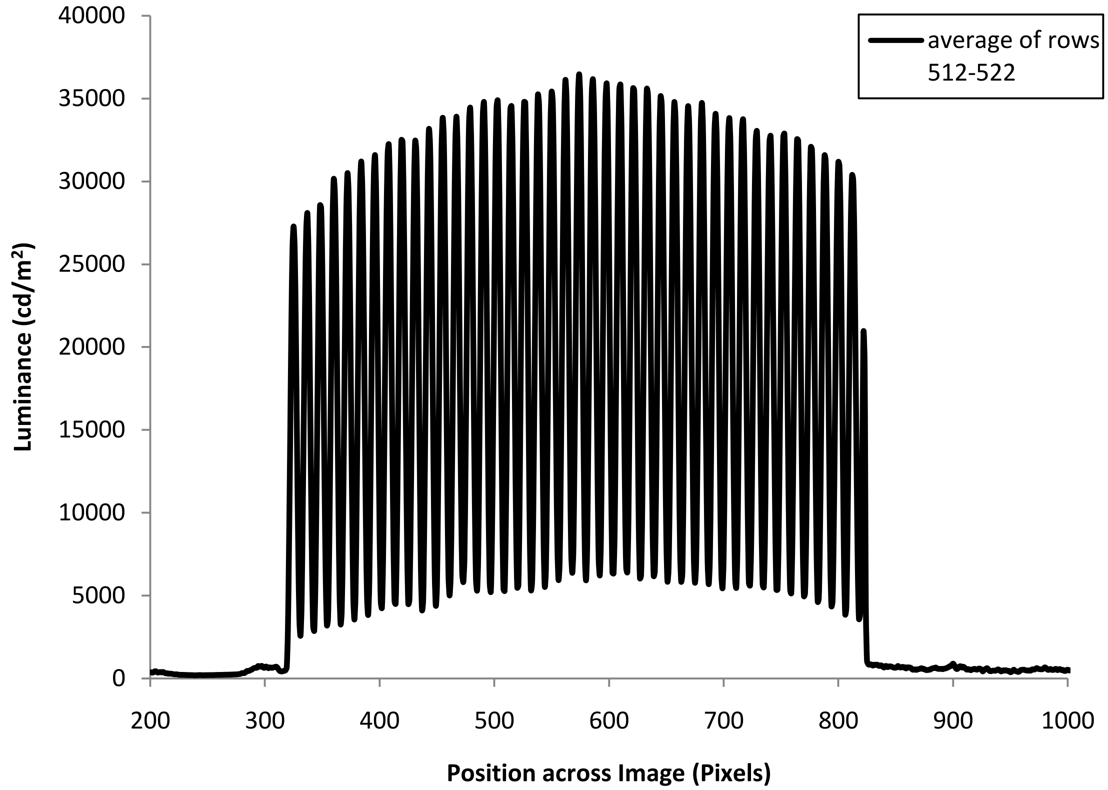
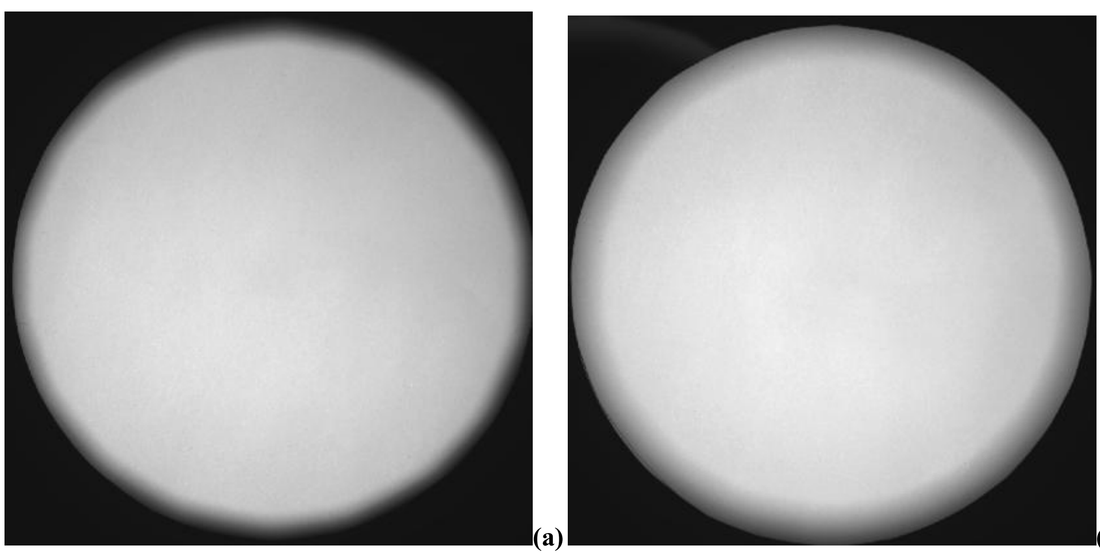


Figure 3-63. Graph of luminance vs. position for 8.0 cycles per degree paper target illuminated by right Robert Juliat fixture

These graphs show that the Robert Juliat fixtures were not uniformly illuminating the stimuli. This was unacceptable as the intent of this study was to determine if differing frequencies have an impact on glare, and if the output from the fixture was not uniform, then the sine wave that was illuminated would not truly be a sine wave (as can be seen from the above graphs).

In addition to the non-uniformity of a single Robert Juliat fixture, the two targets (left and right) did not have the same average luminance. This was obviously a problem as the right fixture always had a higher luminance than the left fixture. For the paired comparison experiment, this was not acceptable.

To attempt to better quantify the problem, the Robert Juliat fixtures were each projected onto a target painted with a flat, high reflectance paint. A digital imaging photometer measurement was taken of those targets (see Figure 3-64). The data from the photometer was also used to generate graphs of the luminance gradient both horizontally and vertically (see Figure 3-65 and Figure 3-66). The following graphs show the plotted luminance from left to right and from top to bottom, where the luminance value at each position for the row data is actually the average of ten rows of data – rows 512 through 522. Those rows were chosen because they are near to the center of the 1024 row matrix of values. The luminance value at each position for the column data is actually the average of ten columns of data – columns SX through TG. Those columns were chosen because they are near to the center of the 1024 column matrix of values. The luminance of the targets ranged from approximately 28,000 cd/m^2 to 40,000 cd/m^2, which was not uniform enough.



(a) (b)

Figure 3-64. Digital Imaging Photometer images of painted targets illuminated from (a) left Robert Juliat fixture and (b) right Robert Juliat fixture

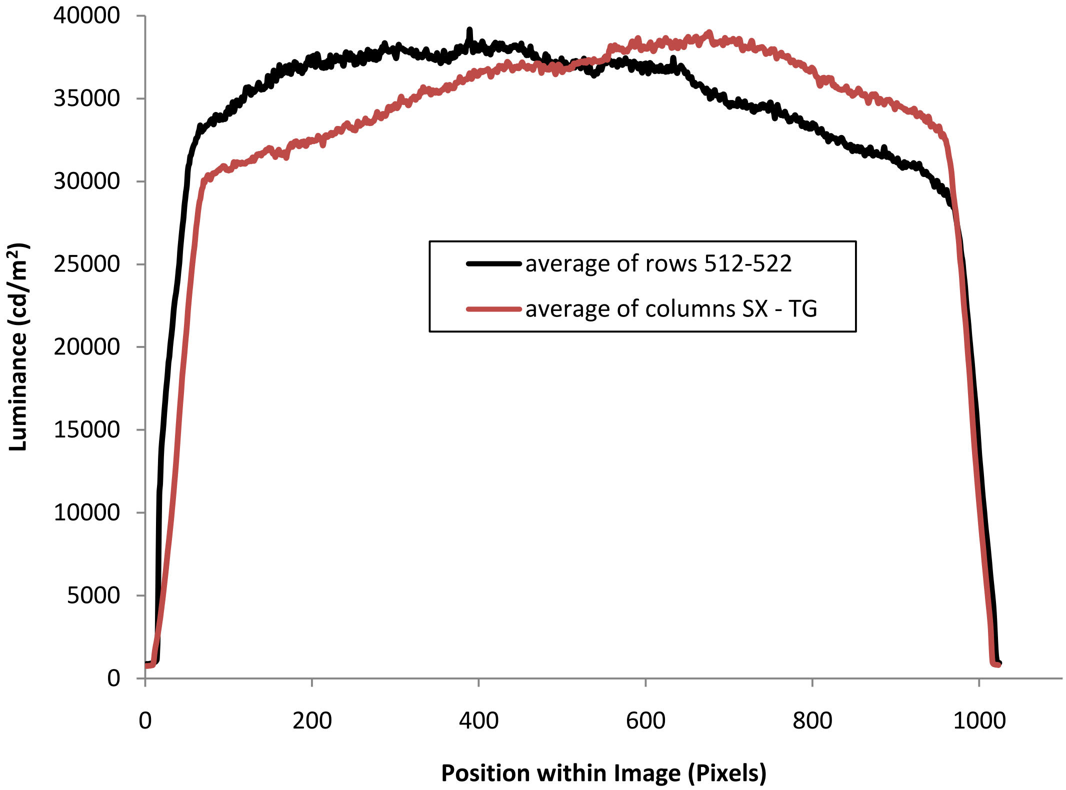


Figure 3-65. Graph of luminance vs. position for painted targets illuminated by left Robert Juliat fixture

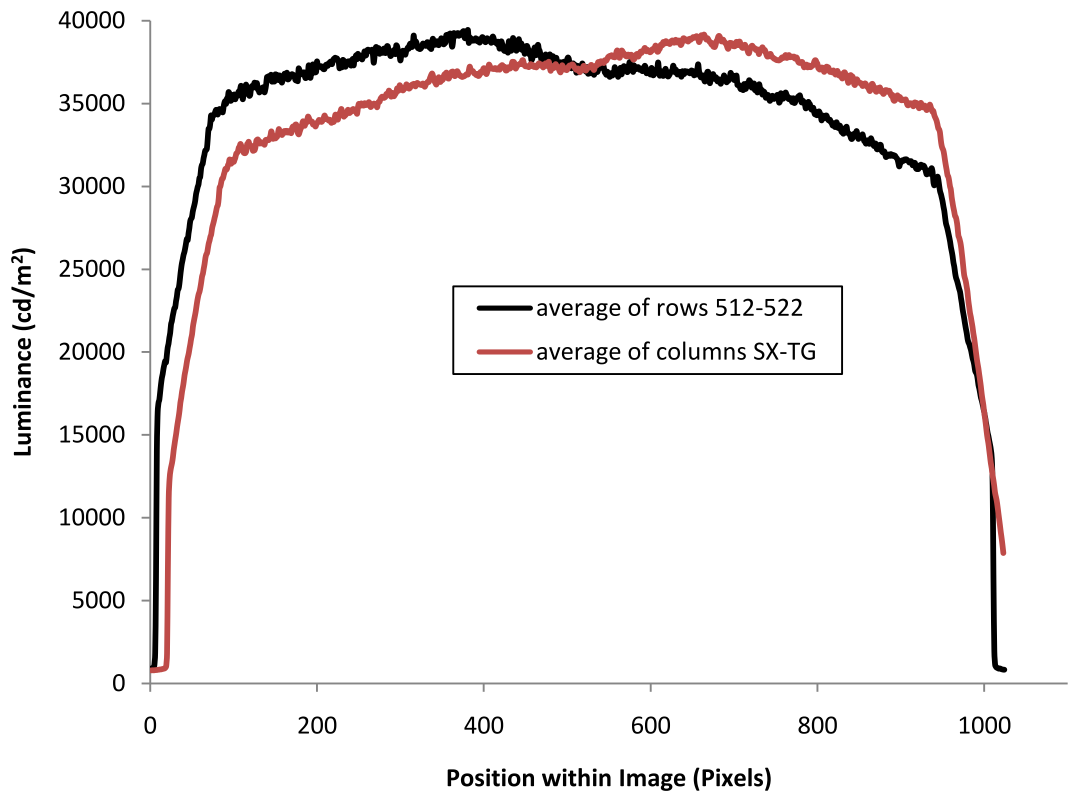


Figure 3-66. Graph of luminance vs. position for painted targets illuminated by right Robert Juliat fixture

Therefore, to attempt to solve both of these problems, a "uniformity" gobo was made for each of the Robert Juliat fixtures. "A" size, 100mm diameter gobos, manufactured by Rosco, were used. The gobos were made from 1.1mm (+/- 0.1mm) thick borosilicate glass (Borofloat glass manufactured by Schott) with a 200 angstrom thick matte aluminum coating. The artwork was custom designed in black and white and was generated in Adobe Illustrator CS2, version 12.0.1. Rosco's precision was 405 lines per inch stochastic line screen.

To generate the "uniformity" gobo, the digital imaging photometer measurements were used as shown above in Figure 3-64. In Matlab, a program was written which

manipulated each pixel of the image, such that if the luminance at the outside edge of the target was 28,000 cd/m^2 and the luminance in the middle of the target was 40,000 cd/m^2, then a multiplication factor was calculated for each pixel which, when multiplied by the original value, forced the value to be 28,000 cd/m^2. This multiplication factor was then converted into a percent black value, and a new image (in .jpg format) was created from those percent black values, making a quasi-negative of the original image, such that where the original image had a high luminance in the middle and lower luminance on the perimeter, the new image had a low luminance in the middle and higher luminance on the perimeter Figure 3-67 and Figure 3-68 show the images generated from the Matlab program. The dark area is almost not visible, but is enough to lower the luminance to make the field more uniform. Figure 3-69 and Figure 3-70 show the contour maps of the percent black values used for these images, which better show the gradient across the gobo. The Matlab code used to generate the "uniformity" gobo images is given in Appendix A – Matlab Code for Uniformity Gobo. The images were sent to Rosco, who manufactured the gobos. One of these gobos was made for each of the two Robert Juliat fixtures, and was placed in the fixture for the duration of the experiment. Figure 3-71 shows the finished product from Rosco, mounted in a gobo holder, ready to be inserted into the Robert Juliat fixture. The text that is barely visible on the gobo was left on intentionally, to indicate the top of the gobo. Otherwise, the correct orientation of the gobo would have been difficult to determine.

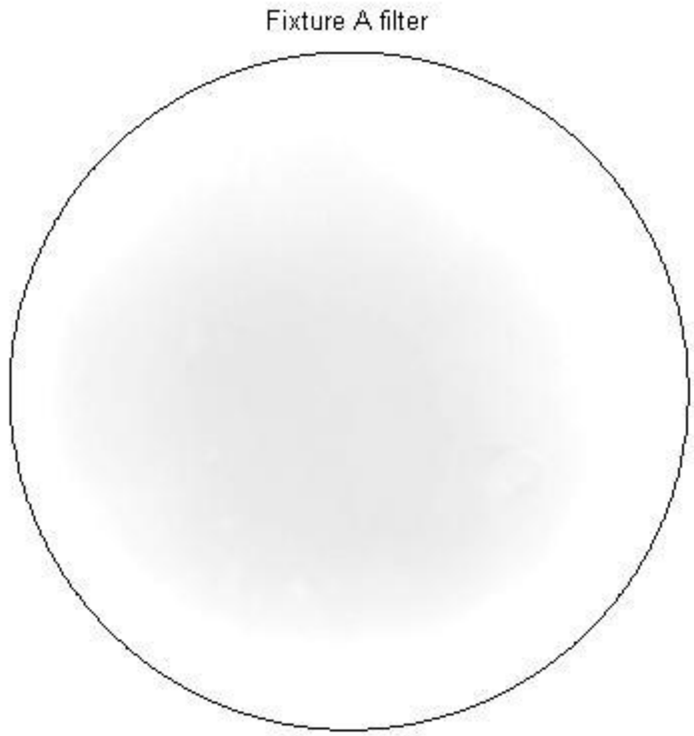


Figure 3-67. Matlab image of “uniformity” gobo for left Robert Juliat fixture (outline circle added for clarity)

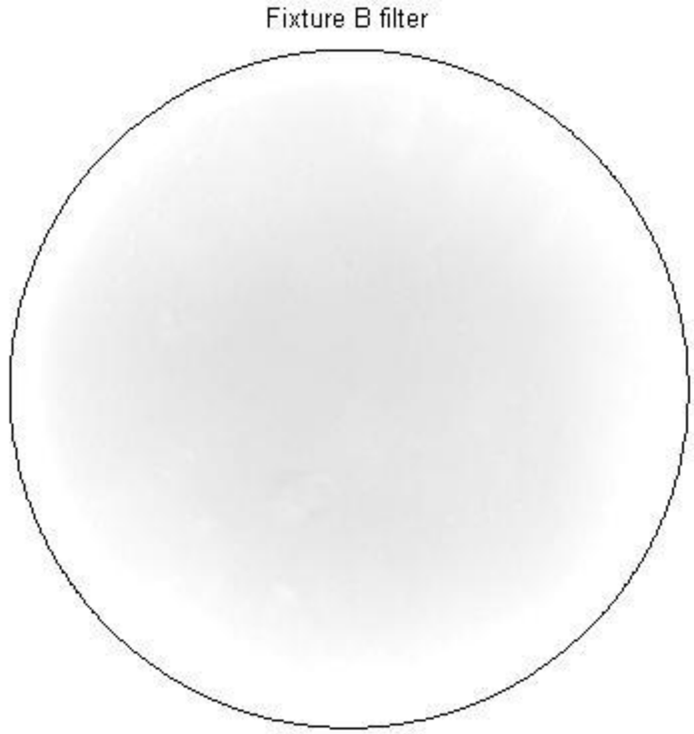


Figure 3-68. Matlab image of "uniformity" gobo for right Robert Juliat fixture (outline circle added for clarity)

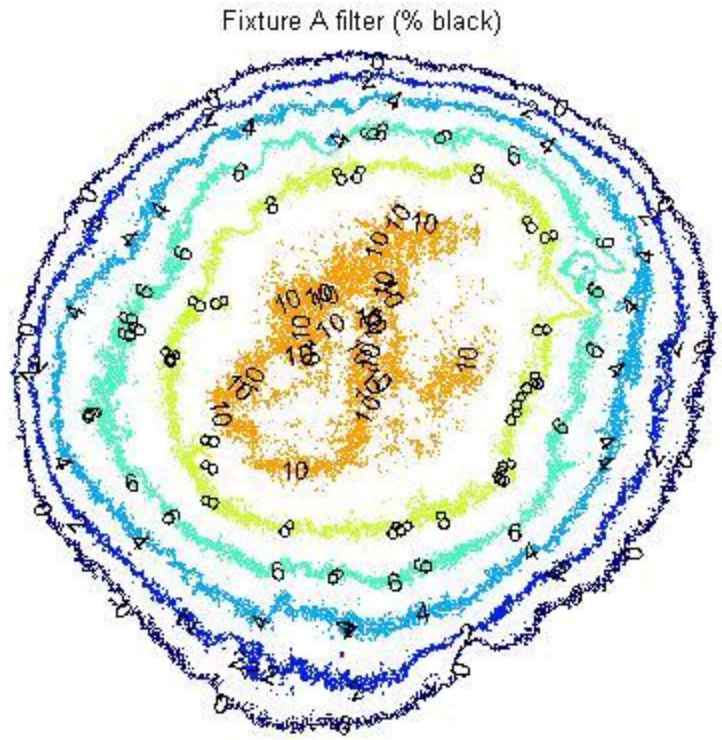


Figure 3-69. Matlab image of contour map of percent grey values for “uniformity” gobo for left Robert Juliat fixture

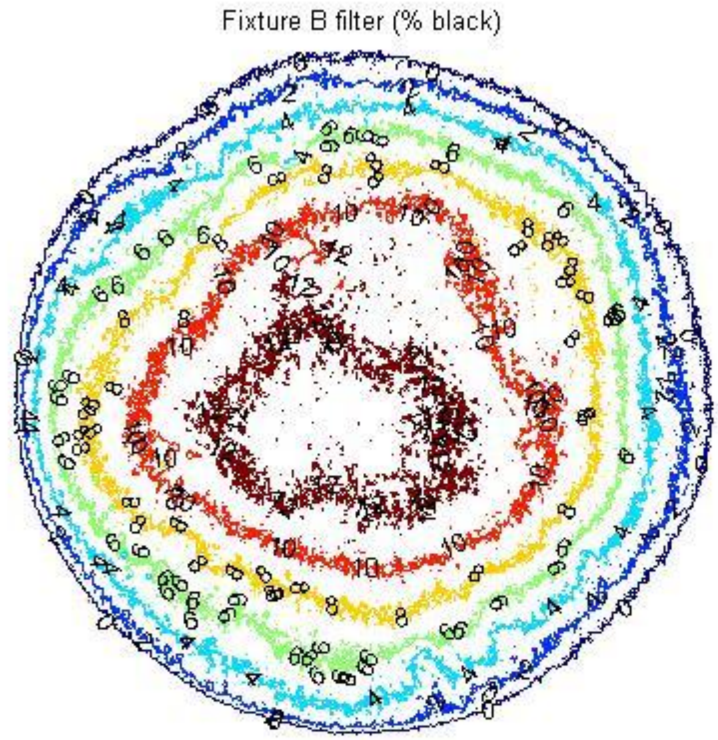


Figure 3-70. Matlab image of contour map of percent grey values for "uniformity" gobo for right Robert Juliat fixture

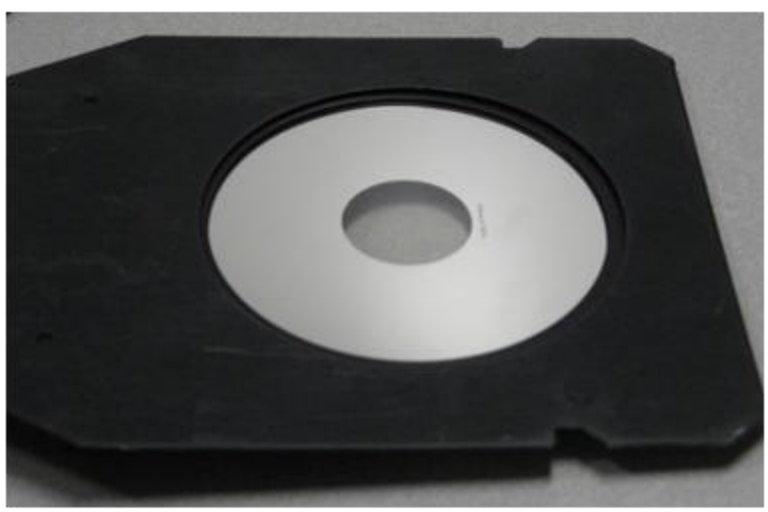


Figure 3-71. "Uniformity" gobo mounted in a gobo holder

When the projected beam from the Robert Juliat fixtures was made to be more uniform due to the “uniformity” gobo, the spatial frequency targets were mounted on the apparatus, and a digital imaging photometer measurement was taken, to verify that the stimuli were now uniformly illuminated (see Figure 3-72 to Figure 3-76).

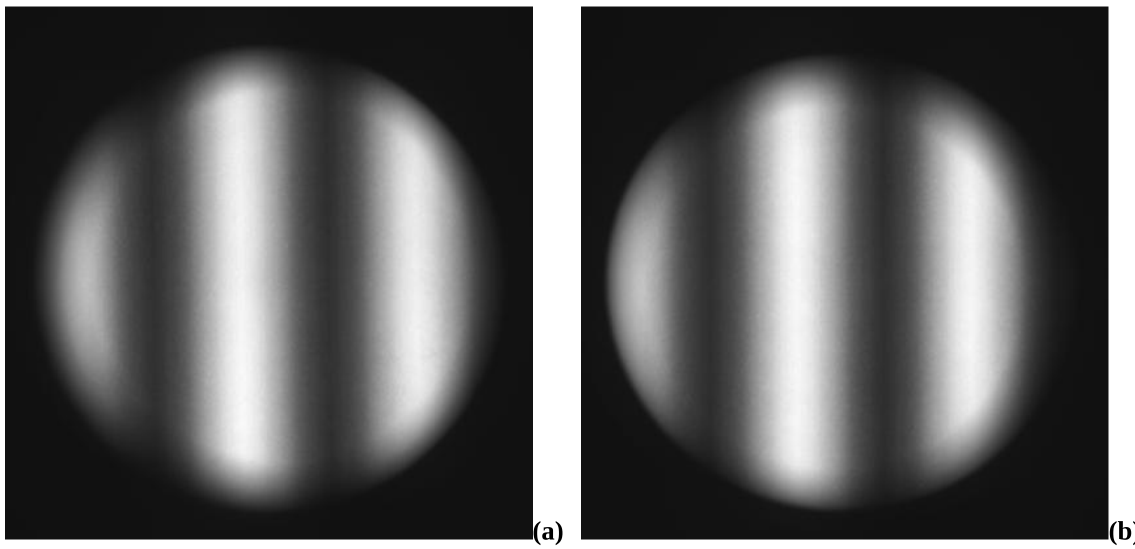


Figure 3-72. Digital Imaging Photometer images of 0.5 cycles per degree paper targets illuminated by (a) left Robert Juliat fixture and (b) right Robert Juliat fixture after “uniformity” gobo was installed

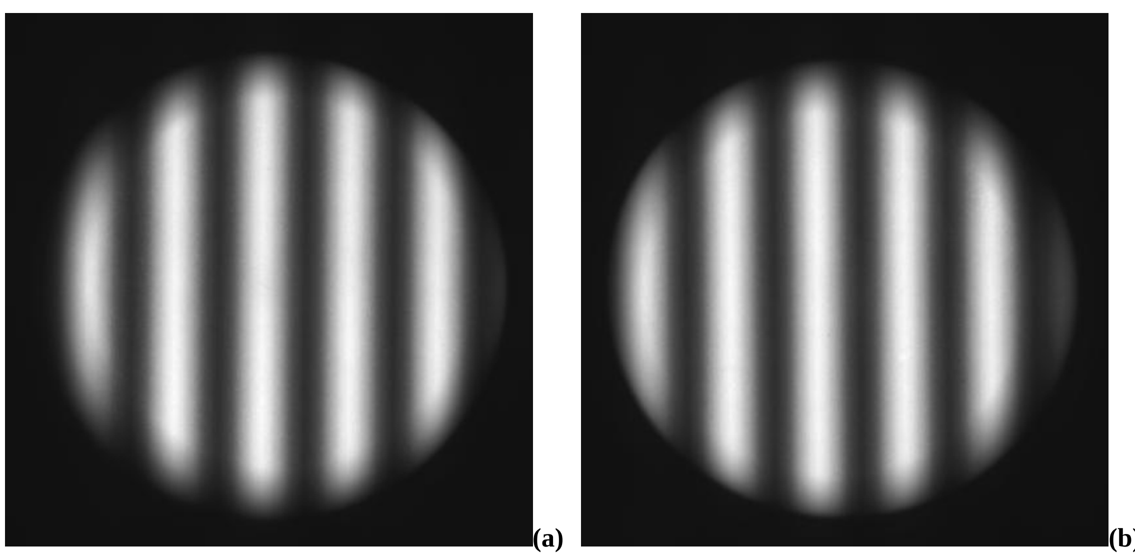
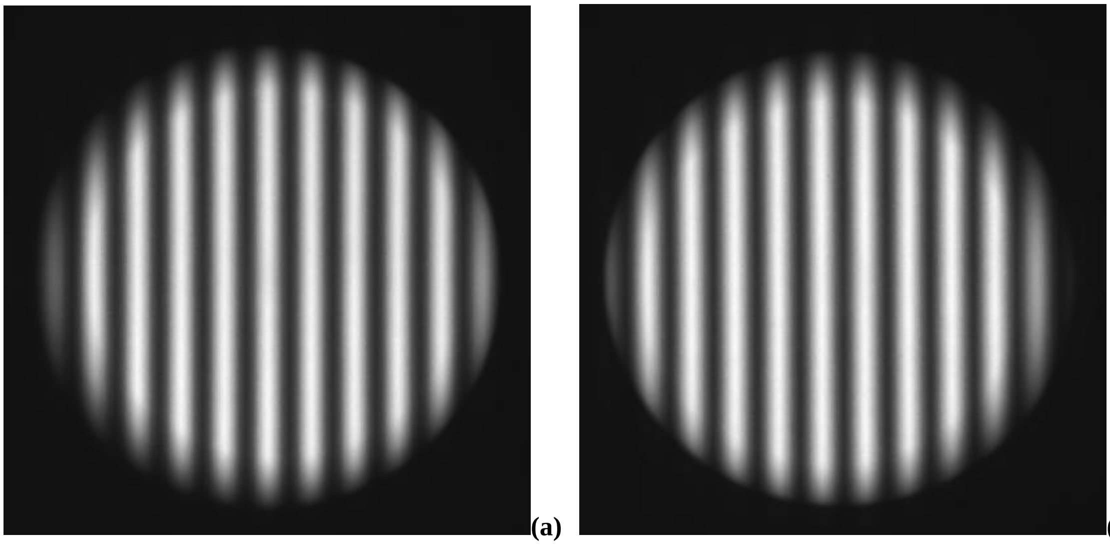
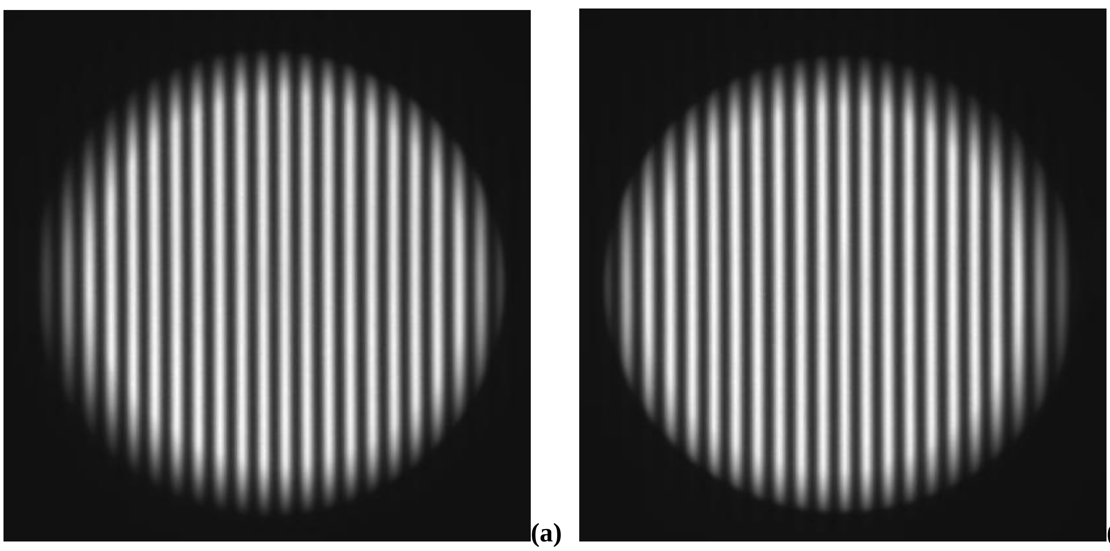


Figure 3-73. Digital Imaging Photometer images of 1.0 cycles per degree paper targets illuminated by (a) left Robert Juliat fixture and (b) right Robert Juliat fixture after “uniformity” gobo was installed



(a) (b)

Figure 3-74. Digital Imaging Photometer images of 2.0 cycles per degree paper targets illuminated by (a) left Robert Juliat fixture and (b) right Robert Juliat fixture after "uniformity" gobo was installed



(a) (b)

Figure 3-75. Digital Imaging Photometer images of 4.0 cycles per degree paper targets illuminated by (a) left Robert Juliat fixture and (b) right Robert Juliat fixture after "uniformity" gobo was installed

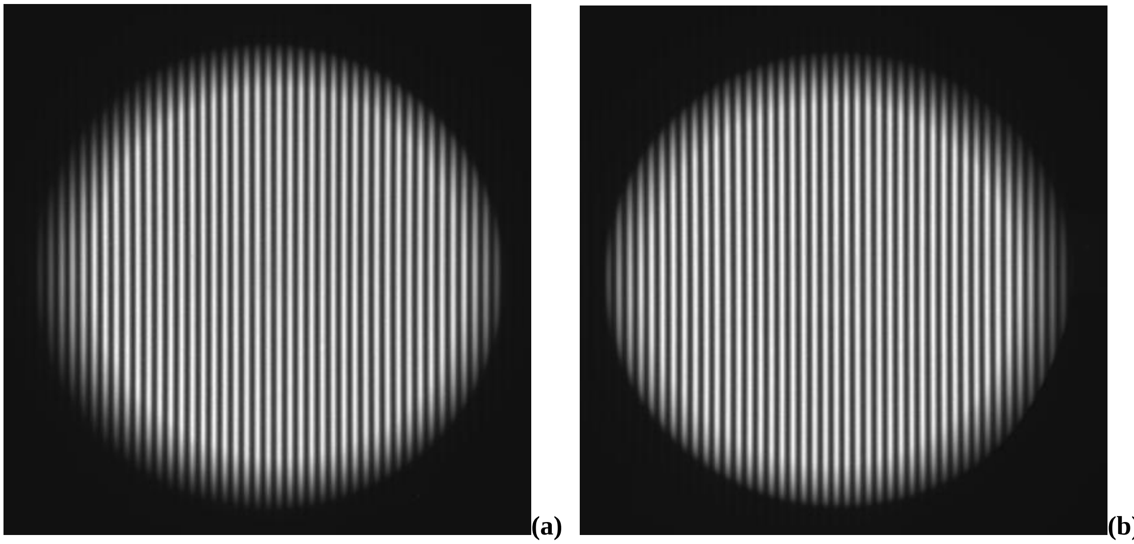


Figure 3-76. Digital Imaging Photometer images of 8.0 cycles per degree paper targets illuminated by (a) left Robert Juliat fixture and (b) right Robert Juliat fixture after "uniformity" gobo was installed

To attempt to better understand the uniformity, the measurement data was used to create the following two and three dimensional graphs, which show the plotted luminance across the images. For the two dimensional graphs, each luminance value is actually the average of ten rows of data (rows 512 through 522). Those rows were chosen because they are near to the center of the 1024 row matrix of values. Also, several columns of data were averaged and plotted on these graphs (columns SX through TG), showing the luminance gradient from the top to the bottom of the stimuli. Those columns were chosen because they are near to the center of the 1024 column matrix of values (which ranges from column A to column AMK). If the stimuli were uniformly illuminated, the average row data would be perfect sine waves, and the average column data would be flat lines (see Figure 3-77 to Figure 3-86). The luminance of the average column data changes from stimulus to stimulus because of where the sine wave was in relation to columns SX through TG.

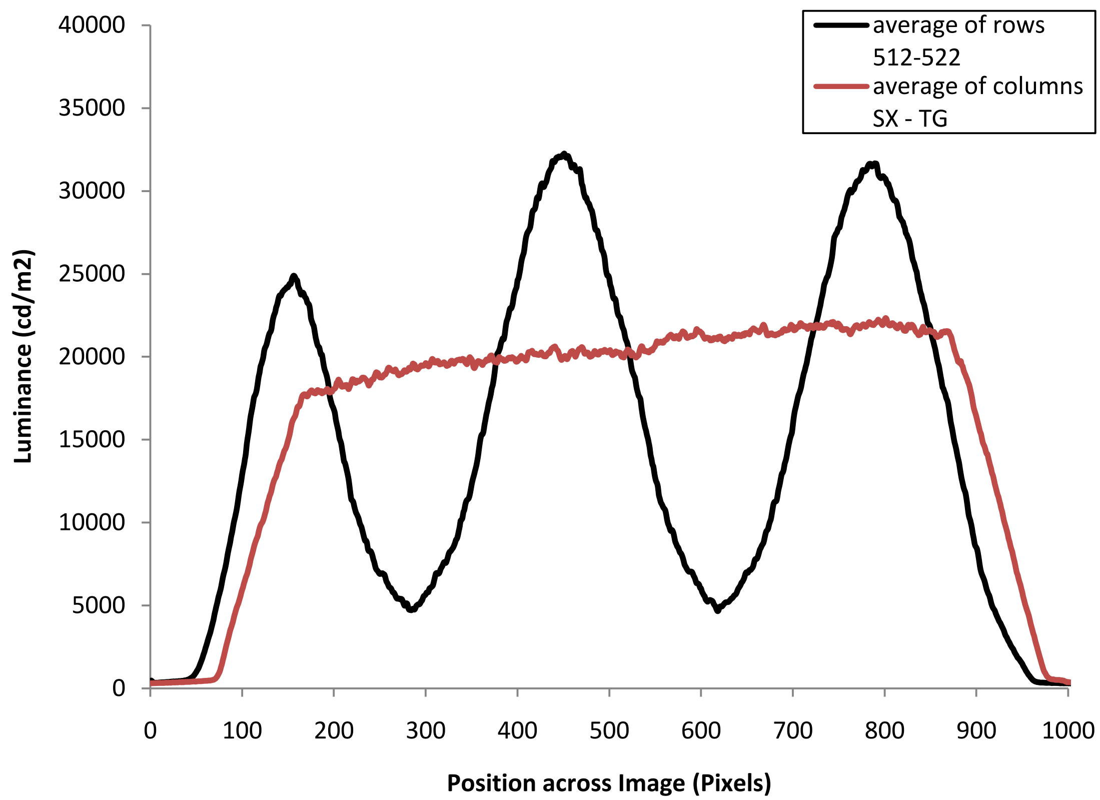


Figure 3-77. Graph of luminance vs. position for 0.5 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

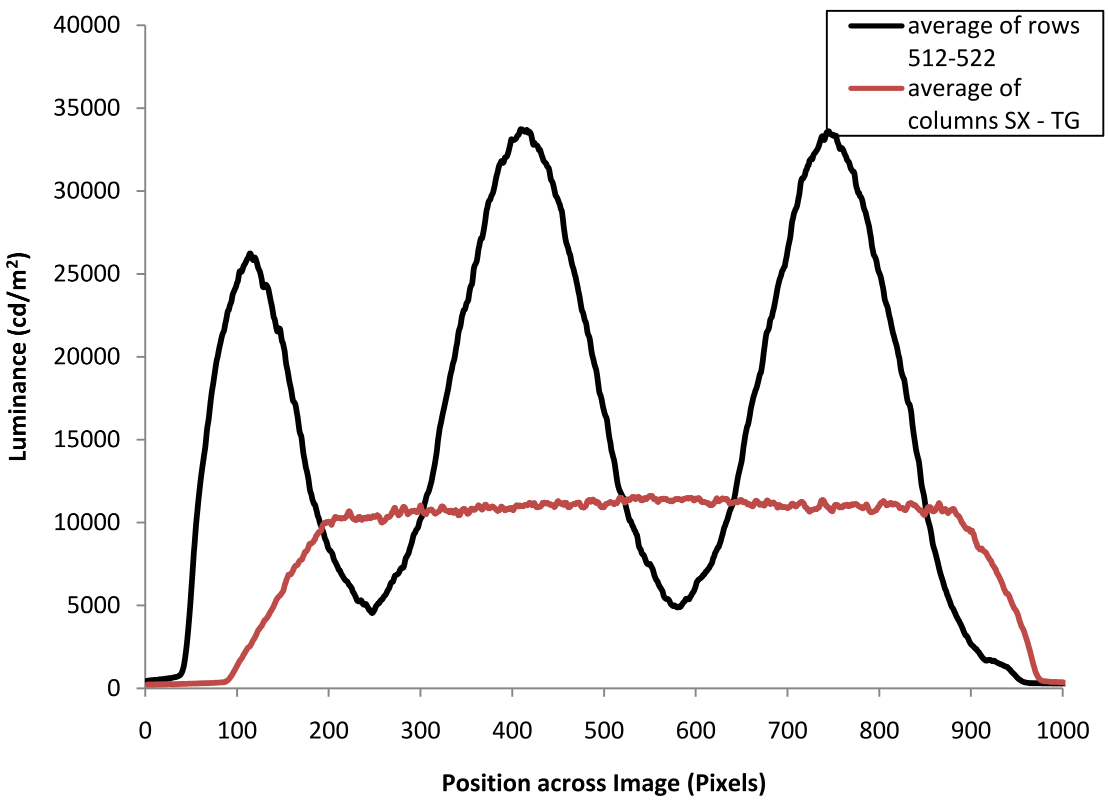


Figure 3-78. Graph of luminance vs. position for 0.5 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

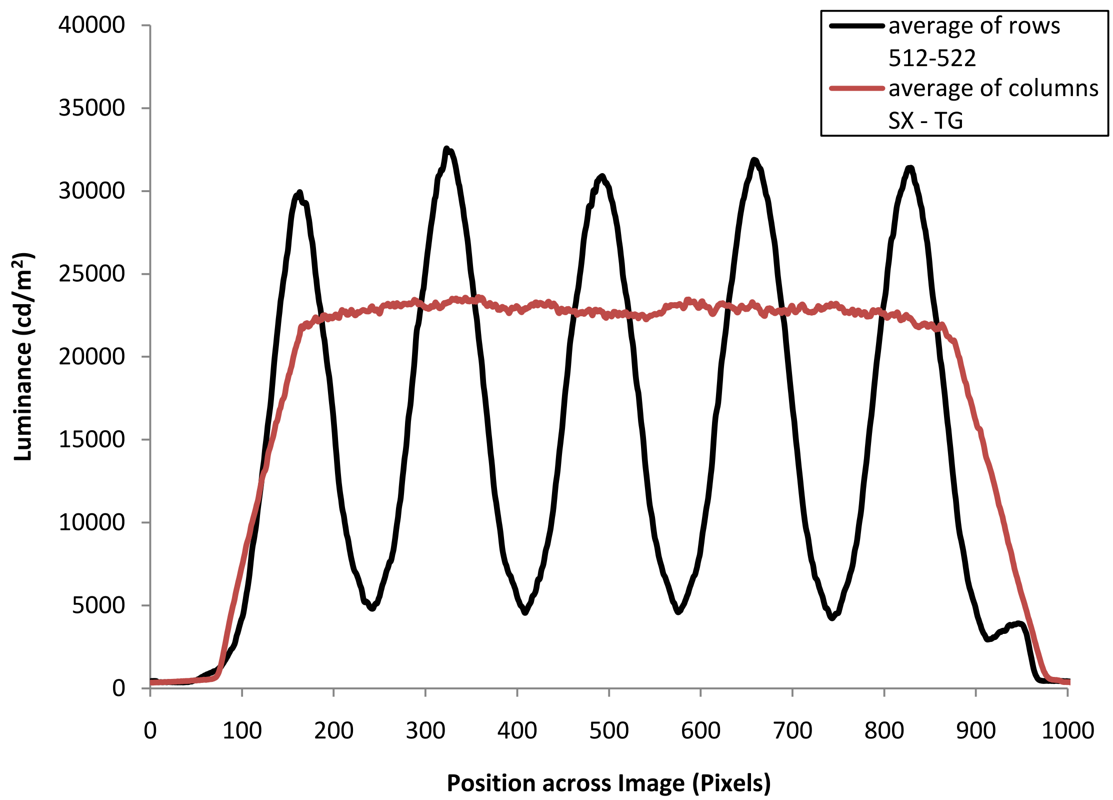


Figure 3-79. Graph of luminance vs. position for 1.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

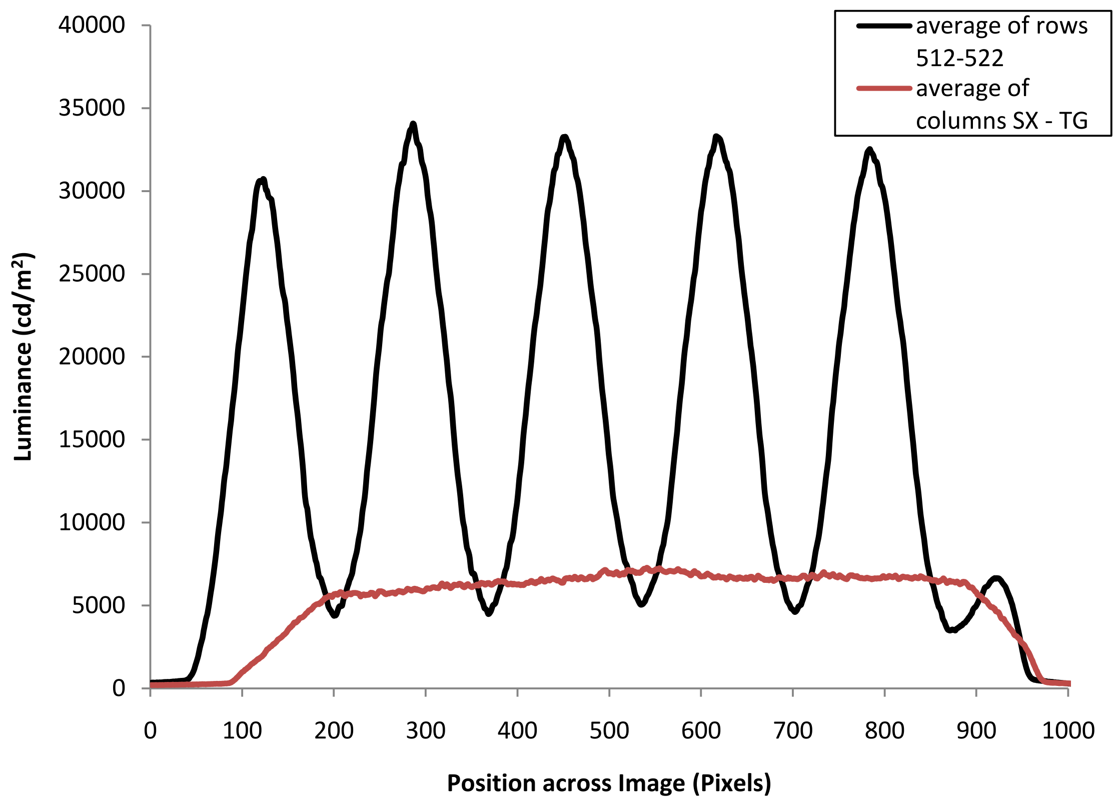


Figure 3-80. Graph of luminance vs. position for 1.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the “uniformity” gobo was installed

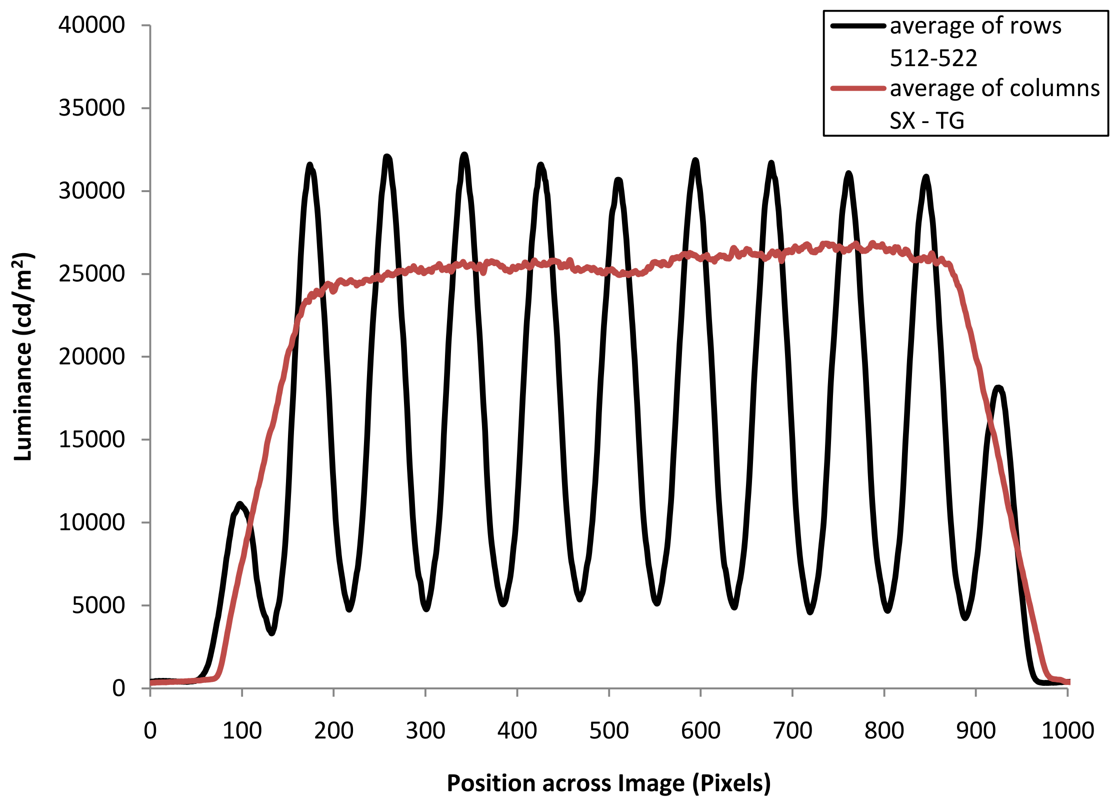


Figure 3-81. Graph of luminance vs. position for 2.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

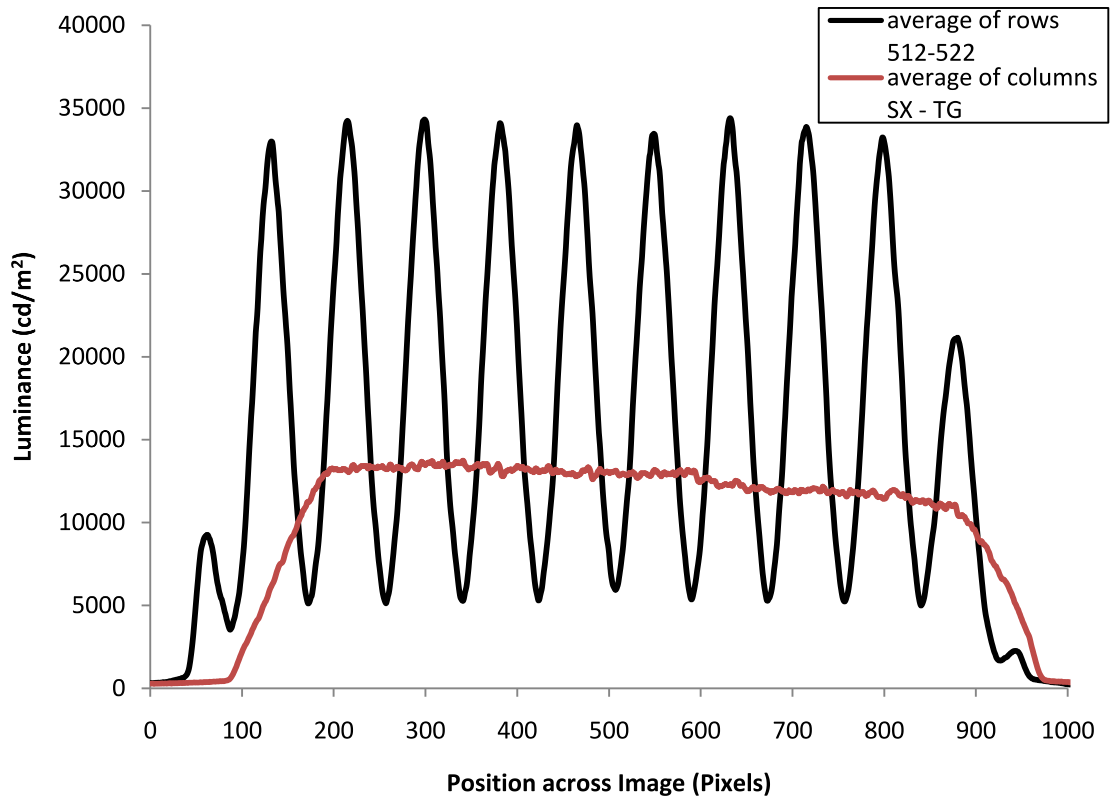


Figure 3-82. Graph of luminance vs. position for 2.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

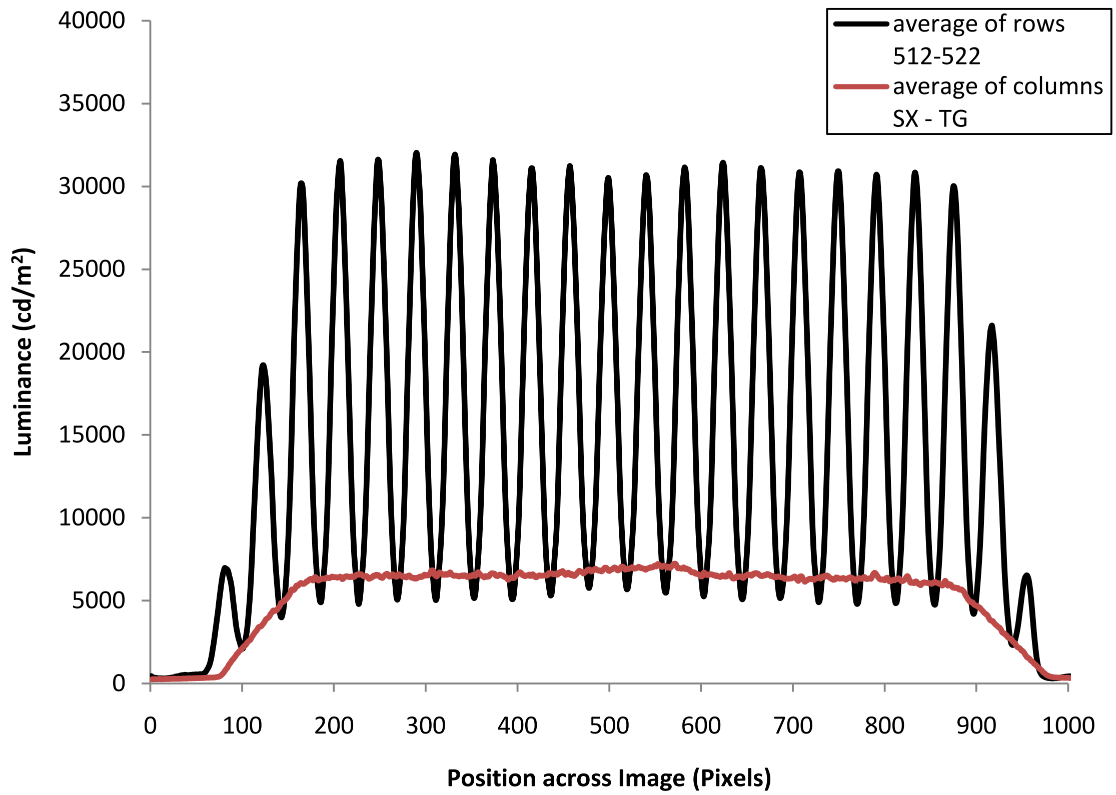


Figure 3-83. Graph of luminance vs. position for 4.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

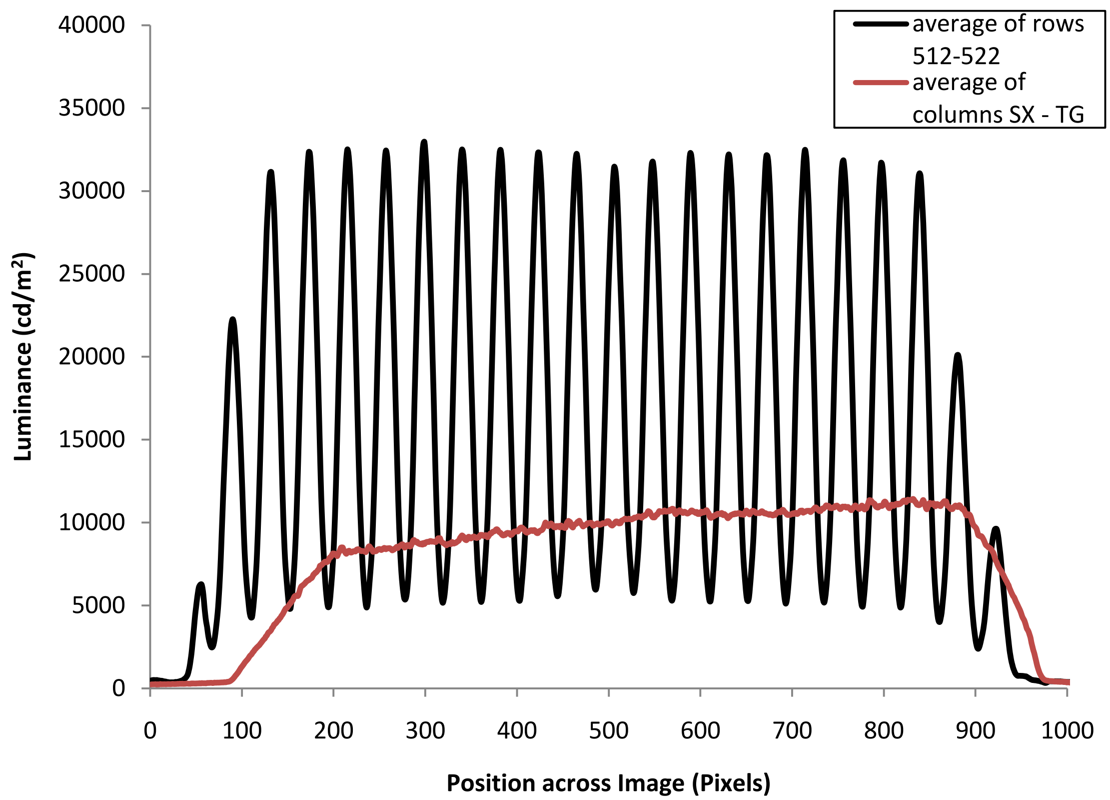


Figure 3-84. Graph of luminance vs. position for 4.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the “uniformity” gobo was installed

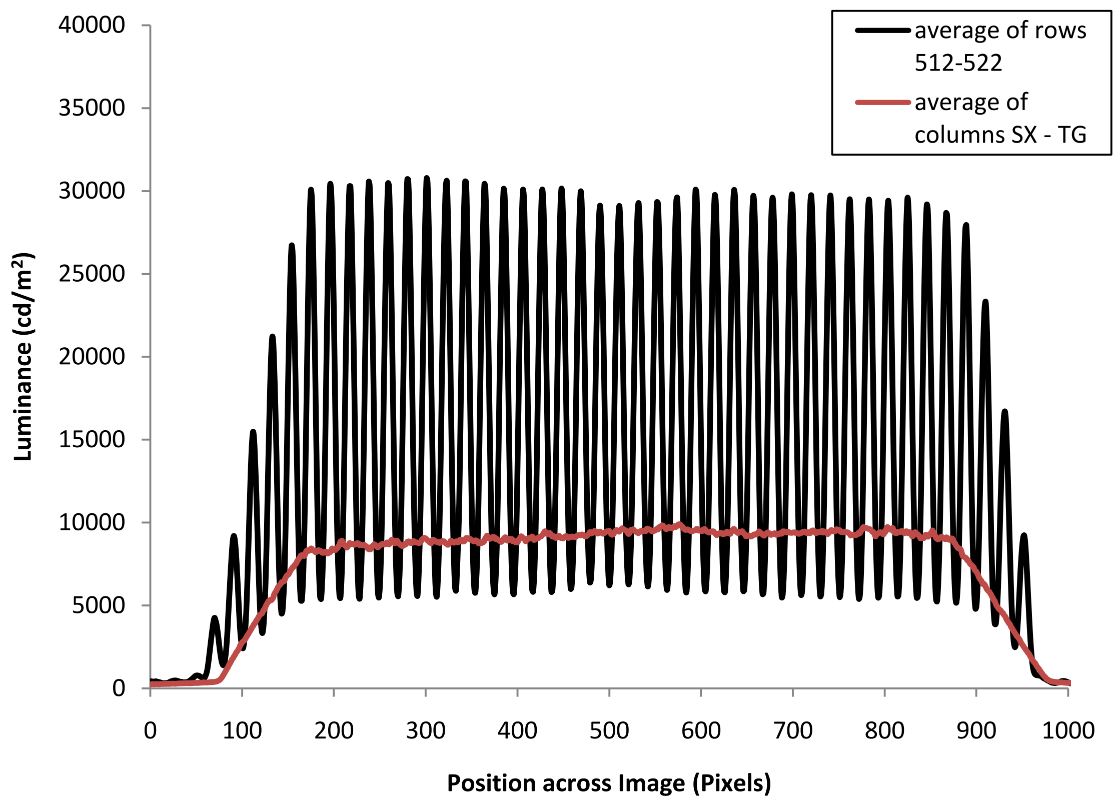


Figure 3-85. Graph of luminance vs. position for 8.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

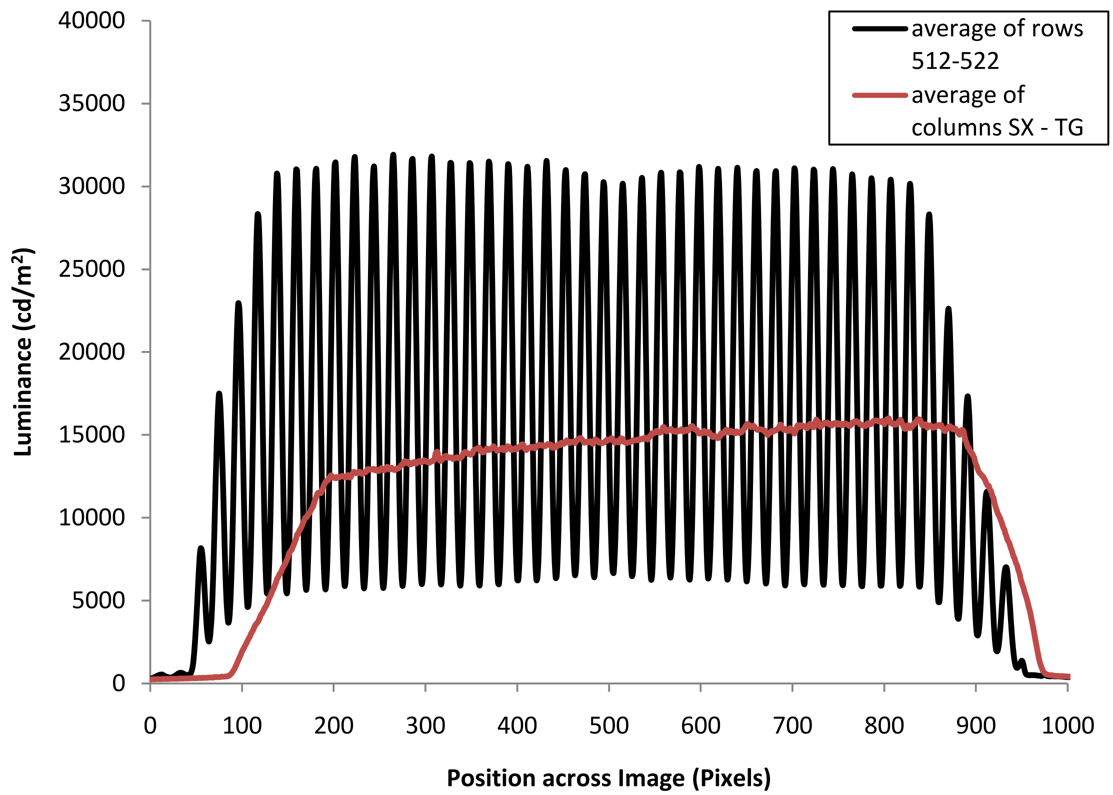


Figure 3-86. Graph of luminance vs. position for 8.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

Note that in several of the figures, the average column data is not flat. This should imply that the "uniformity" gobo did not correct the non-uniformity. However, there is another potential explanation. To generate the average column data, ten columns from the 1024 x 1024 matrix were averaged together and plotted. If the target was not perfectly vertical when the image was taken, then each column of data would actually be cutting across a sine wave, rather than representing a single point on a sine wave (see Figure 3-87). This is an exaggerated case, but it shows how a slight rotation could significantly affect the data.

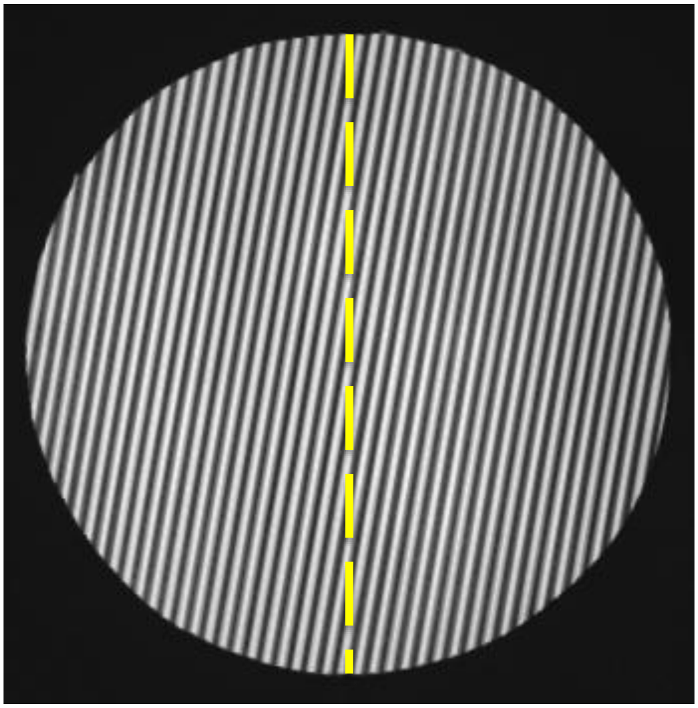


Figure 3-87. Illustration of how a slight rotation of the target (10 degrees) could significantly affect the average column data.

For the three dimensional graphs, a 254 by 254 matrix of luminance values was exported from the digital imaging photometer and plotted (see Figure 3-88 to Figure 3-97). The photometer has the capability to output a 1024 by 1024 matrix, but Excel limits the number of series for a 3D graph to 254, so that was what was exported from the photometer. To output a 254 by 254 matrix means that the photometer software averaged several pixels together from the original 1024 by 1024 array.

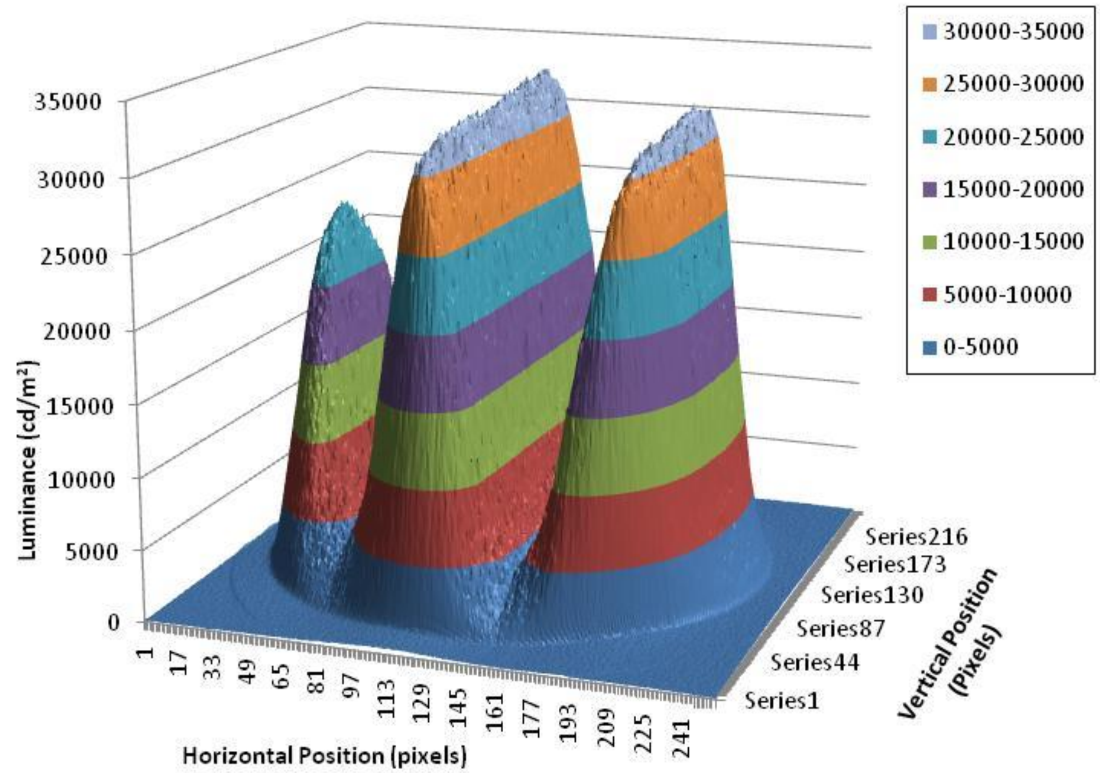


Figure 3-88. 3D graph of luminance vs. position for 0.5 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

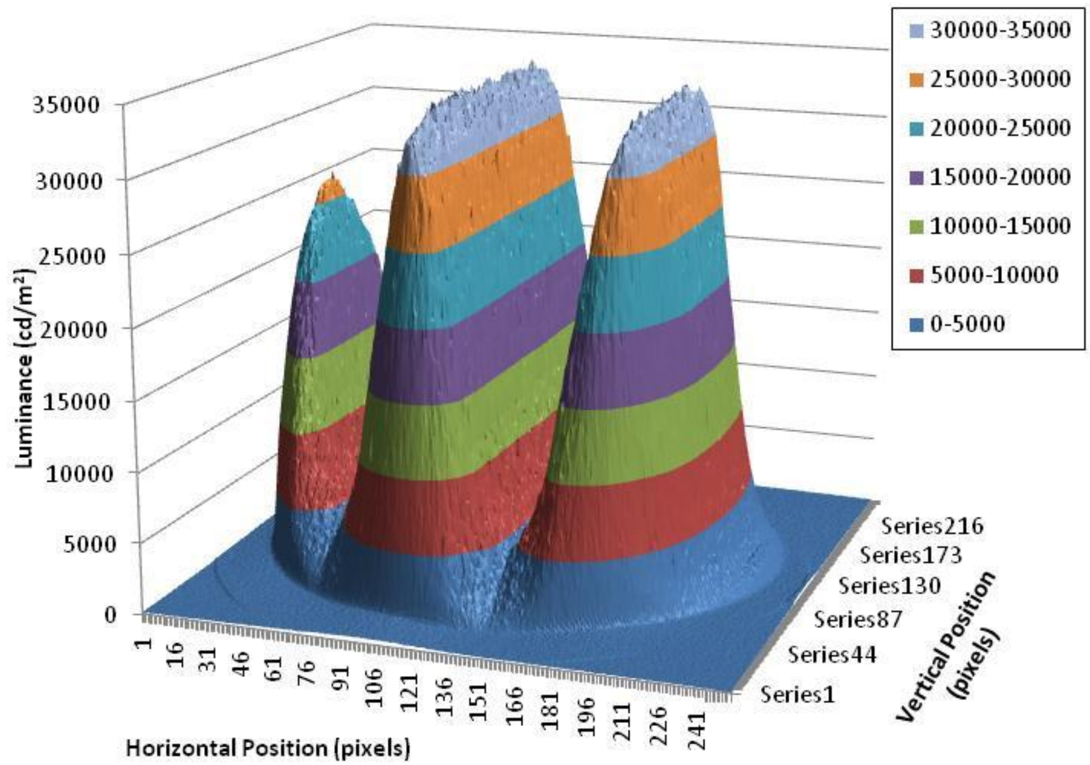


Figure 3-89. 3D graph of luminance vs. position for 0.5 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

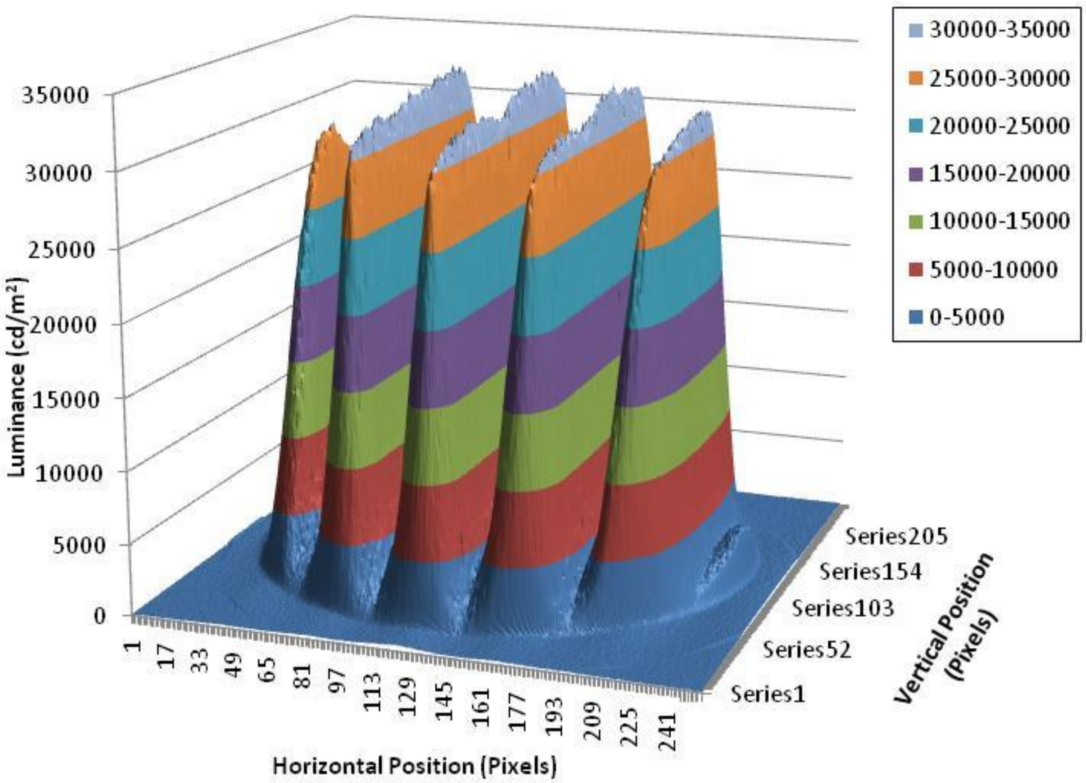


Figure 3-90. 3D graph of luminance vs. position for 1.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

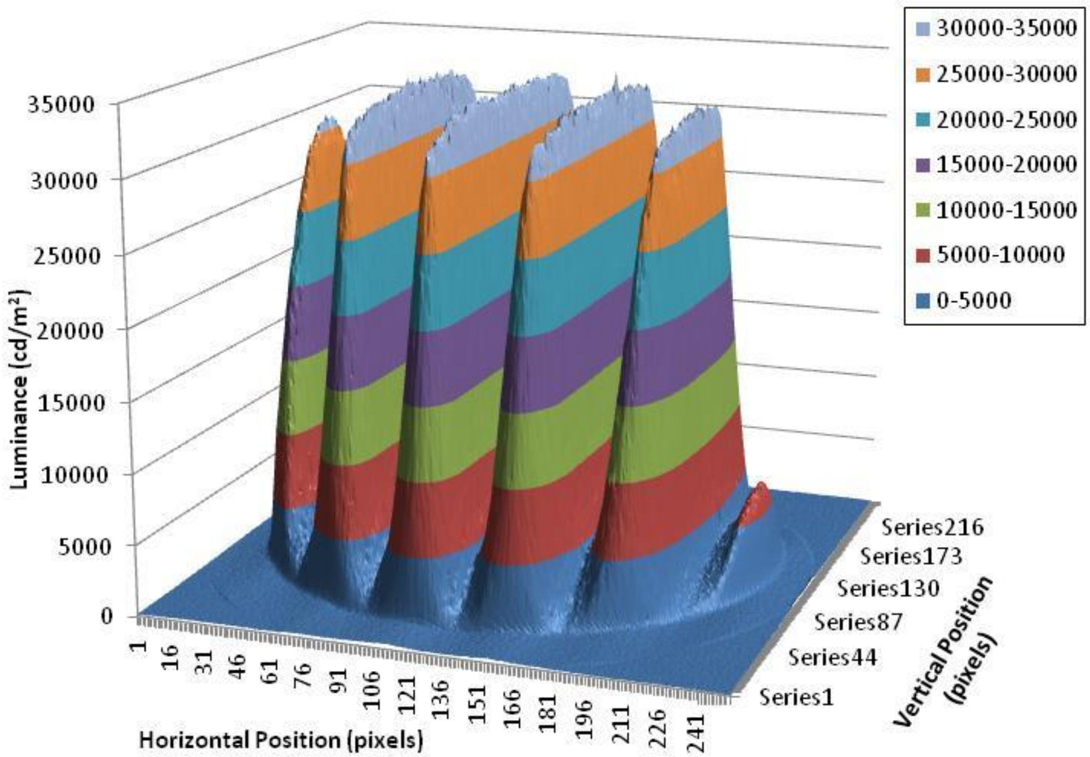


Figure 3-91. 3D graph of luminance vs. position for 1.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

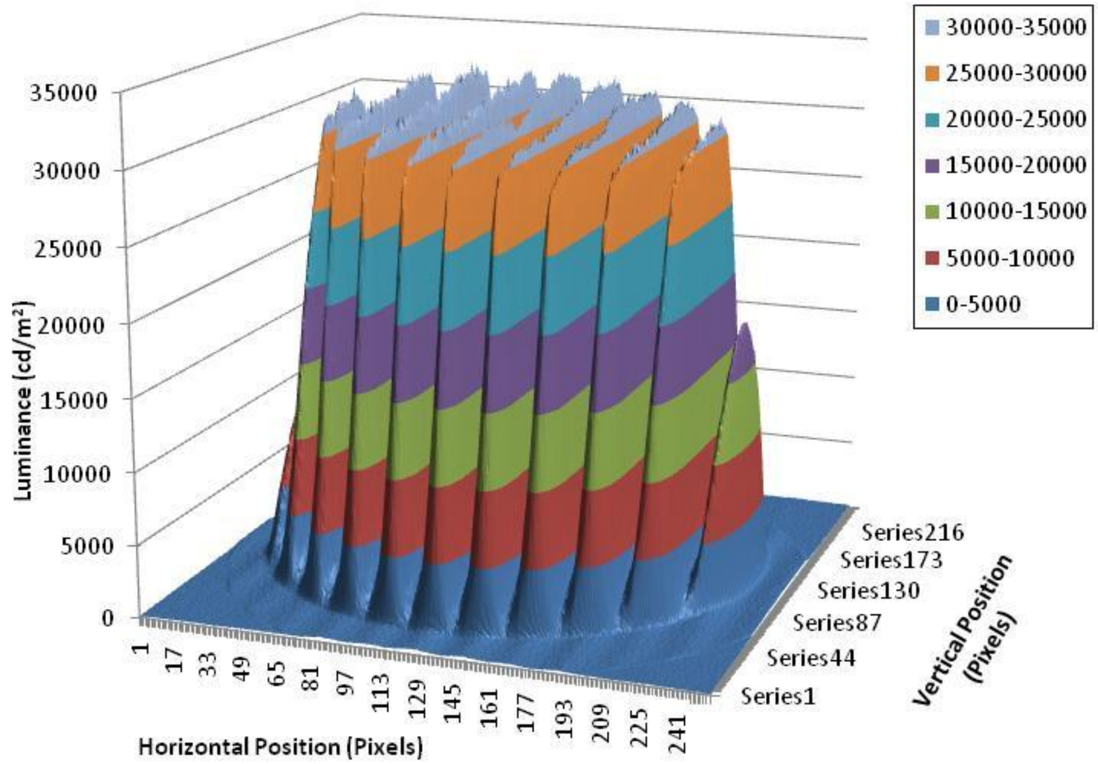


Figure 3-92. 3D graph of luminance vs. position for 2.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

.

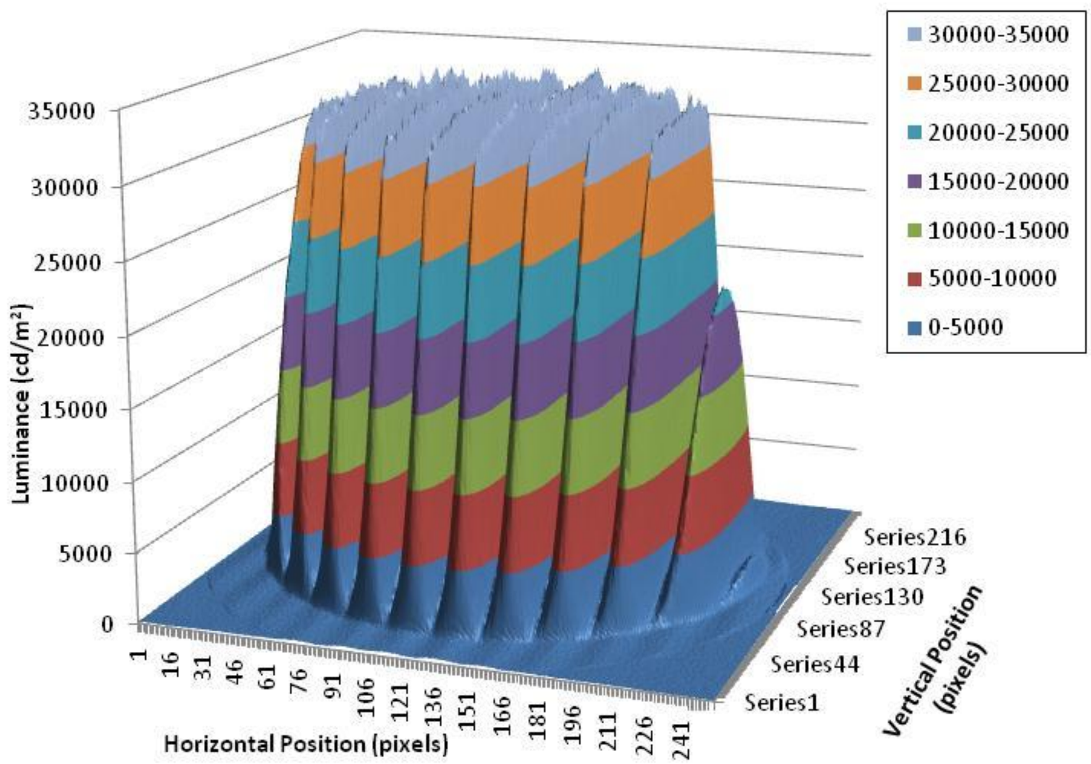


Figure 3-93. 3D graph of luminance vs. position for 2.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

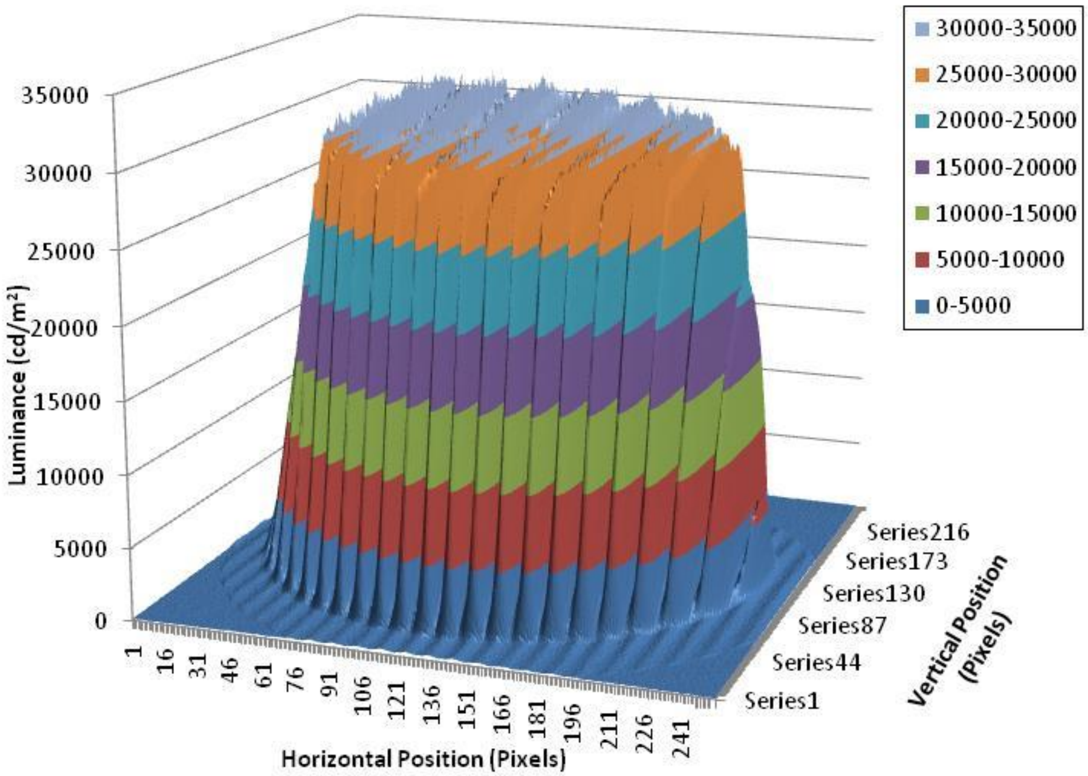


Figure 3-94. 3D graph of luminance vs. position for 4.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

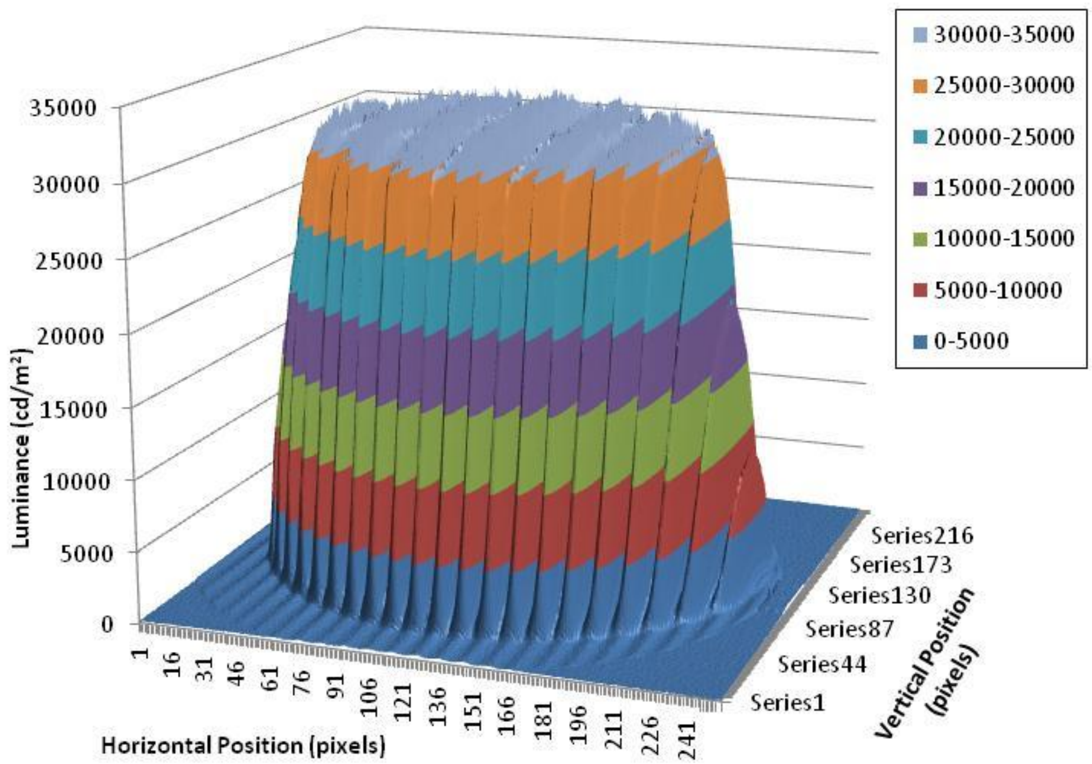


Figure 3-95. 3D graph of luminance vs. position for 4.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

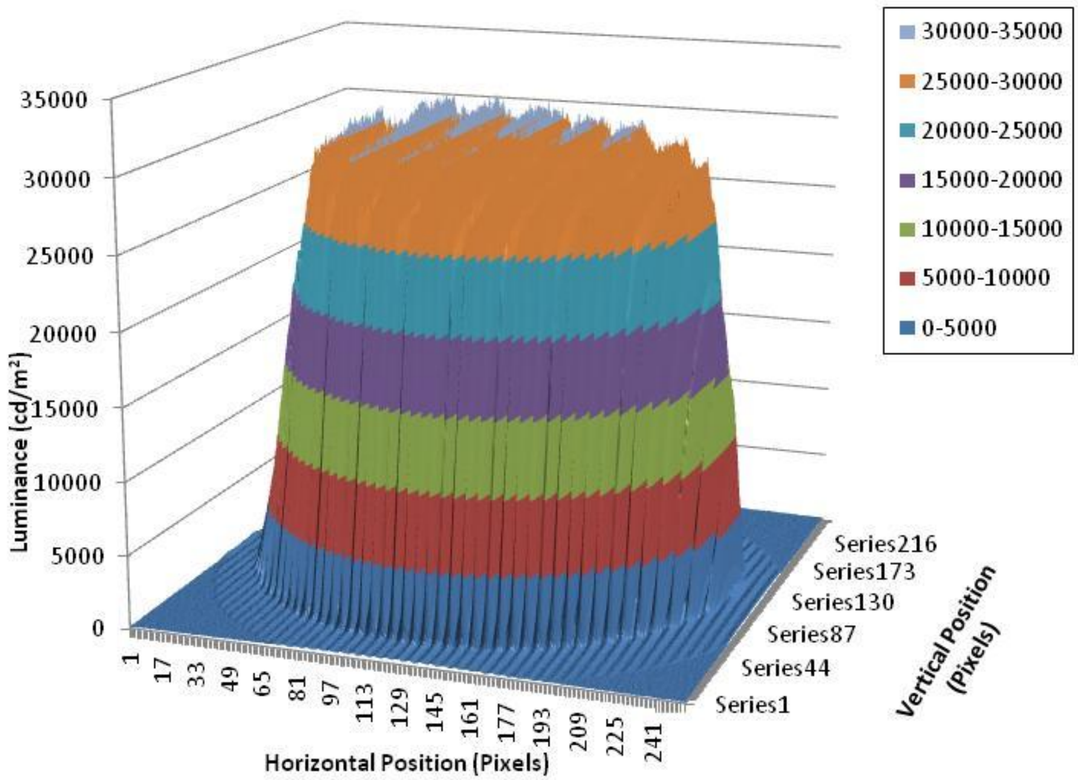


Figure 3-96. 3D graph of luminance vs. position for 8.0 cycles per degree paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

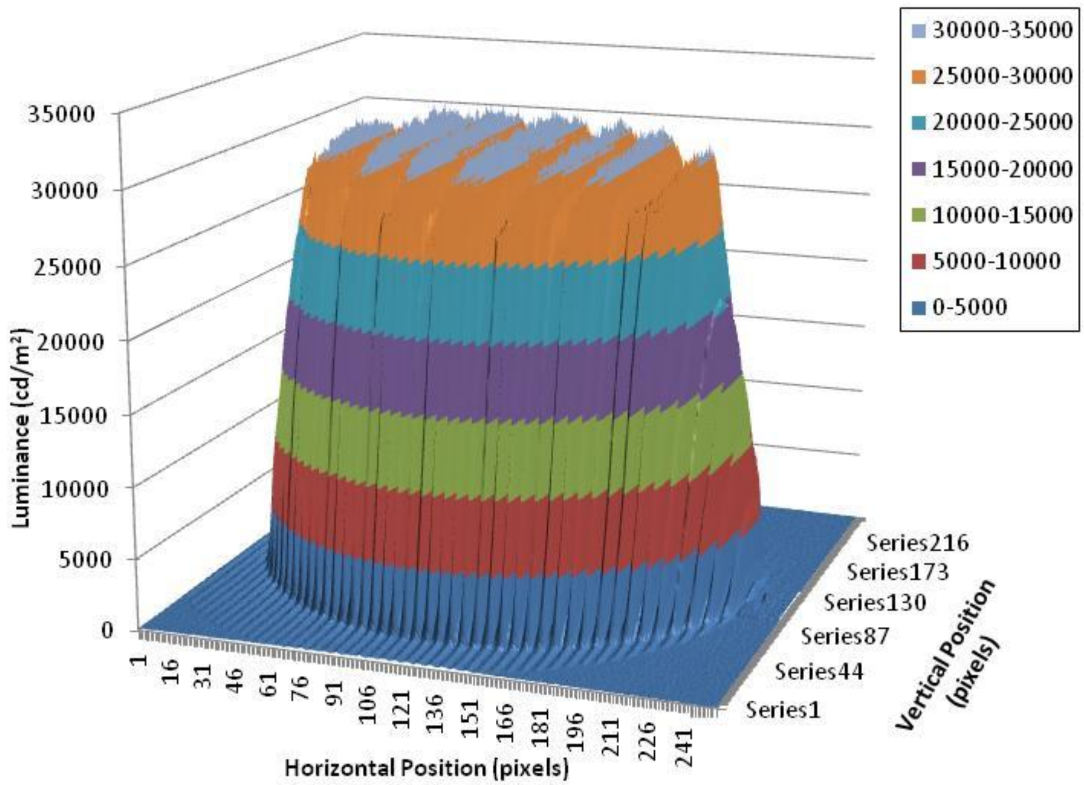
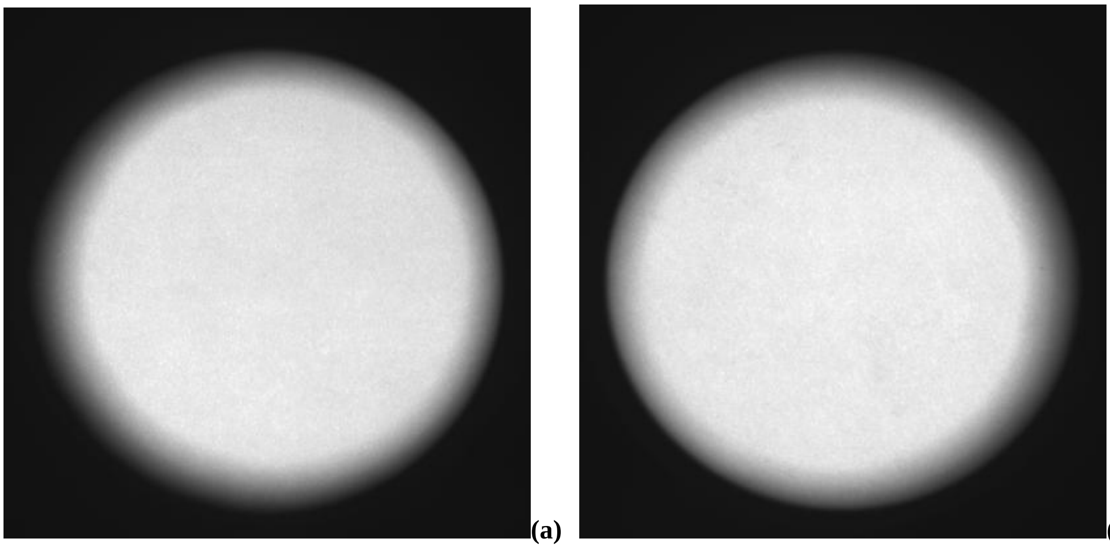


Figure 3-97. 3D graph of luminance vs. position for 8.0 cycles per degree paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

The uniform stimuli were then generated, with the intention of matching the average luminance of the sine wave stimuli. A number of different percent grey values were tested before a good match was found using 39% grey (see Figure 3-98).

Table 3-8 shows the average luminances of all of the stimuli. The digital imaging photometer was again used to generate 2D and 3D plots showing the luminance across the uniform stimuli (see Figure 3-99 to Figure 3-102) and also to determine the average luminance. The average luminance of each of the stimuli was determined from the digital imaging photometer software, Prometric. Once an image had been taken with the photometer, the software allowed the user to generate a "point of interest." All pixels

within the point of interest were averaged, which gave an average luminance (see Figure 3-103 for a screen shot from the Prometric software).



(a) (b)

Figure 3-98. Digital Imaging Photometer images of Uniform paper target illuminated by (a) left Robert Juliat fixture and (b) right Robert Juliat fixture after the "uniformity" gobo was installed

Table 3-8. Table of luminance values for each stimulus

Frequency (cycles per degree)	Right Side Average Luminance (cd/m^2)	Left Side Average Luminance (cd/m^2)
Uniform	15115	15111
0.5	15637	15245
1.0	15343	14988
2.0	16215	15028
4.0	15242	14967
8.0	15249	14864
Average	15467	15034
Standard Deviation	406	131
Overall Average	15250	
Overall Std. Dev.	366	

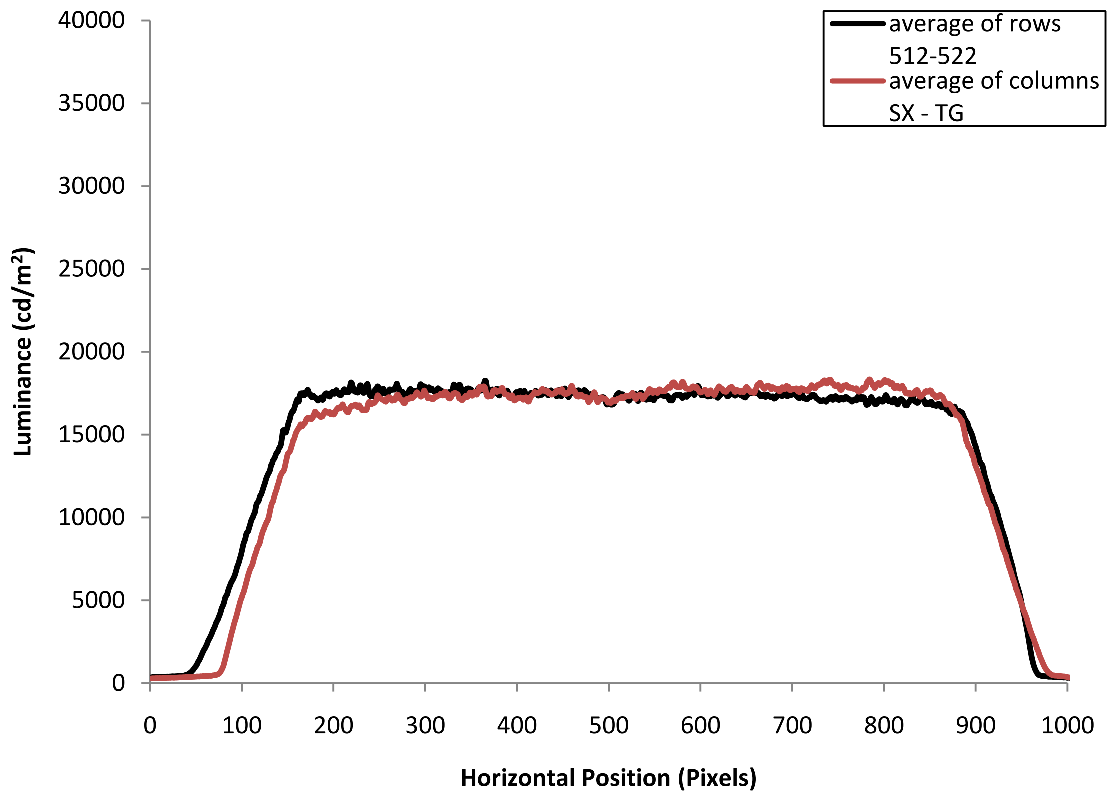


Figure 3-99. Graph of luminance vs. position for Uniform paper target illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed

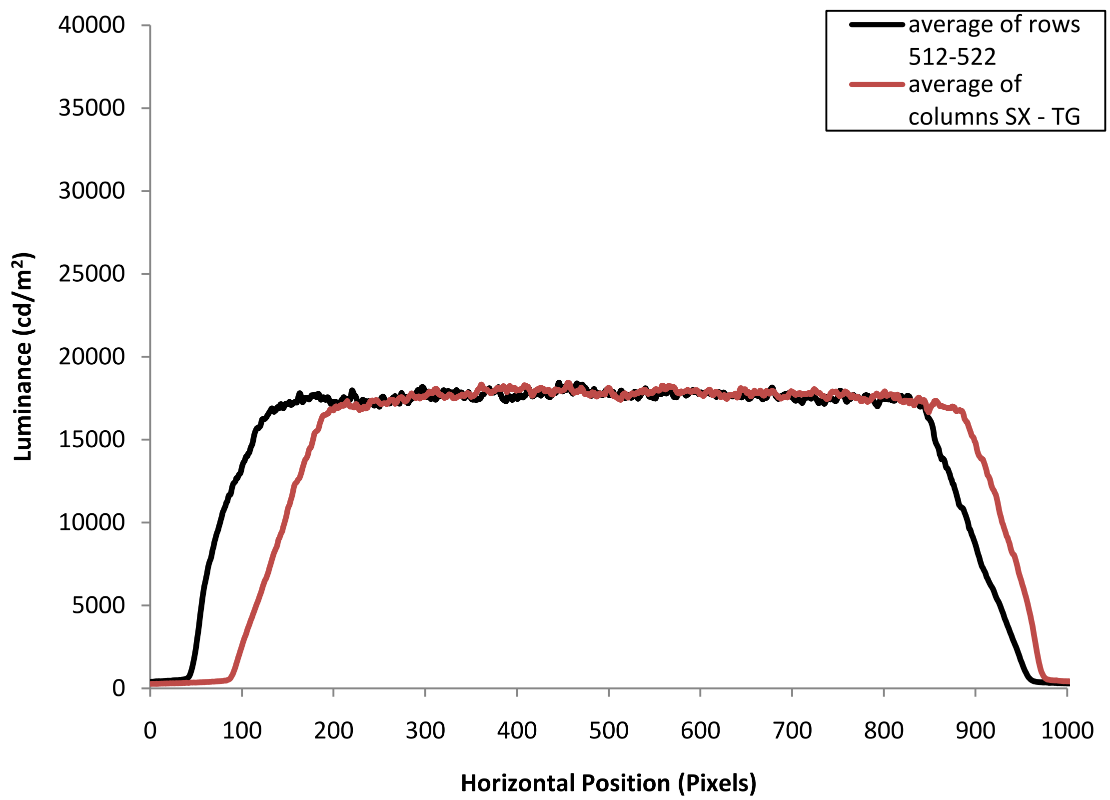


Figure 3-100. Graph of luminance vs. position for Uniform paper target illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

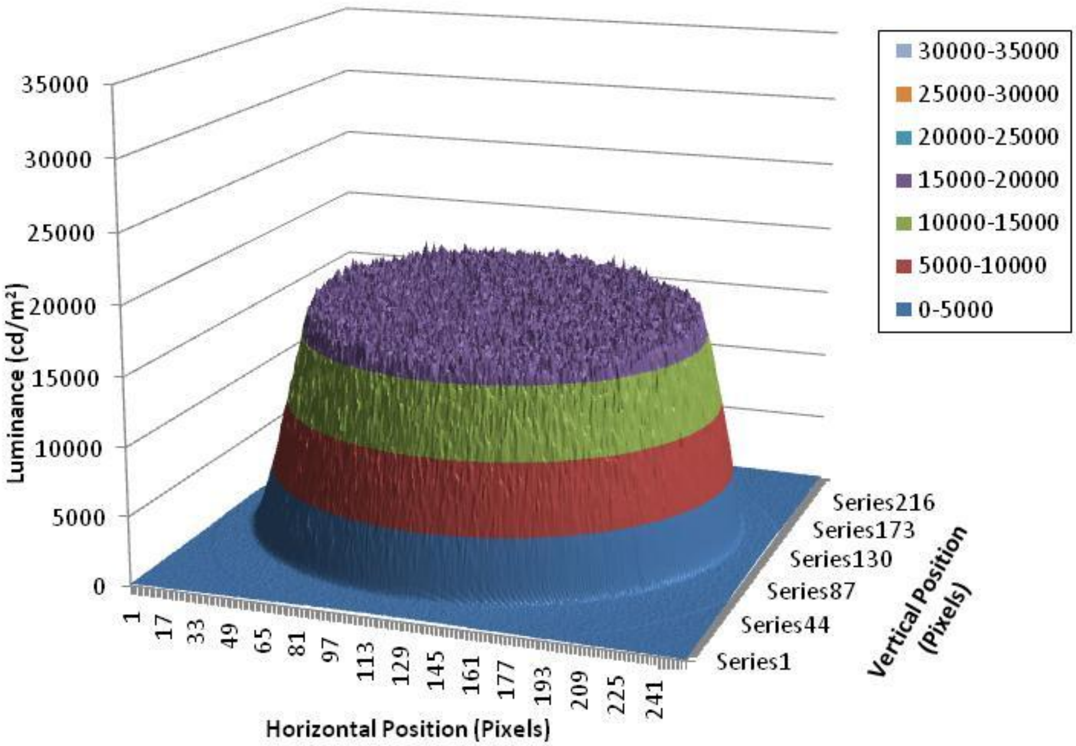


Figure 3-101. 3D graph of luminance vs. position for Uniform paper target illuminated by left Robert Juliat fixture after the “uniformity” gobo was installed

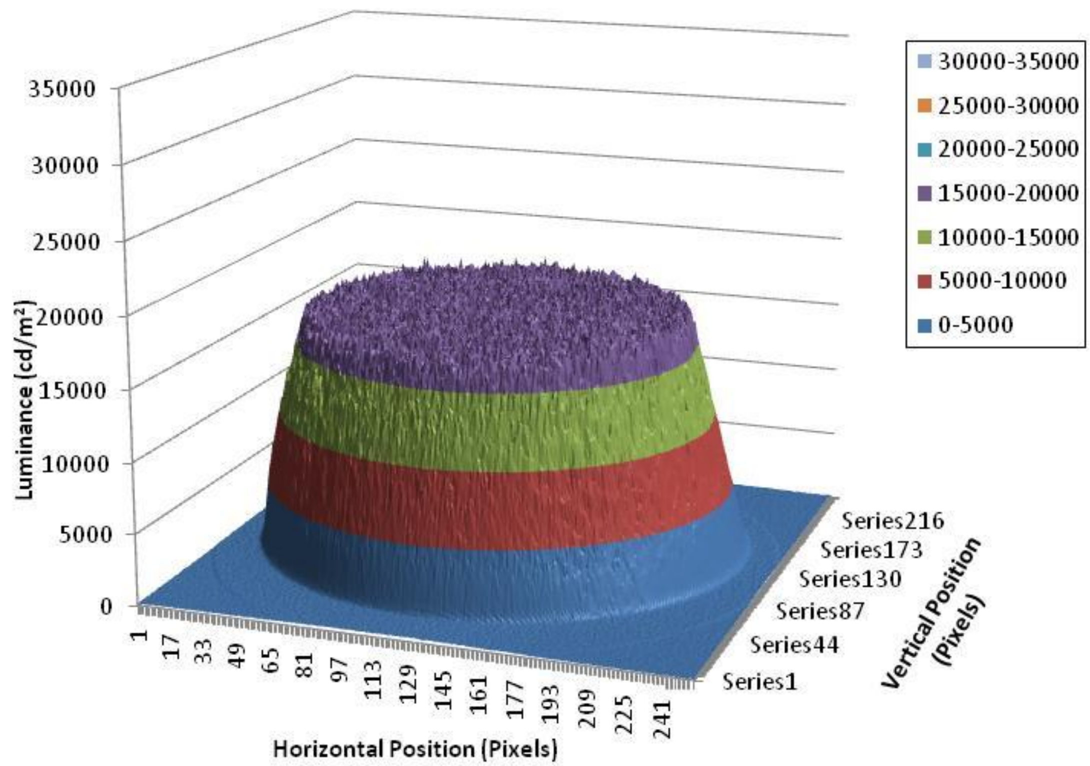


Figure 3-102. 3D graph of luminance vs. position for Uniform paper target illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed

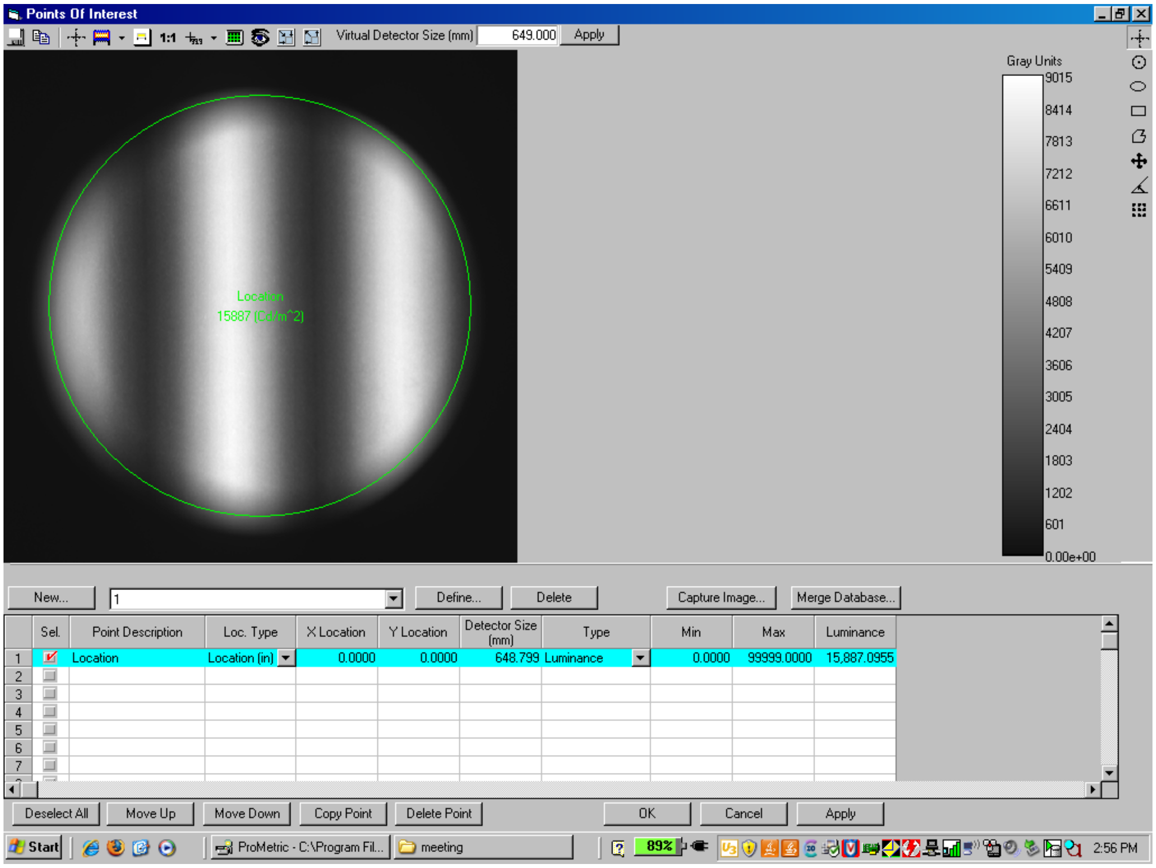


Figure 3-103. Screen shot from Prometric software showing "points of interest", which allows the user to determine average luminance of an area

The question then became, how uniform did it need to be? The output from a theatrical fixture will never be perfectly uniform, so how close to uniform is reasonable to expect for scientific research? The CIE (1981) reported that Blackwell performed several studies looking at the detectability of a briefly flashed luminous disk on a uniform background (Blackwell & Blackwell 1971). This research showed that there is a threshold below which the disk cannot be differentiated from its background. From this data, Bodmann (1973) fit a curve which can be written as the following equation:

$$\Delta L = 0.05936 \ \left((1.219 + L)^{0.4}\right)^{2.5} \tag{3.2}$$

Where ΔL = the minimum discernible difference in luminance

L = the background luminance (cd/m^2)

This suggests that for a background of approximately 30,000 cd/m^2, the minimum discernable difference in luminance (or Just Noticeable Difference – JND) would be 1780 cd/m^2. So if it were possible to attain a range of luminances from the Robert Juliat fixtures between 30,000 cd/m^2 and 30,000 +/- 1780 cd/m^2, that would be ideal, and it would be very unlikely that the subjects could notice any change. It is important to note that Blackwell's work was performed using foveal tasks, and using a uniform disk on a uniform background. In the current study, neither of these was the case. The subjects here were doing peripheral tasks, and the stimuli were gradually changing in luminance. The ability of a subject to detect a difference in luminance will certainly be lower in the periphery than in the fovea. In addition, it will be much more difficult for a subject to detect a gradually changing luminance than an abruptly changing luminance.

Research performed to study gradually changing luminances and the ability to detect changes in brightness confirms that. Wallace and Lockhead (1987) examined how subjects would perceive the brightness of gradually changing luminances, both changing from lighter to darker and darker to lighter on a disk of fixed diameter. Three different ways of changing the luminance were used: a linear change, a quadratic change, and a cubic change. Subjects were asked to match the brightness of the inside and outside of the disk to a matching scale that was presented to them simultaneously. They found that subjects could not recognize a change in brightness until the contrast between the inside and outside of the disk was 20% or higher (where contrast is defined as maximum luminance minus minimum luminance divided by maximum luminance plus minimum luminance). Using that definition of contrast and applying it to the stimuli used in the

current study, the contrast between the highest and lowest luminance stimuli (using average luminances) was calculated to be 4.4% (calculated from values in Table 3-8). From Wallace and Lockhead's research (1987), that suggests that subjects would not be able to tell a difference between the highest and lowest luminance stimuli.

After determining that the stimuli were acceptably uniform, it was also important that the stimuli, which were intended to be sine wave gratings, were good representations of real sine waves. To determine that, ten rows of the measured data were averaged and plotted (as in previous figures) from the digital imaging photometer measurements, and a true sine wave was superimposed over the averaged data (see Figure 3-104 to Figure 3-113). A correlation was calculated and is reported on each figure, showing the fit of the sine wave to the measured data. The correlation values show that the measured stimuli were a good approximation to a true sine wave.

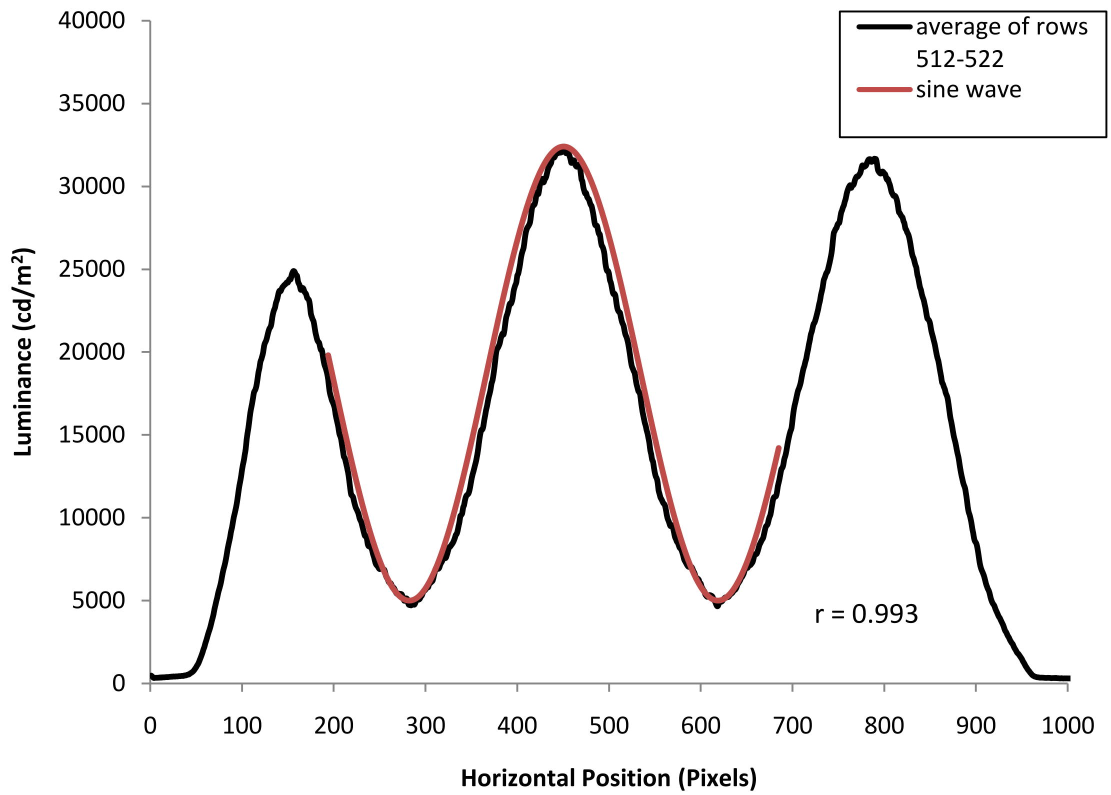


Figure 3-104. Graph of luminance vs. position for 0.5 cycles per degree paper target illuminated by left Robert Juliat fixture after the “uniformity” gobo was installed, with a true sine wave superimposed

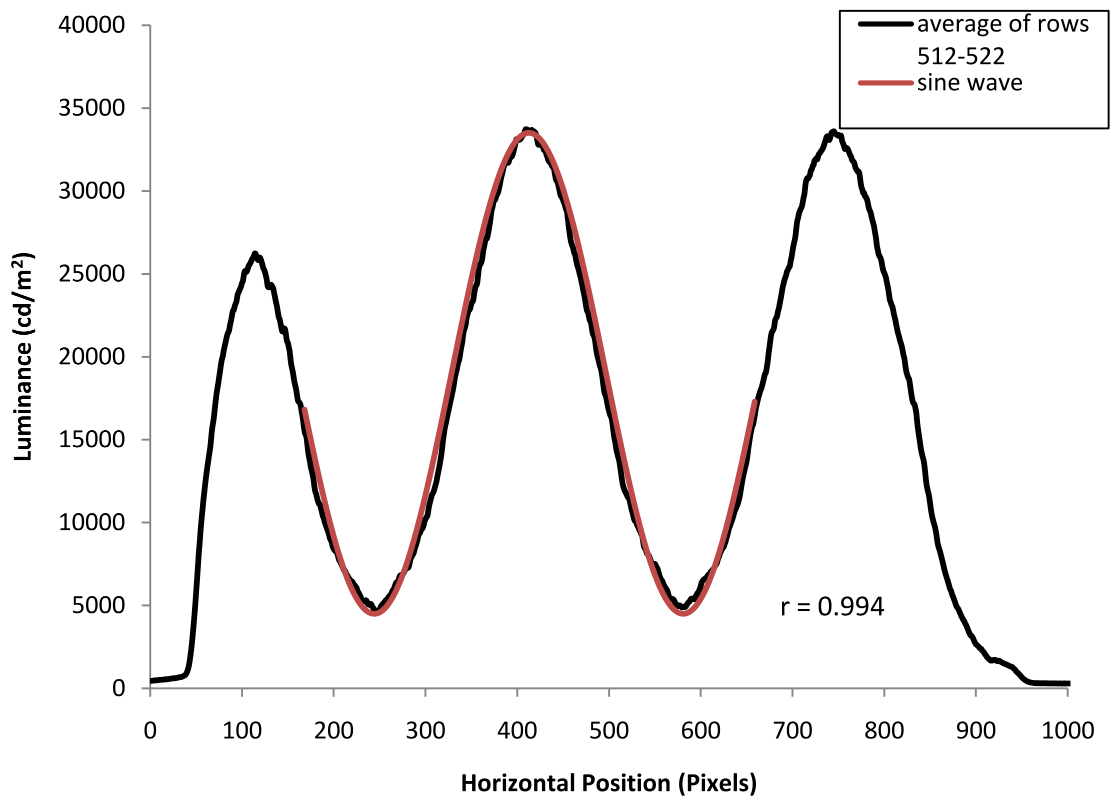


Figure 3-105. Graph of luminance vs. position for 0.5 cycles per degree paper target illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

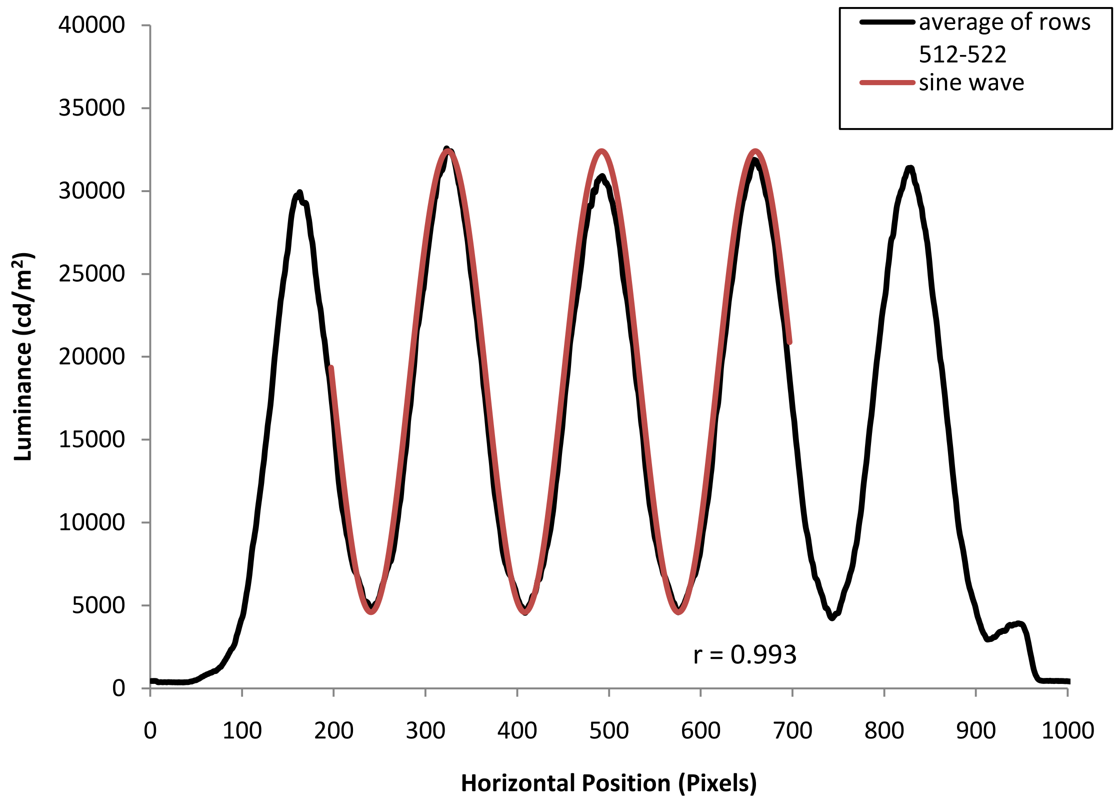


Figure 3-106. Graph of luminance vs. position for 1.0 cycles per degree paper target illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

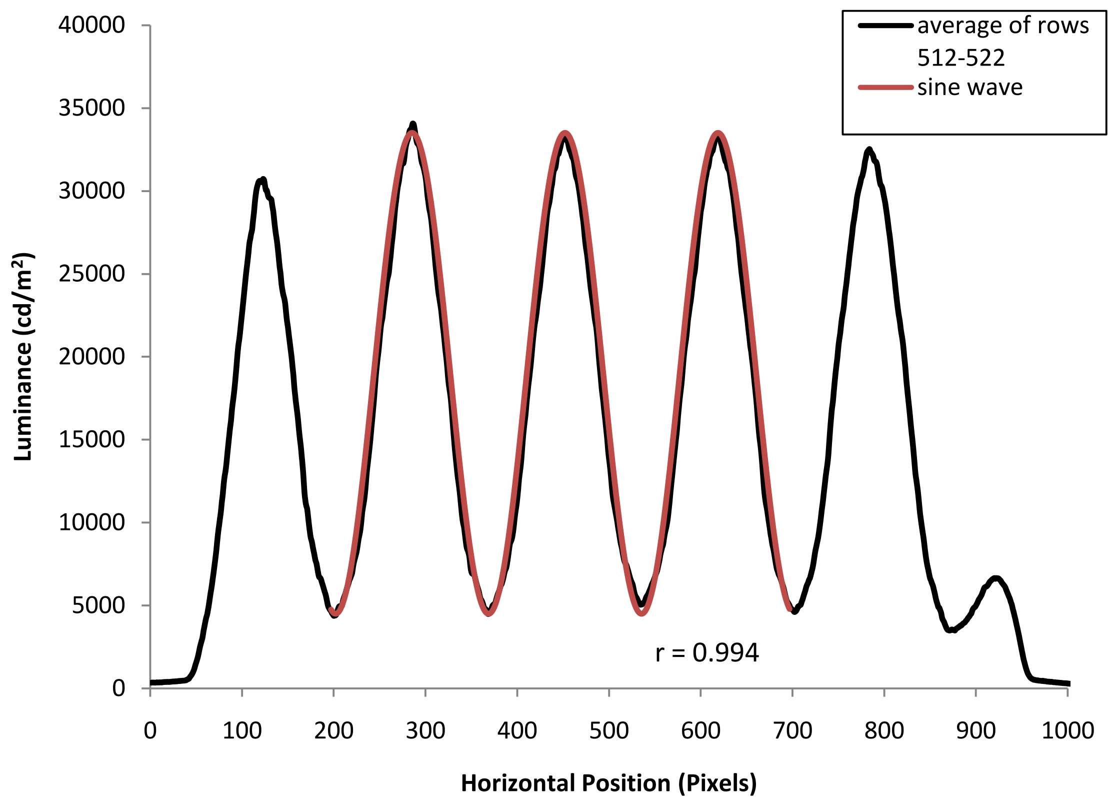


Figure 3-107. Graph of luminance vs. position for 1.0 cycles per degree paper target illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

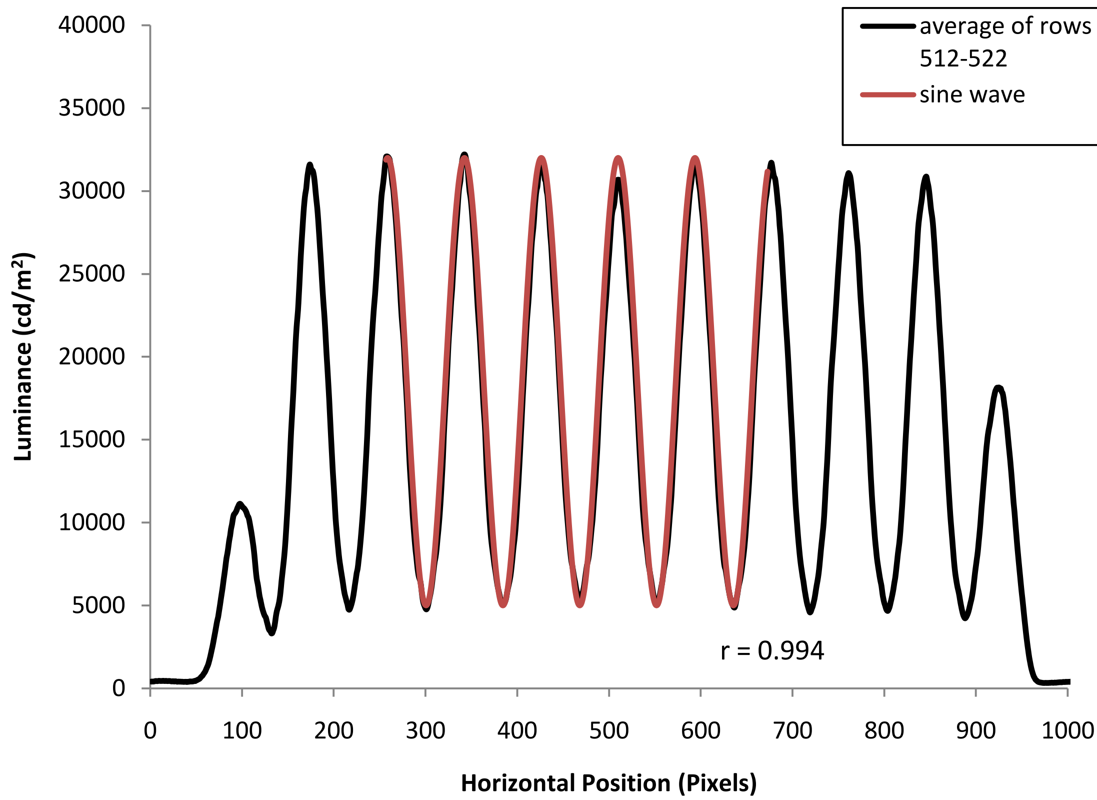


Figure 3-108. Graph of luminance vs. position for 2.0 cycles per degree paper target illuminated by left Robert Juliat fixture after the “uniformity” gobo was installed, with a true sine wave superimposed

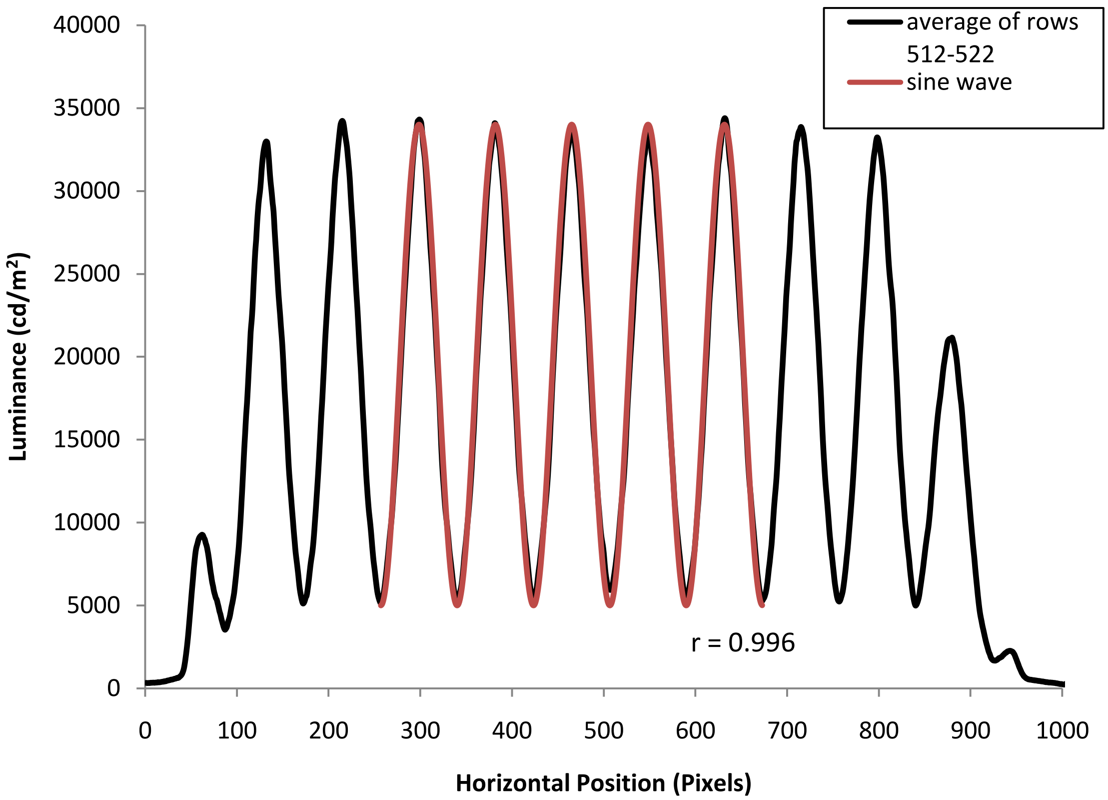


Figure 3-109. Graph of luminance vs. position for 2.0 cycles per degree paper target illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

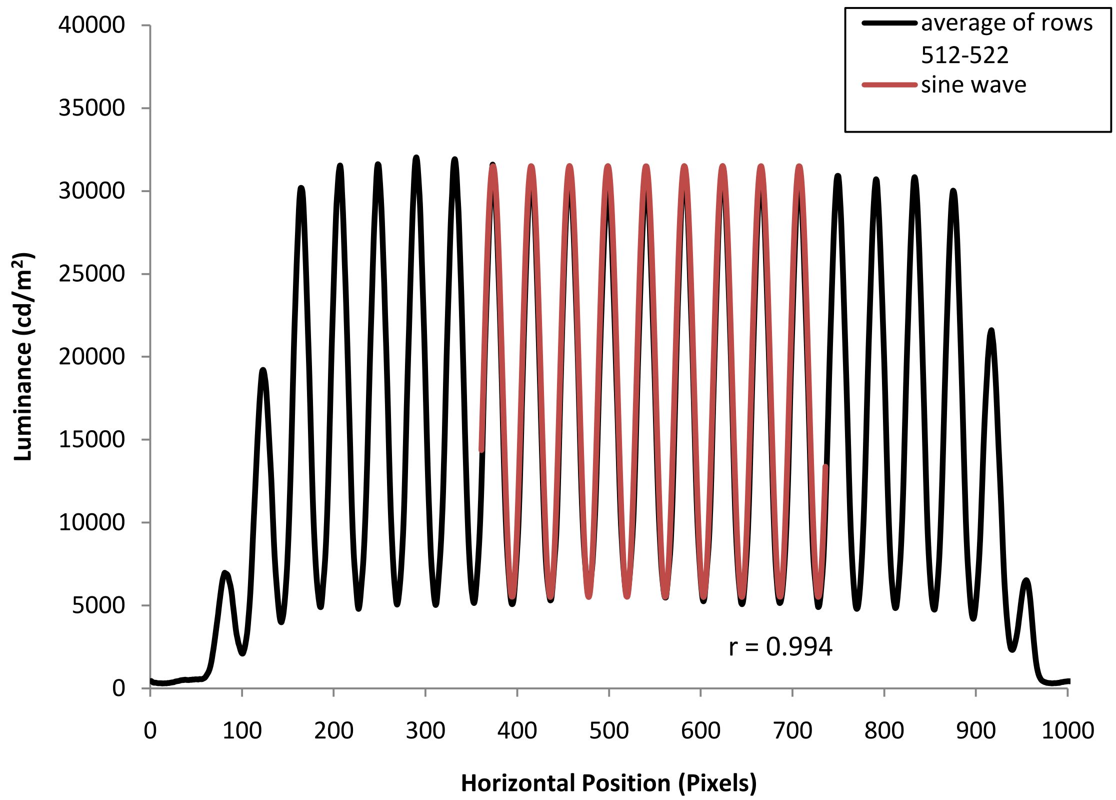


Figure 3-110. Graph of luminance vs. position for 4.0 cycles per degree paper target illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

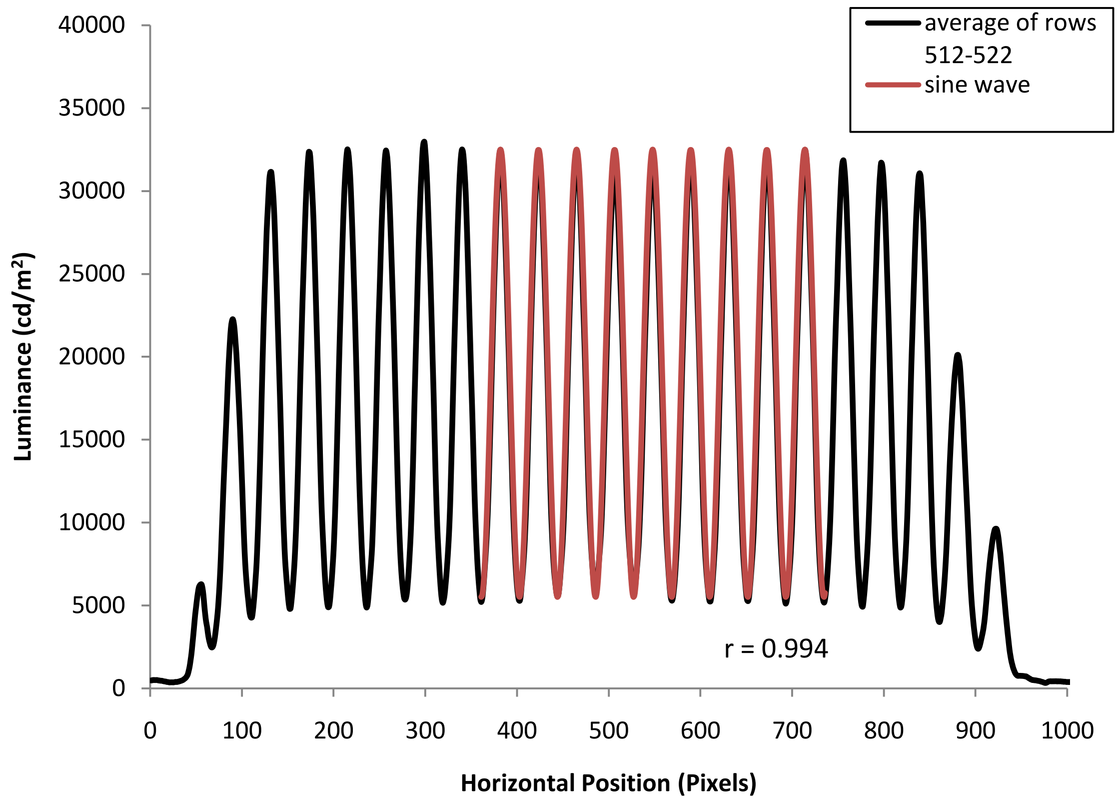


Figure 3-111. Graph of luminance vs. position for 4.0 cycles per degree paper target illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

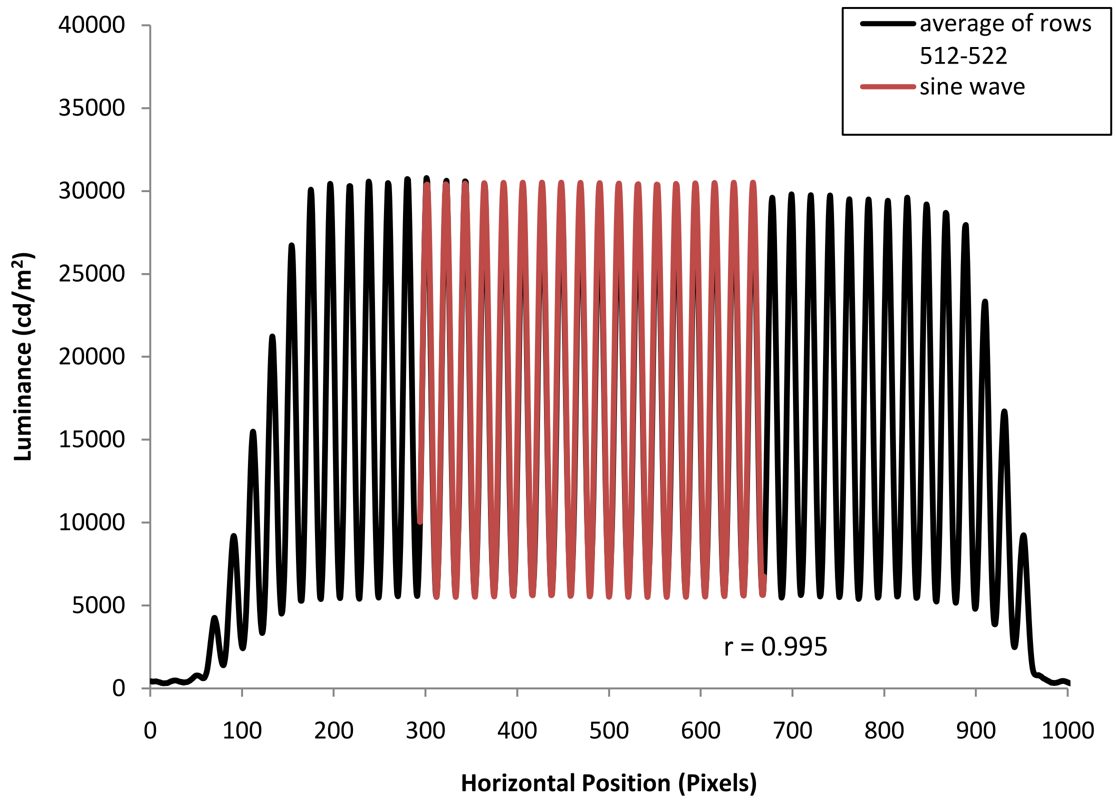


Figure 3-112. Graph of luminance vs. position for 8.0 cycles per degree paper target illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

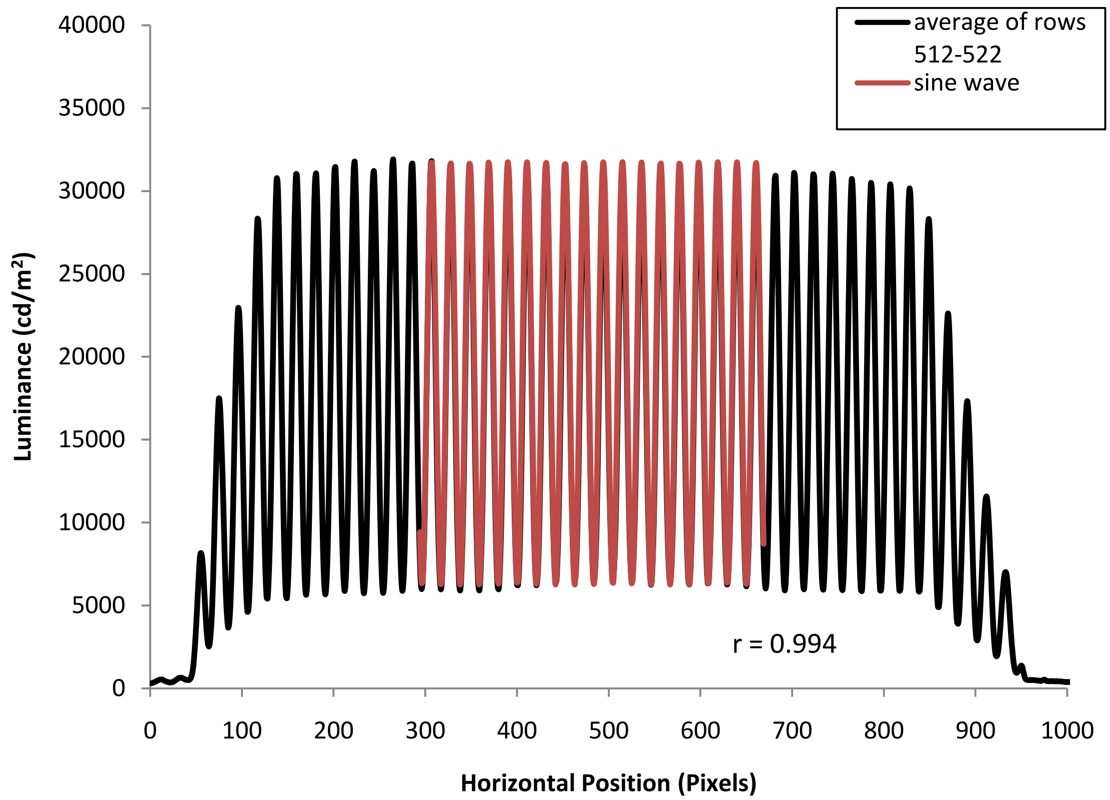


Figure 3-113. Graph of luminance vs. position for 8.0 cycles per degree paper target illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed, with a true sine wave superimposed

It was also important to know that not only were the stimuli similar in luminance at 0 degrees (which is how all of the above measurements were taken), but also that the luminance did not change significantly as the stimuli were moved to 10, 20, 30, and 40 degrees above the line of sight, which are the positions utilized in the experiment. The uniform and 0.5 cycles per degree stimuli were mounted to the apparatus, and moved to those positions. The digital imaging photometer was again used to take images of the stimuli, and used to obtain average luminance values (see Figure 3-114). The angle of the photometer was changed for each position so that the photometer line of sight was

normal to the stimuli. It is clear from this figure that the average luminance of the stimuli was not affected as the position of the apparatus was changed.

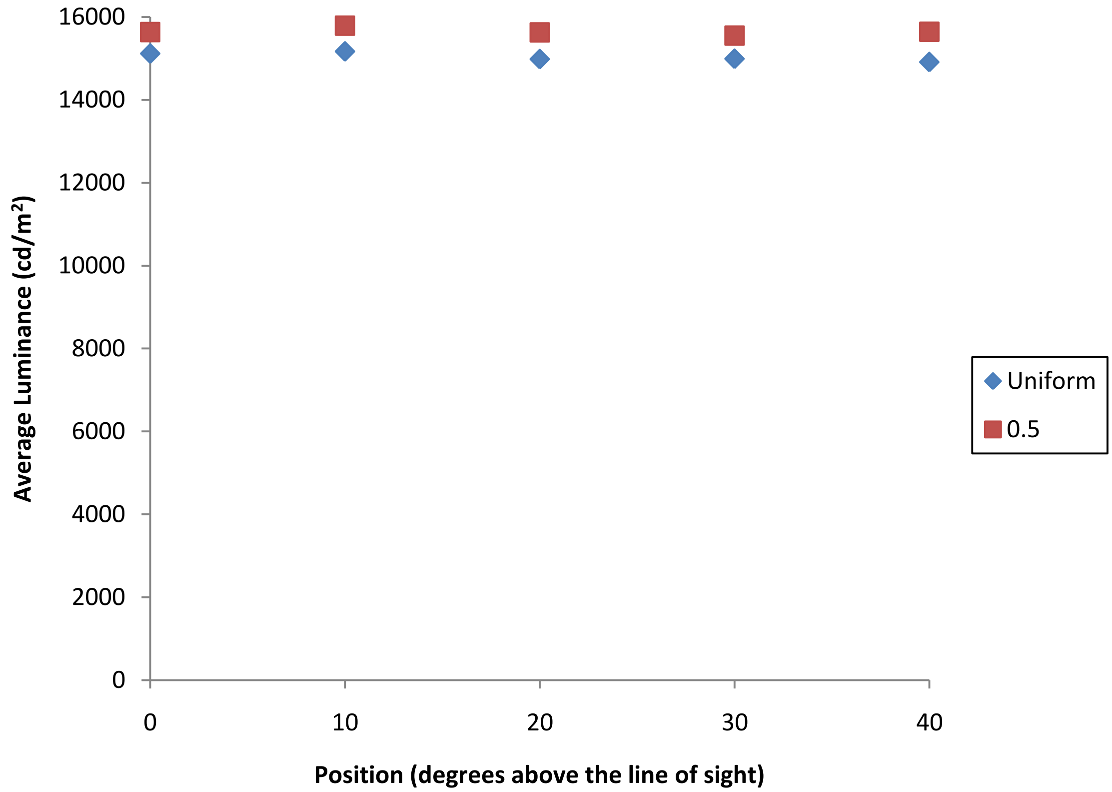


Figure 3-114. Graph of average luminance vs. position above the line of sight for Uniform and 0.5 cycles per degree paper targets illuminated by right Robert Juliat fixture after "uniformity" gobo was installed

From the measured data, the actual frequencies of the sine wave stimuli were also calculated. The stimuli were designed to be 0.5, 1.0, 2.0, 4.0, and 8.0 cycles per degree. To calculate the frequencies of the actual printed stimuli, a digital imaging photometer measurement was taken of an object of known dimensions. It was then possible to determine how many pixels were in each inch of the image. The digital imaging photometer was also used to take images of the stimuli at that same position. Then the number of pixels per cycle of sine wave was counted for each of the sine wave stimuli.

By multiplying the number of pixels per cycle of sine wave by the inverse of the number of pixels per inch, it was possible to determine how many inches each cycle of sine wave was. Then it was simple geometry to determine the cycles per degree (at the viewing distance of 1.5m). The following chart shows the nominal and actual measured frequencies for the five sine wave frequency stimuli (see Table 3-9).

Table 3-9. Table of nominal and actual frequencies of paper targets at viewing distance of 1.5m

Nominal Frequency (cycles per degree)	Left Side Actual Frequency (cycles per degree)	Right Side Actual Frequency (cycles per degree)
0.5	0.496	0.491
1.0	0.999	0.987
2.0	1.974	1.961
4.0	4.047	3.927
8.0	7.853	7.853

It was also important to make sure that the uniform stimulus (at the different positions) would give a reasonable range of UGR values. The intent of including the uniform stimulus in the experiment was to act as a control, and so that anchor points of UGR values would be available when the stimuli were ordered in terms of discomfort. UGR effectively ranges from a low of 10 to a high of 30 (CIE 1995). Therefore, it was important that the four uniform stimuli (uniform stimulus at each of four positions) covered the glare spectrum fairly well. Using the source luminance as that provided by the Robert Juliat fixtures, and the geometry utilized for this experiment, the UGR values were calculated for the uniform stimulus at each of the four positions (see Table 3-10). This range of UGR values covers the glare range well, and is also interesting in that there are about three UGR points between each one. One UGR point is the least detectable

difference, whereas three UGR points represents a recommended step in glare criteria (CIE 1995).

Table 3-10. Table of UGR values for the Uniform stimulus at different positions for right Robert Juliat fixture (left side is almost identical)

	Stimulus			
	Uniform at 10 degrees	Uniform at 20 degrees	Uniform at 30 degrees	Uniform at 40 degrees
UGR Value	27.7	24.6	21.5	18.4

Stimuli Size

The size of the stimuli also needed to be determined. The overriding consideration for size was from the CIE (2002), which defines "normal" sources as those with a projected area between 0.005m^2 (7.75in^2) and 1.5m^2 (2325 in^2). Within this range, the actual size was chosen to match past research. In Luckiesh & Guth's (1949) size study, they used five different solid angles ranging from 0.0001steradians to 0.126steradians. One of the sizes was 0.0079steradians, which is within the "normal" source range as outlined by the CIE (2002). In addition, Petherbridge and Hopkinson (1950) used several different solid angles ranging from 0.00027steradians to 0.027steradians in their discomfort glare study. They used a size of 0.0085steradians, which closely matches Luckiesh & Guth's value of 0.0079steradians. Most compelling in determining stimulus size was the work of Hilz and Cavonius (1974) in their study relating contrast sensitivity to spatial frequency at different eccentricities. They reported that they used three different test fields: 2º, 2.45º, and 5º. Both 2º and 2.45º would not be "normal" sources, so the 5º test field size was chosen. At the 1.5m viewing distance in

the current experiment, 5º yielded a stimulus diameter of 0.131m (5.16in). The 0.008steradians used by both Luckiesh & Guth (1949) and Petherbridge and Hopkinson (1950) also would generate approximately the same diameter for the stimulus: 0.151m (5.96in). It was therefore determined that a stimulus size of approximately 0.140m (5.5in) would be ideal. Because of the limitation of the internal iris (see Figure 3-13), the smallest stimulus achievable that still gave good uniformity was 0.143m (5.625in), which equates to 0.007steradians. In an attempt to minimize spill light on the wall beyond the apparatus, the actual printed target was made a little larger than the 0.143m (5.625in). The diameter of the actual printed targets was 0.165m (6.5in). The stimulus was the circle of light illuminating the printed targets, which was 0.143m (5.625in) in diameter (see Figure 3-115), which equates to 0.00714steradians.

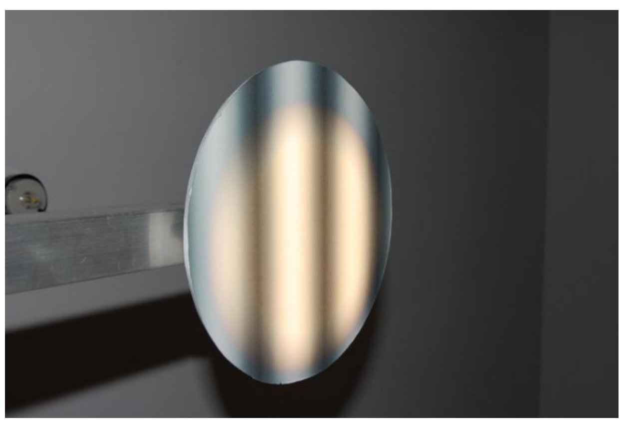


Figure 3-115. Printed target mounted to apparatus with Robert Juliat fixture illuminating only a portion of the target.

Changes to the Stimuli Over Time

During the course of the experiment, there were slight changes to the stimuli caused by lamp lumen depreciation of the incandescent lamps in the Robert Juliat fixtures. The first subject was run on November 17, 2007, and the last subject was run on

January 22, 2008. The stimuli were measured before Subject #1, and after Subjects #10, #19, #24, #26, #28, #30, #34, #38, and #41 (see Figure 3-116, Figure 3-117, and Figure 3-118) with the digital imaging photometer. It was determined that after Subject #19, there was a significant difference between the left and right stimuli. Therefore, a dimmer was installed for the left fixture, as it was projecting a higher luminance. The dimmer was used to lower the luminance of the left fixture until it better matched the right fixture.

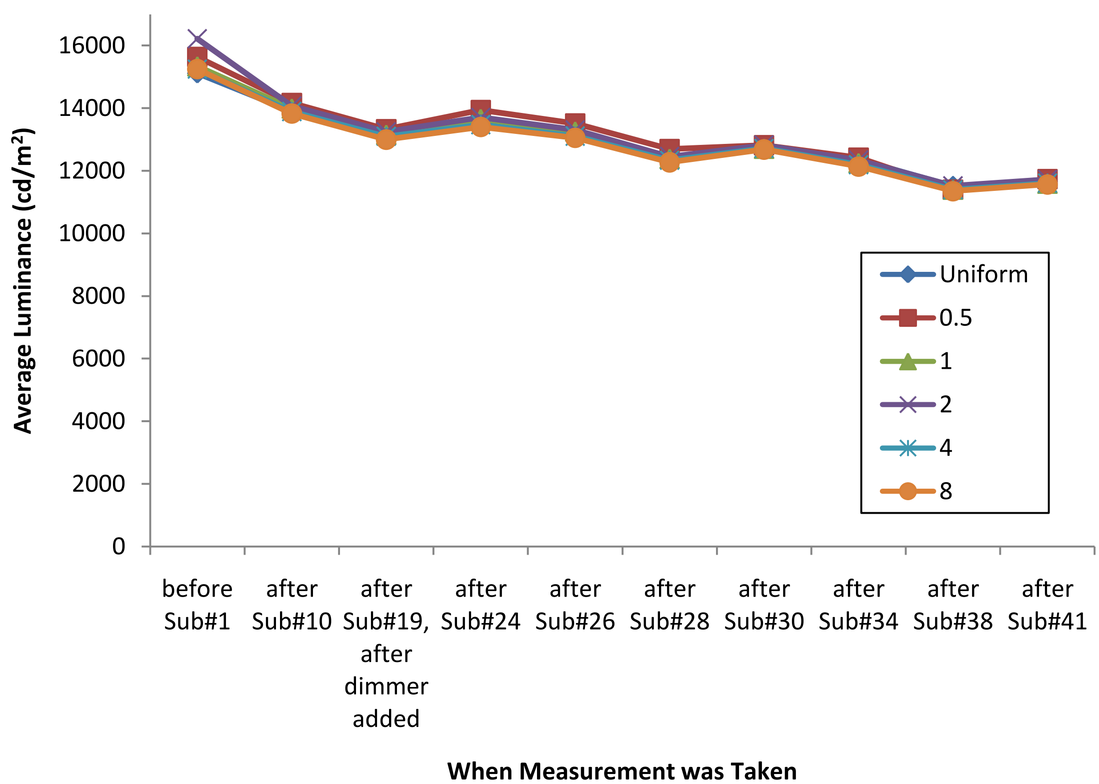


Figure 3-116. Graph of average luminance vs. duration of experiment for paper targets illuminated by right Robert Juliat fixture.

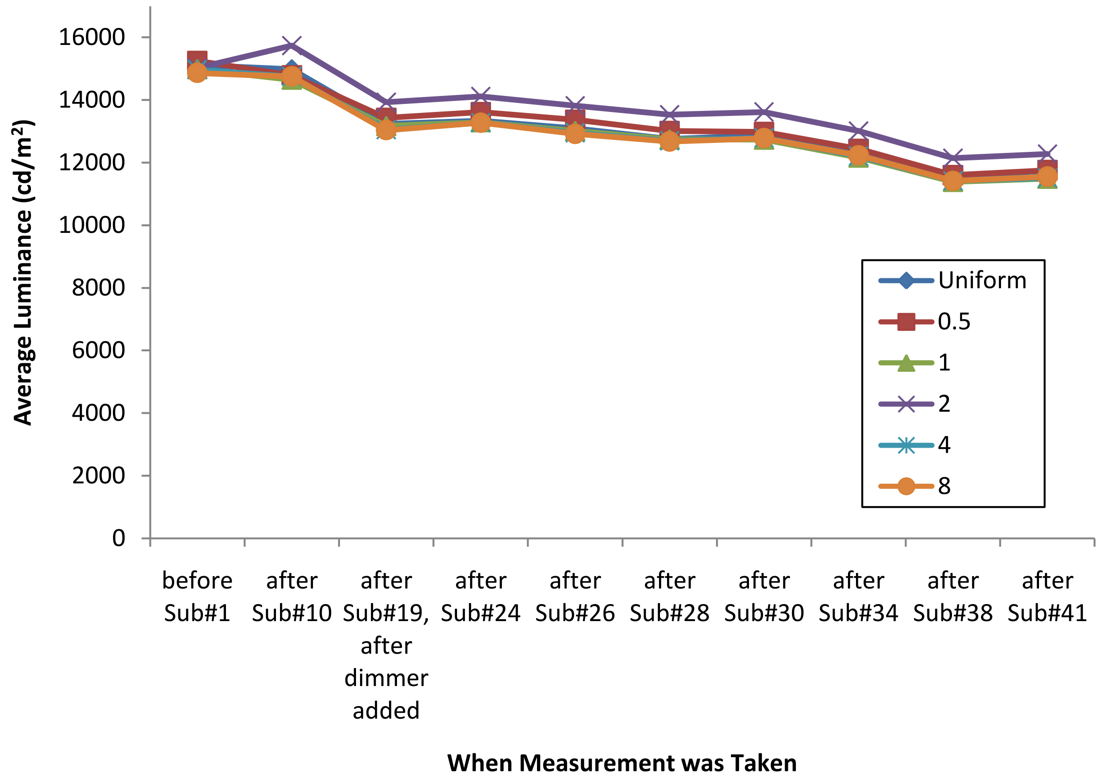


Figure 3-117. Graph of average luminance vs. duration of experiment for paper targets illuminated by left Robert Juliat fixture

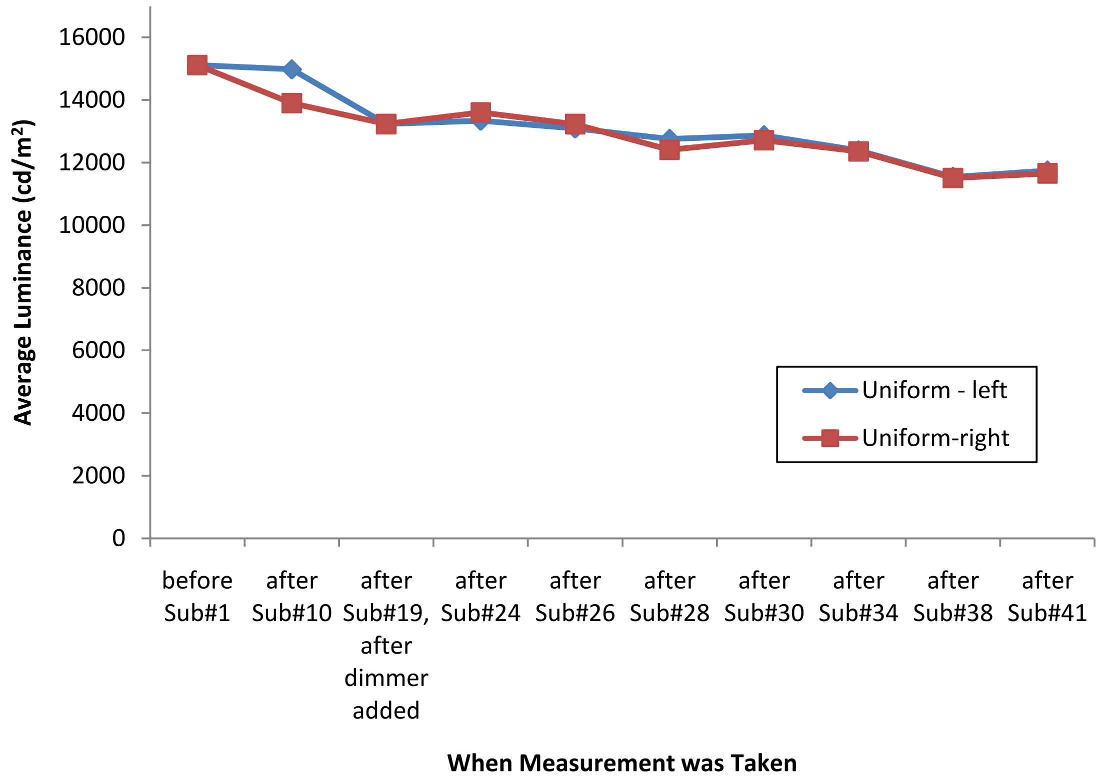


Figure 3-118. Graph of average luminance vs. duration of experiment for Uniform paper targets illuminated by either left or right Robert Juliat fixture

Because the two sides of the apparatus were discovered to be unequal, it was necessary to determine when the luminance from the two fixtures became unequal, so that any subject who was shown stimuli of unequal luminance could be eliminated from the data analysis. From examining the data from the several subjects before the discovery was made, it appeared that only two subjects needed to be eliminated. That decision was made simply by looking at the total number of times the subject said the right caused him more discomfort versus the number of times he said the left caused more discomfort. If the two sides of the apparatus were equal in luminance, the total right and total left counts should have been approximately equal. For two subjects

(subject #18 and #19), it was obvious that was not the case, so those two subjects were eliminated from the data set.

It is not certain why the right fixture's luminance was lower than the left. There are several possibilities. One possibility is that, because the second experiment (the rating scale experiment) was run using only the right fixture, it was on for a longer period of time than the left fixture. It is unlikely, however, that the extra 20 minutes per subject affected the lamp lumen depreciation that significantly. Another possibility is that there was something inherently wrong with the lamp in the right fixture. But, again, that is unlikely the cause. If there had been something wrong with the lamp, it most likely would have shown a problem before subject #19. The most likely explanation has to do with the routine used to change from trial to trial in the paired comparison experiment. When one trial was completed, both fixtures were turned off, each arm was moved back to the 10 degree position, the stimuli were changed, and each arm was moved to the position required for the next trial. When the fixtures were turned off via the contactor, the arms were moved almost immediately after, while the lamps were still hot, but the right arm was always moved first. So of the two, the right lamp had less time to cool off before it was moved. Even though incandescent lamps typically have a high resistance to shock, it seems that this movement while the lamps were hot might have had an effect on the lumen output over time.

After subject #28, all stimuli were measured on each side at all positions, to verify that the stimuli luminances were still relatively consistent. Figure 3-119 and Figure 3-120 show the average luminance of the different spatial frequencies at different positions, which show that the luminance was fairly constant across positions.

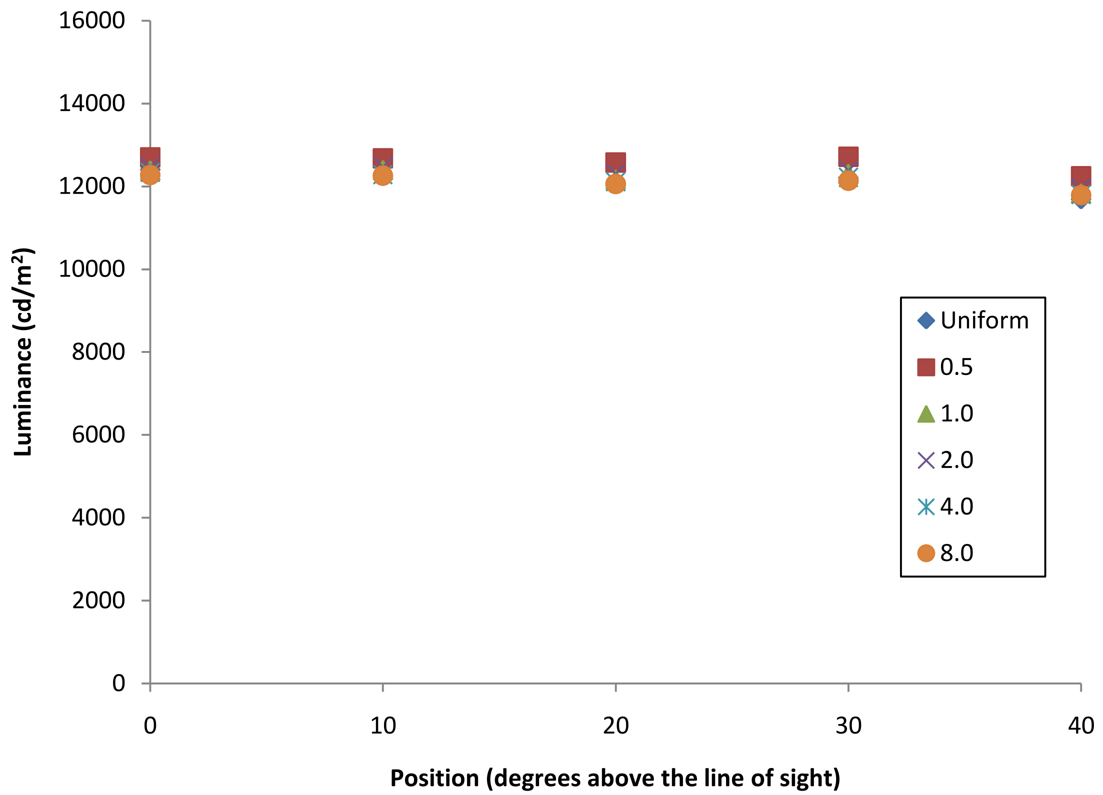


Figure 3-119. Graph of luminance vs. position above the line of sight for paper targets illuminated by right Robert Juliat fixture after the "uniformity" gobo was installed, measured after subject #28

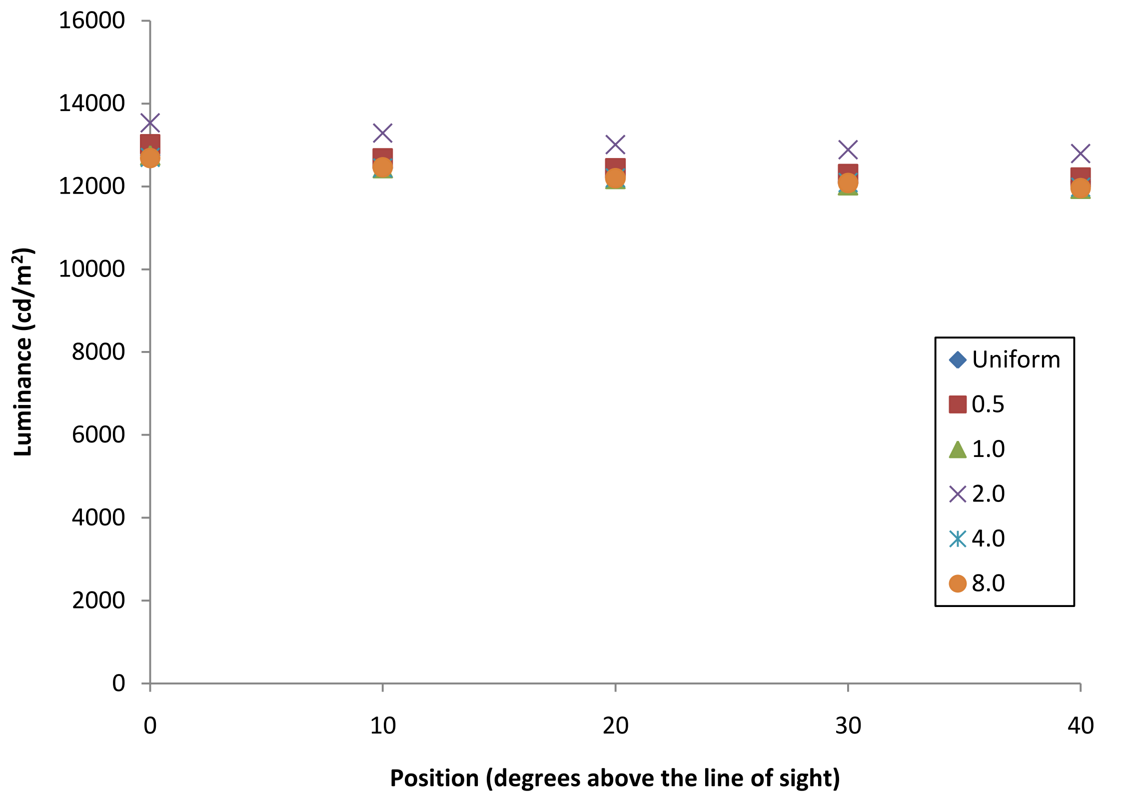


Figure 3-120. Graph of luminance vs. position above the line of sight for paper targets illuminated by left Robert Juliat fixture after the "uniformity" gobo was installed, measured after subject #28

As the stimuli luminances changed over time, so did the UGR values of the uniform stimuli. Those values are given in Table 3-11. Although the UGR values certainly did decrease over the duration of the experiment, the change was similar for all of the positions, and therefore should not affect the results significantly (note that at the end of the experiment, after subject #41, there was still approximately a three UGR point difference between each position – just as it was at subject #1). For the remainder of the discussion, when UGR values are discussed, values at subject #1 will be given, unless noted otherwise.

Table 3-11. UGR values for Uniform stimulus at different positions, at different times over the duration of the experiment

	Stimulus			
	Uniform at 10 degrees	Uniform at 20 degrees	Uniform at 30 degrees	Uniform at 40 degrees
UGR Value at Subject #1	27.7	24.6	21.5	18.4
UGR Value at Subject #10	27.1	24.0	20.9	17.8
UGR Value at Subject #19	26.8	23.7	20.5	17.5
UGR Value at Subject #24	27.0	23.8	20.7	17.7
UGR Value at Subject #26	26.8	23.6	20.5	17.5
UGR Value at Subject #28	26.3	23.2	20.1	17.0
UGR Value at Subject #30	26.5	23.4	20.3	17.2
UGR Value at Subject #34	26.3	23.2	20.1	17.0
UGR Value at Subject #38	25.8	22.7	19.6	16.5
UGR Value at Subject #41	25.9	22.8	19.7	16.6

Subjects

A total of 41 subjects participated in the experiments. The subjects were obtained by several methods: from a direct relationship with the principal investigator; from a direct relationship with one of the students assisting the principal investigator; from an email sent to all Architectural Engineering students; and, from a flier posted on bulletin boards within the Peter Kiewit Institute. Subjects were paid $10 per hour for their participation. Due to several factors, only 35 subjects were included in the data analysis for the paired comparison experiment, and only 32 subjects were included in the data

analysis for the rating scale experiment. Two of the 41 subjects did not complete the testing due to what appeared to the principal investigator to be a very significant left or right eye bias and were therefore not included in the data analysis. When shown two identical stimuli, those two subjects always said the right was more discomforting than the left, and were therefore asked not to finish the study. Two more of the 41 subjects were not included in the data analysis because the left stimulus had a significantly higher luminance than the right during those two subject's trials (as was discussed in Changes to the Stimuli Over Time). Two more of the 41 subjects were not included in the data analysis because they were the first two subjects run and there were minor modifications made to the apparatus after they completed the study. Those six subjects were not included in the paired comparison analysis. For the rating scale analysis, those six plus three other subjects were not included. The other three were not included because the background luminance was lowered after they completed the study (the background luminance was lowered simply to increase the discomfort). For the paired comparison analysis, changing the background luminance did not affect the results, but for the rating scale experiment, it would have.

Of the 35 subjects included in the paired comparison analysis, 15 were female, and 20 were male. The average age was 23, ranging from 19 to 33. Fifteen of the 35 were wearing contact lenses at the time of the experiment, and 20 were not wearing any correction. Subjects were not allowed to wear glasses during the experiment, as the stimuli were positioned in the periphery. Depending on the size of the frames, the stimuli may or may not have been within the projected area of the glasses. Therefore, individuals who wore glasses were not allowed to be subjects. Thirty-four of the 35 were

right handed, and only one was left handed. All 35 of the subjects had indoor occupations.

Of the 32 subjects included in the rating scale analysis, 14 were female, and 18 were male. The average age was 23, ranging from 19 to 33. Fourteen of the 32 were wearing contact lenses at the time of the experiment, and 18 were not wearing any correction. Thirty-one of the 32 were right handed, and only one was left handed. All 32 of the subjects had indoor occupations.

Procedure

Each subject reported to room 116 in the Peter Kiewit Institute on the Omaha campus of the University of Nebraska-Lincoln. Upon arrival, each subject was given an Informed Consent form to read and sign (see Appendix B – Informed Consent Form for a copy of the Informed Consent form), a personal information survey to complete (see Appendix C – General Information Survey for a copy of the information survey), and a payment voucher to sign. Each subject was also given a copy of the informed consent form for his own use. After filling out the necessary paperwork, the subject was seated in front of the Keystone Visual Skills Telebinocular, where instructions were read for the screening test (see Appendix D – Keystone Visual Skills Screening Test Subject Instructions for Screening Instructions). After the subject completed the Keystone Visual Skills screening test (see Keystone Visual Skills Test), he was informed whether he passed or failed. If he passed, he was allowed to continue with the experiment. If not, he was not allowed to participate in the experiment. The instructions were then read to the

subject for the paired comparison experiment, and the experiment was begun (see Appendix E – Paired Comparison Subject Instructions for Subject Instructions).

In addition to the principal investigator, two additional experimenters were used to run each subject. They changed the stimuli from one trial to the next, while the PI moved the arms and recorded the subject's response. Each subject saw 300 pairs of stimuli. In a complete paired comparison experiment, each of the 24 stimuli (4 levels of position x 6 levels of frequency) must be compared to the other 23 stimuli. That computes to 276 pairs (24 choose 2). Plus, to be able to evaluate each subject's potential for a left or right bias, each stimulus was compared to itself, adding 24 additional pairs, for a total of 300. Because each subject saw each pair only once, it had to be determined which of the pair would be on the right and which would be on the left. A matrix of left-right orientations for each pair was developed (see Table 3-12). The intent of the matrix was simply to alternate left, right, left, right, etc. for each row, so that each stimulus would be on the left half the time and on the right half the time when it was paired with the other 23 stimuli. The L on the first row, second column, indicates that the 0.5 cycles per degree stimulus at 10 degrees above the line of sight was on the left when it was paired with 0.5 cycles per degree at 20 degrees. Half of the subjects saw this orientation, and half of the subjects saw the reverse, for counterbalancing. The order of the 300 stimuli pairs was also randomized for each subject, so no two subjects saw the same order.

Table 3-12. Matrix of left right orientation for each stimulus pair

		Stimulus																						
		0.5-10	0.5-20	0.5-30	0.5-40	1.0-10	1.0-20	1.0-30	1.0-40	2.0-10	2.0-20	2.0-30	2.0-40	4.0-10	4.0-20	4.0-30	4.0-40	8.0-10	8.0-20	8.0-30	8.0-40	U-10	U-20	U-30	U-40
Stimulus	0.5-10		L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L
	0.5-20			L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
	0.5-30				L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L
	0.5-40					L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
	1.0-10						L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L
	1.0-20							L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
	1.0-30								L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L
	1.0-40									L	R	L	R	L	R	L	R	L	R	L	R	L	R	L	R
	2.0-10										L	R	L	R	L	R	L	R	L	R	L	R	L	R	L
	2.0-20											L	R	L	R	L	R	L	R	L	R	L	R	L	R
	2.0-30												L	R	L	R	L	R	L	R	L	R	L	R	L
	2.0-40													L	R	L	R	L	R	L	R	L	R	L	R
	4.0-10														L	R	L	R	L	R	L	R	L	R	L
	4.0-20															L	R	L	R	L	R	L	R	L	R
	4.0-30																L	R	L	R	L	R	L	R	L
	4.0-40																	L	R	L	R	L	R	L	R
	8.0-10																		L	R	L	R	L	R	L
	8.0-20																			L	R	L	R	L	R
	8.0-30																				L	R	L	R	L
	8.0-40																					L	R	L	R
	U-10																						L	R	L
	U-20																							L	R
	U-30																								L
	U-40																								

To begin the experiment, the principal investigator turned on the yellow flashing LED showing the position where the subject should fixate his gaze. The principal investigator explained that a pair of stimuli would be shown to the subject, which could vary in position and in pattern. It was the subject’s task to determine which stimulus caused more discomfort, left or right, and how much more discomfort on a scale of 1 through 5, where 1 meant not much more discomfort at all and 5 meant much, much more discomfort. An example pair was shown to the subject, to give him an idea of the scale to be used. The uniform stimulus at 10 degrees was shown on the right and the uniform stimulus at 40 degrees was shown on the left. The subject was reminded that he should be looking only at the fixation point and instructed that for this pair, the right should cause him more discomfort at a level of three. It was important to give the subject an example on which to base his responses. Not giving him an example would be like

asking him to measure many different items, but not telling him what scale to use. After the example trial, a warm-up trial was conducted. For the warm-up trial, two randomly selected stimuli were chosen. The fixtures were turned off, the visual shield was lowered so the subject did not have a view of the stimuli being changed but could still see the fixation point clearly, and the principal investigator lowered each arm to the ten degree position. Then each of the two additional investigators mounted the correct frequency stimulus on the apparatus and the principal investigator changed the position of each arm, which put the correct frequencies at the correct positions. The subject was reminded to look only at the fixation point, the two Robert Juliat fixtures were turned on, the visual shield was raised, and the principal investigator asked the subject to determine which caused more discomfort, left or right, and how much more discomfort on a scale of 1 though 5. After the warm-up, the subject was asked if he felt comfortable with the procedure. If he was, then the 300 trials were initiated. If he was not, the warm-up trial procedure was repeated. For each trial (as in the warm-up trial), the two stimuli were read from the pre-randomized list, and the investigators put the correct frequencies on the apparatus at the correct positions. The subject was reminded to look only at the fixation point, the fixtures were turned on, the visual shield was raised, and the principal investigator asked the subject to determine which caused more discomfort, left or right, and how much more discomfort on a scale of 1 through 5. The principal investigator recorded the subject's response, and then moved to the next trial, at which point the process was repeated. Breaks were taken after every 75 trials. After the 300 trials were completed, the paired comparison experiment was complete.

Before the rating scale experiment was begun, the apparatus was modified slightly to allow for only one stimulus. The left Robert Juliat fixture was disconnected from the lighting contactor, so that only the right fixture would operate. In addition, the chin/forehead rest was relocated so that the subject was positioned directly in line with the right stimulus.

The subject was then given the instructions for the rating scale experiment (see Appendix F – Rating Scale Subject Instructions for subject instructions). The principal investigator explained that only one stimulus would be shown to the subject. It was the subject's task to rate the level of discomfort on a scale of 1 through 7 (see Figure 3-121 for the rating scale). For this experiment, an example trial was not conducted, as the words associated with the rating scale values were deemed to be enough guidance. A warm-up trial was conducted, where a randomly selected stimulus was chosen. One of the additional investigators mounted the correct frequency stimulus on the apparatus and the principal investigator changed the position of the arm, which put the correct frequency at the correct position. The subject was reminded to look only at the fixation point, the Robert Juliat fixture was turned on, the visual shield was raised, and the principal investigator asked the subject to rate the level of discomfort on a scale of 1 through 7. After the warm-up, the subject was asked if he felt comfortable with the procedure. If he was, then the 24 trials were initiated. If he was not, the warm-up procedure was repeated. For each trial (as in the warm-up trial), the stimulus was read from the pre-randomized list, and the investigators put the correct frequency on the apparatus at the correct position. The subject was reminded to look only at the fixation point, the fixture was turned on, the visual shield was raised, and the principal

investigator asked the subject to rate the level of discomfort on a scale of 1 through 7. The principal investigator recorded the subject's response, and then moved to the next trial, at which point the process was repeated. No breaks were taken during the rating scale experiment. After the 24 trials were completed, the rating scale experiment was complete, and the subject was allowed to leave.

GLARE RATING SCALE	
1	Just Perceptible
2	Perceptible
3	Just Acceptable
4	Unacceptable
5	Just Uncomfortable
6	Uncomforable
7	Intolerable

Figure 3-121. Glare rating scale as given to the subjects for the rating scale experiment

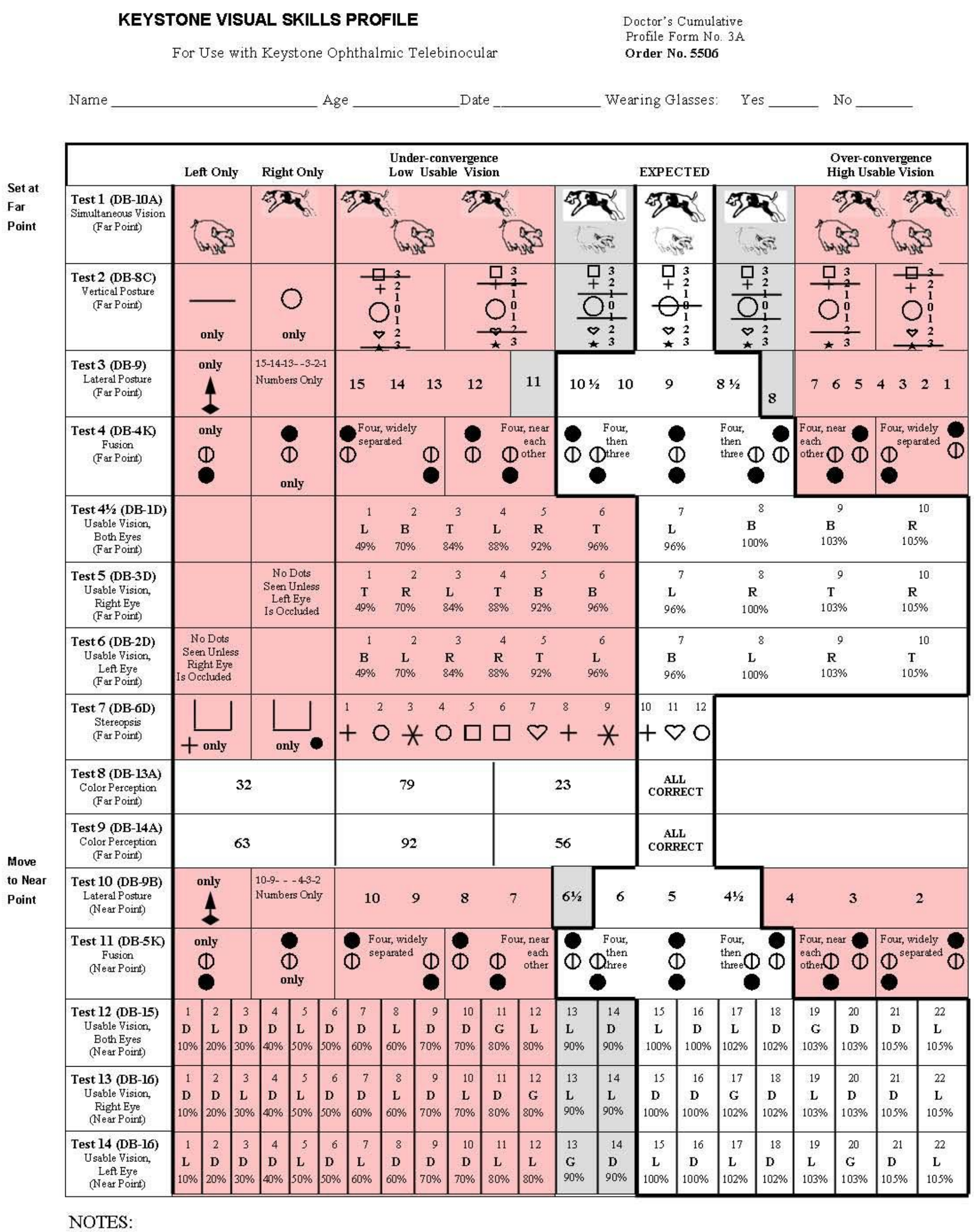
The subjects did not perform a critical task (such as reading or a visual acuity task as in Guth 1951) during the experiment. Even though Guth showed a slight difference in BCD with and without a task (at positions of 20 degrees or less off the line of sight, subjects have more discomfort with a task than without; and, at positions higher than 20 degrees off the line of sight, there is more discomfort without a task), a task was not used in this experiment as it was believed that the effect of frequency could be small (if there was an effect at all), and if the subject had to perform a task, it would be more difficult for him to make the determination between two frequencies. This would most likely have led to increased variability, as some of the subjects would focus more on the task

and less on the stimuli, and others would do the opposite. Therefore, a task was not included.

Keystone Visual Skills Test

It was important that the vision of each subject was verified. Significant vision deficiencies or left/right biases would have been detrimental to the validity of the results of the experiments performed. The Keystone Visual Skills Test was chosen as an appropriate metric to verify each subject's visual acuity. According to the Keystone Visual Skills test instruction manual, the tests reveal problems with specific visual skills. "They serve uniquely to demonstrate that a visual problem exists; to indicate the degree of elimination of that problem; to make clear to the patient that a visual problem exists and to convey to him/her what is meant by the term visual achievement" (Keystone 2003). For the test, each subject is asked to put his head against a forehead rest in the Telebinocular, so the distance from the stereotargets to the eye is the same. The subject should have good posture during the test. There are special instructions for how to position the subject within the Telebinocular if he is wearing bifocals, but as none of the subjects were wearing glasses, this was not a concern. Each subject saw fifteen different stereotargets in the test, each of which was testing a different visual competency. The first nine stereotargets were shown to the subject at the far point; the other five were shown at the near point. The tests at the far point were checking for whether or not both eyes see at the same time, vertical imbalance, whether or not both eyes work together, right and left eye independent operation, depth perception, and color perception. At the near point, the tests were checking for binocular coordination and acuity and right and

left eye independent operation. There was a small incandescent lamp within the Telebinocular, which illuminated the stereotargets so that each subject was exposed to the same luminances during the test. The intent of the test for this research was simply to verify that each subject had vision within the normal range (see Figure 3-122 for the scoring sheet). The normal range is anywhere within the white area. Even if subjects were slightly outside of the normal range, they were still allowed to participate in the experiment. Two potential subjects, however, were not allowed to participate in the experiment because of their failure to score within the normal range during the tests which determine if both eyes are working together (tests #5 and #6 – they only saw dots when one eye was occluded). As this experiment was primarily a paired comparison experiment, a subject whose two eyes don’t work together would have been detrimental.



KEYSTONE VISUAL SKILLS PROFILE

For Use with Keystone Ophthalmic Telebinocular

Doctor's Cumulative Profile Form No. 3A
Order No. 5506

Name ______________ Age ________ Date ________ Wearing Glasses: Yes ______ No ______

	Left Only	Right Only	Under-convergence Low Usable Vision	EXPECTED	Over-convergence High Usable Vision
Set at Far Point — Test 1 (DB-10A) Simultaneous Vision (Far Point)					
Test 2 (DB-8C) Vertical Posture (Far Point)	only	only			
Test 3 (DB-9) Lateral Posture (Far Point)	only	15-14-13- -3-2-1 Numbers Only	15 14 13 12 11	10½ 10 9 8½ 8	7 6 5 4 3 2 1
Test 4 (DB-4K) Fusion (Far Point)	only	only	Four, widely separated; Four, near each other	Four, then three; Four, then three	Four, near each other; Four, widely separated
Test 4½ (DB-1D) Usable Vision, Both Eyes (Far Point)			1 L 49%; 2 B 70%; 3 T 84%; 4 L 88%; 5 R 92%; 6 T 96%	7 L 96%; 8 B 100%; 9 B 103%; 10 R 105%	
Test 5 (DB-3D) Usable Vision, Right Eye (Far Point)		No Dots Seen Unless Left Eye Is Occluded	1 T 49%; 2 R 70%; 3 L 84%; 4 T 88%; 5 B 92%; 6 B 96%	7 L 96%; 8 R 100%; 9 T 103%; 10 R 105%	
Test 6 (DB-2D) Usable Vision, Left Eye (Far Point)	No Dots Seen Unless Right Eye Is Occluded		1 B 49%; 2 L 70%; 3 R 84%; 4 R 88%; 5 T 92%; 6 L 96%	7 B 96%; 8 L 100%; 9 R 103%; 10 T 105%	
Test 7 (DB-6D) Stereopsis (Far Point)	only	only	1 2 3 4 5 6 7 8 9	10 11 12	
Test 8 (DB-13A) Color Perception (Far Point)	32		79; 23	ALL CORRECT	
Test 9 (DB-14A) Color Perception (Far Point)	63		92; 56	ALL CORRECT	
Move to Near Point — Test 10 (DB-9B) Lateral Posture (Near Point)	only	10-9- - -4-3-2 Numbers Only	10 9 8 7	6½ 6 5 4½	4 3 2
Test 11 (DB-5K) Fusion (Near Point)	only	only	Four, widely separated; Four, near each other	Four, then three; Four, then three	Four, near each other; Four, widely separated
Test 12 (DB-15) Usable Vision, Both Eyes (Near Point)	1 D 10%; 2 L 20%; 3 D 30%	4 D 40%; 5 L 50%; 6 D 50%	7 D 60%; 8 L 60%; 9 D 70%; 10 D 70%; 11 G 80%; 12 L 80%; 13 L 90%; 14 D 90%	15 L 100%; 16 D 100%; 17 L 102%; 18 D 102%	19 G 103%; 20 D 103%; 21 D 105%; 22 L 105%
Test 13 (DB-16) Usable Vision, Right Eye (Near Point)	1 D 10%; 2 D 20%; 3 L 30%	4 D 40%; 5 L 50%; 6 D 50%	7 D 60%; 8 L 60%; 9 D 70%; 10 L 70%; 11 D 80%; 12 G 80%; 13 L 90%; 14 L 90%	15 D 100%; 16 D 100%; 17 G 102%; 18 D 102%	19 L 103%; 20 D 103%; 21 D 105%; 22 L 105%
Test 14 (DB-16) Usable Vision, Left Eye (Near Point)	1 L 10%; 2 D 20%; 3 D 30%	4 D 40%; 5 L 50%; 6 D 50%	7 L 60%; 8 D 60%; 9 D 70%; 10 D 70%; 11 L 80%; 12 L 80%; 13 G 90%; 14 D 90%	15 L 100%; 16 D 100%; 17 L 102%; 18 D 102%	19 L 103%; 20 G 103%; 21 D 105%; 22 L 105%

NOTES:

Keystone View - 866-874-6360 - www.keystoneview.com - sales@keystoneview.com

Figure 3-122. Keystone Visual Skills Test scoring sheet where white area indicates "normal" scoring range

Chapter 4 – Results

The two experiments generated three different bodies of data. The first is the body of data that was collected from the rating scale experiment. This is simply a rating (1 through 7) for each of the 24 stimuli, for each of the 32 subjects (see Table 4-1).

Table 4-1. Results of rating scale experiment for Subject #1

0.5-10	0.5-20	0.5-30	0.5-40	1.0-10	1.0-20	1.0-30	1.0-40	2.0-10	2.0-20	2.0-30	2.0-40	4.0-10	4.0-20	4.0-30	4.0-40	8.0-10	8.0-20	8.0-30	8.0-40	U-10	U-20	U-30	U-40
7	5	5	2	7	7	6	4	7	6	4	3	7	6	5	3	7	6	3	2	6	6	4	2

The second and third are the data that were collected from the paired comparison experiment. The first of those is the data collected from the subject choosing which stimulus was more discomforting, left or right. For each subject, a matrix was generated with 1's and 0's, simply showing which was more discomforting (see Table 4-2). A "1" in the cell means that the row stimulus was considered more discomforting than the column stimulus. A "0" means that the column stimulus was considered more discomforting than the row stimulus. For example, 0.5 cycles per degree at 20 degrees above the line of sight was considered more discomforting than 0.5 cycles per degree at 30 degrees, but less discomforting than 0.5 cycles per degree at 10 degrees. Note that the diagonal is blank. When two identical stimuli were compared, either the left or the right was chosen, which could be either a "0" or a "1", therefore it was left blank. This body of data is referred to as the paired comparison "choice" data.

Table 4-2. Results of paired comparison experiment for Subject #1 showing only the preference between stimuli. A "1" in the cell means the row was more discomforting than the column. A "0" in the cell means the column was more discomforting than the row.

	Stimulus	0.5-10	0.5-20	0.5-30	0.5-40	1.0-10	1.0-20	1.0-30	1.0-40	2.0-10	2.0-20	2.0-30	2.0-40	4.0-10	4.0-20	4.0-30	4.0-40	8.0-10	8.0-20	8.0-30	8.0-40	U-10	U-20	U-30	U-40
Stimulus	0.5-10		1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1
	0.5-20	0		1	1	0	0	1	1	0	1	1	1	0	1	1	1	0	1	0	1	0	0	1	1
	0.5-30	0	0		1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	1
	0.5-40	0	0	0		0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1
	1.0-10	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1.0-20	0	1	1	1	0		1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0	1
	1.0-30	0	0	1	1	0	0		1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0
	1.0-40	0	0	0	1	0	0	0		0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
	2.0-10	0	1	1	1	0	1	1	1		1	1	1	1	1	1	1	0	1	1	1	0	1	1	1
	2.0-20	0	0	1	1	0	1	1	1	0		0	1	0	0	1	1	0	0	1	1	0	0	1	1
	2.0-30	0	0	1	0	0	0	1	1	0	1		1	0	0	0	1	0	0	1	0	0	0	0	1
	2.0-40	0	0	0	1	0	0	0	0	0	0	0		0	0	0	1	0	0	0	0	0	0	0	0
	4.0-10	0	1	1	1	0	1	1	1	0	1	1	1		1	1	1	0	1	1	1	1	1	1	1
	4.0-20	0	0	1	1	0	1	1	1	0	1	1	1	0		0	1	0	1	1	1	0	0	1	1
	4.0-30	0	0	1	1	0	0	1	1	0	0	1	1	0	1		1	0	0	0	1	0	0	1	1
	4.0-40	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0		0	0	0	1	0	0	0	1
	8.0-10	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1		1	1	1	0	1	1	1
	8.0-20	0	0	1	1	0	1	1	1	0	1	1	1	0	0	1	1	0		1	1	0	0	0	1
	8.0-30	0	1	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	0		1	0	0	0	1
	8.0-40	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0		0	0	0	0
	U-10	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1		1	1	1
	U-20	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0		0	1
	U-30	0	0	1	1	0	1	1	1	0	0	1	1	0	0	0	1	0	1	1	1	0	1		1
	U-40	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	

The second body of data collected from the paired comparison experiment is the "magnitude" data. After subjects were asked which they considered more discomforting, left or right, they were also asked how much more discomforting on a scale of 1 through 5. For each subject, a matrix was generated with values of 1 through 5, showing how much more discomforting one stimulus was than another (see Table 4-3). Where there is a number in the cell, it means the row stimulus was considered more discomforting than the column stimulus, and the magnitude is what was given by the subject. Where the cell is blank, it means the row is less discomforting than the column. For example, 0.5 cycles per degree at 20 degrees above the line of sight is considered more discomforting than 0.5 cycles per degree at 30 degrees above the line of sight at a magnitude of 2. Note also that not every cell has a number in it. That is because every comparison was made only once, so only half of the matrix is completed. Note also that along the diagonal, there are letters in the cells. An "r" in the cell means that when the two identical stimuli were compared, the subject said that the right was more discomforting than the left. An "***ℓ***" in the cell means that the subject said the left was more discomforting. This body of data is referred to as the paired comparison "magnitude" data.

Table 4-3. Results of paired comparison experiment for Subject #1 showing the magnitude difference between stimuli

		Stimulus																							
		0.5-10	0.5-20	0.5-30	0.5-40	1.0-10	1.0-20	1.0-30	1.0-40	2.0-10	2.0-20	2.0-30	2.0-40	4.0-10	4.0-20	4.0-30	4.0-40	8.0-10	8.0-20	8.0-30	8.0-40	U-10	U-20	U-30	U-40
Stimulus	0.5-10	r	2	4	4		3	3	4	2	2	3	3	3	2	4	4		3	4	4		2	4	4
	0.5-20		r	2	4			2	4		2	3	3		2	3	3		2		3			2	2
	0.5-30			r	1				2				2				3			2	2				2
	0.5-40				ℓ							2									2				1
	1.0-10	3	4	3	5	ℓ	3	4	4	2	3	4	5	3	2	4	4	2	3	3	2	2	4	4	4
	1.0-20		1	3	3		r	1	3			4	3			2	3			3	3				3
	1.0-30			2	2			r	2				2				1			1					
	1.0-40				2				ℓ				1				1								
	2.0-10		3	4	5		4	3	5	r	2	4	4	2	2	4	5		2	4	4		2	4	4
	2.0-20			1	3		2	2	3		r		4			2	2			3	2			2	4
	2.0-30			1				2	2		1	r	1				2			2					1
	2.0-40				1								r				2								
	4.0-10		4	4	5		3	3	5		3	3	4	r	3	3	4		3	4	4	3	3	4	4
	4.0-20			2	3		3	3	3		2	2	3		ℓ		5		2	2	3			2	2
	4.0-30			1	2			2	3			1	2		1	r	2				2			1	2
	4.0-40				1												r				1				2
	8.0-10	3	3	3	5		3	4	5	2	3	5	4	3	2	4	5	r	2	3	5		3	4	5
	8.0-20			3	3		2	3	3		2	2	3			2	3		r	3	4				4
	8.0-30		2		1				2				3			1	2			ℓ	2				2
	8.0-40							1	1			1	2								r				
	U-10	2	3	4	5		4	5	4	2	4	4	5		3	2	4	2	3	4	5	r	1	5	4
	U-20		2	3	4		2	3	3		1	2	4		2	3	3		2	3	3		r		3
	U-30			1	2		1	2	3			1	2				3		2	2	2		1	ℓ	2
	U-40							2	1				1								1				r

Rating Scale Analysis

For the rating scale data, the data was analyzed with a repeated measures ANOVA. Prior to that discussion, however, there is much to be gained simply by looking at descriptive statistics. The discomfort glare ratings from the 32 subjects who were included in the rating scale analysis were averaged to determine an average rating for each of the 24 stimuli. Those values are plotted in Figure 4-1, which shows 95% confidence intervals as error bars and the calculated UGR values for the four uniform stimuli. It is clear from the error bars that there is no statistically significant difference in the average rating between frequencies within any single position, but there does seem to be a significant difference in rating between positions. This would imply that position does have a significant impact on discomfort glare: as position increases, discomfort glare decreases. However, frequency does not appear to significantly impact discomfort glare.

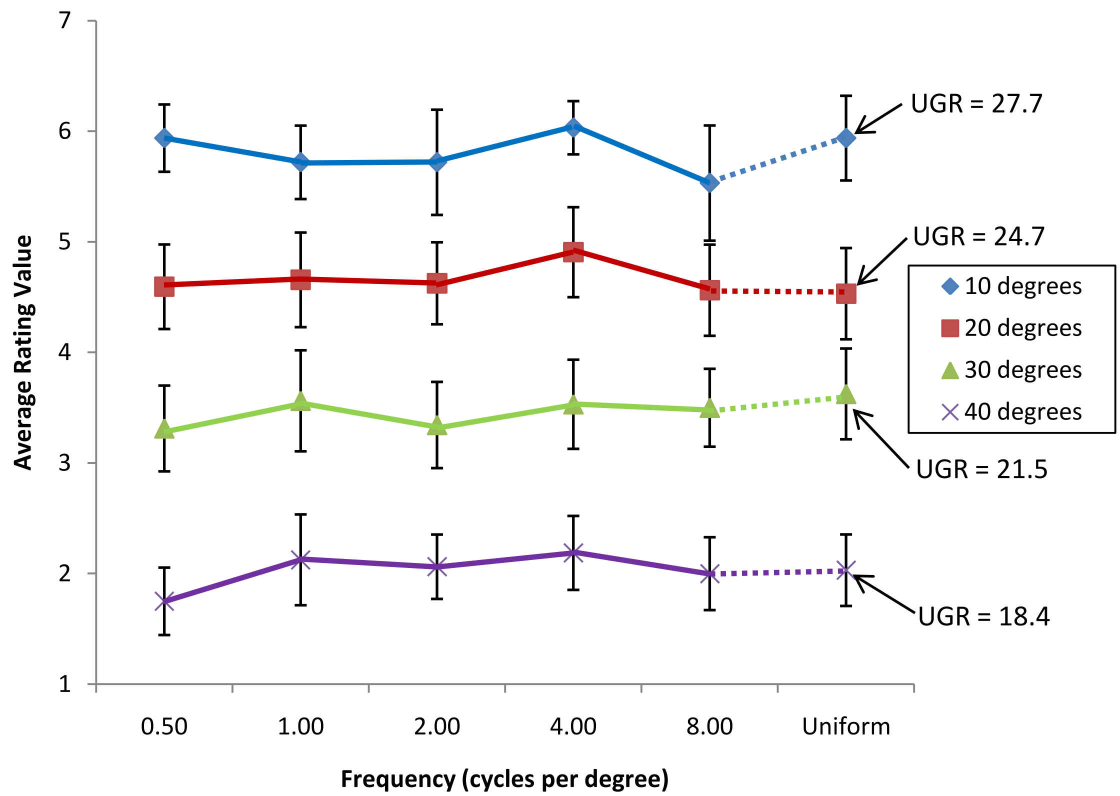


Figure 4-1. Graph of average rating vs stimulus for all 34 subjects combined

Repeated Measures ANOVA

The statistical analysis technique that was used for the rating scale data was a Repeated Measures Analysis of Variance (ANOVA). An ANOVA is a simple extension of the t-test statistic, but where the t-test is used to compare means of only two different groups, ANOVA can be used to compare the means of more than two groups. With ANOVA, the test statistic is the F statistic. An F value is calculated for each group, and is compared to a critical value, based on a predetermined alpha (α) level and appropriate degrees of freedom. If the calculated F value exceeds the critical F value, the null hypothesis that the means are the same is rejected. With multiple groups, ANOVA tests for not only the main effects, but also the interactions between the main effects. The

rating scale experiment was considered a repeated measures type because subjects were asked for their discomfort rating for all levels of position and all levels of frequency. One concern with repeated measures is that order effects will impact the results (Levin 1999). Therefore, the order of the 24 trials was randomized for each subject.

The design was a 6 x 4, full-factorial, repeated measures experiment. The null hypothesis for the effect of position in the rating scale analysis is as follows:

$$\mu_{10\,degrees} = \mu_{20\,degrees} = \mu_{30\,degrees} = \mu_{40\,degrees} \tag{4.1}$$

Similarly, the null hypothesis for the effect of frequency is as follows:

$$\mu_{0.5cpd} = \mu_{1.0cpd} = \mu_{2.0cpd} = \mu_{4.0cpd} = \mu_{8.0cpd} = \mu_{Uniform} \tag{4.2}$$

The data for this analysis are shown in Table 4-1, for each subject. Each subject saw each of the 24 stimuli, and simply rated the level of discomfort from 1 to 7, therefore the range of values for each stimulus (for each subject) is 1 through 7. The data were entered into SAS for the analysis. The SAS commands and output are shown in Appendix G – SAS Command File and Output File for Rating Scale Analysis.

The means for each of the positions and for each frequency are given in Table 4-4.

Table 4-4. Mean values of each level of the two independent variables, position and spatial frequency

Independent Variable	Level	Mean (μ) Discomfort Rating	Standard Deviation (σ) of Discomfort Rating
Position	10 degrees	5.8125	0.738
	20 degrees	4.6458	0.852
	30 degrees	3.4792	0.861
	40 degrees	2.026	0.683
Spatial Frequency	0.5 cpd	3.8984	0.684
	1.0 cpd	4.0156	0.757
	2.0 cpd	3.9375	0.838
	4.0 cpd	4.1641	0.692
	8.0 cpd	3.8984	0.695
	Uniform	4.0313	0.726

Overall, the effect of position on discomfort glare is significant, meaning there are group differences in discomfort among 10, 20, 30, and 40 degrees above the line of sight ($F(3,93)=257.16$, $p<0.05$). This is to be expected from previous glare studies, which have also shown that as position increases, discomfort decreases (Lukiesh & Guth 1949, Guth 1951). There is a significant linear trend in the data ($F(1,31)=467.79$, $p<0.05$), with higher positions showing lower discomfort. This result confirms the graphical result shown in Figure 4-1. Neither the quadratic ($p = 0.1657$) nor the cubic ($p=0.2972$) trends are significant. This is interesting, because the glare metrics, specifically UGR and VCP, differ on the exponent applied to position. In the UGR formula, the position factor is squared (indicating a quadratic relationship to discomfort) (CIE 1995), while in the VCP formula, the position factor has an exponent of 1.0 (indicating a linear relationship to discomfort) (Rea 2000). The data from the current study seems to coincide with the VCP formula on this point.

The effect of frequency on discomfort glare is not significant (p=0.1380), nor is the interaction of position by frequency (p=0.7187). This is somewhat disappointing, as the hypothesis is that frequency will have a significant effect on glare. Of the two experiments, this one was definitely more difficult for the subject. Rating a series of stimuli on a scale of 1 through 7, even with descriptions attached to each value, was a more difficult task than determining which of two stimuli, when presented together, was more discomforting. It was expected that the effect of frequency would not be as large as the effect of position, and it appears that the experiment simply was not powerful enough to find the effect (if, indeed, there is an effect of frequency).

As position was found to be a significant factor of discomfort, Scheffe post-hoc tests were run to determine where the differences are in the four levels. Table 4-5 shows that each of the levels of position was significantly different from every other level.

Table 4-5. Scheffe groupings for the different levels of Position

		Stimulus Position (degrees above line of sight)			
		10	20	30	40
Stimulus	10				
Position	20	Different			
(degrees above	30	Different	Different		
line of sight)	40	Different	Different	Different	

Paired Comparison Analysis

Several different analysis techniques were used to analyze the two different bodies of data gathered from the paired comparison experiment. Prior to the discussion of those methods, descriptive statistics were calculated to better understand the raw data.

For each subject, the number of times each stimulus was considered more discomforting than what it was compared to was summed. These sums were then averaged across subject, to generate Figure 4-2 and Figure 4-3. These graphs suggest that there is an obvious difference in discomfort between positions, but the difference between frequencies is less pronounced. There does appear to be a slight increase in discomfort as frequency increases, within any position.

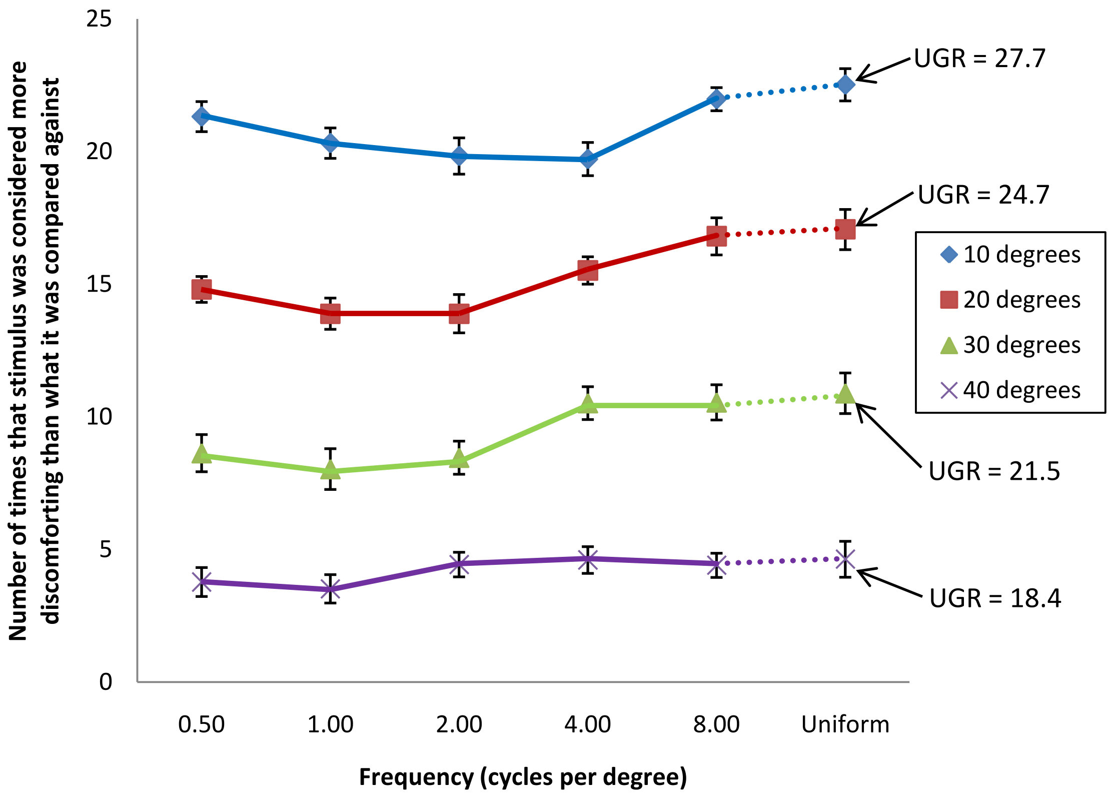


Figure 4-2. Graph from "choice" data showing number of times each stimulus was considered more discomforting than everything it was compared to, averaged across subjects

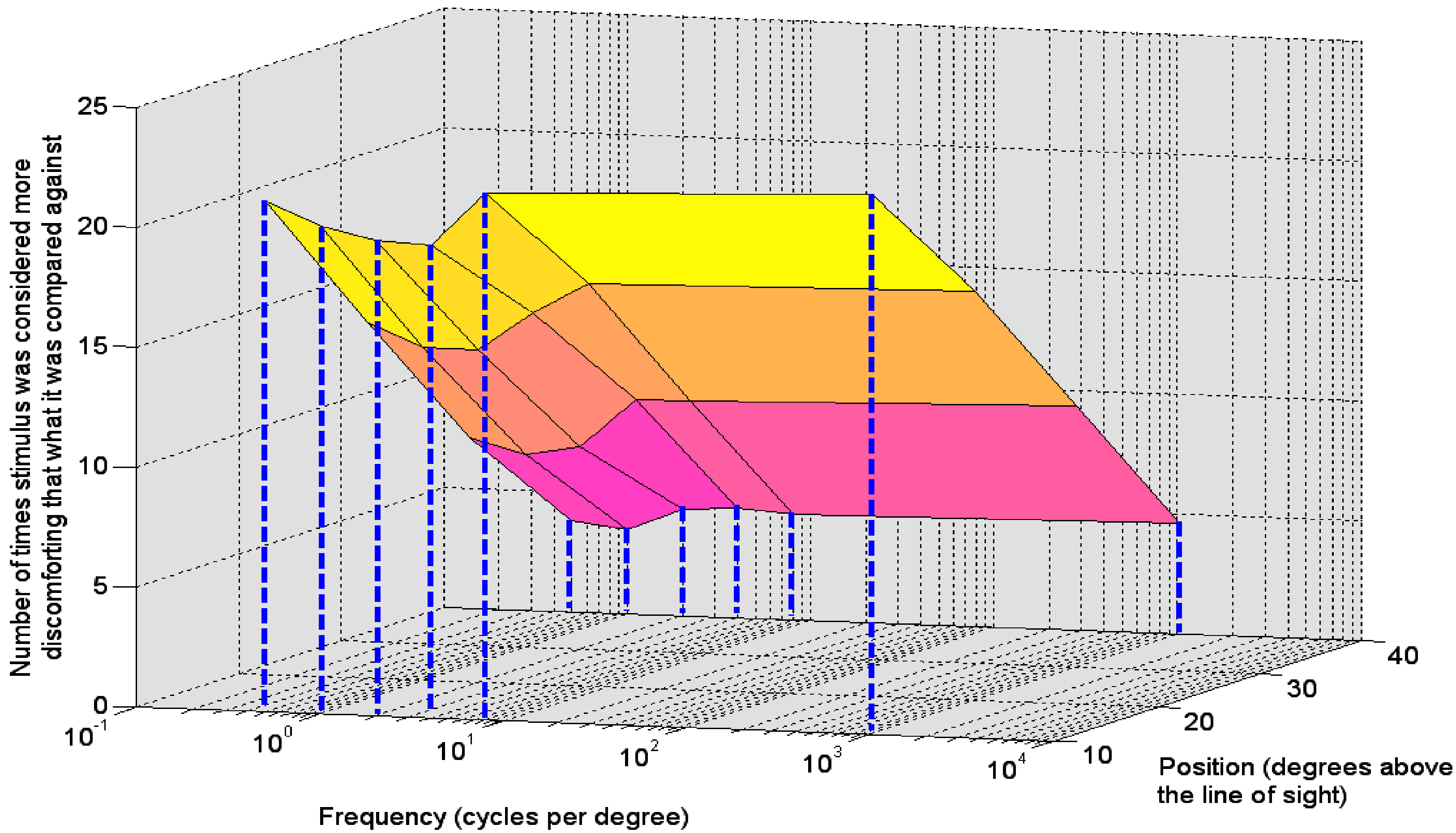


Figure 4-3. 3D graph from "choice" data showing number of times each stimulus was considered more discomforting than everything it was compared to, averaged across subjects with dashed lines shown for clarification (uniform stimulus was arbitrarily assigned a large value of frequency for graphing)

Another descriptive tool used to look at the data was simply to sum the raw data across subjects. Table 4-6 shows how many subjects rated each stimulus over every other stimulus. Note that the table has been rearranged from previous tables to be in position groupings, rather than frequency groupings, to show the significant effect of position.

Table 4-6. Results of Paired Comparison “choice” experiment summed across subjects. The number in the cell indicates how many of the 35 subjects rated the row stimulus as more discomforting than the column stimulus.

		Stimulus																							
		U-10	8.0-10	4.0-10	2.0-10	1.0-10	0.5-10	U-20	8.0-20	4.0-20	2.0-20	1.0-20	0.5-20	U-30	8.0-30	4.0-30	2.0-30	1.0-30	0.5-30	U-40	8.0-40	4.0-40	2.0-40	1.0-40	0.5-40
Stimulus	U-10		22	25	28	30	27	34	35	33	34	33	34	35	35	35	35	34	35	35	35	34	35	35	35
	8.0-10	13		26	25	28	20	34	34	33	35	34	35	35	35	35	35	33	34	35	35	35	35	35	35
	4.0-10	10	9		16	11	7	28	28	30	33	35	31	35	34	34	35	35	34	35	35	35	35	35	35
	2.0-10	7	10	19		12	9	30	29	29	35	34	33	31	34	35	35	34	34	34	35	35	35	35	35
	1.0-10	5	7	24	23		12	27	30	33	35	34	31	34	34	34	35	35	35	35	35	34	35	35	34
	0.5-10	8	15	28	26	23		30	28	32	33	35	35	34	35	35	34	35	35	35	35	35	35	35	35
	U-20	1	1	7	5	8	5		21	23	29	27	28	30	34	34	35	33	34	35	35	35	35	33	34
	8.0-20	0	1	7	6	5	7	14		25	25	29	26	32	31	35	35	33	33	35	35	35	35	34	35
	4.0-20	2	2	5	6	2	3	12	10		26	23	17	32	29	31	33	33	35	35	35	35	35	33	34
	2.0-20	1	0	2	0	0	2	6	10	9		17	13	28	31	25	33	33	32	35	35	34	35	35	35
	1.0-20	2	1	0	1	1	0	8	6	12	18		14	26	27	28	34	34	32	33	35	34	35	35	35
	0.5-20	1	0	4	2	4	0	7	9	18	23	21		30	29	27	32	34	33	35	35	35	35	34	35
	U-30	0	0	0	4	1	1	5	3	3	7	9	5		16	17	26	26	24	33	34	34	33	34	31
	8.0-30	0	0	1	1	1	0	1	4	6	4	8	6	19		22	22	25	20	33	30	34	34	32	31
	4.0-30	0	0	1	0	1	0	1	0	4	11	7	8	18	13		21	26	25	31	33	32	33	34	34
	2.0-30	0	0	0	0	0	1	0	0	2	2	1	2	9	13	14		16	16	31	27	29	32	33	33
	1.0-30	1	2	0	1	0	0	2	2	2	2	1	1	9	10	9	19		14	28	28	25	26	30	34
	0.5-30	0	1	1	1	0	0	1	2	0	3	3	2	11	15	10	19	21		26	29	27	29	33	33
	U-40	0	0	0	1	0	0	0	0	0	0	2	0	2	2	4	4	7	9		17	15	20	21	23
	8.0-40	0	0	0	0	0	0	0	0	0	0	0	0	1	5	2	8	7	6	18		16	15	24	17
	4.0-40	1	0	0	0	1	0	0	0	0	0	1	0	1	1	3	6	10	7	20	19		17	19	20
	2.0-40	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	3	9	6	15	20	18		23	21
	1.0-40	0	0	0	0	0	0	2	1	2	0	0	1	1	3	1	2	5	2	14	11	16	12		15
	0.5-40	0	0	0	0	1	0	1	0	1	0	0	0	4	4	1	2	1	2	12	18	16	14	20	

Unidimensional Analysis – "Choice" Data

The first statistical analysis tool used to analyze the paired comparison "choice" data was a method of Unidimensional Analysis called Thurstone's Case V method. Thurstone developed a method for arranging stimuli on a psychological continuum. He suggests that for any psychological stimuli, two statements are true. First, reactions to the stimuli are subjective; and, second, how the subject views the stimuli can vary from one instance to another. For any stimuli, there will be a typical reaction, called a modal reaction. This mode can be determined from multiple reactions to stimuli from a single subject, or from a frequency of reactions from more than one subject. He assumed that the reactions follow the normal curve distribution, so the mode is equivalent to the mean, therefore the mean can represent the value for each stimuli. The frequency of reactions can be used to generate distances between stimuli based on the proportions of preference, so that not only can it be determined which is more preferable, but also by how much. For example, if 50 subjects are asked which of two statements is considered more positive, and 40 of them say the latter is more positive, then the proportion of preference for the latter is 80% (or 40/50). Thurstone suggests that those proportions be converted into z-scores, which can be plotted on a one-dimensional scale to show where the two statements are on a continuum of positive effect (Dunn-Rankin et al 2004).

Prior to performing a Unidimensional Analysis, Dunn-Rankin et al (2004) recommend first analyzing the number of circular triads in the data, which assists in determining how consistent the subjects were in making their judgments. Circular triads are a problem specific to paired comparison experiments, because they result from multiple pair wise choices. A circular triad results when a subject chooses stimulus A as

more discomforting than stimulus B, stimulus B more discomforting than stimulus C, and stimulus C more discomforting than stimulus A, i.e. A>B, B>C, and C>A. If a subject showed this pattern, it would be impossible to determine which of the three stimuli he feels is the most (or least) discomforting. If a subject has a significant number of circular triads ($p>0.05$), he should be eliminated from the data analysis, as it could be assumed that the subject was either guessing or did not understand the task asked of him.

For the paired comparison data, a circular triad analysis was conducted for each of the 35 subjects with Dunn-Rankin's (2004) TRICIR program. For each subject the total number of circular triads was calculated (out of a total possible 575), along with the coefficient of consistency. This is a measure of subject consistency based on the number of circular triads, which ranges from 0 (meaning the subject had the maximum possible number of circular triads) to 1.0 (meaning the subject had no circular triads). The p value associated with that coefficient of consistency was also calculated (see Table 4-7). The number of circular triads for each subject was calculated from the following formula:

$$Number\ of\ Circular\ Triads\ (NCT) = \frac{K(K-1)(2K-1)}{12} - \frac{\sum a_j^2}{2} \qquad \textbf{(4.3)}$$

Where: K = Number of objects being compared

a_j = number of times the subject preferred object "j"

Similarly, the coefficient of consistency was calculated from the following formula:

$$Coefficient\ of\ Consistency\ (\zeta) = 1 - \frac{24(NCT)}{(K^3 - K)} \qquad \textbf{(4.4)}$$

Table 4-7. Circular triad results for each subject

Subject	No. of Circular Triads	ζ	p statistic
1	60	0.895	$p < 0.05$
2	7	0.988	$p < 0.05$
3	74	0.871	$p < 0.05$
4	120	0.790	$p < 0.05$
5	35	0.939	$p < 0.05$
6	67	0.883	$p < 0.05$
7	35	0.939	$p < 0.05$
8	65	0.886	$p < 0.05$
9	68	0.881	$p < 0.05$
10	14	0.976	$p < 0.05$
11	65	0.886	$p < 0.05$
12	14	0.976	$p < 0.05$
13	36	0.937	$p < 0.05$
14	120	0.790	$p < 0.05$
15	28	0.951	$p < 0.05$
16	66	0.885	$p < 0.05$
17	79	0.862	$p < 0.05$
18	31	0.946	$p < 0.05$
19	29	0.949	$p < 0.05$
20	130	0.773	$p < 0.05$
21	30	0.948	$p < 0.05$
22	86	0.850	$p < 0.05$
23	28	0.951	$p < 0.05$
24	49	0.914	$p < 0.05$
25	18	0.969	$p < 0.05$
26	132	0.769	$p < 0.05$
27	37	0.935	$p < 0.05$
28	22	0.962	$p < 0.05$
29	3	0.995	$p < 0.05$
30	11	0.981	$p < 0.05$
31	112	0.804	$p < 0.05$
32	37	0.935	$p < 0.05$
33	56	0.902	$p < 0.05$
34	44	0.923	$p < 0.05$
35	54	0.906	$p < 0.05$

None of the subjects had a significant number of circular triads, so all of them were included in the data analysis.

For the paired comparison data, the frequency of responses was used as the input for the Unidimensional Analysis. Thurstone's Case V process was followed, and the result is shown in Figure 4-4, which shows where the 24 stimuli rank on a psychological continuum from least to most discomforting. Note that there appear to be four "groupings" of stimuli, which correspond to the four levels of the independent variable of position (10, 20, 30, and 40 degrees above the line of sight). The least discomforting is the 40 degrees above the line of sight grouping, and the most discomforting is the 10 degrees above the line of sight grouping, as one would expect from past research that shows as position increases, discomfort decreases. The different frequencies within those groupings are not as easy to interpret. But there are several interesting trends with respect to frequency. First, note that the Uniform stimulus is always the most discomforting in each position grouping. Note also that 8.0 cycles per degree (the highest sinusoidal frequency studied) is the second most discomforting stimulus in all of the position groupings except for the 40 degree grouping, where it falls to fourth most discomforting. Also, 4.0 cycles per degree is always the third most discomforting stimulus in all of the position groupings except for the 10 degree grouping.

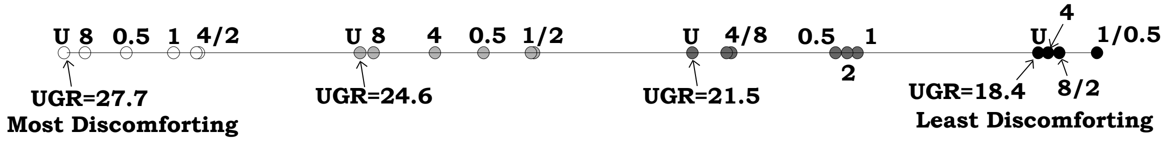


Figure 4-4. Results of Unidimensional Scaling for 24 stimuli, from most discomforting to least discomforting. Numbers indicate frequency of stimuli (in cycles per degree). Clear circles indicate stimuli at 10 degrees above the line of sight, light grey indicates 20 degrees, dark grey indicates 30 degrees, and black indicates 40 degrees.

It is interesting to take this analysis one step further. As these 24 stimuli are arranged on a one-dimensional scale based on distances between stimuli, and because UGR values can be calculated for the uniform stimuli, a UGR value can therefore be calculated for each of the stimuli. This calculation works well in this case because the distance between each pair of the four uniform stimuli is nearly consistent, which is to be expected because the difference in UGR values between them is consistent at 3.1. A ratio of UGR value to distance can be calculated for the uniform stimuli, which can then be applied to all of the other stimuli. And since a distance between each stimuli is now known because of the Unidimensional Analsys, using the ratio gives a UGR difference between stimuli. Therefore, a UGR value can be calculated for each stimulus (see Table 4-8 and Figure 4-5). According to the CIE (1995), one UGR point is the least detectable step. Table 4-8 shows that the different positions are certainly detectably different from each other, and even the frequencies within a position seem to be detectably different from each other.

Table 4-8. Calculated UGR Values for each Stimulus. Values for uniform stimuli are calculated from the original CIE UGR formula; values for non-uniform stimuli are calculated from the distances generated from the Unidimensional Analysis

Stimulus	UGR Value
1.0-40	17.8
0.5-40	17.8
8.0-40	18.2
2.0-40	18.2
4.0-40	18.3
U-40	18.4
1.0-30	20.1
2.0-30	20.2
0.5-30	20.3
4.0-30	21.3
8.0-30	21.4
U-30	21.5
1.0-20	23.0
2.0-20	23.0
0.5-20	23.5
4.0-20	24.0
8.0-20	24.6
U-20	24.6
4.0-10	26.1
2.0-10	26.2
1.0-10	26.4
0.5-10	26.8
8.0-10	27.2
U-10	27.7

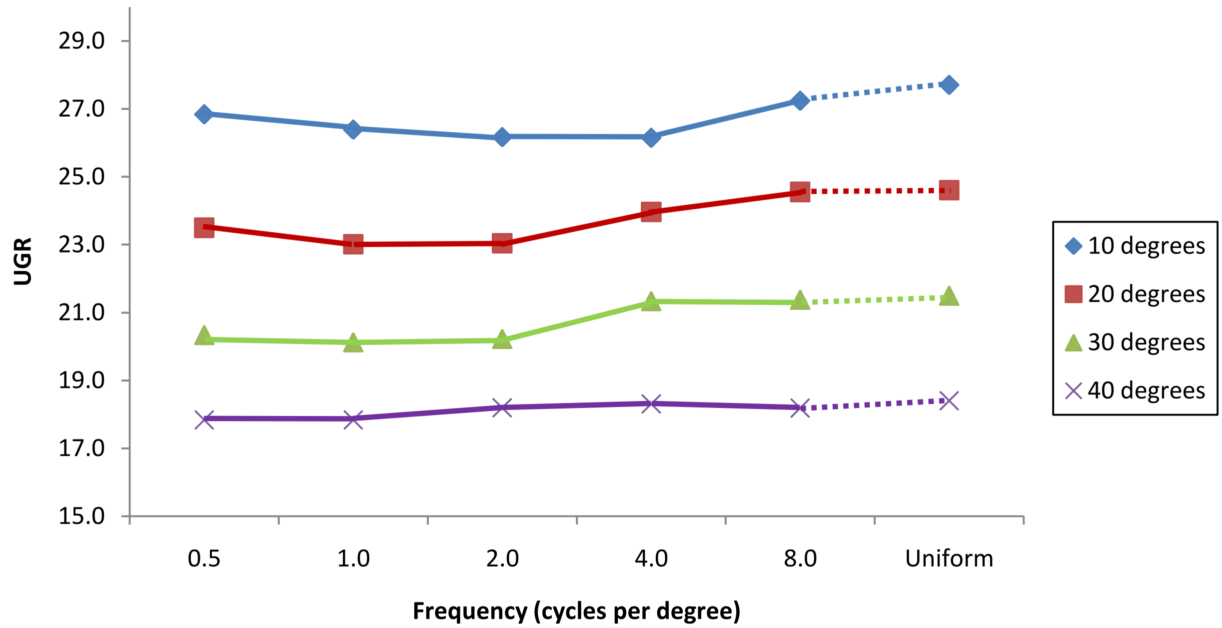


Figure 4-5. Graph from paired comparison "choice" data showing calculated UGR values vs. frequency of stimuli for different positions, based on Unidimensional Analysis

Repeated Measures ANOVA – "Choice" Data

The second statistical method used to analyze the paired comparison "choice" data was a repeated measures ANOVA. The experiment was a 6 x 4, repeated measures. The null hypothesis for the effect of position in the rating scale analysis is as follows:

$$\mu_{10\ degrees} = \mu_{20\ degrees} = \mu_{30\ degrees} = \mu_{40\ degrees} \tag{4.5}$$

Similarly, the null hypothesis for the effect of frequency is as follows:

$$\mu_{0.5cpd} = \mu_{1.0cpd} = \mu_{2.0cpd} = \mu_{4.0cpd} = \mu_{8.0cpd} = \mu_{Uniform} \tag{4.6}$$

This analysis differs from the Rating Scale ANOVA because with this experiment, the data was simply counts – for each subject, how many times was each

stimulus more discomforting than what it was compared against. Since each stimulus was compared with all other stimuli, the total count for a stimulus (for each subject) ranges from 1 (if the only time stimulus A was said to be more discomforting was when it was compared to itself) to 24 (if every time stimulus A was shown to the subject, it was said to be more discomforting, including when it was compared to itself). Those values for Subject #1 were determined by simply summing each row of Table 4-2. This process was repeated for the additional 34 subjects. The data was then entered into SAS for the analysis. The SAS commands and output are given in Appendix H – SAS Command File and Output File for Paired Comparison Analysis.

The means for each of the positions and for each frequency are given in Table 4-9.

Table 4-9. Mean values of each level of the two independent variables, position and spatial frequency

Independent Variable	Level	Mean (μ) Discomfort Count	Standard Deviation (σ) of Discomfort Count
Position	10 degrees	20.9381	0.720
	20 degrees	15.3238	0.422
	30 degrees	9.5095	0.781
	40 degrees	4.2286	0.700
Spatial Frequency	0.5 cpd	12.1286	1.162
	1.0 cpd	11.4357	1.413
	2.0 cpd	11.65	1.162
	4.0 cpd	12.5857	0.836
	8.0 cpd	13.4286	1.158
	Uniform	13.7714	1.604

There was a concern about whether or not a Repeated Measures ANOVA was the proper choice of analysis technique for this data, as it is “count” data, and therefore would most likely violate ANOVA’s two primary assumptions. The first assumption is that the population scores would be normally distributed about the mean. The second is that the population variances of each group would be equal. However, ANOVA is typically fairly robust to violations of both normality and homogeneous variance. According to Maxwell & Delaney (2004), “ANOVA is generally robust to violations of the normality assumption, in that even when data are nonnormal, the actual Type I error rate is usually close to the nominal value. Thus, many do not regard lack of normality as a serious impediment to the use of ANOVA.” According to Howell (1999), “if the populations can be assumed to be either symmetric or at least similar in shape (e.g., all negatively skewed) and if the largest variance is no more than four or five times the smallest, the analysis of variance is most likely to be valid.”

SAS was used to explore these issues of normality and homogeneity of variance. Figure 4-6 shows a histogram with a normal curve superimposed, showing that the data is very nearly normal.

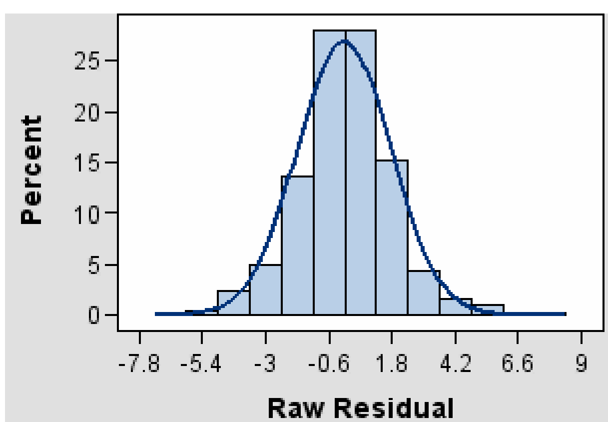


Figure 4-6. SAS graph showing the raw data with a normal curve superimposed

The issue of homogeneity of variance can be examined by looking at a scatter plot of the residuals, to see whether or not there appears to be a pattern (see Figure 4-7). If there truly is homogeneity of variance (or homoscedasticity), the plot should appear as a random scatter of points. If there is not homogeneity of variance (or heteroscedasticity), there would be some pattern to the scatter, a funnel shape for instance, where the spread of the errors increases with increasing predictors (Judd & McClelland 2001). In this case, there is no pattern to the data, indicating homogeneity of variance. In addition to plots, the variances can be examined numerically to determine if Howell's (1999) test of homogeneity is met. As can be seen from Table 4-10, the largest variance is 5.527, and the smallest is 1.734. The ratio of the former to the latter is approximately 3.2, which does meet his requirement.

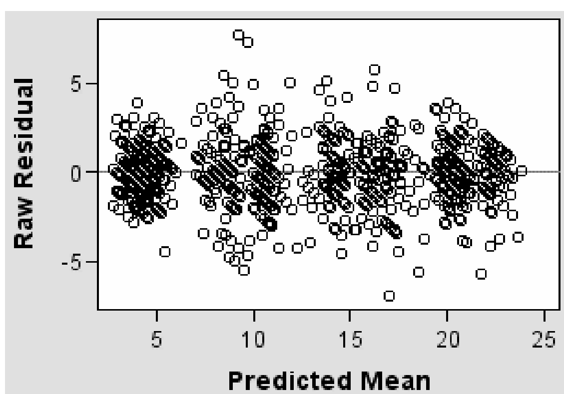


Figure 4-7. SAS scatterplot of the residuals vs the predicted means, showing no pattern to the data, indicating homogeneity of variance

Table 4-10. Mean values, standard deviations, and variances of each of the 24 stimuli

Stimulus	Mean (μ) Discomfort Count	Standard Deviation (σ) of Discomfort Count	Variance (σ^2) of Discomfort Count
0.5 cpd at 10deg	21.314	1.711	2.928
1.0 cpd at 10deg	20.286	1.708	2.917
2.0 cpd at 10deg	19.829	2.065	4.264
4.0 cpd at 10deg	19.714	1.888	3.565
8.0 cpd at 10deg	21.971	1.317	1.734
Uniform at 10deg	22.514	1.837	3.375
0.5 cpd at 20deg	14.800	1.471	2.164
1.0 cpd at 20deg	13.886	1.778	3.161
2.0 cpd at 20deg	13.886	2.180	4.752
4.0 cpd at 20deg	15.514	1.560	2.434
8.0 cpd at 20deg	16.800	2.112	4.461
Uniform at 20deg	17.057	2.287	5.230
0.5 cpd at 30deg	8.600	2.089	4.364
1.0 cpd at 30deg	8.057	2.351	5.527
2.0 cpd at 30deg	8.457	1.884	3.549
4.0 cpd at 30deg	10.514	1.869	3.493
8.0 cpd at 30deg	10.543	2.005	4.020
Uniform at 30deg	10.886	2.311	5.341
0.5 cpd at 40deg	3.800	1.659	2.752
1.0 cpd at 40deg	3.514	1.616	2.611
2.0 cpd at 40deg	4.429	1.399	1.957
4.0 cpd at 40deg	4.600	1.519	2.307
8.0 cpd at 40deg	4.400	1.376	1.893
Uniform at 40deg	4.629	2.045	4.182

In addition, Judd et al (in press), suggest that if the data violates the normality assumption, it should be transformed. The generally acceptable transformation for count data is the square root transformation. Other transformations include the log transformation and the power transformations (where the choice of the exponent is a trial and error process of reducing the nonnormality and increasing the homogeneity of

variance). Several transformations were applied to this data. The square root transformation was tested first. Unfortunately, the square root transformation actually made the homogeneity of variance worse rather than better. The largest variance changed to 0.271 and the smallest was 0.0207, with a ratio of the former to the latter of approximately 13. The log transformation also made the homogeneity of variance worse (ratio of maximum to minimum was 84.109), as did several different power transformations (an exponent of 2.0 yielded a ratio of 38.86, an exponent of 1.5 yielded a ratio of 9.607, and an exponent of -0.5 yielded a ratio of 506.31).

Because the non-transformed data appear to be normal and have a maximum to minimum ratio of less than 5, ANOVA was considered to be an appropriate analysis technique, even though it is "count" data.

Overall, the effect of position on discomfort glare is significant, meaning there are group differences in discomfort among 10, 20, 30, and 40 degrees above the line of sight ($F(3,102)=3050$, $p<0.05$). This is to be expected from previous glare studies, which have also shown that as position increases, discomfort decreases. There is a significant linear trend in the data ($F(1,34)=8921$, $p<0.05$), with higher positions showing less discomfort. This result confirms the graphical result shown in Figure 4-2. Neither the quadratic ($p=0.2561$) nor the cubic ($p=0.1587$) trends is significant. As stated above, this tends to support the VCP equation's exponent on position, rather than the UGR's exponent.

The effect of frequency on discomfort glare is significant, meaning there are group differences in discomfort among Uniform, 0.5, 1.0, 2.0, 4.0, and 8.0 cycles per degree ($F(5,170)=16.84$, $p<0.05$). This was hypothesized, but was not confirmed with

the Rating Scale experiment. It is encouraging that the paired comparison data does indeed show a significant impact of frequency on discomfort. There is a significant linear trend in the data ($F(1,34)=20.24$, $p<0.05$), with higher frequencies showing more discomfort. There is also a significant quadratic trend ($F(1,34)=17.30$, $p<0.05$), and a significant cubic trend ($F(1,34)=13.61$, $p<0.05$). Neither the quartic ($p=0.6941$) nor the quintic ($p=0.6075$) trends is significant. These results are not obvious from Figure 4-2, but are nonetheless interesting. The linear trend implies that as frequency increases, discomfort increases. The quadratic trend implies that discomfort is minimized for the middle frequencies and is higher for the higher and lower frequencies. The cubic trend implies that discomfort increases, then decreases, then increases again as frequency increases.

The interaction between position and frequency is also significant ($F(15,510)=7.64$, $p<0.05$), suggesting that the effect of frequency on discomfort is different at different positions. This interesting effect can be seen both from the graphical results in Figure 4-2 and from the results of the Unidimensional Scaling analysis in Figure 4-4. From both of these figures, it appears that the effect of frequency on discomfort is more pronounced at lower positions. In Figure 4-2, the discomfort from the stimuli at 40 degrees is fairly flat across frequency, while the discomfort from the stimuli at 10 degrees is more variable with frequency. In Figure 4-4, the frequencies are more spread out at lower positions, and more bunched together (meaning not as much difference in discomfort between the frequencies) at higher positions. This means that frequency differences affect the perception of glare much more at positions closer to the

line of sight. This is to be expected, as the fovea is where different patterns are more likely to be differentiated (than in the periphery).

As position was found to be a significant factor of discomfort, Scheffe post-hoc tests were run to determine where the differences are in the four levels. Table 4-11 shows that each of the levels of position was significantly different from every other level.

Table 4-11. Scheffe groupings for the different levels of position

		Stimulus Position (degrees above line of sight)			
		10	20	30	40
Stimulus	10				
Position	20	Different			
(degrees above	30	Different	Different		
line of sight)	40	Different	Different	Diferent	

As frequency was also found to be a significant factor of discomfort, Scheffe post-hoc tests were run to determine where the differences are in the six levels. Table 4-12 shows where the differences are. In every level, the frequency is the same as the next higher level of frequency (i.e. 0.5 is the same as 1.0). This analysis primarily shows that lower frequencies are different from higher frequencies.

Table 4-12. Scheffe groupings for the different levels of frequency

		Stimulus Frequency (cpd)					
		0.5	1.0	2.0	4.0	8.0	Uniform
Stimulus	0.5						
Frequency	1.0	Same					
(cpd)	2.0	Same	Same				
	4.0	Same	Different	Same			
	8.0	Different	Different	Different	Same		
	Uniform	Different	Different	Different	Different	Same	

There were two potential problems with the experimental design. First, it was discovered during the data analysis phase of the project that, while the intent of the design was that the experiment was balanced, it actually was not. As shown in Table 3-12, the investigator intended to balance which stimulus would be on the left, and which would be on the right for all pairs, for each subject. However, closer inspection of this table shows that for each stimulus, when it is paired with a different frequency at the same position, the lower frequency is always on the left. Because there was a significant effect of increasing discomfort as frequency increases, this suggests that a subject would be right biased (as the higher frequency is always on the right at equal positions, which should cause more discomfort). This issue of “presentation bias” was a major concern, and was therefore added into the ANOVA as a factor. If the “presentation bias” was truly affecting the subjects’ perceptions of discomfort, then the factor would be significant. To add it to the ANOVA, each subject was categorized with either a left or a right “presentation bias” based on which presentation matrix was used for that subject. The effect of presentation bias was not significant (p=0.1675). Ideally, this issue would have been discovered before the subjects were run, and it would have been corrected.

The other potential problem was the "subject's bias." For 24 of the 300 trials, the exact same stimuli were paired against one another. It was expected that the subjects would say that for 12 of those pairs, the left was more discomforting, and for the other 12, the right was more discomforting. This result was expected because if the two sides of the apparatus truly are identical, then the subject is randomly choosing the left or the right, which should be 50% left and 50% right. Only two of the 35 subjects were 12 and 12. The other 33 were either left or right biased in their perceptions, based on those 24 comparisons. This issue of "subject's bias" was also a major concern, as it cast doubt on whether the two sides of the apparatus truly were the same, and was discovered about halfway through running subjects when the investigator noticed that subjects seemed to be biased in their responses. In fact, two subjects seemed to be so biased that they were not allowed to complete the experiment, as they were consistently choosing the right when two identical stimuli were shown to them. For the remaining 35 subjects, each was categorized with either a left or a right "subject's bias", which was added to the ANOVA as a factor. Similarly to the "presentation bias", if this "subject's bias" was affecting the perception of discomfort, it would be a significant factor. The effect of "subject's bias" was not significant (p=0.0810). The two subjects who did not complete the experiment cast so much doubt on the equality of the two halves of the apparatus that measurements were immediately taken to ensure the luminances were equal (which they were). Of the 33 subjects who were not 50% left and 50% right, 17 were right biased and 16 were left biased. The fact that these values were approximately equal suggests that this is an effect of individual differences, and has nothing to do with the apparatus or with the experimental design.

These two potential problems, when discussed together, actually bring up a third potential problem. Is it possible that the "subject's bias" was due to the "presentation bias"? Even though neither issue was found to have a significant impact on discomfort, could there still be a serious experimental design problem? If one was due to the other, there should be a significant number of subjects who had both a left (or right) "presentation bias" and "subject's bias." Of the 35 subjects, 17 had both a left (or right) "presentation bias" and a left (or right) "subject's bias." As this value makes up for only approximately half of the subjects, it seems safe to say that this is not a serious problem with the experimental design.

Multidimensional Scaling – "Magnitude" Data

There were two methods used to analyze the paired comparison "magnitude" data. The first was multidimensional scaling (MDS). This method is used to determine a spatial representation of proximity data between stimuli. It uses proximity data among a group of objects and generates a configuration of those objects which optimizes the proximities (Kruskal & Wish 1984). The classic MDS example is distances between cities. Imagine a map of the United States. From the map, it would be a simple task to create a matrix of distances between several cities: New York; Seattle; Los Angeles; and, Houston, for example. One would simply measure the distance between the cities on the map, and scale the measurement based on the scale of the map. That would generate a distance between the cities (see Table 4-13).

Table 4-13. Distances between cities

		Stimulus			
		New York	Seattle	Los Angeles	Houston
Stimulus	New York		2408	2451	1420
	Seattle	2408		959	1891
	Los Angeles	2451	959		1374
	Houston	1420	1891	1374	

The reverse procedure is a little more difficult to do; that is, take the matrix of distances and create the map. With this simple example of only four cities, it would still be a fairly simple task, but it would become increasingly more difficult as more cities are added. It would also more difficult if the matrix of values had some error (imagine asking 5 different people to measure the distances between the four cities and then averaging their responses). Also, with just the matrix, it is not obvious if the solution is indeed a two-dimensional picture, or whether the data is better represented by a one, three, four, or more dimensional picture, which also makes the procedure more difficult (Kruskal & Wish 1984). This is where MDS is best utilized. The MDS procedure takes the proximity information gathered for *n* stimuli. The user decides how many dimensions, *t*, will be used to try to describe the data. The *n* stimuli are randomly placed in the *t* dimensions, and the MDS program moves the stimuli around in iterational steps, searching for a solution where the distances between the stimuli match the original proximity data. The output from an MDS procedure is typically a graph, showing where the stimuli ended up, and a measure of how well the final distances match the proximity data. That measure is called stress, or “badness of fit.” Typically, stress will decrease as more dimensions are added to the solution, i.e. a three-dimensional solution will typically have a lower stress value than a two-dimensional solution of the same data. The user

must balance the stress values with the number of dimensions to find the best solution (lowest stress) with a reasonable number of dimensions. When too many dimensions are added, the solution becomes much more difficult to interpret (Dunn-Rankin et al 2004). The difficult part of an MDS analysis is that it is not always obvious what the axes of the graph should be. With the cities example, it is obvious that the axes are north-south and east-west. But for more complicated analyses, it is left up to the user to “name” the axes.

The magnitude data is a sort of proximity data. If a subject said the left was more discomforting than the right, at a level of 4 (out of 5), those two stimuli must be very dissimilar. However, if the subject said the level was only 1, then the two stimuli must be very similar. MDS is the ideal choice to analyze this magnitude data. MDS was also considered as a method to analyze the “choice” data, but it is not proximity data. To attempt to obtain some sort of proximity information, the data could be added (or averaged) across subject. However, consider what would happen if the data were added. If every subject said that stimulus x was more discomforting than stimulus y, then the total for that pair would be the total number of subjects, in this case 35. That would suggest that the two stimuli are very different. However, consider if none of the subjects said that stimulus x was more discomforting than stimulus y. Then the total for that pair would be zero. This too would suggest that the two stimuli are very different. Similarly, if half of the subjects said that stimulus x was more discomforting than stimulus y, then the total for that pair would be half the number of subjects, or 17.5. This would suggest that the two stimuli are very similar (as half of the subjects said x was more discomforting than y and the other half said the reverse). But that means that in the MDS analysis, a value of 35 and a value of 0 would mean the same thing, that the two stimuli

are very different, and a value of 17.5 would mean the two stimuli are similar, which is not true proximity data. Therefore, MDS was not used to analyze the “choice” data.

The typical MDS analysis assumes that there is a single matrix of similarity or dissimilarity information (as in the distances between cities example). However, in this experiment, there were 35 different matrices (one for each subject). The different matrices could have been added or averaged together to obtain one matrix, but in doing that, information is lost regarding the differences between subjects. A standard MDS can analyze multiple matrices, but it assumes that the reason the matrices are different is due to random error (Kruskal & Wish 1984). For this reason, a modification to the standard MDS, called Individual Differences Scaling (IDS), was used to analyze the magnitude data. Its premise is that each subject is using the same dimensions to make his determinations, but he may not use the dimensions in the same degree as every other subject. In other words, each subject may weight the dimensions differently. The output from the IDS differs from a standard MDS in that it not only gives a single underlying configuration (called a group stimulus space), showing where on the map the stimuli are located (as an MDS does), but it also gives subjects’ weightings, showing how much weight each subject gave to each dimension. It is a compromise between one map that accurately represents similarity for all subjects and a separate map for each subject (Lattin et al 2003). Another difference between IDS and MDS is that IDS does not use stress as a measure of fit. Rather, it uses percentage of variance accounted for (Kruskal & Wish 1984).

The computer program used for performing the IDS was Dunn-Rankin’s (2004) SINDSCAL. It requires that a half matrix is generated for each subject. Therefore, the

data in Table 4-3 were rearranged to match the requirements of the program (see Table 4-14). Thirty-five separate half-matrices were input into the SINDSCAL program, which was run for one, two, three, four, and five dimensions. As shown in Figure 4-8, both the first and second dimensions account for a significant amount of variance, so it seems appropriate to use a two-dimensional solution. However, for the two-dimensional solution, the total percentage of variance accounted for is still only 43% (26% from the first dimension plus 17% from the second dimension), which is fairly low. This means that there is still 57% of the variance in the data that is not accounted for in the solution. However, additional dimensions do not account for much more variance. Therefore, the two dimensional solution was chosen as the best compromise between significant variance accounted for and an interpretable number of dimensions.

Table 4-14. Results of paired comparison experiment for Subject #1 showing the magnitude difference between stimuli, rearranged into a complete half-matrix

		Stimulus																							
		0.5-10	0.5-20	0.5-30	0.5-40	1.0-10	1.0-20	1.0-30	1.0-40	2.0-10	2.0-20	2.0-30	2.0-40	4.0-10	4.0-20	4.0-30	4.0-40	8.0-10	8.0-20	8.0-30	8.0-40	U-10	U-20	U-30	U-40
Stimulus	0.5-10																								
	0.5-20	2																							
	0.5-30	4	2																						
	0.5-40	4	4	1																					
	1.0-10	3	4	3	5																				
	1.0-20	3	1	3	3	3																			
	1.0-30	3	2	2	2	4	1																		
	1.0-40	4	4	2	2	4	3	2																	
	2.0-10	2	3	4	5	2	4	3	5																
	2.0-20	2	2	1	3	3	2	2	3	2															
	2.0-30	3	3	1	2	4	4	2	2	4	1														
	2.0-40	3	3	2	1	5	3	2	1	4	4	1													
	4.0-10	3	4	4	5	3	3	3	5	2	3	3	4												
	4.0-20	2	2	2	3	2	3	3	3	2	2	2	3	3											
	4.0-30	4	3	1	2	4	2	2	3	4	2	1	2	3	1										
	4.0-40	4	3	3	1	4	3	1	1	5	2	2	2	4	5	2									
	8.0-10	3	3	3	5	2	3	4	5	2	3	5	4	3	2	4	5								
	8.0-20	3	2	3	3	3	2	3	3	2	2	2	3	3	2	2	3	2							
	8.0-30	4	2	2	1	3	3	1	2	4	3	2	3	4	2	1	2	3	3						
	8.0-40	4	3	2	2	2	3	1	1	4	2	1	2	4	3	2	1	5	4	2					
	U-10	2	3	4	5	2	4	5	4	2	4	4	5	3	3	2	4	2	3	4	5				
	U-20	2	2	3	4	4	2	3	3	2	1	2	4	3	2	3	3	3	2	3	3	1			
	U-30	4	2	1	2	4	1	2	3	4	2	1	2	4	2	1	3	4	2	2	2	5	1		
	U-40	4	2	2	1	4	3	2	1	4	4	1	1	4	2	2	2	5	4	2	1	4	3	2	

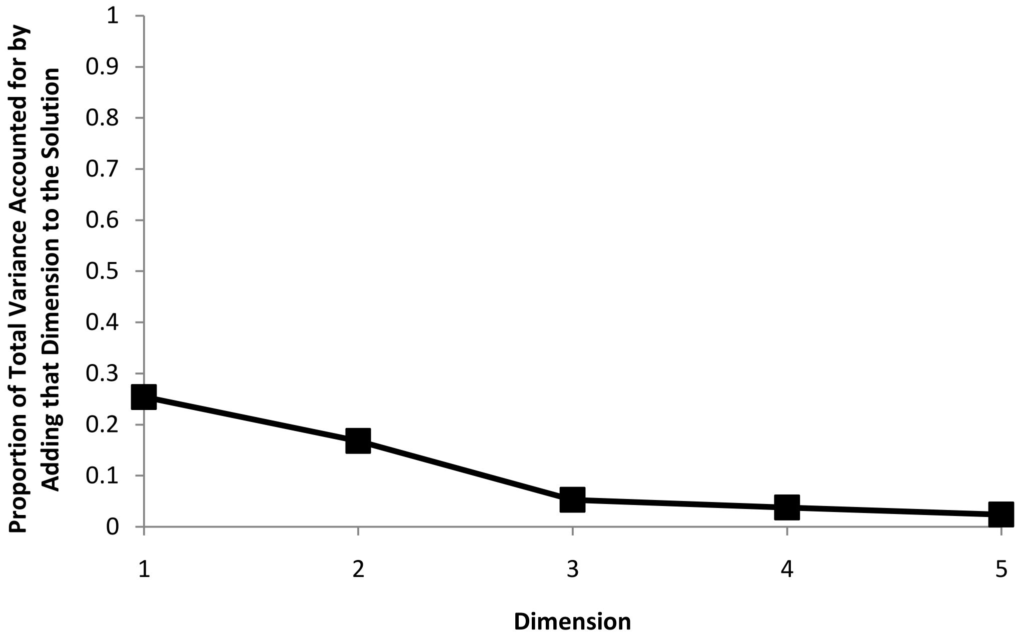


Figure 4-8. Graph of variance accounted for vs. dimension for the Individual Differences MDS

As mentioned above, an IDS analysis results in two separate graphs. The first is the group stimulus space (see Figure 4-9) and the second is the subjects' weights space (see Figure 4-10). The group stimulus space is the map produced from the data. It was difficult to name the axes of the IDS based on Figure 4-9, so another tool was used to evaluate the solution. The different frequency targets were printed and arranged on a large piece of plastic in the same manner they are arranged in Figure 4-9 (see Figure 4-11). With the actual targets, it was much easier to see trends in the data.

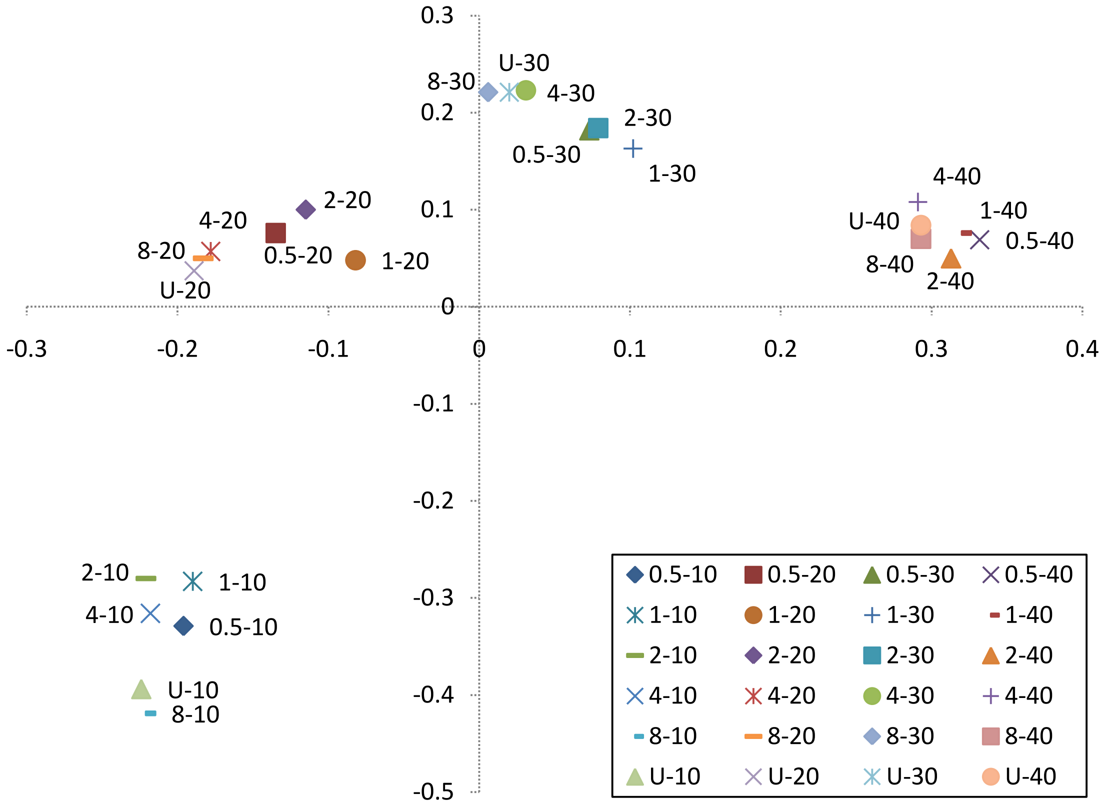


Figure 4-9. Two dimensional Group Stimulus Space generated from IDS for magnitude data from paired comparison experiment

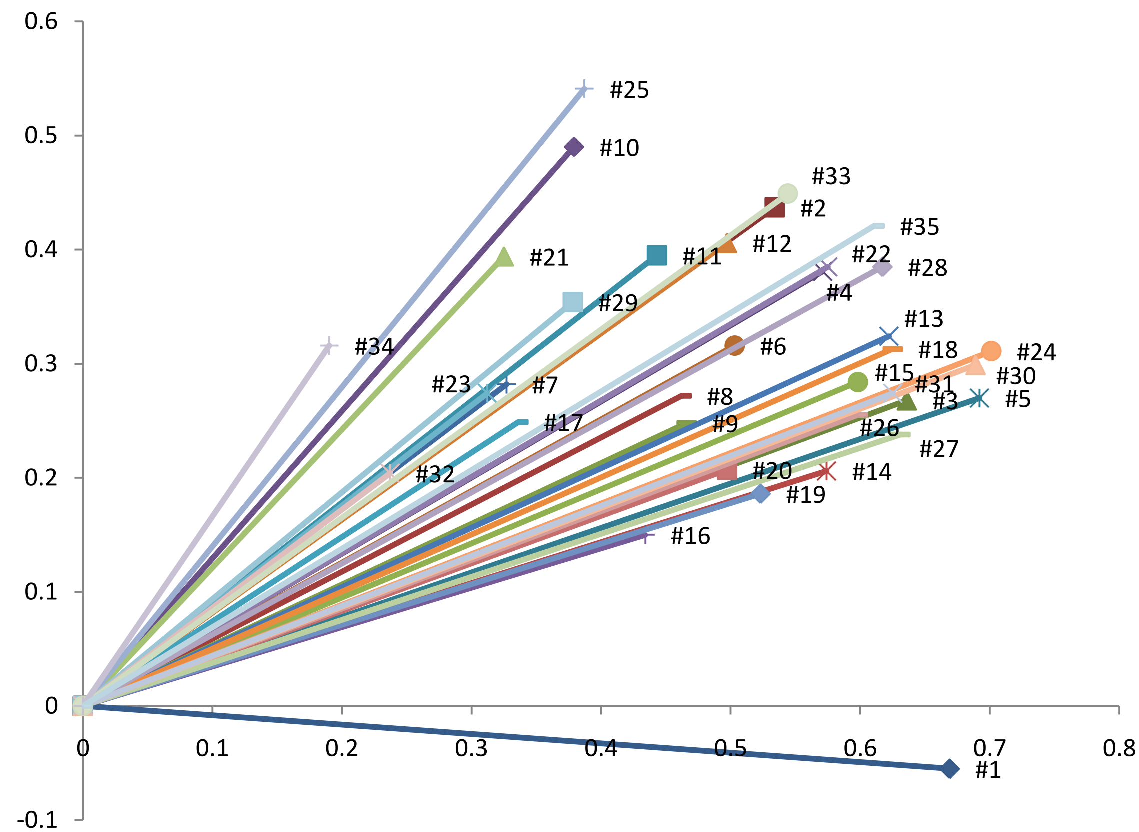


Figure 4-10. Two dimensional Subjects' Weights Space generated from IDS for magnitude data from paired comparison experiment

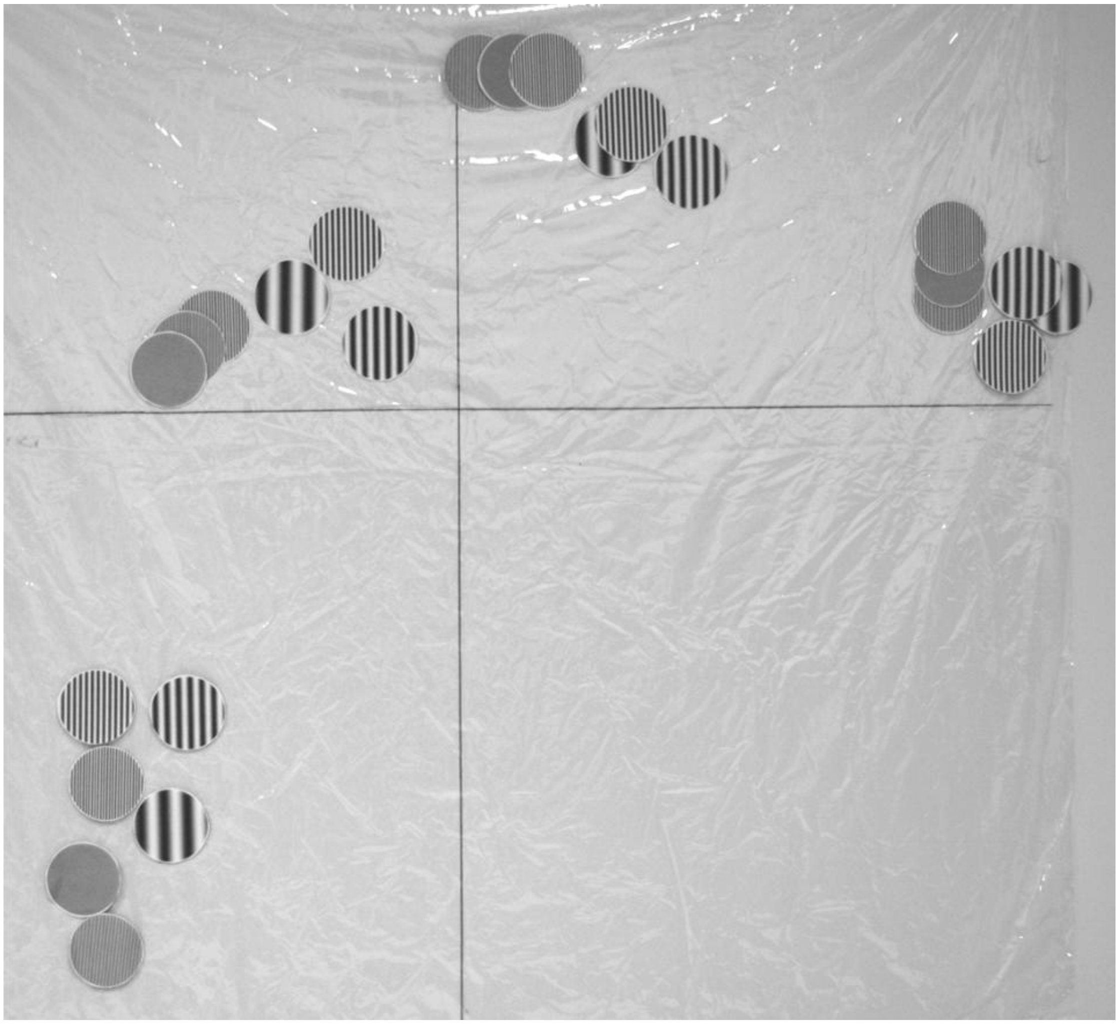


Figure 4-11. MDS solution shown with printed targets and mounted on plastic.

Note the groupings that form in Figure 4-9 and Figure 4-11. These appear to be groupings based on position, where all of the 10 degrees above the line of sight stimuli are grouped together (as well as the other positions). It was originally thought that this would imply that the horizontal axis would be named “position”, with values closer to the line of sight toward the left end. However, if that were true, then each of the different frequencies at any one position would be aligned vertically, which is not the case here. After further thought, it appears that the horizontal axis should actually be “discomfort”, with more discomfort toward the left end. This would explain the position groupings, but

also accounts for the fact that within a grouping, the frequencies are not aligned. In fact, note that the uniform, 8.0, and 4.0 cycles per degree stimuli are toward the left within each position grouping. This supports the decision that the horizontal axis would be named “discomfort”, as it has been shown above that the uniform, 8.0 and 4.0 cycles per degree stimuli typically cause the most discomfort.

The vertical axis is much more difficult to interpret, however. It would have been ideal if it had been frequency. Unfortunately, it clearly is not frequency. The most interesting effect in the vertical direction is that the 10 degree grouping appears to be much more spread out than the other groupings. This suggests that maybe the vertical axis should be named “discriminability”, with easy to discriminate toward the bottom, and difficult to discriminate toward the top. That would suggest that it is much easier for a subject to discriminate between frequencies at 10 degrees above the line of sight than at 20, 30, and 40 degrees.

The weights space (see Figure 4-10) shows to what extent each subject uses the two dimensions to make his determinations. For instance, subject #1 used primarily the x dimension criteria to make his judgments, whereas subject #34 used more of the y dimension criteria to make his judgments, and subject #7 used both approximately equally.

All of the coordinates (both x and y) in the weights space should be positive. The fact that the y coordinate value for subject #1 is negative is most likely a statistical fluctuation rather than an error with the model because the negative value is near zero (Kruskal & Wish 1984). The length of each line in the subjects’ weight space from the

origin to the plotted point indicates the amount of variance accounted for by the group stimulus space (Dunn-Rankin et al 2004) for each subject. For example, subject #1 agrees with the group stimulus space much more than does subject #32. All of the subjects' correlations to the group stimulus space are given in Table 4-15, which shows a range from a low of 0.360 for Subject #32 to a high of 0.856 for Subject #24. In some cases, there can be an obvious explanation for some subjects having a higher correlation or being more prone to use one dimension more than the other. Lattin et al (2003) give an example of an IDS where the subjects were asked to give their impressions on different breakfast foods. The authors label the two dimensions of the group stimulus space "sweetness" and "preparation method – prepared at home vs. purchased." Two of the four subjects seem to use both dimensions equally, while the other two seem to almost solely rely on the "sweetness" dimension. In their example, the two subjects who rely on the "sweetness" dimension are both men, while the two who use both equally are women. Their assumption is that men don't see as much of a difference in preparation method because maybe they are not the ones who are preparing the food. Unfortunately, in the case of Figure 4-10, there are no such clear explanations with the information gathered from the subjects. Gender is evenly distributed through the graph, as is the presence or absence of contact lenses, and age. It is assumed that the differences are simply based on differences among individuals.

Table 4-15. Correlations of subjects' responses with the Group Stimulus Space for two dimensional solution

Subject #	Correlation
1	0.653
2	0.794
3	0.768
4	0.784
5	0.822
6	0.677
7	0.497
8	0.609
9	0.596
10	0.712
11	0.684
12	0.738
13	0.791
14	0.671
15	0.742
16	0.504
17	0.480
18	0.786
19	0.610
20	0.598
21	0.588
22	0.790
23	0.479
24	0.856
25	0.762
26	0.723
27	0.744
28	0.829
29	0.598
30	0.836
31	0.761
32	0.360
33	0.812
34	0.419
35	0.849

Even though Figure 4-8 clearly shows that the two dimensional solution is the most appropriate, it may be interesting to look at the one dimensional solution as well (see Figure 4-12). With the one dimensional solution, it appears that the dimension should be named "discomfort", with more discomforting stimuli toward the left end of

the scale. There appear to be four distinct groupings of stimuli, which correspond to the different positions, where lower positions (closer to the line of sight) are more discomforting. Higher frequencies are, within each position grouping, more discomforting, with the 8 cycles per degree and the Uniform stimuli causing the most discomfort in all cases. This should look very similar to the results from the unidimensional scaling solution shown in Figure 4-4, which it does.

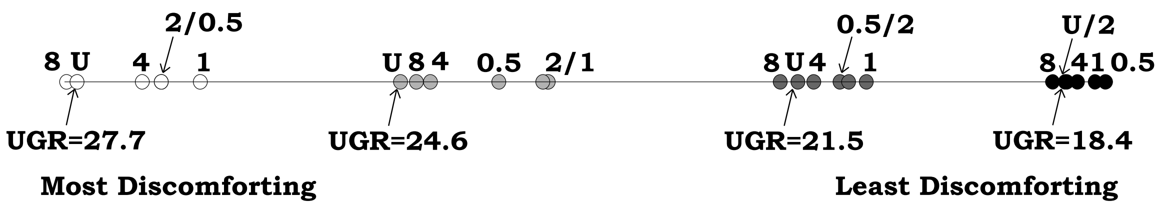


Figure 4-12. One dimensional Group Stimulus Space generated from IDS for magnitude data from paired comparison experiment. Numbers indicate frequency of stimuli (in cycles per degree). Clear circles indicate stimuli at 10 degrees above the line of sight, light grey indicates 20 degrees, dark grey indicates 30 degrees, and black indicates 40 degrees.

Analysis of Variance – "Magnitude" Data

While the MDS gave interesting results, there was no indication of significance with that method. Therefore, the second method used to analyze the paired comparison "magnitude" data was an analysis of variance. The actual ANOVA is fairly straightforward, but manipulating the data to get it into a form to perform the ANOVA is not as straightforward because the data are actually differences between two stimuli (i.e. a value of 5 in Table 4-3 really means that the row stimulus is 5 "discomfort units" higher than the column stimulus). The procedure was developed by University of Nebraska-Lincoln Statistics faculty members Dr. Kent Eskridge and Dr. Daryl Travnicek (2008). This method is based on the fundamental equation for ANOVA, which is shown here.

$$y_{ij} = \mu + S_j + P + \tau_i + \varepsilon \tag{4.7}$$

Where: y_{ij} = True effect of a particular stimulus for a particular subject

μ = Grand mean of all stimuli for all subjects

S_j = Effect of a particular subject

P = Effect of time (subjects get tired, etc.)

τ_i = Effect of a particular stimulus

ε = Error

The intent here is to get an estimate of y_{ij}. However, in this data set, y_{ij}'s are not available. Instead, the data collected is really a set of $(y_{1j} – y_{2j})$'s for each subject. So the challenge is to get estimates for the y_{ij}'s from the data set collected. This is done by matrix manipulation. Starting with the data in Table 4-3, a complete upper-half matrix was generated for each subject. Values above the diagonal were left untouched. Values below the diagonal were mirrored to above the diagonal, and a negative sign was added. Then each value in the complete upper-half matrix tells how much more discomforting the row stimulus was than the column stimulus. A negative value simply means the column was more discomforting (see Table 4-16).

Table 4-16. Paired Comparison magnitude data for Subject #1, with all the values under the diagonal mirrored to above the diagonal with a negative sign. The number in the cell indicates how much more discomforting the row stimulus was than the column stimulus. A negative value means the column was considered more discomforting.

		Stimulus																							
		0.5-10	0.5-20	0.5-30	0.5-40	1.0-10	1.0-20	1.0-30	1.0-40	2.0-10	2.0-20	2.0-30	2.0-40	4.0-10	4.0-20	4.0-30	4.0-40	8.0-10	8.0-20	8.0-30	8.0-40	U-10	U-20	U-30	U-40
Stimulus	0.5-10		2	4	4	-3	3	3	4	2	2	3	3	3	2	4	4	-3	3	4	4	-2	2	4	4
	0.5-20			2	4	-4	-1	2	4	-3	2	3	3	-4	2	3	3	-3	2	-2	3	-3	-2	2	2
	0.5-30				1	-3	-3	-2	2	-4	-1	-1	2	-4	-2	-1	3	-3	-3	2	2	-4	-3	-1	2
	0.5-40					-5	-3	-2	-2	-5	-3	2	-1	-5	-3	-2	-1	-5	-3	-1	2	-5	-4	-2	1
	1.0-10						3	4	4	2	3	4	5	3	2	4	4	2	3	3	2	2	4	4	4
	1.0-20							1	3	-4	-2	4	3	-3	-3	2	3	-3	-2	3	3	-4	-2	-1	3
	1.0-30								2	-3	-2	-2	2	-3	-3	-2	1	-4	-3	1	-1	-5	-3	-2	-2
	1.0-40									-5	-3	-2	1	-5	-3	-3	1	-5	-3	-2	-1	-4	-3	-3	-1
	2.0-10										2	4	4	2	2	4	5	-2	2	4	4	-2	2	4	4
	2.0-20											-1	4	-3	-2	2	2	-3	-2	3	2	-4	-1	2	4
	2.0-30												1	-3	-2	-1	2	-5	-2	2	-1	-4	-2	-1	1
	2.0-40													-4	-3	-2	2	-4	-3	-3	-2	-5	-4	-2	-1
	4.0-10														3	3	4	-3	3	4	4	3	3	4	4
	4.0-20															-1	5	-2	2	2	3	-3	-2	2	2
	4.0-30																2	-4	-2	-1	2	-2	-3	1	2
	4.0-40																	-5	-3	-2	1	-4	-3	-3	2
	8.0-10																		2	3	5	-2	3	4	5
	8.0-20																			3	4	-3	-2	-2	4
	8.0-30																				2	-4	-3	-2	2
	8.0-40																					-5	-3	-2	-1
	U-10																						1	5	4
	U-20																							-1	3
	U-30																								2
	U-40																								

Once the complete half matrix of $(y_{1j} - y_{2j})$'s was generated for each subject, it was reorganized into a 1 by 276 matrix (the values are organized vertically rather than horizontally into one string). That matrix is then equated to the product of two matrices: the first is a 24 by 276 matrix of 1's, -1's, and 0's; and, the second is a 1 by 24 matrix of y_{ij}'s.

$$\begin{bmatrix} y_1 - y_2 \\ y_1 - y_3 \\ y_1 - y_4 \\ y_1 - y_5 \\ \ldots \\ y_1 - y_{24} \\ y_2 - y_3 \\ y_2 - y_4 \\ \ldots \\ y_2 - y_{24} \\ y_3 - y_4 \\ \ldots \\ y_3 - y_{24} \\ \ldots \\ y_{23} - y_{24} \end{bmatrix} = \begin{bmatrix} 1 - 1\,0\,0\,0\,0\,0\,0\,0\,0 & \ldots & 0\,0 \\ 1\,0 - 1\,0\,0\,0\,0\,0\,0\,0 & \ldots & 0\,0 \\ 1\,0\,0 - 1\,0\,0\,0\,0\,0\,0 & \ldots & 0\,0 \\ 1\,0\,0\,0 - 1\,0\,0\,0\,0\,0 & \ldots & 0\,0 \\ & \ldots & \\ 1\,0\,0\,0\,0\,0\,0\,0\,0\,0 & \ldots & 0\ -1 \\ 0\,1 - 1\,0\,0\,0\,0\,0\,0\,0 & \ldots & 0\,0 \\ 0\,1\,0 - 1\,0\,0\,0\,0\,0\,0 & \ldots & 0\,0 \\ & \ldots & \\ 0\,1\,0\,0\,0\,0\,0\,0\,0\,0 & \ldots & 0\ -1 \\ 0\,0\,1 - 1\,0\,0\,0\,0\,0\,0 & \ldots & 0\,0 \\ & \ldots & \\ 0\,0\,1\,0\,0\,0\,0\,0\,0\,0 & \ldots & 0\ -1 \\ & \ldots & \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0 & \ldots & 1\ -1 \end{bmatrix} \text{ X } \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ \ldots \\ y_{10} \\ y_{11} \\ y_{12} \\ \ldots \\ y_{16} \\ y_{17} \\ \ldots \\ y_{20} \\ \ldots \\ y_{24} \end{bmatrix}$$

Through matrix multiplication, y_1-y_2 = $1*y_1 + (-1)*y_2 + 0*y_3 + 0*y_4 + \ldots + 0*y_{24}$, and so forth for each of the pairs of y's. The values of the $(y_{1j} - y_{2j})$'s are known, the value of the 0, 1 and -1 matrix is known, so the value of the y_{ij}'s can be calculated, which is

effectively an estimate of how discomforting that stimulus is. Those data are averaged across subjects and plotted in Figure 4-13. An Analysis of Variance was also performed on those data.

Overall, the effect of position on discomfort glare is significant, meaning there are group differences in discomfort among 10, 20, 30, and 40 degrees above the line of sight ($F(3,102)=2887$, $p<0.05$). This is to be expected from the UGR equation, which shows that as position increases, discomfort decreases.

Similarly, the effect of frequency on discomfort glare is significant, meaning there are group differences in discomfort among Uniform, 0.5, 1.0, 2.0, 4.0, and 8.0 cycles per degree ($F(5,170)=21.5$, $p<0.05$). In addition, the interaction between position and frequency is also significant ($F(15,510)=2.51$, $p<0.05$). These results are similar to those found from the analysis of the paired comparison “choice” data.

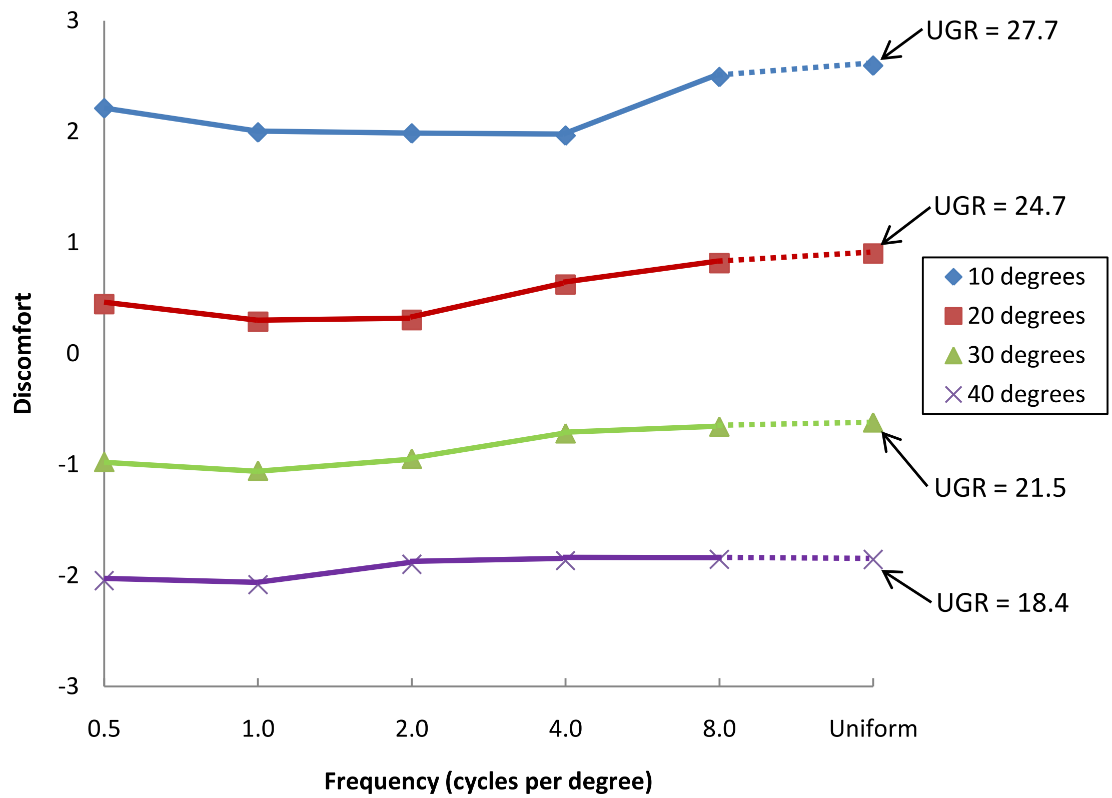


Figure 4-13. Graph of discomfort, averaged across subjects, versus frequency

As position was found to be a significant factor of discomfort, Scheffe post-hoc tests were run to determine where the differences are in the four levels. Table 4-17 shows that each of the levels of position was significantly different from every other level.

Table 4-17. Scheffe groupings for the different levels of position

		Stimulus Position (degrees above line of sight)			
		10	20	30	40
Stimulus	10				
Position	20	Different			
(degrees above	30	Different	Different		
line of sight)	40	Different	Different	Different	

As frequency was also found to be a significant factor of discomfort, Scheffe post-hoc tests were run to determine where the differences are in the six levels. Table 4-18 shows where the differences are. In every level, the frequency is the same as the next higher level of frequency (i.e. 0.5 is the same at 1.0 cycles per degree) except that 4.0 and 8.0 cycles per degree are different. This analysis primarily shows that lower frequencies are different from higher frequencies.

Table 4-18. Scheffe groupings for the different levels of frequency

		Stimulus Frequency (cpd)					
		0.5	1.0	2.0	4.0	8.0	Uniform
Stimulus	0.5						
Frequency	1.0	Same					
(cpd)	2.0	Same	Same				
	4.0	Same	Different	Same			
	8.0	Different	Different	Different	Different		
	Uniform	Different	Different	Different	Different	Same	

This analysis yielded almost identical results to those obtained from the paired comparison “choice” data analysis above.

Chapter 5 – Conclusions

Both the rating scale and the paired comparison experiment confirmed what glare research has been showing for years – that position has a significant effect on discomfort (as position increases, discomfort decreases). The fact that these experiments confirmed that suggests that the experimental procedures utilized were valid. More importantly, though, the paired comparison experiment showed that spatial frequency also has a significant effect on discomfort. Previous research on this topic showed that frequency was not a significant predictor of discomfort (Waters et al 1995). Specifically, this research shows that as frequency increases, discomfort increases, and that a non-uniform stimulus is considered less discomforting than a uniform one. The fact that Waters et al (1995) did not find a significant impact of frequency is most likely due to the fact that only two levels of frequency were studied, but in this research, there were five levels of frequency plus the uniform stimulus (for a total of six levels). In addition, this research shows that the interaction of position with spatial frequency is also significant. This corroborates research done by Waters et al (1995) which also showed a significant interaction. This suggests that the effect of spatial frequency on discomfort changes based on where the stimulus is in the field of view. The effect of spatial frequency is more pronounced at lower positions (closer to the line of sight).

UGR Complex Extension

The original idea for this dissertation came from the CIE's extension for complex sources to the UGR (CIE 2002). The actual extension was not explored, but rather the

issue behind the extension – how does a luminance gradient affect discomfort? It appears from this research that sinusoidal frequencies in the range of those studied (0.5 to 8 cycles per degree) cause less discomfort than a uniform stimulus with the same average luminance. This would suggest that a practitioner could use the original UGR formula and know that he is conservative in his calculations. It would be ideal if this research could be used to verify or disprove the UGR extension for complex sources (CIE 2002) as this extension did not appear to have any research behind it. However, the equation given for complex sources requires the maximum intensity from the luminaire and the angle at which that intensity projects from the luminaire, which is not applicable for the stimuli used in this experiment. So a direct check cannot be done, but a modified check can be.

The CIE explains that the calculation of UGR for a complex source should be different from the uniform source in its $L^2\omega$ term, because for a complex source, they define luminance as

$$L = \frac{I_{max}}{A_o \cos \gamma_{max}} \tag{5.1}$$

where I_{max} and γ_{max} are the maximum intensity and angle of full flash (presumably from a specular parabolic louver). Substituting that definition of L (from (5.1)) into the definition of $L^2\omega$, where

$$L^2\omega = \frac{L\,I}{r^2} \tag{5.2}$$

yields the following as the $L^2\omega$ term for complex sources:

$$L^2\omega = \frac{I\, I_{max}}{r^2\, A_o\, cos\gamma_{max}} \tag{5.3}$$

Because I_{max} and γ_{max} really are not applicable in the current research, it seems appropriate to go back to what appears to be their intent, which is that for complex sources

$$L^2\omega = L_{average}\, L_{maximum}\omega \tag{5.4}$$

where $L_{average}$ is the average luminance across the luminaire, but $L_{maximum}$ would be the maximum luminance. Using that logic, $L^2\omega$ can easily be calculated for the different spatial frequency stimuli used in this experiment.

To determine if the CIE suggestion for complex sources agrees with the findings outlined in this paper, the uniform stimulus must be calculated first. Using the "standard" form of $L^2\omega$, where

$$L^2\omega = L_{average}\, L_{average}\omega \tag{5.5}$$

the uniform stimulus is found to have an $L^2\omega$ value of 1630781 $sr(cd/m^2)^2$. Using this to calculate UGR at 10 degrees above the line of sight, we find a value of 27.32. Using equation (5.4), the 2.0 cycles per degree stimulus is found to have an $L^2\omega$ value of 3467125 $sr(cd/m^2)^2$. Using this to calculate UGR, we find a value of 29.94. This confirms the CIE's statement that the complex source $L^2\omega$ will always predict more glare than the uniform source (and will therefore have a higher UGR value, meaning more discomfort). However, this is not what the current research shows, which shows that the complex sources were always considered less discomforting (lower UGR value) than the uniform sources. Therefore, this research does not support the CIE's recommendations

for complex sources. However, this research did not utilize real luminaires, and it may be that when real luminaires are used, the CIE's recommendation is correct. Future research should be done to see if real non-uniform luminaires cause more discomfort than uniform luminaires.

Relations to Contrast Sensitivity Research

One interesting comparison to make is how this research compares to the research that has been done involving contrast sensitivity and its relationship to spatial frequency. From Figure 3-1, it appears that humans are less sensitive to contrast at points farther from the fovea. So, as eccentricity increases (farther into the periphery), contrast sensitivity decreases. This is to be expected, in that the density of cones decreases from the fovea out to the periphery. A similar finding is true with discomfort: as eccentricity increases, discomfort decreases. This suggests that where we are most sensitive to contrast (in the fovea), we also encounter the most discomfort.

The relation to spatial frequency is a little more complicated in that contrast sensitivity peaks at a particular frequency, and it drops on either side of that peak frequency (at any given eccentricity). And that peak frequency decreases as eccentricity increases (from Figure 3-1). In this research, it has been shown that discomfort is minimized at a particular frequency, and it increases on either side of that minimum frequency (at any given position), which is contrary to the effect of contrast sensitivity. However, similar to the effect of contrast sensitivity, the minimum frequency decreases as eccentricity increases (from Figure 4-2). The interesting part of this is that the

frequency where discomfort is minimized is the same frequency at which contrast sensitivity is maximized, at a particular position (eccentricity). This suggests that where we are most sensitive to contrast, we have the least amount of discomfort (at any given position). Specifically, note that the highest contrast sensitivity at a 23 degree eccentricity is found for a spatial frequency of approximately 2 cycles per degree (from Figure 3-1), and in this research, it was found that the least amount of discomfort at a 20 degree eccentricity is for a spatial frequency of 2 cycles per degree (from Figure 4-2). Similarly, the highest contrast sensitivity at an 8 degree eccentricity is found for a spatial frequency of approximately 4 cycles per degree (from Figure 3-1), and in this research, it was found that the least amount of discomfort at a 10 degree eccentricity is for a spatial frequency of 4 cycles per degree (from Figure 4-2).

This finding is surprising. It was expected that the spatial frequencies that were most detectable (highest contrast sensitivity) would cause the most discomfort; however, that is not the case. It is possible that the fact that the frequency is discernible is what makes it less discomforting than one that is not discernible. This question of why this happens may be answered by looking to the structure of receptive fields in the retina.

Receptive Field Size

It seems that the fact that the response to spatial frequencies differs at different positions must have something to do with the different receptive field sizes encountered in the retina. It is commonly known that receptive field size increases with eccentricity (Sekuler & Blake 1990). Can the receptive field size somehow explain the findings

reported here? It has been shown that peak spatial resolution occurs in the fovea, which corresponds to the distance between foveal cones. Each of these cones has a direct connection to ganglion cells. In the periphery, several cones converge onto one ganglion cell, and it is believed that the density of ganglion cells in this area is what limits resolution (Popovic & Sjostrand 2005). More specifically, it is now believed that it is the midget class of ganglion cells that limits spatial resolution (Popovic & Sjostrand 2005, Thibos et al 1987, Anderson et al 2002, Dacey 1993). Dacey (1993) measured how the field diameter of these midget ganglion cells increases with eccentricity in humans (see Figure 5-1). The vertical axes of the graphs are field diameter, which was measured by Dacey as follows: "A measure of dendritic field diameter was acquired for the intracellularly filled ganglion cells by tracing a convex polygon around the perimeter of the traced dendritic tree. The area of this polygon was then calculated by entering the outline into a computer via a graphics tablet. Dendritic field diameter was expressed as the diameter of a circle with the same area as that of the polygon" (Dacey 1993).

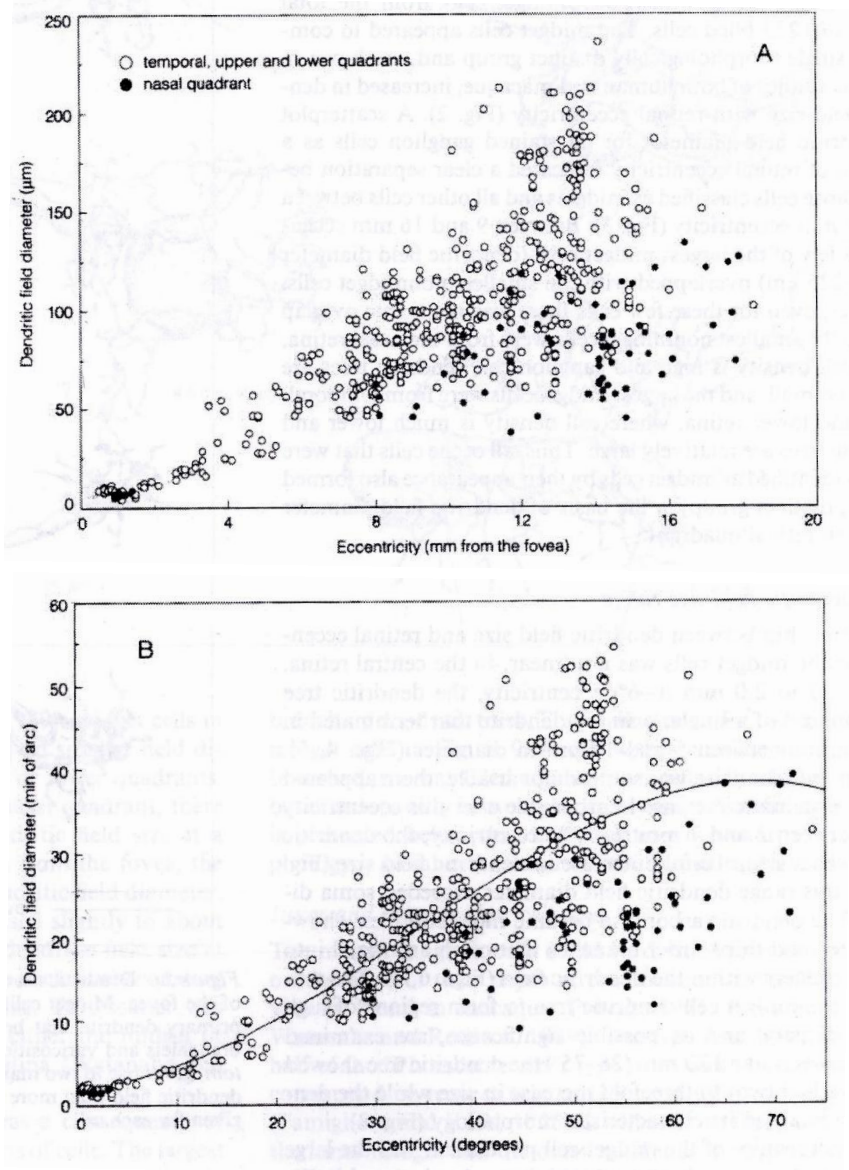


Figure 5-1. Graphs of field size for human midget ganglion cells (plotted in both µm and minutes of arc) vs. eccentricity (plotted in both mm from the fovea and degrees from the fovea). The best fit curve has the following equation: $y = 2.1 + 0.058x + 0.022x^2 - 0.00022x^3$ (from Dacey 1993).

Using the best fit curve from these graphs, the size of the midget fields can be calculated at each of the positions (eccentricities) used in this research, which can be superimposed over the particular spatial frequencies used (at the size they would be when

they are projected back onto the retina) to see if this helps to explain the findings of this discomfort glare research (see Figure 5-2 through Figure 5-6).

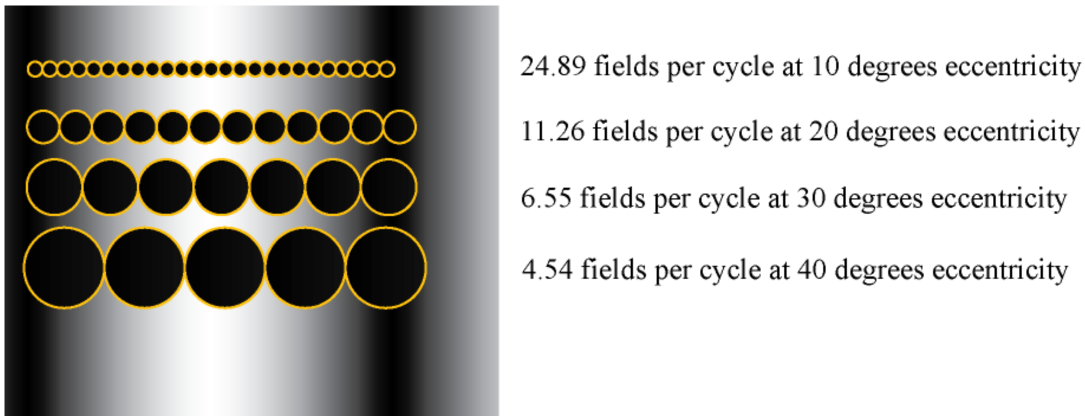


Figure 5-2. Midget ganglion fields overlaid on 0.5 cycles per degree sinusoidal stimulus as it would be projected onto the retina

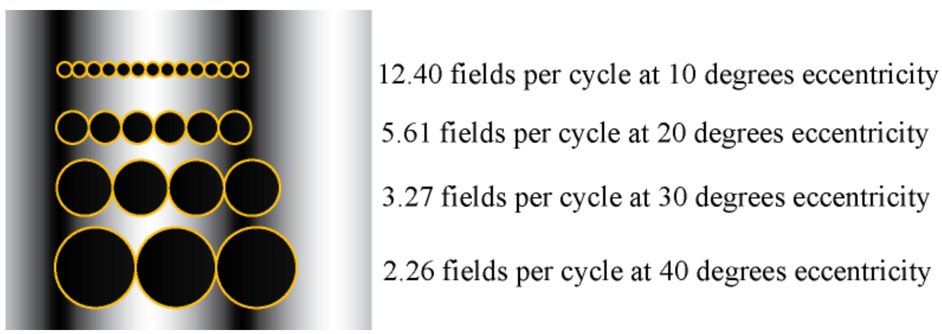


Figure 5-3. Midget ganglion fields overlaid on 1.0 cycle per degree sinusoidal stimulus as it would be projected onto the retina

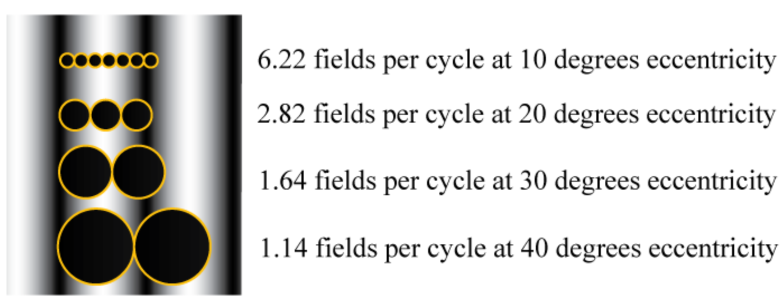


Figure 5-4. Midget ganglion fields overlaid on 2.0 cycle per degree sinusoidal stimulus as it would be projected onto the retina

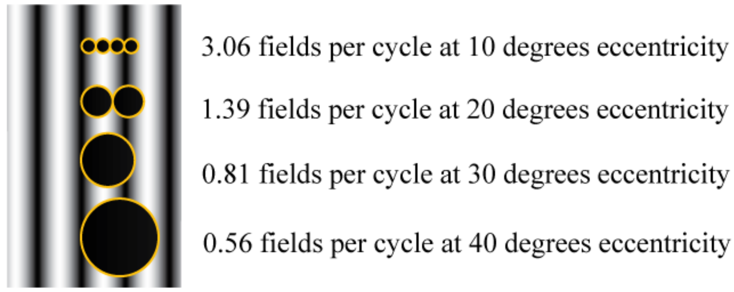


Figure 5-5. Midget ganglion fields overlaid on 4.0 cycle per degree sinusoidal stimulus as it would be projected onto the retina

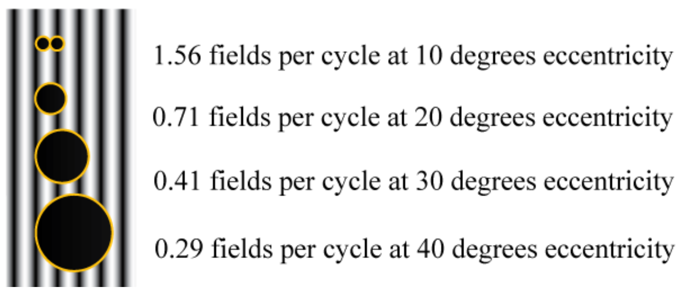


Figure 5-6. Midget ganglion fields overlaid on 8.0 cycle per degree sinusoidal stimulus as it would be projected onto the retina

Unfortunately, at first glance, this exercise does not seem to explain why different frequencies at any one position affect the perception of discomfort differently. One would expect that if this did explain the differences, then at 10 degrees eccentricity (for example), the number of midget ganglion fields per cycle for the 4.0 cycles per degree stimulus would align better with the sine wave than the fields would for the other stimuli (because the 4.0 cycle per degree stimulus caused the least amount of discomfort of the stimuli at ten degrees – see Figure 4-2). And this just does not appear to be the case. Similarly, at 20 degrees, one would expect the 2.0 cycle per degree stimulus to align better than the others.

Note from Figure 5-1 that there is significant variability in the field size at any given eccentricity (Dacey 1993), especially at higher eccentricities. So it is possible that in using the best fit curve, the problem is being simplified too much, and therefore the results that were expected are not being found.

However, at closer inspection, an interesting pattern seems to occur. Notice that for each position, the lowest glare perception occurs for the frequency which has approximately 3 receptive fields per cycle (see Figure 5-7 through Figure 5-10). Is this simply a coincidence, or could there be a physiological reason that 3 fields per cycle somehow causes less discomfort? With this research, that answer is unknown. Further investigation into this issue would be warranted. If it is found that 3 fields per cycle really do cause less discomfort, for some reason, then it would be possible to predict, at any position, what frequency would be less discomforting than other frequencies. This would be a significant contribution to the industry, as fixture manufacturers could incorporate that information into their fixtures, avoiding those frequencies.

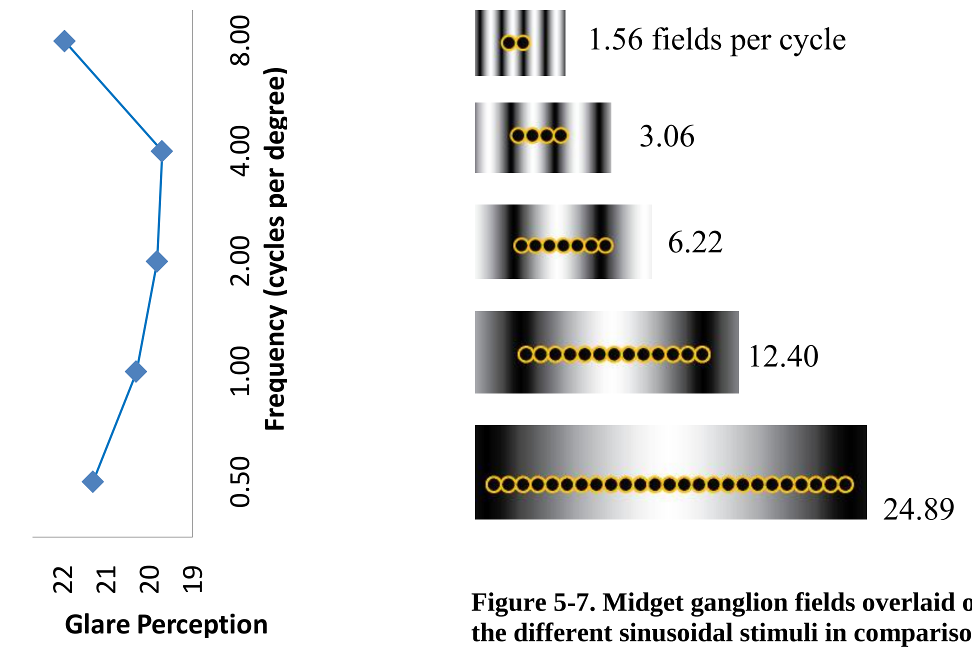


Figure 5-7. Midget ganglion fields overlaid on the different sinusoidal stimuli in comparison with the graph of glare perception vs frequency (from Figure 4-2) at 10 degrees above the line of sight

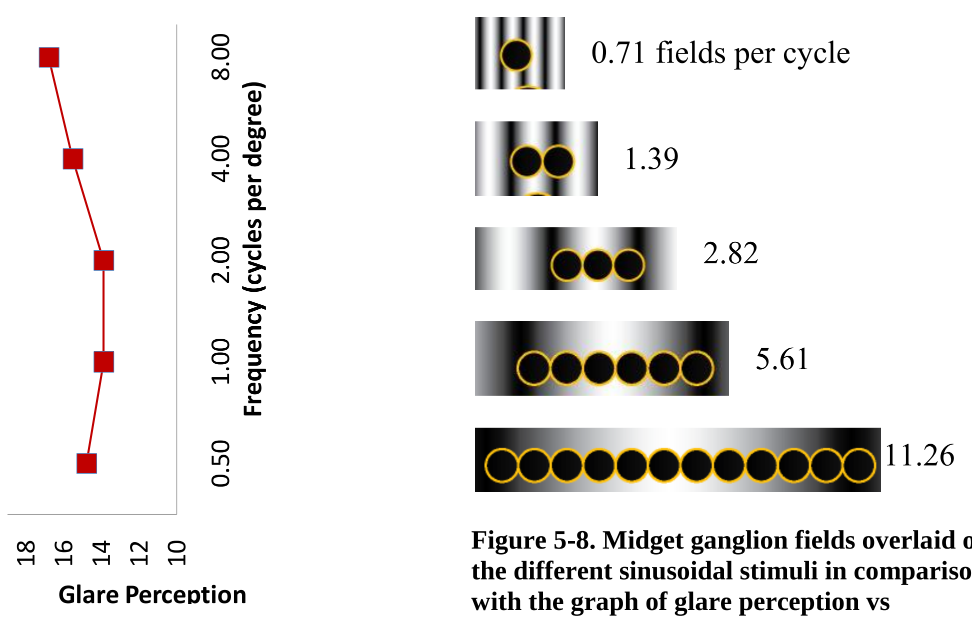


Figure 5-8. Midget ganglion fields overlaid on the different sinusoidal stimuli in comparison with the graph of glare perception vs frequency (from Figure 4-2) at 20 degrees above the line of sight

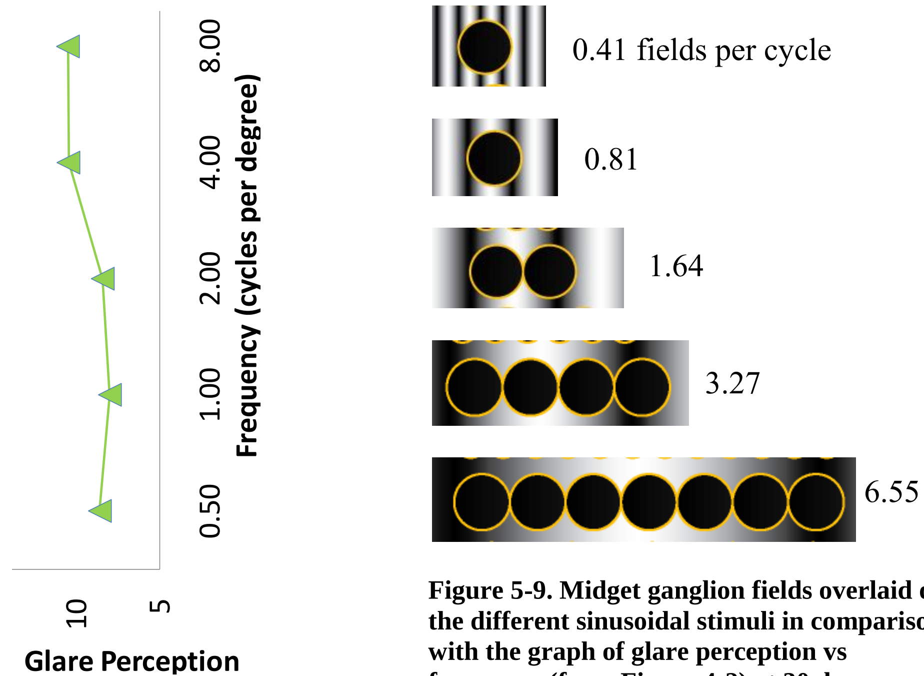


Figure 5-9. Midget ganglion fields overlaid on the different sinusoidal stimuli in comparison with the graph of glare perception vs frequency (from Figure 4-2) at 30 degrees above the line of sight

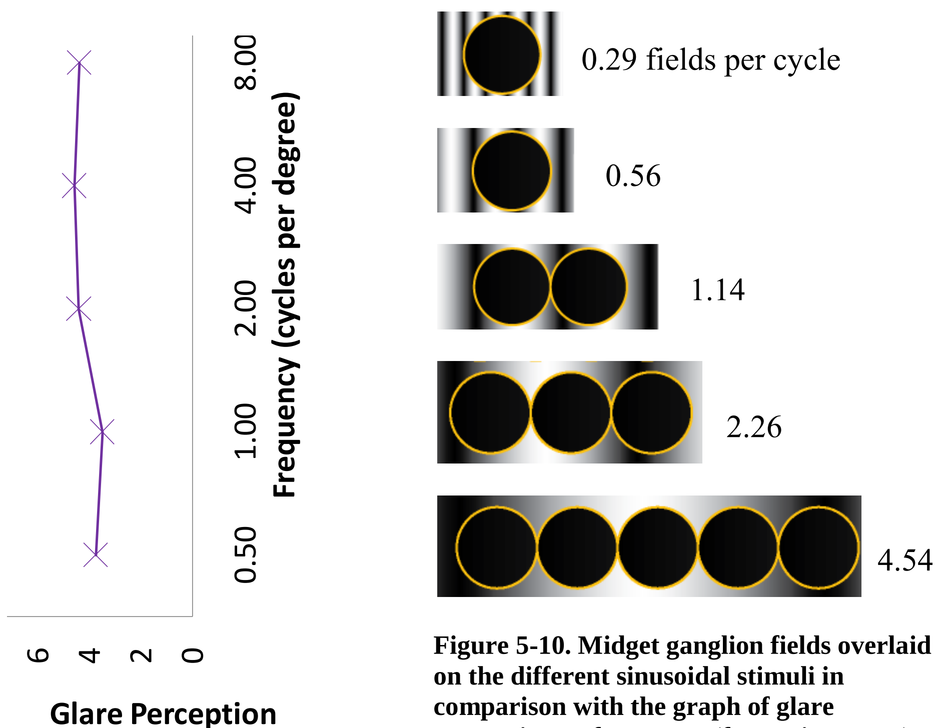


Figure 5-10. Midget ganglion fields overlaid on the different sinusoidal stimuli in comparison with the graph of glare perception vs frequency (from Figure 4-2) at 40 degrees above the line of sight

Future Research

The next step in this research topic is to connect the theoretical with the practical. This research has shown that spatial frequency does have a significant impact on the subjective impression of discomfort. But it does not make a connection between the printed paper targets used in the experiments and real-world luminaires. The true impact this research will have on the lighting community cannot happen until this connection is made. Therefore, the next step needs to be a thorough documentation of complex luminaires used in buildings today, primarily parabolic troffers, which have a luminance gradient across the luminaire. Through the use of Fourier Analysis, any complex scene can be broken down into a sum of sinusoids. Therefore, images could be taken of standard parabolic troffers and analyzed using Fourier Analysis, which would determine what spatial frequencies make up the luminance gradients. These images would have to be taken from many different angles and distances to get a comprehensive understanding of the frequencies involved. These images could be used with the apparatus developed for this experiment to confirm the conclusions drawn here, but with images of real luminaires (preferably again using some sort of paired comparison experimental design). Assuming the results of that experiment confirm what has been stated above, luminaire manufacturers could be educated as to which luminaires (because of the specific luminance gradient) cause more discomfort. The luminaires could be redesigned to include primarily lower frequency components (as higher frequencies tend to cause more discomfort), thus providing a more comfortable environment. Certainly, the higher frequencies cannot be eliminated, as it is the higher frequencies that allow for sharp

edges. But the primary frequency in the luminaire could be designed to be as low as possible.

When the luminaire research has been completed, the next step is to develop a metric for discomfort glare which incorporates this issue of spatial frequency. It seems that it would be appropriate to start with the original UGR formula and revise it to include a frequency component. It was hoped that this modification to the UGR formula could be made based on the results obtained from the current study, but it was determined that there were not enough data points to perform this modification with a high degree of comfort.

The significant effect of frequency was found primarily at positions near to the line of sight. While this still has merit in interior lighting situations, it may have even more value in tasks where the glare source is closer to the line of sight. Car headlights are a primary example of this, as they are typically very close to the line of sight while driving. The findings of this research would suggest that if car headlights were manufactured to have a low frequency luminance gradient (rather than a uniform luminance as they are currently designed), they would be less discomforting, which could be extremely beneficial. Similarly, airport runway lighting would have the same potential, as it is typically near to the pilot's line of sight.

This effect of frequency was found when the subjects were not performing a task. Guth (1951) found that, when subjects were performing a task, discomfort was worse than when they weren't performing a task at positions less than 20 degrees from the line of sight. It is to be expected that this would be true for sources with non-uniform

luminances too, but this should be studied. Therefore, a similar study to the paired comparison experiment performed in this research could be designed, but with the added element of having the subject perform some sort of visual acuity task, like determining where the opening is on a Landolt ring.

Another potential area for research is to go back to the UGR extensions outlined by the CIE (2002) and design an experiment specifically to test those extensions. As several of them have little or no experimental backing, it seems an obvious step to simply test those extensions. From that research, the metrics that the CIE proposed (2002) could be either confirmed or improved.

One other research area to come from this project is a further investigation of midget ganglion field sizes and how they may relate to the perception of glare from non-uniform sources. This area would require collaboration with a neurology or biology expert, where a specific experiment could be designed to test the relationship.

Another research area for discomfort glare (not specifically for non-uniform stimuli, but for all stimuli) is to look at objective measures of glare. Everything discussed in this paper has been subjective measures, but it would be ideal if there were an objective measure of discomfort, which would take out all of the variability that deals with different subjects and their moods, feelings, fatigue, etc. The principal investigator had wanted to include an objective measure of discomfort in this research, but it did not seem like there was one that had been proven to be effective in discomfort studies; therefore, no objective measures were used. This issue of an objective measure is not a new idea when it comes to lighting. Researchers have been looking at pupil size and

pupillary oscillations to see if they can be used as a measure of discomfort since the 1950s (for example, Fugate & Fry 1956, Hopkinson 1956, Fry & King 1975, Howarth et al 1993). At the First International Symposium on Glare, Berman et al (1991) reported that they were looking at facial muscle movement as an objective measure for discomfort. They later reported that analyzing electromyographic (EMG) measurements taken of the muscles on the forehead correlate well with subjective ratings of discomfort given by subjects (Berman et al 1994). More recently, Murray et al (2002) reported that "Regardless of its origin, discomfort glare is always accompanied by a strong flinch reflex in the extra-ocular (facial) muscles surrounding the eye." They have developed a semi-portable device which attaches to the face to record the muscle movement around the eyes and can be moved to any desired location. They claim that "The signal amplitude is proportional to the vertical illuminance at the eye and can therefore be used as an objective index of the discomfort induced." This certainly sounds promising, although it seems that it might be oversimplifying the issue of discomfort, which is more than just vertical illuminance at the eye.

Researchers are looking at additional objective measures of discomfort, such as salivary cortisol production. Smyth et al (1998) studied the effects of stressful situations and mood on the production of salivary cortisol. They found that salivary cortisol levels increased with a stressful situation (either undergoing one currently or even anticipating one). This was not a study about lighting, but it is interesting to consider that if discomfort is considered a stressful situation, then it is possible that salivary cortisol might be a possible measure for discomfort. Kuller and Wetterberg (1993) did look at the effects of different lighting conditions (including different illuminance levels and

different sources) on several objective measures. This was not a discomfort study, but the authors found that there were some interesting correlations between objective measures and lighting conditions. Heart rate, cortisol production, and melatonin levels did not show significant differences between lighting conditions; however, the EEG did show significant differences. The EEG delta rhythm decreased with increased illuminance levels. The EEG theta and alpha rhythms increased with “daylight” fluorescent lamps (as compared with warm-white fluorescent lamps). The EEG beta rhythm increased in the afternoon. This concept seems to have some promise.

The subjective impression of discomfort glare is still not well understood. Additional research on this topic would certainly be a worthwhile effort.

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Appendix A – Matlab Code for Uniformity Gobo

```
'Lamp_luminance_scaling.m
'This program uses measured luminance data from two fixtures, as well
as
'the measured relationship between transmission and % blackness to
create
'two graphical outputs which will create a light of uniform luminance

clear all   'Clears all of the variables in the workspace
close all   'Closes all open figures

'Import the luminance files from fixture A & fixture B
'The two files are measured luminance over a 1025X1025 matrix and are
'taken of the two fixtures
load fixtureA.mat
load fixtureB.mat

'Find the maximum value of luminance and location of maximum for
fixture
'A and Fixture B

%Max luminance - Fixture A
    [C,I]=max(fixtureA);
    [D,J]=max(C);



    max_A=D;
    max_A_index=[I(J) J];

'Max luminance - Fixture B
    [C,I]=max(fixtureB);
    [D,J]=max(C);

    max_B=D;
    max_B_index=[I(J) J];

'Crop image around exterior threshold (where the luminance goes from
zero
'to something to crop out background.
'define a threshold luminance around which to crop the image
'setting a luminance value below which the image gets thrown away

crop_thres=10000;

'find the value where the luminance goes above 10000 for both fixtures
and in
'4 directions for each

Crop index - Fixture A
   for i=1:max_A_index(1)
      if fixtureA(i,max_A_index(2))>crop_thres
         min_x_A=i;
```

```
            break
         end
      end

       for i=max_A_index(1):length(fixtureA)
           if fixtureA(i,max_A_index(2))<crop_thres
               max_x_A=i;
               break
           end
       end

       for i=1:max_A_index(2)
           if fixtureA(max_A_index(1),i)>crop_thres
               min_y_A=i;
               break
           end
       end

       for i=max_A_index(2):length(fixtureA)
           if fixtureA(max_A_index(1),i)<crop_thres
               max_y_A=i;
               break
           end
       end

       fixtureA_crop=fixtureA(min_x_A:max_x_A,min_y_A:max_y_A);


   'Crop index - Fixture B
       for i=1:max_B_index(1)
           if fixtureB(i,max_B_index(2))>crop_thres
               min_x_B=i;
               break
           end
       end

       for i=max_B_index(1):length(fixtureB)
           if fixtureB(i,max_B_index(2))<crop_thres
               max_x_B=i;
               break
          end
       end

       for i=1:max_B_index(2)
           if fixtureB(max_B_index(1),i)>crop_thres
               min_y_B=i;
              break
           end
       end

      for i=max_B_index(2):length(fixtureB)
           if fixtureB(max_B_index(1),i)<crop_thres
               max_y_B=i;
               break
           end
```

```
    end

    fixtureB_crop=fixtureB(min_x_B:max_x_B,min_y_B:max_y_B);


‘To visualize the images, show both a simulated image of the output as
well
‘as a contour map.
fixtureA_crop_scale=fixtureA_crop/1000;
fixtureB_crop_scale=fixtureB_crop/1000;

‘create a color map of gray scales
    map=zeros(1,3);
    n=round((max([max_A max_B]))/1000)+1;
    for i=1:n
        map(i,:)=(i/n);
    end



‘plot figures of lamp intensity and contour maps of intensity
    figure(1)
    image(fixtureA_crop_scale)
    colormap(map)      ‘Assigning the "map" colormap we just created to
image
    axis off           ‘Remove axis ticks and numbers
    axis image         ‘Set aspect ratio to obtain square pixels
    title('Fixture A intensity')

    figure(2)
    [C,h]=contour(fixtureA_crop_scale);
    set(h,'ShowText','on','TextStep',get(h,'LevelStep')*1);
    axis off           % Remove axis ticks and numbers
    axis image         % Set aspect ratio to obtain square pixels
    title('Fixture A intensity')

    figure(3)
    image(fixtureB_crop_scale)
    colormap(map)
    axis off
    axis image
    title('Fixture B intensity')

    figure(4)
    [C,h]=contour(fixtureB_crop_scale);
    set(h,'ShowText','on','TextStep',get(h,'LevelStep')*1);
    axis off           % Remove axis ticks and numbers
    axis image         % Set aspect ratio to obtain square pixels
    title('Fixture B intensity')


‘Load the transmission loss data
load transmission.mat  ‘([percent_black  transmission])

‘fit a polynomial line to the measured data
    n=2;    ‘polynomial order
```

```
    p=polyfit(transmission(:,2),transmission(:,1),n);

‘evaluate the polynomial line
    f = polyval(p,transmission(:,2));
    T_0 = roots(p);     ‘finds the value of transmission at zero black
    T_0 = T_0(n);       ‘only the last root is useful

‘plot the measured data and fit line
    figure(5)
    plot(transmission(:,2),transmission(:,1),'o',transmission(:,2),f,'-
')
    xlabel('Transmission')
    ylabel('% black')
    legend('Measured data','Fit line')
    axis([0 1 0 100])

%Set a maximum luminance around which to even the lamp luminance
%This is the maximum value of luminance that will be obtained.
%This will set both lamps to the max value


max_inten=30000;

‘calculate the required percentage of black required to even out the
lamp
    for i=1:length(fixtureA_crop(:,1))
        for j=1:length(fixtureA_crop(1,:))
            ‘intensity above max
            Temp_A=max([0 fixtureA_crop(i,j)-max_inten]);
            ‘required transmission of point
            trans=T_0-temp_A/fixtureA_crop(i,j);
            ‘get % black value from the polyline
            black_A(i,j)=polyval(p,trans);
        end
    end
'repeat for fixture B
    for i=1:length(fixtureB_crop(:,1))
        for j=1:length(fixtureB_crop(1,:))
            temp_B=max([0 fixtureB_crop(i,j)-max_inten]);
            trans=T_0-temp_B/fixtureB_crop(i,j);
            black_B(i,j)=polyval(p,trans);
        end
    end

‘create a color map of gray scales for % black
‘ zero is clear (white), 100 is 100% black
    map1=zeros(100,3);
    for i=1:100
        map1(i,:)=(101-i)/100;
    end

‘define the center and radius of the circle that you want
‘this can be calculated from the cropped image
‘ Center is mid-point in the x and y directions
‘ Radius is smaller of (length(x)/2) and (length(y)/2)
```

```
center_x_A=(length(fixtureA_crop(1,:)))/2;
center_y_A=(length(fixtureA_crop(:,1)))/2;
center_x_B=(length(fixtureB_crop(1,:)))/2;
center_y_B=(length(fixtureB_crop(:,1)))/2;

radius_x_A=((max_x_A-min_x_A)/2)-2;     'the constant at the end
accounts for
radius_y_A=((max_y_A-min_y_A)/2)-2;     'the thickness of the drawn
line
radius_x_B=((max_x_B-min_x_B)/2)-2;
radius_y_B=((max_y_B-min_y_B)/2)-2;


'set radius for A to be the same as radius for B
if radius_x_A<radius_y_A
    radius_A=radius_x_A;
else radius_A=radius_y_A;
end


if radius_x_B<radius_y_B
    radius_B=radius_x_B;
else radius_B=radius_y_B;
end


'create a vector from min x to max x and from max x to the min x
x_A=[center_x_A-radius_A:1:center_x_A+radius_A center_x_A+radius_A:-
1:center_x_A-radius_A]';
x_B=[center_x_B-radius_B:1:center_x_B+radius_B center_x_B+radius_B:-
1:center_x_B-radius_B]';


'calculate the y "above the center" that corresponds to the first half
of the 'x

for i=1:length(x_B)/2
    y_B(i,1)=sqrt(radius_B^2-(center_x_B-x_B(i))^2)+center_y_B;
end

for i=1:length(x_A)/2
    y_A(i,1)=sqrt(radius_A^2-(center_x_A-x_A(i))^2)+center_y_A;
end

'calculate the y "below the center" that corresponds to the second half
of 'the x

for i=length(x_B)/2:length(x_B)
    y_B(i,1)=center_y_B-sqrt(radius_B^2-(x_B(i)-center_x_B)^2);
end

for i=length(x_A)/2:length(x_A)
```

```
    y_A(i,1)=center_y_A-sqrt(radius_A^2-(x_A(i)-center_x_A)^2);
end

‘plot the final filtering image, as well as a contour map of %black
    figure(6)
    image(black_A)
    colormap(map1)
    axis off
    axis image
    title('Fixture A filter')


 hold on    ‘this lets you plot additional data on top of the previous
plot
‘plots the x and y data calculated for the circle
 plot(x_A,y_A,'k','linewidth',1)
‘ "k" makes it black, and the number at the end is the width of the
line

hold off
‘it is good to turn hold off so the next plots don't get superimposed

     figure(7)
     [C,h]=contour(black_A);
     set(h,'ShowText','on','TextStep',get(h,'LevelStep')*1);
     axis off
     axis image
     title('Fixture A filter (% black)')

    figure(8)
    image(black_B)
    colormap(map1)
    axis off
    axis image
    title('Fixture B filter')

    hold on
    plot(x_B,y_B,'k','linewidth',1)
    hold off


     figure(9)
     [C,h]=contour(black_B);
     set(h,'ShowText','on','TextStep',get(h,'LevelStep')*1);
     axis off
     axis image
     title('Fixture B filter (% black)')


‘ stretch the vertical aspect of the images because of the eccentricity
‘ of the apparatus
 temp_A=zeros(221);

for i=1:230       ‘percentage of stretch
```

```
  temp_A(i,:)=black_A((round((i-111)*0.96)+107),:);
end

figure(10)
    image(temp_A)
    colormap(map1)
    axis image
    axis off
    title('Fixture A filter')

temp_B=zeros(221);

for i=1:230


  temp_B(i,:)=black_B((round((i-111)*0.96)+107),:);
end

figure(11)
    image(temp_B)
    colormap(map1)
    axis image
    axis off
    title('Fixture B filter')
```

Appendix B – Informed Consent Form

IRB#2007-02-216 EP
Date Approved: 06/29/07
Valid Until: 03/25/08

Nebraska Lincoln

ARCHITECTURAL ENGINEERING
100 Peter Kiewit Institute
1110 South 67 Street
Omaha, NE 68182-0681
(402) 554-3856
FAX (402) 554-2080

INFORMED CONSENT FORM

Project Title:
Subjective Evaluation of Discomfort Glare from Sources of Non-Uniform Luminance

Identification of this Research:
This research project involves determining which stimuli cause more discomfort, which will allow the experimenters to better understand how spatial frequency and position affect our impression of discomfort glare. You will be asked to complete one pre-screening session which lasts approximately ten minutes. If you meet the vision requirement you will be included in the experiment and your experimental session will begin immediately following the pre-screening. If selected, the research will take place in one session lasting approximately four hours. This research is funded through an educational stipend of the Industry Experienced Graduate Student Fellowship. You must be at least 19 years old to participate.

Procedures:
This research will take place in the Peter Kiewit Institute building at the Omaha campus of the University of Nebraska. At the start of the session you will be given a brief training sheet to read and have the opportunity to ask any questions.

For the initial pre-screening you will take the *Keystone Visual Skills Test* to confirm you have normal vision. If your vision is acceptable for these experiments, you will be included in the study. For the first part of the study, you will sit in a chair with your head positioned in a head rest, focusing your gaze at a fixation point on the wall. Two stimuli will be shown to you in your periphery, and you will be asked which causes more discomfort, left or right, and also how much more discomfort on a scale of 1 through 5. This process will be repeated as the lighting conditions are changed. You will be able to stop and rest as necessary between trials and 5 minute breaks are scheduled approximately every hour. For the second part of the study, you will again sit in a chair with your head positioned in a head rest, focusing your gaze at a fixation point on the wall. One stimulus will be shown to you in your periphery, and you will be asked to rate the level of discomfort on a scale of 1 through 7.

Risk and/or Discomforts:
There are no known risks associated with this research. The *Keystone Visual Skills* test is a non-invasive screening test for vision. In the glare experiment, the lighting levels are carefully monitored and controlled in a non-harmful range.

Confidentiality:
Any information obtained from this study which can identify you from other participants will be stored

Initials: __________

IRB#2007-02-216 EP
Date Approved: 06/29/07
Valid Until: 03/25/08

in a locked cabinet in the faculty advisor's office and will be seen only by the investigator and faculty advisor. Records will be kept for 10 years following the study. You will be asked to fill out a general information survey that includes demographic information such as your gender, age, and city of residence. Data from individual subjects may be reported, but in no case will you be personally connected to the data.

Benefits:
Your involvement in this study will not directly benefit you, but it may help to improve the future of light fixture design by minimizing glare from those fixtures.

Compensation:
You will not be compensated for the pre-screening session. However, if selected for the full experiment, you will be compensated at $10 per hour.

Right to Ask Questions:
You have the right to ask any question about this research and have your questions answered at any time before, during, or after the experiments. You may contact the investigators at (402) 554-2038. If you have any questions about your rights as a research participant that have not been answered by the investigator or to report any concerns about the study, you may contact the University of Nebraska-Lincoln Institutional Review Board, telephone (402) 472-6965.

Participation is Voluntary:
You are free to decide not to participate in this study or to withdraw at any time without adversely affecting your relationship with the investigators or the University of Nebraska. Your decision will not result in any loss of benefits to which you are otherwise entitled. You were selected to participate because you volunteered.

Consent, Right to Receive a Copy:
You are voluntarily making a decision whether or not to participate in this research study. Your signature certifies that you have decided to participate having read and understood the information presented. You will be given a copy of this consent form to keep.

____________________________ ________________________
Signature of Research Participant Date

____
Printed Name

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Appendix C – General Information Survey

University of Nebraska Lincoln

ARCHITECTURAL ENGINEERING
100 Peter Kiewit Institute
1110 South 67 Street
Omaha, NE 68182-0681
(402) 554-3856
FAX (402) 554-2080

For Experimenter's Use
Subject #: _________
Date: _____________

GENERAL INFORMATION SURVEY

Project Title:
Subjective Evaluation of Discomfort Glare from Sources of Non-Uniform Luminance

Your answers to the following questions will help us make a more adequate interpretation of the results of this research.

1. Gender: ☐ male ☐ female
2. Age: ___________
3. Are you wearing: ☐ glasses ☐ contacts ☐ neither
4. Are you: ☐ left handed ☐ right handed
5. Residence (city and state only): ____________________
6. What is your major (if applicable)? ____________________
7. What is your semester standing (if applicable)? ____________________
8. Have you ever participated in an experiment before? ____________________

Appendix D – Keystone Visual Skills Screening Test Subject Instructions

Experimenter: "This first test will be done as a pre-screening for the main experiment. After you have completed the pre-screening you will be informed whether you have been selected for the main experiment. At that time you may choose to continue with the experiment or decline. If you are included in the full experiment you will be compensated by earning $10 per hour. Now let us begin the pre-screening. This test is to confirm you have normal vision."

(Experimenter runs the subject through the Keystone Visual Skills Test as directed in the manual.)

"Ok, we've finished the test."

Appendix E – Paired Comparison Subject Instructions

1. General Introduction

(Neither of the fixtures is on, and both are adjusted such that the targets are directly at the line of sight of the subject)

Experimenter: "The research you will be participating in today involves glare from sources of non-uniform brightnesses. In the first part of the study, the task is to tell me which of the two stimuli causes more discomfort and how much more. In the second part of the study, the task is to tell me how much discomfort is caused by a single stimulus."

"First, I will explain the apparatus, and make sure you are comfortable. Next, I will explain the experimental procedures to you and then we will start the experiment."

"Please ask questions if anything is unclear."

2. Equipment Introduction

"Now, I will introduce this apparatus to you."

(The experimenter stands in the middle of the apparatus and points to the two targets)

"The stimuli will be mounted on these targets. The stimuli will vary in their pattern and in their position. The pattern will be changed by changing the targets, and the position will be changed by rotating the fixtures."

(The experimenter then moves beyond the apparatus to the wall behind)

"This is the fixation point. It is where I want you to fixate your gaze. I never want you to look directly at the targets, always look at the fixation point."

(The experimenter then moves to the front of the apparatus)

"This is called a chin rest and forehead rest."

(The experimenter points to the chin rest and forehead rest)

"When you are doing the experiments, you will sit in the chair and put your chin on the chin rest and your forehead against the forehead rest. It is important that your eyes are positioned in line with the fixation point. We will raise or lower the chair so that the chin rest and forehead rest are comfortable and you are in a comfortable position."

(To demonstrate, the experimenter put his chin on the chin rest and adjusts the chair height and makes him comfortable with the chair and chin rest.)

"If you have any question please ask me at any time."

3. ***Familiarize the subject with the apparatus***

"Now it is your turn to become familiar with this apparatus, but first I will clean the forehead rest and chin rest."

(Experimenter wipes down chinrest and forehead rest with household cleaner.)

"Please sit in the chair and put your chin on the chin rest and lean your forehead against the forehead rest. We may need to adjust the chair so that your eyes are positioned at the center of the targets. Please do not adjust the chin rest. Are you comfortable?"

(If subject answers yes, move on. If subject answers no, readjust. Repeat as needed.)

4. ***Instructions of experiment procedures***

(Give out an experiment instruction paper sheet for part 1 to the subject.)

"Now I will read the experiment instructions to you. At this time just listen to me; later I will give you a chance to try this step by step."

“Remember, the goal of this experiment is to determine which of the two stimuli causes more discomfort. I will show you the two stimuli, which may have different patterns and may be at different positions. When you have decided which causes more discomfort, tell me. I will also ask you to tell me how much more discomfort it causes, on a scale of 1 through 5, where 1 means not that much more discomfort at all, and 5 means much, much more discomfort. Let me show you an example to give you an idea of the scale.”

(Show the subject the uniform stimulus at 40 degrees on the left and the uniform stimulus at 10 degrees on the right.)

“For this trial, the right should cause you more discomfort at a level of 3. Use this example to make your future judgements.”

“In total there will be 406 trials. After you finish all the trials the experimental session will be complete. Also, please do not touch the fixtures, as they can get warm.”

(Read subject’s instructions – part 1.)

5. Warm-up trial

“Now, let us begin our warm-up trial. In this trial, I will remind you of the procedures step-by-step.

“First, I will show you the two stimuli. Remember to stay focused on the fixation point at all times. Do not look directly at the stimuli.”

(Turn on the two fixtures.)

“Now, please tell me which causes more discomfort. And please tell me how much more discomfort it causes, on a scale of 1 through 5, where 1 means not that much more discomfort at all, and 5 means much, much more discomfort. ”

(Wait for the subject to tell me which causes more discomfort)

"Are you familiar with the procedure? In the later trials I will not give this much detail. If you are not confident, let us do another warm-up trial."

(The experimenter repeats warm-up trial as needed)

6. Trials

(The experimenter changes the gobos and positions of the fixtures.)

"Now let's begin the experiment."

(After the subject finishes this trial, the experimenter changes the gobos and positions and proceeds to next trial until finish all the trials)

"Now, I will begin the next trial."

(Between every 5 trials, remind the subject to stay focused on the fixation point.)

"That completes this portion of the study. Now we will move to the next part."

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Instructions for Glare Experiments – ***Part 1***

Project Title:
Subjective Evaluation of Discomfort Glare from Sources of Non-Uniform Luminance

Identification of this Research:
This part of the experiment involves determining which of two stimuli causes more discomfort.

Instructions:
This experiment deals with your perception of glare. Your task is to tell me which of two stimuli causes more discomfort, left or right, and how much discomfort on a scale of 1 through 5. The two stimuli may differ in pattern and position. Your head will be held in a fixed position using a chin/forehead rest, and I will ask that you always look directly at the fixation point on the wall beyond the apparatus. This ensures that everyone views the same visual stimulus. When you have decided which causes more discomfort and how much more, we will move to the next trial, and repeat until all trials are completed.

Appendix F – Rating Scale Subject Instructions

1. Instructions for experiment procedures (part 2)

(Give out an experiment instruction paper sheet for part 2 to the subject.)

"Now I will read the experiment instructions to you. At this time just listen to me; later I will give you a chance to try this step by step."

"I will show you only one stimulus. Please tell me how much discomfort it causes, on a scale of 1 through 7, where this is the scale I want you to use."

(Give subject a copy of the rating scale.)

"In total there will be 28 trials. After you finish all the trials the experimental session will be complete. Also, please do not touch the fixtures, as they can get warm."

(Read subject's instructions.)

2. Warm-up trial

"Now, let us begin our warm-up trial. In this trial, I will remind you of the procedures step-by-step."

"First, I will show you the stimulus. Remember to stay focused on the fixation point at all times. Do not look directly at the stimulus."

(Turn on the fixture.)

"Now, please tell rate the level of discomfort on a scale of 1 through 7.

(Wait for the subject to tell me the level of discomfort)

“Are you familiar with the procedure? In the later trials I will not give this much detail. If you are not confident, let us do another warm-up trial.”

(The experimenter repeats warm-up trial as needed)

3. Trials

(The experimenter changes the gobo and position of the fixture.)

“Now let’s begin the experiment.”

(After the subject finishes this trial, the experimenter changes the gobo and position and proceeds to next trial until finish all the trials)

“Now, I will begin the next trial.”

(Between every 5 trials, remind the subject to stay focused on the fixation point.)

“That completes the study.”

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Instructions for Glare Experiments – Part 2

Project Title:
Subjective Evaluation of Discomfort Glare from Sources of Non-Uniform Luminance

Identification of this Research:
This part of the experiment involves rating the level of discomfort of a single stimulus.

Instructions:
This experiment deals with your perception of glare. Your task is to rate the level of discomfort caused by the single stimulus on a scale of 1 through 7. Your head will be held in a fixed position using a chin/forehead rest, and I will ask that you always look directly at the fixation point on the wall beyond the apparatus. This ensures that everyone views the same visual stimulus. When you have rated the level of discomfort, we will move to the next trial, and repeat until all trials are completed.

Glare Rating Scale

1	Just perceptible
2	Perceptible
3	Just acceptable
4	Unacceptable
5	Just uncomfortable
6	Uncomfortable
7	Just intolerable

Appendix G – SAS Command File and Output File for Rating Scale Analysis

```
options ls=78 ps=55;
libname rating '';
data rating.temp;
input subjid a1-a24;
posit1 = mean(of a1-a6);
posit2 = mean(of a7-a12);
posit3 = mean(of a13-a18);
posit4 = mean(of a19-a24);
freq1 = mean(a1, a7, a13, a19);
freq2 = mean(a2, a8, a14, a20);
freq3 = mean(a3, a9, a15, a21);
freq4 = mean(a4, a10, a16, a22);
freq5 = mean(a5, a11, a17, a23);
freq6 = mean(a6, a12, a18, a24);
cards;
6     7     7     7     7     7     6     5     7     6     6     6
      6     5     6     4     5     3     4     2     4     3     3
      2     2
7     6     6     4     6     6     6     5     6     5     6     4
      5     5     4     4     5     4     5     2     4     3     4
      3     3
8     7     6     7     5     7     7     6     7     6     6     5
      4     6     4     5     6     4     5     3     1     4     2
      5     5
9     6     5     4     5     5     5     4     4     4     4     4
      4     4     4     3     4     3     3     3     3     2     4
      3     2
10    6     5     7     7     7     7     3     5     4     6     5
      6     3     4     5     3     4     4     2     3     2     3
      3     3
11    7     7     6     7     7     6     6     5     4     5     5
      4     3     4     3     4     3     4     1     1     2     2
      2     2
12    6     5     6     6     6     6     5     5     5     5     4
      5     2     3     3     4     3     2     2     2     2     2
      2     1
13    6     7     6     5     7     7     5     4     4     5     4
      5     2     3     3     3     4     5     1     3     1     2
      2     2
14    6     6     7     6     7     7     5     5     6     6     4
      6     3     2     3     5     4     3     1     1     1     2
      1     2
15    3     6     6     6     3     3     6     4     5     3     3
      2     2     5     3     2     3     3     3     3     2     2
      2     2
16    5     6     5     6     4     5     3     5     6     2     5
      4     3     4     3     5     6     5     1     3     3     3
      2     4
17    5     4     7     5     2     6     5     6     5     6     7
      5     4     6     6     3     5     5     1     1     3     1
      1     2
```

20	6	5	1	6	7	3	5	7	6	6	4
	5	3	2	2	2	4	7	1	5	2	4
	4	3									
21	6	6	6	6	3	6	5	5	5	6	4
	4	4	4	5	4	5	4	4	4	3	2
	2	2									
22	6	6	7	7	7	7	6	5	4	5	6
	6	4	4	4	5	5	5	2	2	2	2
	2	3									
23	5	4	4	5	3	6	3	2	3	3	5
	3	3	2	2	3	3	2	1	2	1	1
	1	2									
24	7	7	6	7	7	6	4	4	6	7	7
	7	6	5	5	3	5	4	2	3	3	2
	2	2									
25	5	7	7	6	7	6	4	5	4	4	4
	5	3	2	2	3	3	2	2	2	1	1
	2	1									
26	6	5	6	6	4	4	3	4	4	4	3
	4	3	3	3	2	4	3	2	3	2	2
	2	2									
27	5	6	4	6	3	6	5	6	4	5	5
	3	3	3	3	3	3	3	1	1	2	2
	2	1									
28	5	6	6	5	5	5	3	3	4	3	3
	4	2	2	2	2	2	3	1	1	1	1
	1	1									
29	6	5	6	6	5	6	3	3	3	5	3
	3	4	3	2	3	2	4	3	3	3	4
	3	2									
30	7	7	7	7	6	7	6	5	6	6	6
	4	4	6	4	5	3	4	3	1	2	2
	1	2									
31	5	6	5	6	5	5	5	3	3	4	3
	3	2	3	2	3	3	2	1	2	2	1
	2	1									
32	6	6	4	6	6	5	5	3	4	5	4
	4	2	3	2	2	2	2	1	1	1	1
	1	1									
33	6	4	5	7	6	7	3	4	4	5	3
	5	2	3	3	3	4	3	1	1	2	2
	1	2									
34	7	7	7	7	6	7	6	5	6	6	6
	4	4	6	4	5	3	4	3	1	2	2
	1	2									
36	7	6	6	6	6	7	4	4	5	4	5
	5	4	1	3	2	2	3	1	1	1	2
	3	1									
37	6	4	5	5	5	7	3	4	3	4	3
	3	2	2	2	2	2	2	1	1	1	1
	1	1									
38	7	5	7	6	7	7	5	6	6	6	6
	7	3	4	5	4	3	4	1	1	3	4
	2	1									
40	6	6	7	6	5	6	6	4	5	4	5
	5	3	4	4	4	4	3	2	3	3	2
	1	3									

```
41      6       5       5       6       6       6       5       4       3       5       5
        5       3       3       3       4       4       4       1       1       1       2
        2       2
;;
proc univariate data=rating.temp;
var posit1-posit4 freq1-freq6;
run;
proc glm data=rating.temp;
model a1-a24 = /nouni;
repeated position 4 polynomial, freq 6 polynomial/nom summary;
run;
/*restructure data set for mixed */
data formixed; set rating.temp;
id=_n_;
 a=a1;  position=1; freq=1; output;
 a=a2;  position=1; freq=2; output;
 a=a3;  position=1; freq=3; output;
 a=a4;  position=1; freq=4; output;
 a=a5;  position=1; freq=5; output;
 a=a6;  position=1; freq=6; output;
 a=a7;  position=2; freq=1; output;
 a=a8;  position=2; freq=2; output;
 a=a9;  position=2; freq=3; output;
 a=a10; position=2; freq=4; output;
 a=a11; position=2; freq=5; output;
 a=a12; position=2; freq=6; output;
 a=a13; position=3; freq=1; output;
 a=a14; position=3; freq=2; output;
 a=a15; position=3; freq=3; output;
 a=a16; position=3; freq=4; output;
 a=a17; position=3; freq=5; output;
 a=a18; position=3; freq=6; output;
 a=a19; position=4; freq=1; output;
 a=a20; position=4; freq=2; output;
 a=a21; position=4; freq=3; output;
 a=a22; position=4; freq=4; output;
 a=a23; position=4; freq=5; output;
 a=a24; position=4; freq=6; output;
 drop a1-a24;
 run;
proc print data=formixed ;
var a id position freq;
run;
proc mixed data=formixed;
class position freq id;
model a= position|freq id/residual;
repeated position freq/type=un@cs subject=id;
lsmeans position freq/pdiff adjust=scheffe;
run;
```

The SAS System 1
11:12 Monday, April 7, 2008

The UNIVARIATE Procedure
Variable: posit1

Moments

N	32	Sum Weights	32
Mean	5.8125	Sum Observations	186
Std Deviation	0.73780407	Variance	0.54435484
Skewness	-0.1872609	Kurtosis	-1.206666
Uncorrected SS	1098	Corrected SS	16.875
Coeff Variation	12.6934033	Std Error Mean	0.13042656

Basic Statistical Measures

Location		Variability	
Mean	5.812500	Std Deviation	0.73780
Median	5.750000	Variance	0.54435
Mode	6.500000	Range	2.33333
		Interquartile Range	1.25000

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	44.56531	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	6.83333
99%	6.83333
95%	6.83333
90%	6.66667
75% Q3	6.50000
50% Median	5.75000
25% Q1	5.25000
10%	4.83333
5%	4.50000
1%	4.50000
0% Min	4.50000

Extreme Observations

------Lowest-----		-----Highest-----	
Value	Obs	Value	Obs
4.50000	16	6.66667	15
4.50000	10	6.66667	17
4.66667	13	6.83333	1

4.83333	12	6.83333	23
5.00000	20	6.83333	27

The UNIVARIATE Procedure
Variable: posit2

Moments

N	32	Sum Weights	32
Mean	4.64583333	Sum Observations	148.666667
Std Deviation	0.85167977	Variance	0.72535842
Skewness	-0.1300582	Kurtosis	-1.0632295
Uncorrected SS	713.166667	Corrected SS	22.4861111
Coeff Variation	18.3321205	Std Error Mean	0.15055713

Basic Statistical Measures

Location		Variability	
Mean	4.645833	Std Deviation	0.85168
Median	4.750000	Variance	0.72536
Mode	4.833333	Range	2.83333
		Interquartile Range	1.41667

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	30.85761	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	6.00000
99%	6.00000
95%	6.00000
90%	5.66667
75% Q3	5.41667
50% Median	4.75000
25% Q1	4.00000
10%	3.33333
5%	3.33333
1%	3.16667
0% Min	3.16667

Extreme Observations

------Lowest-----		-----Highest-----	
Value	Obs	Value	Obs
3.16667	16	5.66667	3
3.33333	29	5.66667	12
3.33333	22	5.83333	17
3.33333	21	6.00000	1
3.50000	24	6.00000	30

The UNIVARIATE Procedure
Variable: posit3

Moments

N	32	Sum Weights	32
Mean	3.47916667	Sum Observations	111.333333
Std Deviation	0.86109666	Variance	0.74148746
Skewness	0.05997177	Kurtosis	-1.1090427
Uncorrected SS	410.333333	Corrected SS	22.9861111
Coeff Variation	24.7500836	Std Error Mean	0.15222182

Basic Statistical Measures

Location		Variability	
Mean	3.479167	Std Deviation	0.86110
Median	3.416667	Variance	0.74149
Mode	3.000000	Range	3.00000
		Interquartile Range	1.41667

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	22.8559	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	5.00000
99%	5.00000
95%	4.83333
90%	4.50000
75% Q3	4.33333
50% Median	3.41667
25% Q1	2.91667
10%	2.50000
5%	2.16667
1%	2.00000
0% Min	2.00000

Extreme Observations

------Lowest-----		-----Highest-----	
Value	Obs	Value	Obs
2.00000	29	4.50000	2
2.16667	25	4.50000	15
2.16667	21	4.66667	17
2.50000	28	4.83333	12
2.50000	24	5.00000	3

The UNIVARIATE Procedure
Variable: posit4

Moments

N	32	Sum Weights	32
Mean	2.02604167	Sum Observations	64.8333333
Std Deviation	0.68274873	Variance	0.46614583
Skewness	0.33086339	Kurtosis	-0.9835623
Uncorrected SS	145.805556	Corrected SS	14.4505208
Coeff Variation	33.6986522	Std Error Mean	0.12069406

Basic Statistical Measures

Location		Variability	
Mean	2.026042	Std Deviation	0.68275
Median	1.833333	Variance	0.46615
Mode	1.500000	Range	2.33333
		Interquartile Range	1.16667

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	16.78659	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	3.33333
99%	3.33333
95%	3.16667
90%	3.00000
75% Q3	2.66667
50% Median	1.83333
25% Q1	1.50000
10%	1.33333
5%	1.00000
1%	1.00000
0% Min	1.00000

Extreme Observations

------Lowest-----		-----Highest-----	
Value	Obs	Value	Obs
1.00000	29	2.83333	14
1.00000	25	3.00000	22
1.00000	21	3.16667	2
1.33333	16	3.16667	13
1.33333	9	3.33333	3

The UNIVARIATE Procedure
Variable: freq1

Moments

N	32	Sum Weights	32
Mean	3.8984375	Sum Observations	124.75
Std Deviation	0.68350194	Variance	0.4671749
Skewness	0.43701205	Kurtosis	-0.4234487
Uncorrected SS	500.8125	Corrected SS	14.4824219
Coeff Variation	17.532715	Std Error Mean	0.12082721

Basic Statistical Measures

Location		Variability	
Mean	3.898438	Std Deviation	0.68350
Median	3.750000	Variance	0.46717
Mode	3.500000	Range	2.75000
		Interquartile Range	0.87500

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	32.26457	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	5.500
99%	5.500
95%	5.000
90%	4.750
75% Q3	4.375
50% Median	3.750
25% Q1	3.500
10%	3.000
5%	3.000
1%	2.750
0% Min	2.750

Extreme Observations

----Lowest----		----Highest---	
Value	Obs	Value	Obs
2.75	21	4.75	14
3.00	29	4.75	17
3.00	26	5.00	23
3.00	16	5.00	27
3.00	11	5.50	3

The UNIVARIATE Procedure
Variable: freq2

Moments

N	32	Sum Weights	32
Mean	4.015625	Sum Observations	128.5
Std Deviation	0.75652404	Variance	0.57232863
Skewness	0.09283694	Kurtosis	0.27533899
Uncorrected SS	533.75	Corrected SS	17.7421875
Coeff Variation	18.8395093	Std Error Mean	0.13373582

Basic Statistical Measures

Location		Variability	
Mean	4.015625	Std Deviation	0.75652
Median	4.125000	Variance	0.57233
Mode	4.250000	Range	3.50000
		Interquartile Range	1.00000

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	30.02655	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	6.000
99%	6.000
95%	5.000
90%	4.750
75% Q3	4.500
50% Median	4.125
25% Q1	3.500
10%	3.000
5%	2.750
1%	2.500
0% Min	2.500

Extreme Observations

----Lowest----		----Highest---	
Value	Obs	Value	Obs
2.50	16	4.75	17
2.75	29	4.75	23
3.00	28	4.75	27
3.00	26	5.00	2
3.00	21	6.00	1

The UNIVARIATE Procedure
Variable: freq3

Moments

N	32	Sum Weights	32
Mean	3.9375	Sum Observations	126
Std Deviation	0.83762337	Variance	0.7016129
Skewness	0.12761888	Kurtosis	-1.0136848
Uncorrected SS	517.875	Corrected SS	21.75
Coeff Variation	21.2729744	Std Error Mean	0.14807229

Basic Statistical Measures

Location		Variability	
Mean	3.937500	Std Deviation	0.83762
Median	3.875000	Variance	0.70161
Mode	3.500000	Range	3.00000
		Interquartile Range	1.50000

NOTE: The mode displayed is the smallest of 2 modes with a count of 4.

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	26.59174	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	5.500
99%	5.500
95%	5.250
90%	5.000
75% Q3	4.750
50% Median	3.875
25% Q1	3.250
10%	2.750
5%	2.750
1%	2.500
0% Min	2.500

Extreme Observations

----Lowest----		----Highest---	
Value	Obs	Value	Obs
2.50	16	5.00	1
2.75	29	5.00	17
2.75	25	5.25	12
2.75	13	5.25	30
3.00	32	5.50	3

The UNIVARIATE Procedure
Variable: freq4

Moments

N	32	Sum Weights	32
Mean	4.1640625	Sum Observations	133.25
Std Deviation	0.69156606	Variance	0.47826361
Skewness	-0.300828	Kurtosis	-0.867885
Uncorrected SS	569.6875	Corrected SS	14.8261719
Coeff Variation	16.6079653	Std Error Mean	0.12225276

Basic Statistical Measures

Location		Variability	
Mean	4.164063	Std Deviation	0.69157
Median	4.250000	Variance	0.47826
Mode	3.500000	Range	2.50000
		Interquartile Range	1.25000

NOTE: The mode displayed is the smallest of 2 modes with a count of 5.

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	34.06109	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	5.25
99%	5.25
95%	5.25
90%	5.00
75% Q3	4.75
50% Median	4.25
25% Q1	3.50
10%	3.25
5%	3.00
1%	2.75
0% Min	2.75

Extreme Observations

----Lowest----		----Highest---	
Value	Obs	Value	Obs
2.75	21	5.00	23
3.00	29	5.00	27
3.00	16	5.00	30
3.25	10	5.25	1
3.50	28	5.25	2

The UNIVARIATE Procedure
Variable: freq5

Moments

N	32	Sum Weights	32
Mean	3.8984375	Sum Observations	124.75
Std Deviation	0.69520064	Variance	0.48330393
Skewness	0.14289641	Kurtosis	-0.5742365
Uncorrected SS	501.3125	Corrected SS	14.9824219
Coeff Variation	17.832802	Std Error Mean	0.12289527

Basic Statistical Measures

Location		Variability	
Mean	3.898438	Std Deviation	0.69520
Median	4.000000	Variance	0.48330
Mode	3.250000	Range	2.50000
		Interquartile Range	1.00000

NOTE: The mode displayed is the smallest of 3 modes with a count of 5.

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	31.72162	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	5.25
99%	5.25
95%	5.25
90%	4.75
75% Q3	4.25
50% Median	4.00
25% Q1	3.25
10%	3.00
5%	2.75
1%	2.75
0% Min	2.75

Extreme Observations

----Lowest----		----Highest---	
Value	Obs	Value	Obs
2.75	29	4.75	5
2.75	21	4.75	13
2.75	10	5.00	15
3.00	16	5.25	3
3.25	25	5.25	17

The UNIVARIATE Procedure
Variable: freq6

Moments

N	32	Sum Weights	32
Mean	4.03125	Sum Observations	129
Std Deviation	0.72609761	Variance	0.52721774
Skewness	-0.2754451	Kurtosis	-0.7718746
Uncorrected SS	536.375	Corrected SS	16.34375
Coeff Variation	18.0117237	Std Error Mean	0.12835714

Basic Statistical Measures

Location		Variability	
Mean	4.031250	Std Deviation	0.72610
Median	4.250000	Variance	0.52722
Mode	3.250000	Range	2.75000
		Interquartile Range	1.12500

NOTE: The mode displayed is the smallest of 3 modes with a count of 5.

Tests for Location: Mu0=0

Test	-Statistic-		-----p Value------	
Student's t	t	31.40651	Pr > |t|	<.0001
Sign	M	16	Pr >= |M|	<.0001
Signed Rank	S	264	Pr >= |S|	<.0001

Quantiles (Definition 5)

Quantile	Estimate
100% Max	5.250
99%	5.250
95%	5.250
90%	4.750
75% Q3	4.500
50% Median	4.250
25% Q1	3.375
10%	3.250
5%	2.750
1%	2.500
0% Min	2.500

Extreme Observations

----Lowest----		----Highest---	
Value	Obs	Value	Obs
2.50	10	4.75	17
2.75	24	4.75	30
3.00	25	5.00	5
3.25	29	5.25	3
3.25	21	5.25	15

The GLM Procedure

Number of Observations Read	32
Number of Observations Used	32

The GLM Procedure
Repeated Measures Analysis of Variance

Repeated Measures Level Information

Dependent Variable	a1	a2	a3	a4	a5	a6
Level of position	1	1	1	1	1	1
Level of freq	1	2	3	4	5	6

Repeated Measures Level Information

Dependent Variable	a7	a8	a9	a10	a11	a12
Level of position	2	2	2	2	2	2
Level of freq	1	2	3	4	5	6

Repeated Measures Level Information

Dependent Variable	a13	a14	a15	a16	a17	a18
Level of position	3	3	3	3	3	3
Level of freq	1	2	3	4	5	6

Repeated Measures Level Information

Dependent Variable	a19	a20	a21	a22	a23	a24
Level of position	4	4	4	4	4	4
Level of freq	1	2	3	4	5	6

The GLM Procedure
Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Source	DF	Type III SS	Mean Square	F Value	Pr > F
position	3	1510.983073	503.661024	257.16	<.0001
Error(position)	93	182.141927	1.958515		

Source	Adj Pr > F G - G	Adj Pr > F H - F
position	<.0001	<.0001
Error(position)		

Greenhouse-Geisser Epsilon	0.7822
Huynh-Feldt Epsilon	0.8503

Source	DF	Type III SS	Mean Square	F Value	Pr > F
freq	5	6.6783854	1.3356771	1.70	0.1380
Error(freq)	155	121.8632813	0.7862147		

Source	Adj Pr > F G - G	Adj Pr > F H - F
freq	0.1617	0.1524
Error(freq)		

Greenhouse-Geisser Epsilon	0.7184
Huynh-Feldt Epsilon	0.8242

Source	DF	Type III SS	Mean Square	F Value	Pr > F
position*freq	15	7.9934896	0.5328993	0.76	0.7187
Error(position*freq)	465	324.6315104	0.6981323		

Source	Adj Pr > F G - G	Adj Pr > F H - F
position*freq	0.6390	0.6823
Error(position*freq)		

The GLM Procedure
Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Greenhouse-Geisser Epsilon	0.5483
Huynh-Feldt Epsilon	0.7645

The GLM Procedure
Repeated Measures Analysis of Variance
Analysis of Variance of Contrast Variables

position_N represents the nth degree polynomial contrast for position

Contrast Variable: position_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	9037.539063	9037.539063	467.79	<.0001
Error	31	598.910938	19.319708		

Contrast Variable: position_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	23.6328125	23.6328125	2.01	0.1657
Error	31	363.6171875	11.7295867		

Contrast Variable: position_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	4.7265625	4.7265625	1.12	0.2972
Error	31	130.3234375	4.2039819		

The GLM Procedure
Repeated Measures Analysis of Variance
Analysis of Variance of Contrast Variables

freq_N represents the nth degree polynomial contrast for freq

Contrast Variable: freq_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	2.1254464	2.1254464	0.38	0.5418
Error	31	173.1459821	5.5853543		

Contrast Variable: freq_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	2.75148810	2.75148810	1.65	0.2088
Error	31	51.77232143	1.67007488		

Contrast Variable: freq_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.95069444	0.95069444	0.36	0.5507
Error	31	80.96041667	2.61162634		

Contrast Variable: freq_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	2.79017857	2.79017857	1.03	0.3191
Error	31	84.35267857	2.72105415		

Contrast Variable: freq_5

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	18.09573413	18.09573413	5.77	0.0225
Error	31	97.22172619	3.13618472		

Obs	a	id	position	freq
1	7	1	1	1
2	7	1	1	2
3	7	1	1	3
4	7	1	1	4
5	7	1	1	5
6	6	1	1	6
7	5	1	2	1
8	7	1	2	2
9	6	1	2	3
10	6	1	2	4
11	6	1	2	5
12	6	1	2	6
13	5	1	3	1
14	6	1	3	2
15	4	1	3	3
16	5	1	3	4
17	3	1	3	5
18	4	1	3	6
19	2	1	4	1
20	4	1	4	2
21	3	1	4	3
22	3	1	4	4
23	2	1	4	5
24	2	1	4	6
25	6	2	1	1
26	6	2	1	2
27	4	2	1	3
28	6	2	1	4
29	6	2	1	5
30	6	2	1	6
31	5	2	2	1
32	6	2	2	2
33	5	2	2	3
34	6	2	2	4
35	4	2	2	5
36	5	2	2	6
37	5	2	3	1
38	4	2	3	2
39	4	2	3	3
40	5	2	3	4
41	4	2	3	5
42	5	2	3	6
43	2	2	4	1
44	4	2	4	2
45	3	2	4	3
46	4	2	4	4
47	3	2	4	5
48	3	2	4	6
49	7	3	1	1
50	6	3	1	2

Obs	a	id	position	freq
51	7	3	1	3
52	5	3	1	4
53	7	3	1	5
54	7	3	1	6
55	6	3	2	1
56	7	3	2	2
57	6	3	2	3
58	6	3	2	4
59	5	3	2	5
60	4	3	2	6
61	6	3	3	1
62	4	3	3	2
63	5	3	3	3
64	6	3	3	4
65	4	3	3	5
66	5	3	3	6
67	3	3	4	1
68	1	3	4	2
69	4	3	4	3
70	2	3	4	4
71	5	3	4	5
72	5	3	4	6
73	6	4	1	1
74	5	4	1	2
75	4	4	1	3
76	5	4	1	4
77	5	4	1	5
78	5	4	1	6
79	4	4	2	1
80	4	4	2	2
81	4	4	2	3
82	4	4	2	4
83	4	4	2	5
84	4	4	2	6
85	4	4	3	1
86	4	4	3	2
87	3	4	3	3
88	4	4	3	4
89	3	4	3	5
90	3	4	3	6
91	3	4	4	1
92	3	4	4	2
93	2	4	4	3
94	4	4	4	4
95	3	4	4	5
96	2	4	4	6
97	6	5	1	1
98	5	5	1	2
99	7	5	1	3
100	7	5	1	4

Obs	a	id	position	freq
101	7	5	1	5
102	7	5	1	6
103	3	5	2	1
104	5	5	2	2
105	4	5	2	3
106	6	5	2	4
107	5	5	2	5
108	6	5	2	6
109	3	5	3	1
110	4	5	3	2
111	5	5	3	3
112	3	5	3	4
113	4	5	3	5
114	4	5	3	6
115	2	5	4	1
116	3	5	4	2
117	2	5	4	3
118	3	5	4	4
119	3	5	4	5
120	3	5	4	6
121	7	6	1	1
122	7	6	1	2
123	6	6	1	3
124	7	6	1	4
125	7	6	1	5
126	6	6	1	6
127	6	6	2	1
128	5	6	2	2
129	4	6	2	3
130	5	6	2	4
131	5	6	2	5
132	4	6	2	6
133	3	6	3	1
134	4	6	3	2
135	3	6	3	3
136	4	6	3	4
137	3	6	3	5
138	4	6	3	6
139	1	6	4	1
140	1	6	4	2
141	2	6	4	3
142	2	6	4	4
143	2	6	4	5
144	2	6	4	6
145	6	7	1	1
146	5	7	1	2
147	6	7	1	3
148	6	7	1	4
149	6	7	1	5
150	6	7	1	6

Obs	a	id	position	freq
151	5	7	2	1
152	5	7	2	2
153	5	7	2	3
154	5	7	2	4
155	4	7	2	5
156	5	7	2	6
157	2	7	3	1
158	3	7	3	2
159	3	7	3	3
160	4	7	3	4
161	3	7	3	5
162	2	7	3	6
163	2	7	4	1
164	2	7	4	2
165	2	7	4	3
166	2	7	4	4
167	2	7	4	5
168	1	7	4	6
169	6	8	1	1
170	7	8	1	2
171	6	8	1	3
172	5	8	1	4
173	7	8	1	5
174	7	8	1	6
175	5	8	2	1
176	4	8	2	2
177	4	8	2	3
178	5	8	2	4
179	4	8	2	5
180	5	8	2	6
181	2	8	3	1
182	3	8	3	2
183	3	8	3	3
184	3	8	3	4
185	4	8	3	5
186	5	8	3	6
187	1	8	4	1
188	3	8	4	2
189	1	8	4	3
190	2	8	4	4
191	2	8	4	5
192	2	8	4	6
193	6	9	1	1
194	6	9	1	2
195	7	9	1	3
196	6	9	1	4
197	7	9	1	5
198	7	9	1	6
199	5	9	2	1
200	5	9	2	2

Obs	a	id	position	freq
201	6	9	2	3
202	6	9	2	4
203	4	9	2	5
204	6	9	2	6
205	3	9	3	1
206	2	9	3	2
207	3	9	3	3
208	5	9	3	4
209	4	9	3	5
210	3	9	3	6
211	1	9	4	1
212	1	9	4	2
213	1	9	4	3
214	2	9	4	4
215	1	9	4	5
216	2	9	4	6
217	3	10	1	1
218	6	10	1	2
219	6	10	1	3
220	6	10	1	4
221	3	10	1	5
222	3	10	1	6
223	6	10	2	1
224	4	10	2	2
225	5	10	2	3
226	3	10	2	4
227	3	10	2	5
228	2	10	2	6
229	2	10	3	1
230	5	10	3	2
231	3	10	3	3
232	2	10	3	4
233	3	10	3	5
234	3	10	3	6
235	3	10	4	1
236	3	10	4	2
237	2	10	4	3
238	2	10	4	4
239	2	10	4	5
240	2	10	4	6
241	5	11	1	1
242	6	11	1	2
243	5	11	1	3
244	6	11	1	4
245	4	11	1	5
246	5	11	1	6
247	3	11	2	1
248	5	11	2	2
249	6	11	2	3
250	2	11	2	4

Obs	a	id	position	freq
251	5	11	2	5
252	4	11	2	6
253	3	11	3	1
254	4	11	3	2
255	3	11	3	3
256	5	11	3	4
257	6	11	3	5
258	5	11	3	6
259	1	11	4	1
260	3	11	4	2
261	3	11	4	3
262	3	11	4	4
263	2	11	4	5
264	4	11	4	6
265	5	12	1	1
266	4	12	1	2
267	7	12	1	3
268	5	12	1	4
269	2	12	1	5
270	6	12	1	6
271	5	12	2	1
272	6	12	2	2
273	5	12	2	3
274	6	12	2	4
275	7	12	2	5
276	5	12	2	6
277	4	12	3	1
278	6	12	3	2
279	6	12	3	3
280	3	12	3	4
281	5	12	3	5
282	5	12	3	6
283	1	12	4	1
284	1	12	4	2
285	3	12	4	3
286	1	12	4	4
287	1	12	4	5
288	2	12	4	6
289	6	13	1	1
290	5	13	1	2
291	1	13	1	3
292	6	13	1	4
293	7	13	1	5
294	3	13	1	6
295	5	13	2	1
296	7	13	2	2
297	6	13	2	3
298	6	13	2	4
299	4	13	2	5
300	5	13	2	6

Obs	a	id	position	freq
301	3	13	3	1
302	2	13	3	2
303	2	13	3	3
304	2	13	3	4
305	4	13	3	5
306	7	13	3	6
307	1	13	4	1
308	5	13	4	2
309	2	13	4	3
310	4	13	4	4
311	4	13	4	5
312	3	13	4	6
313	6	14	1	1
314	6	14	1	2
315	6	14	1	3
316	6	14	1	4
317	3	14	1	5
318	6	14	1	6
319	5	14	2	1
320	5	14	2	2
321	5	14	2	3
322	6	14	2	4
323	4	14	2	5
324	4	14	2	6
325	4	14	3	1
326	4	14	3	2
327	5	14	3	3
328	4	14	3	4
329	5	14	3	5
330	4	14	3	6
331	4	14	4	1
332	4	14	4	2
333	3	14	4	3
334	2	14	4	4
335	2	14	4	5
336	2	14	4	6
337	6	15	1	1
338	6	15	1	2
339	7	15	1	3
340	7	15	1	4
341	7	15	1	5
342	7	15	1	6
343	6	15	2	1
344	5	15	2	2
345	4	15	2	3
346	5	15	2	4
347	6	15	2	5
348	6	15	2	6
349	4	15	3	1
350	4	15	3	2

Obs	a	id	position	freq
351	4	15	3	3
352	5	15	3	4
353	5	15	3	5
354	5	15	3	6
355	2	15	4	1
356	2	15	4	2
357	2	15	4	3
358	2	15	4	4
359	2	15	4	5
360	3	15	4	6
361	5	16	1	1
362	4	16	1	2
363	4	16	1	3
364	5	16	1	4
365	3	16	1	5
366	6	16	1	6
367	3	16	2	1
368	2	16	2	2
369	3	16	2	3
370	3	16	2	4
371	5	16	2	5
372	3	16	2	6
373	3	16	3	1
374	2	16	3	2
375	2	16	3	3
376	3	16	3	4
377	3	16	3	5
378	2	16	3	6
379	1	16	4	1
380	2	16	4	2
381	1	16	4	3
382	1	16	4	4
383	1	16	4	5
384	2	16	4	6
385	7	17	1	1
386	7	17	1	2
387	6	17	1	3
388	7	17	1	4
389	7	17	1	5
390	6	17	1	6
391	4	17	2	1
392	4	17	2	2
393	6	17	2	3
394	7	17	2	4
395	7	17	2	5
396	7	17	2	6
397	6	17	3	1
398	5	17	3	2
399	5	17	3	3
400	3	17	3	4

Obs	a	id	position	freq
401	5	17	3	5
402	4	17	3	6
403	2	17	4	1
404	3	17	4	2
405	3	17	4	3
406	2	17	4	4
407	2	17	4	5
408	2	17	4	6
409	5	18	1	1
410	7	18	1	2
411	7	18	1	3
412	6	18	1	4
413	7	18	1	5
414	6	18	1	6
415	4	18	2	1
416	5	18	2	2
417	4	18	2	3
418	4	18	2	4
419	4	18	2	5
420	5	18	2	6
421	3	18	3	1
422	2	18	3	2
423	2	18	3	3
424	3	18	3	4
425	3	18	3	5
426	2	18	3	6
427	2	18	4	1
428	2	18	4	2
429	1	18	4	3
430	1	18	4	4
431	2	18	4	5
432	1	18	4	6
433	6	19	1	1
434	5	19	1	2
435	6	19	1	3
436	6	19	1	4
437	4	19	1	5
438	4	19	1	6
439	3	19	2	1
440	4	19	2	2
441	4	19	2	3
442	4	19	2	4
443	3	19	2	5
444	4	19	2	6
445	3	19	3	1
446	3	19	3	2
447	3	19	3	3
448	2	19	3	4
449	4	19	3	5
450	3	19	3	6

Obs	a	id	position	freq
451	2	19	4	1
452	3	19	4	2
453	2	19	4	3
454	2	19	4	4
455	2	19	4	5
456	2	19	4	6
457	5	20	1	1
458	6	20	1	2
459	4	20	1	3
460	6	20	1	4
461	3	20	1	5
462	6	20	1	6
463	5	20	2	1
464	6	20	2	2
465	4	20	2	3
466	5	20	2	4
467	5	20	2	5
468	3	20	2	6
469	3	20	3	1
470	3	20	3	2
471	3	20	3	3
472	3	20	3	4
473	3	20	3	5
474	3	20	3	6
475	1	20	4	1
476	1	20	4	2
477	2	20	4	3
478	2	20	4	4
479	2	20	4	5
480	1	20	4	6
481	5	21	1	1
482	6	21	1	2
483	6	21	1	3
484	5	21	1	4
485	5	21	1	5
486	5	21	1	6
487	3	21	2	1
488	3	21	2	2
489	4	21	2	3
490	3	21	2	4
491	3	21	2	5
492	4	21	2	6
493	2	21	3	1
494	2	21	3	2
495	2	21	3	3
496	2	21	3	4
497	2	21	3	5
498	3	21	3	6
499	1	21	4	1
500	1	21	4	2

Obs	a	id	position	freq
501	1	21	4	3
502	1	21	4	4
503	1	21	4	5
504	1	21	4	6
505	6	22	1	1
506	5	22	1	2
507	6	22	1	3
508	6	22	1	4
509	5	22	1	5
510	6	22	1	6
511	3	22	2	1
512	3	22	2	2
513	3	22	2	3
514	5	22	2	4
515	3	22	2	5
516	3	22	2	6
517	4	22	3	1
518	3	22	3	2
519	2	22	3	3
520	3	22	3	4
521	2	22	3	5
522	4	22	3	6
523	3	22	4	1
524	3	22	4	2
525	3	22	4	3
526	4	22	4	4
527	3	22	4	5
528	2	22	4	6
529	7	23	1	1
530	7	23	1	2
531	7	23	1	3
532	7	23	1	4
533	6	23	1	5
534	7	23	1	6
535	6	23	2	1
536	5	23	2	2
537	6	23	2	3
538	6	23	2	4
539	6	23	2	5
540	4	23	2	6
541	4	23	3	1
542	6	23	3	2
543	4	23	3	3
544	5	23	3	4
545	3	23	3	5
546	4	23	3	6
547	3	23	4	1
548	1	23	4	2
549	2	23	4	3
550	2	23	4	4

Obs	a	id	position	freq
551	1	23	4	5
552	2	23	4	6
553	5	24	1	1
554	6	24	1	2
555	5	24	1	3
556	6	24	1	4
557	5	24	1	5
558	5	24	1	6
559	5	24	2	1
560	3	24	2	2
561	3	24	2	3
562	4	24	2	4
563	3	24	2	5
564	3	24	2	6
565	2	24	3	1
566	3	24	3	2
567	2	24	3	3
568	3	24	3	4
569	3	24	3	5
570	2	24	3	6
571	1	24	4	1
572	2	24	4	2
573	2	24	4	3
574	1	24	4	4
575	2	24	4	5
576	1	24	4	6
577	6	25	1	1
578	6	25	1	2
579	4	25	1	3
580	6	25	1	4
581	6	25	1	5
582	5	25	1	6
583	5	25	2	1
584	3	25	2	2
585	4	25	2	3
586	5	25	2	4
587	4	25	2	5
588	4	25	2	6
589	2	25	3	1
590	3	25	3	2
591	2	25	3	3
592	2	25	3	4
593	2	25	3	5
594	2	25	3	6
595	1	25	4	1
596	1	25	4	2
597	1	25	4	3
598	1	25	4	4
599	1	25	4	5
600	1	25	4	6

Obs	a	id	position	freq
601	6	26	1	1
602	4	26	1	2
603	5	26	1	3
604	7	26	1	4
605	6	26	1	5
606	7	26	1	6
607	3	26	2	1
608	4	26	2	2
609	4	26	2	3
610	5	26	2	4
611	3	26	2	5
612	5	26	2	6
613	2	26	3	1
614	3	26	3	2
615	3	26	3	3
616	3	26	3	4
617	4	26	3	5
618	3	26	3	6
619	1	26	4	1
620	1	26	4	2
621	2	26	4	3
622	2	26	4	4
623	1	26	4	5
624	2	26	4	6
625	7	27	1	1
626	7	27	1	2
627	7	27	1	3
628	7	27	1	4
629	6	27	1	5
630	7	27	1	6
631	6	27	2	1
632	5	27	2	2
633	6	27	2	3
634	6	27	2	4
635	6	27	2	5
636	4	27	2	6
637	4	27	3	1
638	6	27	3	2
639	4	27	3	3
640	5	27	3	4
641	3	27	3	5
642	4	27	3	6
643	3	27	4	1
644	1	27	4	2
645	2	27	4	3
646	2	27	4	4
647	1	27	4	5
648	2	27	4	6
649	7	28	1	1
650	6	28	1	2

Obs	a	id	position	freq
651	6	28	1	3
652	6	28	1	4
653	6	28	1	5
654	7	28	1	6
655	4	28	2	1
656	4	28	2	2
657	5	28	2	3
658	4	28	2	4
659	5	28	2	5
660	5	28	2	6
661	4	28	3	1
662	1	28	3	2
663	3	28	3	3
664	2	28	3	4
665	2	28	3	5
666	3	28	3	6
667	1	28	4	1
668	1	28	4	2
669	1	28	4	3
670	2	28	4	4
671	3	28	4	5
672	1	28	4	6
673	6	29	1	1
674	4	29	1	2
675	5	29	1	3
676	5	29	1	4
677	5	29	1	5
678	7	29	1	6
679	3	29	2	1
680	4	29	2	2
681	3	29	2	3
682	4	29	2	4
683	3	29	2	5
684	3	29	2	6
685	2	29	3	1
686	2	29	3	2
687	2	29	3	3
688	2	29	3	4
689	2	29	3	5
690	2	29	3	6
691	1	29	4	1
692	1	29	4	2
693	1	29	4	3
694	1	29	4	4
695	1	29	4	5
696	1	29	4	6
697	7	30	1	1
698	5	30	1	2
699	7	30	1	3
700	6	30	1	4

Obs	a	id	position	freq
701	7	30	1	5
702	7	30	1	6
703	5	30	2	1
704	6	30	2	2
705	6	30	2	3
706	6	30	2	4
707	6	30	2	5
708	7	30	2	6
709	3	30	3	1
710	4	30	3	2
711	5	30	3	3
712	4	30	3	4
713	3	30	3	5
714	4	30	3	6
715	1	30	4	1
716	1	30	4	2
717	3	30	4	3
718	4	30	4	4
719	2	30	4	5
720	1	30	4	6
721	6	31	1	1
722	6	31	1	2
723	7	31	1	3
724	6	31	1	4
725	5	31	1	5
726	6	31	1	6
727	6	31	2	1
728	4	31	2	2
729	5	31	2	3
730	4	31	2	4
731	5	31	2	5
732	5	31	2	6
733	3	31	3	1
734	4	31	3	2
735	4	31	3	3
736	4	31	3	4
737	4	31	3	5
738	3	31	3	6
739	2	31	4	1
740	3	31	4	2
741	3	31	4	3
742	2	31	4	4
743	1	31	4	5
744	3	31	4	6
745	6	32	1	1
746	5	32	1	2
747	5	32	1	3
748	6	32	1	4
749	6	32	1	5
750	6	32	1	6

Obs	a	id	position	freq
751	5	32	2	1
752	4	32	2	2
753	3	32	2	3
754	5	32	2	4
755	5	32	2	5
756	5	32	2	6
757	3	32	3	1
758	3	32	3	2
759	3	32	3	3
760	4	32	3	4
761	4	32	3	5
762	4	32	3	6
763	1	32	4	1
764	1	32	4	2
765	1	32	4	3
766	2	32	4	4
767	2	32	4	5
768	2	32	4	6

The Mixed Procedure

Model Information

Data Set	WORK.FORMIXED
Dependent Variable	a
Covariance Structure	Unstructured @ Compound Symmetry
Subject Effect	id
Estimation Method	REML
Residual Variance Method	None
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

Class Level Information

Class	Levels	Values
position	4	1 2 3 4
freq	6	1 2 3 4 5 6
id	32	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Dimensions

Covariance Parameters	11
Columns in X	67
Columns in Z	0
Subjects	32
Max Obs Per Subject	24

Number of Observations

Number of Observations Read	768
Number of Observations Used	768
Number of Observations Not Used	0

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	2111.85136141	
1	2	2047.13787459	0.00341880
2	1	2045.74651637	0.00011776
3	1	2045.70206868	0.00000020
4	1	2045.70199382	0.00000000

The Mixed Procedure

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm		Subject	Estimate
position	UN(1,1)	id	1.1627
	UN(2,1)	id	-0.03501
	UN(2,2)	id	0.9459
	UN(3,1)	id	0.03504
	UN(3,2)	id	-0.07411
	UN(3,3)	id	0.9028
	UN(4,1)	id	0.1718
	UN(4,2)	id	0.002778
	UN(4,3)	id	0.09143
	UN(4,4)	id	0.8055
freq	Corr	id	0.2520

Fit Statistics

-2 Res Log Likelihood	2045.7
AIC (smaller is better)	2067.7
AICC (smaller is better)	2068.1
BIC (smaller is better)	2083.8

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
10	66.15	<.0001

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
position	3	93	279.30	<.0001
freq	5	155	1.70	0.1377
position*freq	15	465	0.79	0.6880
id	31	0	4.19	.

The Mixed Procedure

Least Squares Means

Effect	position	freq	Estimate	Standard Error	DF	t Value	Pr > |t|
position	1		5.8125	0.1170	93	49.68	<.0001
position	2		4.6458	0.1055	93	44.03	<.0001
position	3		3.4792	0.1031	93	33.75	<.0001
position	4		2.0260	0.09737	93	20.81	<.0001
freq		1	3.8984	0.09058	155	43.04	<.0001
freq		2	4.0156	0.09058	155	44.33	<.0001
freq		3	3.9375	0.09058	155	43.47	<.0001
freq		4	4.1641	0.09058	155	45.97	<.0001
freq		5	3.8984	0.09058	155	43.04	<.0001
freq		6	4.0313	0.09058	155	44.51	<.0001

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Estimate	Standard Error	DF	t Value
position	1		2		1.1667	0.1601	93	7.29
position	1		3		2.3333	0.1533	93	15.22
position	1		4		3.7865	0.1383	93	27.38
position	2		3		1.1667	0.1533	93	7.61
position	2		4		2.6198	0.1434	93	18.27
position	3		4		1.4531	0.1340	93	10.84
freq		1		2	-0.1172	0.1108	155	-1.06
freq		1		3	-0.03906	0.1108	155	-0.35
freq		1		4	-0.2656	0.1108	155	-2.40
freq		1		5	-557E-18	0.1108	155	-0.00

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Pr > |t|	Adjustment	Adj P
position	1		2		<.0001	Scheffe	<.0001
position	1		3		<.0001	Scheffe	<.0001
position	1		4		<.0001	Scheffe	<.0001
position	2		3		<.0001	Scheffe	<.0001
position	2		4		<.0001	Scheffe	<.0001
position	3		4		<.0001	Scheffe	<.0001
freq		1		2	0.2918	Scheffe	0.9518
freq		1		3	0.7249	Scheffe	0.9997
freq		1		4	0.0177	Scheffe	0.3366
freq		1		5	1.0000	Scheffe	1.0000

The Mixed Procedure

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Estimate	Standard Error	DF	t Value
freq		1		6	-0.1328	0.1108	155	-1.20
freq		2		3	0.07812	0.1108	155	0.71
freq		2		4	-0.1484	0.1108	155	-1.34
freq		2		5	0.1172	0.1108	155	1.06
freq		2		6	-0.01563	0.1108	155	-0.14
freq		3		4	-0.2266	0.1108	155	-2.05
freq		3		5	0.03906	0.1108	155	0.35
freq		3		6	-0.09375	0.1108	155	-0.85
freq		4		5	0.2656	0.1108	155	2.40
freq		4		6	0.1328	0.1108	155	1.20
freq		5		6	-0.1328	0.1108	155	-1.20

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Pr > |t|	Adjustment	Adj P
freq		1		6	0.2324	Scheffe	0.9194
freq		2		3	0.4818	Scheffe	0.9921
freq		2		4	0.1823	Scheffe	0.8758
freq		2		5	0.2918	Scheffe	0.9518
freq		2		6	0.8880	Scheffe	1.0000
freq		3		4	0.0425	Scheffe	0.5257
freq		3		5	0.7249	Scheffe	0.9997
freq		3		6	0.3987	Scheffe	0.9818
freq		4		5	0.0177	Scheffe	0.3366
freq		4		6	0.2324	Scheffe	0.9194
freq		5		6	0.2324	Scheffe	0.9194

Appendix H – SAS Command File and Output File for Paired Comparison Analysis

```
options ls=78 ps=55;
libname paired '';
data paired.temp;
input subjid present $ bias $ a1-a24;
presentx1=-1*(present='L')+ 1*(present='R');
biasx2=-1*(bias='L') + 1*(bias='R');
interx3 = presentx1*biasx2;
posit1 = mean(of a1-a6);
posit2 = mean(of a7-a12);
posit3 = mean(of a13-a18);
posit4 = mean(of a19-a24);
freq1 = mean(a1, a7, a13, a19);
freq2 = mean(a2, a8, a14, a20);
freq3 = mean(a3, a9, a15, a21);
freq4 = mean(a4, a10, a16, a22);
freq5 = mean(a5, a11, a17, a23);
freq6 = mean(a6, a12, a18, a24);

cards;
1 R R 21      24      20      20      22      22      15      13      13      15      14
      17      8       7       9       12      9       14      4       4       3       4
      5       5
2 L L 21      20      21      21      22      24      13      14      15      16      17
      18      9       7       9       11      12      9       1       2       4       4
      5       5
3 R R 19      18      20      18      22      24      17      12      11      11      19
      20      7       6       9       9       12      17      2       3       5       7
      4       8
4 L L 23      21      19      18      23      21      14      15      17      15      15
      16      8       6       10      8       9       10      6       5       4       6
      5       6
5 R R 21      18      19      21      23      24      15      10      11      16      18
      19      4       4       4       13      14      12      3       4       6       8
      5       8
6 L R 21      21      20      21      22      23      16      13      12      17      14
      16      11      7       9       12      11      10      4       4       6       4
      4       2
7 R R 21      21      19      22      23      23      15      15      12      15      18
      17      11      7       8       9       9       10      2       3       6       4
      6       4
8 L L 22      20      24      20      21      21      13      16      14      13      14
      19      8       8       9       11      11      11      5       5       3       3
      5       4
9 R L 23      19      19      20      21      24      16      13      14      16      17
      16      8       7       8       10      11      12      5       2       3       7
      3       6
10 L R 18     21      23      21      23      22      13      16      19      16      16
      14      9       11      12      8       9       8       4       4       3       4
      3       3
11 R L 23     21      18      21      21      21      17      17      16      15      15
      13      12      11      10      10      8       10      4       2       5       4
      4       2
```

12 L L 24	21	22	19	21	22	15	14	13	17	17
17	9	10	9	9	11	9	6	2	3	3
3	4									
13 R R 21	22	21	20	21	24	17	16	12	14	16
14	10	11	11	9	10	10	5	3	5	3
4	1									
14 L R 24	23	22	18	20	16	15	16	18	14	14
10	11	13	9	10	9	10	6	6	3	6
4	3									
15 R L 22	21	20	21	22	22	14	14	15	17	16
17	7	8	9	11	11	12	3	2	3	4
5	4									
16 L R 20	18	17	19	21	24	12	12	13	15	20
22	6	4	5	15	13	11	2	2	7	7
7	8									
17 R L 21	20	18	19	23	22	13	13	14	18	17
18	7	7	8	12	12	13	3	6	5	5
3	3									
18 L L 23	21	19	20	23	23	15	14	13	16	18
17	8	7	8	11	9	8	2	2	5	5
8	5									
19 R L 19	20	22	23	22	23	14	12	16	17	16
17	5	5	8	11	11	10	5	3	6	7
7	1									
20 L R 21	19	19	19	23	21	16	14	14	15	18
19	9	8	10	7	8	10	6	3	6	4
3	8									
21 R R 21	20	19	19	23	24	14	15	12	17	18
19	8	7	8	11	10	12	3	3	2	4
4	7									
22 L R 23	21	20	19	22	23	15	14	14	16	16
16	8	8	7	11	7	9	5	7	6	6
4	3									
23 R L 23	23	23	21	17	18	16	18	15	14	12
17	14	14	8	8	8	9	7	5	4	2
3	1									
24 L R 21	21	20	19	24	22	15	12	15	14	18
17	10	11	9	10	11	9	2	3	6	3
3	5									
25 R L 23	21	20	19	22	24	16	14	14	15	18
15	11	9	9	10	11	8	2	5	4	5
3	2									
26 L R 20	19	22	17	21	21	16	14	11	16	17
20	11	7	6	11	11	8	6	2	7	6
7	4									
27 R L 21	20	19	18	21	24	17	12	11	15	21
18	8	9	8	12	11	13	3	1	4	5
3	6									
28 R R 21	19	19	20	23	24	14	15	15	15	19
18	9	8	9	11	9	11	1	4	4	3
4	5									
29 L L 20	22	19	21	23	24	14	14	14	16	17
18	7	8	9	10	11	12	2	3	5	2
3	6									
30 L L 23	20	21	20	21	24	13	14	15	16	17
18	6	8	10	10	10	12	4	3	2	5
3	5									

```
31 R R 17     20     22     24     23     20     12     16     18     13     14
       13     9      11     13     7      9      8      5      8      3      4
       5      6
32 R R 22     18     20     21     22     24     15     13     13     15     17
       17     7      7      9      10     14     13     3      1      4      4
       4      7
34 L R 19     15     13     13     24     23     16     11     9      19     22
       18     10     9      7      15     17     17     2      3      3      6
       4      5
35 R L 24     22     17     20     21     23     17     14     14     16     15
       18     10     8      5      11     9      10     4      4      4      4
       6      4
36 L L 20     20     18     18     23     24     13     11     14     18     18
       19     6      4      5      13     12     14     6      4      6      3
       5      6
;;

proc glm data=paired.temp;
model a1-a24 = /nouni;
repeated position 4 polynomial, freq 6 polynomial/nom summary;
run;

/*restructure data set for mixed */
data formixed; set paired.temp;
id=_n_;
 a=a1;  position=1; freq=1; output;
 a=a2;  position=1; freq=2; output;
 a=a3;  position=1; freq=3; output;
 a=a4;  position=1; freq=4; output;
 a=a5;  position=1; freq=5; output;
 a=a6;  position=1; freq=6; output;
 a=a7;  position=2; freq=1; output;
 a=a8;  position=2; freq=2; output;
 a=a9;  position=2; freq=3; output;
 a=a10; position=2; freq=4; output;
 a=a11; position=2; freq=5; output;
 a=a12; position=2; freq=6; output;
 a=a13; position=3; freq=1; output;
 a=a14; position=3; freq=2; output;
 a=a15; position=3; freq=3; output;
 a=a16; position=3; freq=4; output;
 a=a17; position=3; freq=5; output;
 a=a18; position=3; freq=6; output;
 a=a19; position=4; freq=1; output;
 a=a20; position=4; freq=2; output;
 a=a21; position=4; freq=3; output;
 a=a22; position=4; freq=4; output;
 a=a23; position=4; freq=5; output;
 a=a24; position=4; freq=6; output;
 drop a1-a24;
 run;

proc print data=formixed ;
var a id presentx1 biasx2 position freq;
run;

 ods html;
```

```
   ods graphics on;
proc mixed data=formixed;
class position freq id;
model a= position|freq|presentx1|biasx2 id/residual;
repeated position freq/type=un@cs subject=id;
lsmeans position freq/pdiff adjust=scheffe;
run;
ods graphics off;
   ods html close;
run;

proc means data=ryan.temp mean n std;
var a1-a24;
run;
```

```
                              The SAS System                                1
                                                        11:22 Monday, April 7, 2008

                             The GLM Procedure

                  Number of Observations Read          35
                  Number of Observations Used          35


                    Repeated Measures Level Information

 Dependent Variable          a1        a2        a3        a4        a5        a6

  Level of position           1         1         1         1         1         1
      Level of freq           1         2         3         4         5         6

                    Repeated Measures Level Information

 Dependent Variable          a7        a8        a9       a10       a11       a12

  Level of position           2         2         2         2         2         2
      Level of freq           1         2         3         4         5         6

                    Repeated Measures Level Information

 Dependent Variable         a13       a14       a15       a16       a17       a18

  Level of position           3         3         3         3         3         3
      Level of freq           1         2         3         4         5         6

                    Repeated Measures Level Information

 Dependent Variable         a19       a20       a21       a22       a23       a24

  Level of position           4         4         4         4         4         4
      Level of freq           1         2         3         4         5         6
```

The GLM Procedure
Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Source	DF	Type III SS	Mean Square	F Value	Pr > F
position	3	32872.31429	10957.43810	3050.78	<.0001
Error(position)	102	366.35238	3.59169		

Source	Adj Pr > F G - G	H - F
position	<.0001	<.0001
Error(position)		

Greenhouse-Geisser Epsilon	0.8193
Huynh-Feldt Epsilon	0.8880

Source	DF	Type III SS	Mean Square	F Value	Pr > F
freq	5	627.100000	125.420000	16.84	<.0001
Error(freq)	170	1265.900000	7.446471		

Source	Adj Pr > F G - G	H - F
freq	<.0001	<.0001
Error(freq)		

Greenhouse-Geisser Epsilon	0.3860
Huynh-Feldt Epsilon	0.4088

Source	DF	Type III SS	Mean Square	F Value	Pr > F
position*freq	15	267.328571	17.821905	7.64	<.0001
Error(position*freq)	510	1189.004762	2.331382		

Source	Adj Pr > F G - G	H - F
position*freq	<.0001	<.0001
Error(position*freq)		

The GLM Procedure
Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Greenhouse-Geisser Epsilon	0.5172
Huynh-Feldt Epsilon	0.6846

The GLM Procedure
Repeated Measures Analysis of Variance
Analysis of Variance of Contrast Variables

position_N represents the nth degree polynomial contrast for position

Contrast Variable: position_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	197165.0057	197165.0057	8921.56	<.0001
Error	34	751.3943	22.0998		

Contrast Variable: position_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	35.0000000	35.0000000	1.33	0.2561
Error	34	892.0000000	26.2352941		

Contrast Variable: position_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	33.8800000	33.8800000	2.08	0.1587
Error	34	554.7200000	16.3152941		

The GLM Procedure
Repeated Measures Analysis of Variance
Analysis of Variance of Contrast Variables

freq_N represents the nth degree polynomial contrast for freq

Contrast Variable: freq_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	1830.989388	1830.989388	20.24	<.0001
Error	34	3076.267755	90.478463		

Contrast Variable: freq_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	394.5336735	394.5336735	17.30	0.0002
Error	34	775.5734694	22.8109844		

Contrast Variable: freq_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	279.5125397	279.5125397	13.61	0.0008
Error	34	698.3374603	20.5393371		

Contrast Variable: freq_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.9806122	0.9806122	0.16	0.6941
Error	34	211.9122449	6.2327131		

Contrast Variable: freq_5

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	2.3837868	2.3837868	0.27	0.6075
Error	34	301.5090703	8.8679138		

The GLM Procedure
Repeated Measures Analysis of Variance
Analysis of Variance of Contrast Variables

position_N represents the nth degree polynomial contrast for position
freq_N represents the nth degree polynomial contrast for freq

Contrast Variable: position_1*freq_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	3.98702041	3.98702041	2.00	0.1668
Error	34	67.90726531	1.99727251		

Contrast Variable: position_1*freq_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	101.6670918	101.6670918	24.57	<.0001
Error	34	140.7001701	4.1382403		

Contrast Variable: position_1*freq_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.82800794	0.82800794	0.37	0.5453
Error	34	75.41560317	2.21810598		

Contrast Variable: position_1*freq_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	32.57147959	32.57147959	17.76	0.0002
Error	34	62.34459184	1.83366447		

Contrast Variable: position_1*freq_5

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	4.66354308	4.66354308	2.08	0.1584
Error	34	76.24856009	2.24260471		

The GLM Procedure
Repeated Measures Analysis of Variance
Analysis of Variance of Contrast Variables

position_N represents the nth degree polynomial contrast for position
freq_N represents the nth degree polynomial contrast for freq

Contrast Variable: position_2*freq_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	76.1730612	76.1730612	20.67	<.0001
Error	34	125.3126531	3.6856663		

Contrast Variable: position_2*freq_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.86743197	0.86743197	0.37	0.5489
Error	34	80.44506803	2.36603141		

Contrast Variable: position_2*freq_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	22.9809921	22.9809921	7.58	0.0094
Error	34	103.0481746	3.0308287		

Contrast Variable: position_2*freq_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.56352041	0.56352041	0.26	0.6139
Error	34	73.87397959	2.17276411		

Contrast Variable: position_2*freq_5

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	21.11023243	21.11023243	7.96	0.0079
Error	34	90.12488662	2.65073196		

The GLM Procedure
Repeated Measures Analysis of Variance
Analysis of Variance of Contrast Variables

position_N represents the nth degree polynomial contrast for position
freq_N represents the nth degree polynomial contrast for freq

Contrast Variable: position_3*freq_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.08359184	0.08359184	0.03	0.8534
Error	34	81.95069388	2.41031453		

Contrast Variable: position_3*freq_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.26573129	0.26573129	0.18	0.6783
Error	34	51.61105442	1.51797219		

Contrast Variable: position_3*freq_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.02762698	0.02762698	0.01	0.9084
Error	34	69.85376190	2.05452241		

Contrast Variable: position_3*freq_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.38617347	0.38617347	0.33	0.5718
Error	34	40.28704082	1.18491297		

Contrast Variable: position_3*freq_5

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	1.15306689	1.15306689	0.79	0.3816
Error	34	49.88125850	1.46709584		

Obs	a	id	presentx1	biasx2	position	freq
1	21	1	1	1	1	1
2	24	1	1	1	1	2
3	20	1	1	1	1	3
4	20	1	1	1	1	4
5	22	1	1	1	1	5
6	22	1	1	1	1	6
7	15	1	1	1	2	1
8	13	1	1	1	2	2
9	13	1	1	1	2	3
10	15	1	1	1	2	4
11	14	1	1	1	2	5
12	17	1	1	1	2	6
13	8	1	1	1	3	1
14	7	1	1	1	3	2
15	9	1	1	1	3	3
16	12	1	1	1	3	4
17	9	1	1	1	3	5
18	14	1	1	1	3	6
19	4	1	1	1	4	1
20	4	1	1	1	4	2
21	3	1	1	1	4	3
22	4	1	1	1	4	4
23	5	1	1	1	4	5
24	5	1	1	1	4	6
25	21	2	-1	-1	1	1
26	20	2	-1	-1	1	2
27	21	2	-1	-1	1	3
28	21	2	-1	-1	1	4
29	22	2	-1	-1	1	5
30	24	2	-1	-1	1	6
31	13	2	-1	-1	2	1
32	14	2	-1	-1	2	2
33	15	2	-1	-1	2	3
34	16	2	-1	-1	2	4
35	17	2	-1	-1	2	5
36	18	2	-1	-1	2	6
37	9	2	-1	-1	3	1
38	7	2	-1	-1	3	2
39	9	2	-1	-1	3	3
40	11	2	-1	-1	3	4
41	12	2	-1	-1	3	5
42	9	2	-1	-1	3	6
43	1	2	-1	-1	4	1
44	2	2	-1	-1	4	2
45	4	2	-1	-1	4	3
46	4	2	-1	-1	4	4
47	5	2	-1	-1	4	5
48	5	2	-1	-1	4	6
49	19	3	1	1	1	1
50	18	3	1	1	1	2

Obs	a	id	presentx1	biasx2	position	freq
51	20	3	1	1	1	3
52	18	3	1	1	1	4
53	22	3	1	1	1	5
54	24	3	1	1	1	6
55	17	3	1	1	2	1
56	12	3	1	1	2	2
57	11	3	1	1	2	3
58	11	3	1	1	2	4
59	19	3	1	1	2	5
60	20	3	1	1	2	6
61	7	3	1	1	3	1
62	6	3	1	1	3	2
63	9	3	1	1	3	3
64	9	3	1	1	3	4
65	12	3	1	1	3	5
66	17	3	1	1	3	6
67	2	3	1	1	4	1
68	3	3	1	1	4	2
69	5	3	1	1	4	3
70	7	3	1	1	4	4
71	4	3	1	1	4	5
72	8	3	1	1	4	6
73	23	4	-1	-1	1	1
74	21	4	-1	-1	1	2
75	19	4	-1	-1	1	3
76	18	4	-1	-1	1	4
77	23	4	-1	-1	1	5
78	21	4	-1	-1	1	6
79	14	4	-1	-1	2	1
80	15	4	-1	-1	2	2
81	17	4	-1	-1	2	3
82	15	4	-1	-1	2	4
83	15	4	-1	-1	2	5
84	16	4	-1	-1	2	6
85	8	4	-1	-1	3	1
86	6	4	-1	-1	3	2
87	10	4	-1	-1	3	3
88	8	4	-1	-1	3	4
89	9	4	-1	-1	3	5
90	10	4	-1	-1	3	6
91	6	4	-1	-1	4	1
92	5	4	-1	-1	4	2
93	4	4	-1	-1	4	3
94	6	4	-1	-1	4	4
95	5	4	-1	-1	4	5
96	6	4	-1	-1	4	6
97	21	5	1	1	1	1
98	18	5	1	1	1	2
99	19	5	1	1	1	3
100	21	5	1	1	1	4

Obs	a	id	presentx1	biasx2	position	freq
101	23	5	1	1	1	5
102	24	5	1	1	1	6
103	15	5	1	1	2	1
104	10	5	1	1	2	2
105	11	5	1	1	2	3
106	16	5	1	1	2	4
107	18	5	1	1	2	5
108	19	5	1	1	2	6
109	4	5	1	1	3	1
110	4	5	1	1	3	2
111	4	5	1	1	3	3
112	13	5	1	1	3	4
113	14	5	1	1	3	5
114	12	5	1	1	3	6
115	3	5	1	1	4	1
116	4	5	1	1	4	2
117	6	5	1	1	4	3
118	8	5	1	1	4	4
119	5	5	1	1	4	5
120	8	5	1	1	4	6
121	21	6	-1	1	1	1
122	21	6	-1	1	1	2
123	20	6	-1	1	1	3
124	21	6	-1	1	1	4
125	22	6	-1	1	1	5
126	23	6	-1	1	1	6
127	16	6	-1	1	2	1
128	13	6	-1	1	2	2
129	12	6	-1	1	2	3
130	17	6	-1	1	2	4
131	14	6	-1	1	2	5
132	16	6	-1	1	2	6
133	11	6	-1	1	3	1
134	7	6	-1	1	3	2
135	9	6	-1	1	3	3
136	12	6	-1	1	3	4
137	11	6	-1	1	3	5
138	10	6	-1	1	3	6
139	4	6	-1	1	4	1
140	4	6	-1	1	4	2
141	6	6	-1	1	4	3
142	4	6	-1	1	4	4
143	4	6	-1	1	4	5
144	2	6	-1	1	4	6
145	21	7	1	1	1	1
146	21	7	1	1	1	2
147	19	7	1	1	1	3
148	22	7	1	1	1	4
149	23	7	1	1	1	5
150	23	7	1	1	1	6

Obs	a	id	presentx1	biasx2	position	freq
151	15	7	1	1	2	1
152	15	7	1	1	2	2
153	12	7	1	1	2	3
154	15	7	1	1	2	4
155	18	7	1	1	2	5
156	17	7	1	1	2	6
157	11	7	1	1	3	1
158	7	7	1	1	3	2
159	8	7	1	1	3	3
160	9	7	1	1	3	4
161	9	7	1	1	3	5
162	10	7	1	1	3	6
163	2	7	1	1	4	1
164	3	7	1	1	4	2
165	6	7	1	1	4	3
166	4	7	1	1	4	4
167	6	7	1	1	4	5
168	4	7	1	1	4	6
169	22	8	-1	-1	1	1
170	20	8	-1	-1	1	2
171	24	8	-1	-1	1	3
172	20	8	-1	-1	1	4
173	21	8	-1	-1	1	5
174	21	8	-1	-1	1	6
175	13	8	-1	-1	2	1
176	16	8	-1	-1	2	2
177	14	8	-1	-1	2	3
178	13	8	-1	-1	2	4
179	14	8	-1	-1	2	5
180	19	8	-1	-1	2	6
181	8	8	-1	-1	3	1
182	8	8	-1	-1	3	2
183	9	8	-1	-1	3	3
184	11	8	-1	-1	3	4
185	11	8	-1	-1	3	5
186	11	8	-1	-1	3	6
187	5	8	-1	-1	4	1
188	5	8	-1	-1	4	2
189	3	8	-1	-1	4	3
190	3	8	-1	-1	4	4
191	5	8	-1	-1	4	5
192	4	8	-1	-1	4	6
193	23	9	1	-1	1	1
194	19	9	1	-1	1	2
195	19	9	1	-1	1	3
196	20	9	1	-1	1	4
197	21	9	1	-1	1	5
198	24	9	1	-1	1	6
199	16	9	1	-1	2	1
200	13	9	1	-1	2	2

Obs	a	id	presentx1	biasx2	position	freq
201	14	9	1	-1	2	3
202	16	9	1	-1	2	4
203	17	9	1	-1	2	5
204	16	9	1	-1	2	6
205	8	9	1	-1	3	1
206	7	9	1	-1	3	2
207	8	9	1	-1	3	3
208	10	9	1	-1	3	4
209	11	9	1	-1	3	5
210	12	9	1	-1	3	6
211	5	9	1	-1	4	1
212	2	9	1	-1	4	2
213	3	9	1	-1	4	3
214	7	9	1	-1	4	4
215	3	9	1	-1	4	5
216	6	9	1	-1	4	6
217	18	10	-1	1	1	1
218	21	10	-1	1	1	2
219	23	10	-1	1	1	3
220	21	10	-1	1	1	4
221	23	10	-1	1	1	5
222	22	10	-1	1	1	6
223	13	10	-1	1	2	1
224	16	10	-1	1	2	2
225	19	10	-1	1	2	3
226	16	10	-1	1	2	4
227	16	10	-1	1	2	5
228	14	10	-1	1	2	6
229	9	10	-1	1	3	1
230	11	10	-1	1	3	2
231	12	10	-1	1	3	3
232	8	10	-1	1	3	4
233	9	10	-1	1	3	5
234	8	10	-1	1	3	6
235	4	10	-1	1	4	1
236	4	10	-1	1	4	2
237	3	10	-1	1	4	3
238	4	10	-1	1	4	4
239	3	10	-1	1	4	5
240	3	10	-1	1	4	6
241	23	11	1	-1	1	1
242	21	11	1	-1	1	2
243	18	11	1	-1	1	3
244	21	11	1	-1	1	4
245	21	11	1	-1	1	5
246	21	11	1	-1	1	6
247	17	11	1	-1	2	1
248	17	11	1	-1	2	2
249	16	11	1	-1	2	3
250	15	11	1	-1	2	4

Obs	a	id	presentx1	biasx2	position	freq
251	15	11	1	-1	2	5
252	13	11	1	-1	2	6
253	12	11	1	-1	3	1
254	11	11	1	-1	3	2
255	10	11	1	-1	3	3
256	10	11	1	-1	3	4
257	8	11	1	-1	3	5
258	10	11	1	-1	3	6
259	4	11	1	-1	4	1
260	2	11	1	-1	4	2
261	5	11	1	-1	4	3
262	4	11	1	-1	4	4
263	4	11	1	-1	4	5
264	2	11	1	-1	4	6
265	24	12	-1	-1	1	1
266	21	12	-1	-1	1	2
267	22	12	-1	-1	1	3
268	19	12	-1	-1	1	4
269	21	12	-1	-1	1	5
270	22	12	-1	-1	1	6
271	15	12	-1	-1	2	1
272	14	12	-1	-1	2	2
273	13	12	-1	-1	2	3
274	17	12	-1	-1	2	4
275	17	12	-1	-1	2	5
276	17	12	-1	-1	2	6
277	9	12	-1	-1	3	1
278	10	12	-1	-1	3	2
279	9	12	-1	-1	3	3
280	9	12	-1	-1	3	4
281	11	12	-1	-1	3	5
282	9	12	-1	-1	3	6
283	6	12	-1	-1	4	1
284	2	12	-1	-1	4	2
285	3	12	-1	-1	4	3
286	3	12	-1	-1	4	4
287	3	12	-1	-1	4	5
288	4	12	-1	-1	4	6
289	21	13	1	1	1	1
290	22	13	1	1	1	2
291	21	13	1	1	1	3
292	20	13	1	1	1	4
293	21	13	1	1	1	5
294	24	13	1	1	1	6
295	17	13	1	1	2	1
296	16	13	1	1	2	2
297	12	13	1	1	2	3
298	14	13	1	1	2	4
299	16	13	1	1	2	5
300	14	13	1	1	2	6

Obs	a	id	presentx1	biasx2	position	freq
301	10	13	1	1	3	1
302	11	13	1	1	3	2
303	11	13	1	1	3	3
304	9	13	1	1	3	4
305	10	13	1	1	3	5
306	10	13	1	1	3	6
307	5	13	1	1	4	1
308	3	13	1	1	4	2
309	5	13	1	1	4	3
310	3	13	1	1	4	4
311	4	13	1	1	4	5
312	1	13	1	1	4	6
313	24	14	-1	1	1	1
314	23	14	-1	1	1	2
315	22	14	-1	1	1	3
316	18	14	-1	1	1	4
317	20	14	-1	1	1	5
318	16	14	-1	1	1	6
319	15	14	-1	1	2	1
320	16	14	-1	1	2	2
321	18	14	-1	1	2	3
322	14	14	-1	1	2	4
323	14	14	-1	1	2	5
324	10	14	-1	1	2	6
325	11	14	-1	1	3	1
326	13	14	-1	1	3	2
327	9	14	-1	1	3	3
328	10	14	-1	1	3	4
329	9	14	-1	1	3	5
330	10	14	-1	1	3	6
331	6	14	-1	1	4	1
332	6	14	-1	1	4	2
333	3	14	-1	1	4	3
334	6	14	-1	1	4	4
335	4	14	-1	1	4	5
336	3	14	-1	1	4	6
337	22	15	1	-1	1	1
338	21	15	1	-1	1	2
339	20	15	1	-1	1	3
340	21	15	1	-1	1	4
341	22	15	1	-1	1	5
342	22	15	1	-1	1	6
343	14	15	1	-1	2	1
344	14	15	1	-1	2	2
345	15	15	1	-1	2	3
346	17	15	1	-1	2	4
347	16	15	1	-1	2	5
348	17	15	1	-1	2	6
349	7	15	1	-1	3	1
350	8	15	1	-1	3	2

Obs	a	id	presentx1	biasx2	position	freq
351	9	15	1	-1	3	3
352	11	15	1	-1	3	4
353	11	15	1	-1	3	5
354	12	15	1	-1	3	6
355	3	15	1	-1	4	1
356	2	15	1	-1	4	2
357	3	15	1	-1	4	3
358	4	15	1	-1	4	4
359	5	15	1	-1	4	5
360	4	15	1	-1	4	6
361	20	16	-1	1	1	1
362	18	16	-1	1	1	2
363	17	16	-1	1	1	3
364	19	16	-1	1	1	4
365	21	16	-1	1	1	5
366	24	16	-1	1	1	6
367	12	16	-1	1	2	1
368	12	16	-1	1	2	2
369	13	16	-1	1	2	3
370	15	16	-1	1	2	4
371	20	16	-1	1	2	5
372	22	16	-1	1	2	6
373	6	16	-1	1	3	1
374	4	16	-1	1	3	2
375	5	16	-1	1	3	3
376	15	16	-1	1	3	4
377	13	16	-1	1	3	5
378	11	16	-1	1	3	6
379	2	16	-1	1	4	1
380	2	16	-1	1	4	2
381	7	16	-1	1	4	3
382	7	16	-1	1	4	4
383	7	16	-1	1	4	5
384	8	16	-1	1	4	6
385	21	17	1	-1	1	1
386	20	17	1	-1	1	2
387	18	17	1	-1	1	3
388	19	17	1	-1	1	4
389	23	17	1	-1	1	5
390	22	17	1	-1	1	6
391	13	17	1	-1	2	1
392	13	17	1	-1	2	2
393	14	17	1	-1	2	3
394	18	17	1	-1	2	4
395	17	17	1	-1	2	5
396	18	17	1	-1	2	6
397	7	17	1	-1	3	1
398	7	17	1	-1	3	2
399	8	17	1	-1	3	3
400	12	17	1	-1	3	4

Obs	a	id	presentx1	biasx2	position	freq
401	12	17	1	-1	3	5
402	13	17	1	-1	3	6
403	3	17	1	-1	4	1
404	6	17	1	-1	4	2
405	5	17	1	-1	4	3
406	5	17	1	-1	4	4
407	3	17	1	-1	4	5
408	3	17	1	-1	4	6
409	23	18	-1	-1	1	1
410	21	18	-1	-1	1	2
411	19	18	-1	-1	1	3
412	20	18	-1	-1	1	4
413	23	18	-1	-1	1	5
414	23	18	-1	-1	1	6
415	15	18	-1	-1	2	1
416	14	18	-1	-1	2	2
417	13	18	-1	-1	2	3
418	16	18	-1	-1	2	4
419	18	18	-1	-1	2	5
420	17	18	-1	-1	2	6
421	8	18	-1	-1	3	1
422	7	18	-1	-1	3	2
423	8	18	-1	-1	3	3
424	11	18	-1	-1	3	4
425	9	18	-1	-1	3	5
426	8	18	-1	-1	3	6
427	2	18	-1	-1	4	1
428	2	18	-1	-1	4	2
429	5	18	-1	-1	4	3
430	5	18	-1	-1	4	4
431	8	18	-1	-1	4	5
432	5	18	-1	-1	4	6
433	19	19	1	-1	1	1
434	20	19	1	-1	1	2
435	22	19	1	-1	1	3
436	23	19	1	-1	1	4
437	22	19	1	-1	1	5
438	23	19	1	-1	1	6
439	14	19	1	-1	2	1
440	12	19	1	-1	2	2
441	16	19	1	-1	2	3
442	17	19	1	-1	2	4
443	16	19	1	-1	2	5
444	17	19	1	-1	2	6
445	5	19	1	-1	3	1
446	5	19	1	-1	3	2
447	8	19	1	-1	3	3
448	11	19	1	-1	3	4
449	11	19	1	-1	3	5
450	10	19	1	-1	3	6

Obs	a	id	presentx1	biasx2	position	freq
451	5	19	1	-1	4	1
452	3	19	1	-1	4	2
453	6	19	1	-1	4	3
454	7	19	1	-1	4	4
455	7	19	1	-1	4	5
456	1	19	1	-1	4	6
457	21	20	-1	1	1	1
458	19	20	-1	1	1	2
459	19	20	-1	1	1	3
460	19	20	-1	1	1	4
461	23	20	-1	1	1	5
462	21	20	-1	1	1	6
463	16	20	-1	1	2	1
464	14	20	-1	1	2	2
465	14	20	-1	1	2	3
466	15	20	-1	1	2	4
467	18	20	-1	1	2	5
468	19	20	-1	1	2	6
469	9	20	-1	1	3	1
470	8	20	-1	1	3	2
471	10	20	-1	1	3	3
472	7	20	-1	1	3	4
473	8	20	-1	1	3	5
474	10	20	-1	1	3	6
475	6	20	-1	1	4	1
476	3	20	-1	1	4	2
477	6	20	-1	1	4	3
478	4	20	-1	1	4	4
479	3	20	-1	1	4	5
480	8	20	-1	1	4	6
481	21	21	1	1	1	1
482	20	21	1	1	1	2
483	19	21	1	1	1	3
484	19	21	1	1	1	4
485	23	21	1	1	1	5
486	24	21	1	1	1	6
487	14	21	1	1	2	1
488	15	21	1	1	2	2
489	12	21	1	1	2	3
490	17	21	1	1	2	4
491	18	21	1	1	2	5
492	19	21	1	1	2	6
493	8	21	1	1	3	1
494	7	21	1	1	3	2
495	8	21	1	1	3	3
496	11	21	1	1	3	4
497	10	21	1	1	3	5
498	12	21	1	1	3	6
499	3	21	1	1	4	1
500	3	21	1	1	4	2

Obs	a	id	presentx1	biasx2	position	freq
501	2	21	1	1	4	3
502	4	21	1	1	4	4
503	4	21	1	1	4	5
504	7	21	1	1	4	6
505	23	22	-1	1	1	1
506	21	22	-1	1	1	2
507	20	22	-1	1	1	3
508	19	22	-1	1	1	4
509	22	22	-1	1	1	5
510	23	22	-1	1	1	6
511	15	22	-1	1	2	1
512	14	22	-1	1	2	2
513	14	22	-1	1	2	3
514	16	22	-1	1	2	4
515	16	22	-1	1	2	5
516	16	22	-1	1	2	6
517	8	22	-1	1	3	1
518	8	22	-1	1	3	2
519	7	22	-1	1	3	3
520	11	22	-1	1	3	4
521	7	22	-1	1	3	5
522	9	22	-1	1	3	6
523	5	22	-1	1	4	1
524	7	22	-1	1	4	2
525	6	22	-1	1	4	3
526	6	22	-1	1	4	4
527	4	22	-1	1	4	5
528	3	22	-1	1	4	6
529	23	23	1	-1	1	1
530	23	23	1	-1	1	2
531	23	23	1	-1	1	3
532	21	23	1	-1	1	4
533	17	23	1	-1	1	5
534	18	23	1	-1	1	6
535	16	23	1	-1	2	1
536	18	23	1	-1	2	2
537	15	23	1	-1	2	3
538	14	23	1	-1	2	4
539	12	23	1	-1	2	5
540	17	23	1	-1	2	6
541	14	23	1	-1	3	1
542	14	23	1	-1	3	2
543	8	23	1	-1	3	3
544	8	23	1	-1	3	4
545	8	23	1	-1	3	5
546	9	23	1	-1	3	6
547	7	23	1	-1	4	1
548	5	23	1	-1	4	2
549	4	23	1	-1	4	3
550	2	23	1	-1	4	4

Obs	a	id	presentx1	biasx2	position	freq
551	3	23	1	-1	4	5
552	1	23	1	-1	4	6
553	21	24	-1	1	1	1
554	21	24	-1	1	1	2
555	20	24	-1	1	1	3
556	19	24	-1	1	1	4
557	24	24	-1	1	1	5
558	22	24	-1	1	1	6
559	15	24	-1	1	2	1
560	12	24	-1	1	2	2
561	15	24	-1	1	2	3
562	14	24	-1	1	2	4
563	18	24	-1	1	2	5
564	17	24	-1	1	2	6
565	10	24	-1	1	3	1
566	11	24	-1	1	3	2
567	9	24	-1	1	3	3
568	10	24	-1	1	3	4
569	11	24	-1	1	3	5
570	9	24	-1	1	3	6
571	2	24	-1	1	4	1
572	3	24	-1	1	4	2
573	6	24	-1	1	4	3
574	3	24	-1	1	4	4
575	3	24	-1	1	4	5
576	5	24	-1	1	4	6
577	23	25	1	-1	1	1
578	21	25	1	-1	1	2
579	20	25	1	-1	1	3
580	19	25	1	-1	1	4
581	22	25	1	-1	1	5
582	24	25	1	-1	1	6
583	16	25	1	-1	2	1
584	14	25	1	-1	2	2
585	14	25	1	-1	2	3
586	15	25	1	-1	2	4
587	18	25	1	-1	2	5
588	15	25	1	-1	2	6
589	11	25	1	-1	3	1
590	9	25	1	-1	3	2
591	9	25	1	-1	3	3
592	10	25	1	-1	3	4
593	11	25	1	-1	3	5
594	8	25	1	-1	3	6
595	2	25	1	-1	4	1
596	5	25	1	-1	4	2
597	4	25	1	-1	4	3
598	5	25	1	-1	4	4
599	3	25	1	-1	4	5
600	2	25	1	-1	4	6

Obs	a	id	presentx1	biasx2	position	freq
601	20	26	-1	1	1	1
602	19	26	-1	1	1	2
603	22	26	-1	1	1	3
604	17	26	-1	1	1	4
605	21	26	-1	1	1	5
606	21	26	-1	1	1	6
607	16	26	-1	1	2	1
608	14	26	-1	1	2	2
609	11	26	-1	1	2	3
610	16	26	-1	1	2	4
611	17	26	-1	1	2	5
612	20	26	-1	1	2	6
613	11	26	-1	1	3	1
614	7	26	-1	1	3	2
615	6	26	-1	1	3	3
616	11	26	-1	1	3	4
617	11	26	-1	1	3	5
618	8	26	-1	1	3	6
619	6	26	-1	1	4	1
620	2	26	-1	1	4	2
621	7	26	-1	1	4	3
622	6	26	-1	1	4	4
623	7	26	-1	1	4	5
624	4	26	-1	1	4	6
625	21	27	1	-1	1	1
626	20	27	1	-1	1	2
627	19	27	1	-1	1	3
628	18	27	1	-1	1	4
629	21	27	1	-1	1	5
630	24	27	1	-1	1	6
631	17	27	1	-1	2	1
632	12	27	1	-1	2	2
633	11	27	1	-1	2	3
634	15	27	1	-1	2	4
635	21	27	1	-1	2	5
636	18	27	1	-1	2	6
637	8	27	1	-1	3	1
638	9	27	1	-1	3	2
639	8	27	1	-1	3	3
640	12	27	1	-1	3	4
641	11	27	1	-1	3	5
642	13	27	1	-1	3	6
643	3	27	1	-1	4	1
644	1	27	1	-1	4	2
645	4	27	1	-1	4	3
646	5	27	1	-1	4	4
647	3	27	1	-1	4	5
648	6	27	1	-1	4	6
649	21	28	1	1	1	1
650	19	28	1	1	1	2

Obs	a	id	presentx1	biasx2	position	freq
651	19	28	1	1	1	3
652	20	28	1	1	1	4
653	23	28	1	1	1	5
654	24	28	1	1	1	6
655	14	28	1	1	2	1
656	15	28	1	1	2	2
657	15	28	1	1	2	3
658	15	28	1	1	2	4
659	19	28	1	1	2	5
660	18	28	1	1	2	6
661	9	28	1	1	3	1
662	8	28	1	1	3	2
663	9	28	1	1	3	3
664	11	28	1	1	3	4
665	9	28	1	1	3	5
666	11	28	1	1	3	6
667	1	28	1	1	4	1
668	4	28	1	1	4	2
669	4	28	1	1	4	3
670	3	28	1	1	4	4
671	4	28	1	1	4	5
672	5	28	1	1	4	6
673	20	29	-1	-1	1	1
674	22	29	-1	-1	1	2
675	19	29	-1	-1	1	3
676	21	29	-1	-1	1	4
677	23	29	-1	-1	1	5
678	24	29	-1	-1	1	6
679	14	29	-1	-1	2	1
680	14	29	-1	-1	2	2
681	14	29	-1	-1	2	3
682	16	29	-1	-1	2	4
683	17	29	-1	-1	2	5
684	18	29	-1	-1	2	6
685	7	29	-1	-1	3	1
686	8	29	-1	-1	3	2
687	9	29	-1	-1	3	3
688	10	29	-1	-1	3	4
689	11	29	-1	-1	3	5
690	12	29	-1	-1	3	6
691	2	29	-1	-1	4	1
692	3	29	-1	-1	4	2
693	5	29	-1	-1	4	3
694	2	29	-1	-1	4	4
695	3	29	-1	-1	4	5
696	6	29	-1	-1	4	6
697	23	30	-1	-1	1	1
698	20	30	-1	-1	1	2
699	21	30	-1	-1	1	3
700	20	30	-1	-1	1	4

Obs	a	id	presentx1	biasx2	position	freq
701	21	30	-1	-1	1	5
702	24	30	-1	-1	1	6
703	13	30	-1	-1	2	1
704	14	30	-1	-1	2	2
705	15	30	-1	-1	2	3
706	16	30	-1	-1	2	4
707	17	30	-1	-1	2	5
708	18	30	-1	-1	2	6
709	6	30	-1	-1	3	1
710	8	30	-1	-1	3	2
711	10	30	-1	-1	3	3
712	10	30	-1	-1	3	4
713	10	30	-1	-1	3	5
714	12	30	-1	-1	3	6
715	4	30	-1	-1	4	1
716	3	30	-1	-1	4	2
717	2	30	-1	-1	4	3
718	5	30	-1	-1	4	4
719	3	30	-1	-1	4	5
720	5	30	-1	-1	4	6
721	17	31	1	1	1	1
722	20	31	1	1	1	2
723	22	31	1	1	1	3
724	24	31	1	1	1	4
725	23	31	1	1	1	5
726	20	31	1	1	1	6
727	12	31	1	1	2	1
728	16	31	1	1	2	2
729	18	31	1	1	2	3
730	13	31	1	1	2	4
731	14	31	1	1	2	5
732	13	31	1	1	2	6
733	9	31	1	1	3	1
734	11	31	1	1	3	2
735	13	31	1	1	3	3
736	7	31	1	1	3	4
737	9	31	1	1	3	5
738	8	31	1	1	3	6
739	5	31	1	1	4	1
740	8	31	1	1	4	2
741	3	31	1	1	4	3
742	4	31	1	1	4	4
743	5	31	1	1	4	5
744	6	31	1	1	4	6
745	22	32	1	1	1	1
746	18	32	1	1	1	2
747	20	32	1	1	1	3
748	21	32	1	1	1	4
749	22	32	1	1	1	5
750	24	32	1	1	1	6

Obs	a	id	presentx1	biasx2	position	freq
751	15	32	1	1	2	1
752	13	32	1	1	2	2
753	13	32	1	1	2	3
754	15	32	1	1	2	4
755	17	32	1	1	2	5
756	17	32	1	1	2	6
757	7	32	1	1	3	1
758	7	32	1	1	3	2
759	9	32	1	1	3	3
760	10	32	1	1	3	4
761	14	32	1	1	3	5
762	13	32	1	1	3	6
763	3	32	1	1	4	1
764	1	32	1	1	4	2
765	4	32	1	1	4	3
766	4	32	1	1	4	4
767	4	32	1	1	4	5
768	7	32	1	1	4	6
769	19	33	-1	1	1	1
770	15	33	-1	1	1	2
771	13	33	-1	1	1	3
772	13	33	-1	1	1	4
773	24	33	-1	1	1	5
774	23	33	-1	1	1	6
775	16	33	-1	1	2	1
776	11	33	-1	1	2	2
777	9	33	-1	1	2	3
778	19	33	-1	1	2	4
779	22	33	-1	1	2	5
780	18	33	-1	1	2	6
781	10	33	-1	1	3	1
782	9	33	-1	1	3	2
783	7	33	-1	1	3	3
784	15	33	-1	1	3	4
785	17	33	-1	1	3	5
786	17	33	-1	1	3	6
787	2	33	-1	1	4	1
788	3	33	-1	1	4	2
789	3	33	-1	1	4	3
790	6	33	-1	1	4	4
791	4	33	-1	1	4	5
792	5	33	-1	1	4	6
793	24	34	1	-1	1	1
794	22	34	1	-1	1	2
795	17	34	1	-1	1	3
796	20	34	1	-1	1	4
797	21	34	1	-1	1	5
798	23	34	1	-1	1	6
799	17	34	1	-1	2	1
800	14	34	1	-1	2	2

Obs	a	id	presentx1	biasx2	position	freq
801	14	34	1	-1	2	3
802	16	34	1	-1	2	4
803	15	34	1	-1	2	5
804	18	34	1	-1	2	6
805	10	34	1	-1	3	1
806	8	34	1	-1	3	2
807	5	34	1	-1	3	3
808	11	34	1	-1	3	4
809	9	34	1	-1	3	5
810	10	34	1	-1	3	6
811	4	34	1	-1	4	1
812	4	34	1	-1	4	2
813	4	34	1	-1	4	3
814	4	34	1	-1	4	4
815	6	34	1	-1	4	5
816	4	34	1	-1	4	6
817	20	35	-1	-1	1	1
818	20	35	-1	-1	1	2
819	18	35	-1	-1	1	3
820	18	35	-1	-1	1	4
821	23	35	-1	-1	1	5
822	24	35	-1	-1	1	6
823	13	35	-1	-1	2	1
824	11	35	-1	-1	2	2
825	14	35	-1	-1	2	3
826	18	35	-1	-1	2	4
827	18	35	-1	-1	2	5
828	19	35	-1	-1	2	6
829	6	35	-1	-1	3	1
830	4	35	-1	-1	3	2
831	5	35	-1	-1	3	3
832	13	35	-1	-1	3	4
833	12	35	-1	-1	3	5
834	14	35	-1	-1	3	6
835	6	35	-1	-1	4	1
836	4	35	-1	-1	4	2
837	6	35	-1	-1	4	3
838	3	35	-1	-1	4	4
839	5	35	-1	-1	4	5
840	6	35	-1	-1	4	6

The Mixed Procedure

Model Information

Data Set	WORK.FORMIXED
Dependent Variable	a
Covariance Structure	Unstructured @ Compound Symmetry
Subject Effect	id
Estimation Method	REML
Residual Variance Method	None
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

Class Level Information

Class	Levels	Values
position	4	1 2 3 4
freq	6	1 2 3 4 5 6
id	35	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

Dimensions

Covariance Parameters	11
Columns in X	175
Columns in Z	0
Subjects	35
Max Obs Per Subject	24

Number of Observations

Number of Observations Read	840
Number of Observations Used	840
Number of Observations Not Used	0

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	3374.50363927	
1	2	3228.58901651	0.00004587
2	1	3228.54394187	0.00000007
3	1	3228.54387456	0.00000000

The Mixed Procedure

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
position UN(1,1)	id	3.3409
UN(2,1)	id	1.7727
UN(2,2)	id	4.4319
UN(3,1)	id	1.5274
UN(3,2)	id	2.9249
UN(3,3)	id	5.3390
UN(4,1)	id	0.4461
UN(4,2)	id	0.7836
UN(4,3)	id	1.0920
UN(4,4)	id	2.6841
freq Corr	id	0.06810

Fit Statistics

-2 Res Log Likelihood	3228.5
AIC (smaller is better)	3250.5
AICC (smaller is better)	3250.9
BIC (smaller is better)	3267.7

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
10	145.96	<.0001

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
position	3	102	3070.84	<.0001
freq	5	170	16.44	<.0001
position*freq	15	510	6.78	<.0001
presentx1	0	.	.	.
presentx1*position	3	710	0.73	0.5323
presentx1*freq	5	710	0.18	0.9716
present*positio*freq	15	710	1.18	0.2780
biasx2	0	.	.	.
biasx2*position	3	710	1.91	0.1262
biasx2*freq	5	710	0.33	0.8919

The Mixed Procedure

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
biasx2*position*freq	15	710	0.95	0.5021
presentx1*biasx2	0	.	.	.
presen*biasx2*positi	3	710	1.64	0.1788
presentx*biasx2*freq	5	710	1.69	0.1353
pres*bias*posit*freq	15	710	0.94	0.5223
id	31	3	0.30	0.9684

Least Squares Means

Effect	position	freq	Estimate	Standard Error	DF	t Value	Pr > |t|
position	1		20.9381	0.1460	102	143.38	<.0001
position	2		15.3238	0.1682	102	91.11	<.0001
position	3		9.5095	0.1846	102	51.51	<.0001
position	4		4.2286	0.1309	102	32.31	<.0001
freq		1	12.1286	0.2423	170	50.05	<.0001
freq		2	11.4357	0.2423	170	47.19	<.0001
freq		3	11.6500	0.2423	170	48.07	<.0001
freq		4	12.5857	0.2423	170	51.93	<.0001
freq		5	13.4286	0.2423	170	55.41	<.0001
freq		6	13.7714	0.2423	170	56.83	<.0001

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Estimate	Standard Error	DF	t Value
position	1		2		5.6143	0.1643	102	34.18
position	1		3		11.4286	0.1895	102	60.31
position	1		4		16.7095	0.1810	102	92.31
position	2		3		5.8143	0.1582	102	36.75

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Pr > |t|	Adjustment	Adj P
position	1		2		<.0001	Scheffe	<.0001
position	1		3		<.0001	Scheffe	<.0001
position	1		4		<.0001	Scheffe	<.0001
position	2		3		<.0001	Scheffe	<.0001

The Mixed Procedure

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Estimate	Standard Error	DF	t Value
position	2		4		11.0952	0.1882	102	58.95
position	3		4		5.2810	0.1931	102	27.35
freq		1		2	0.6929	0.3309	170	2.09
freq		1		3	0.4786	0.3309	170	1.45
freq		1		4	-0.4571	0.3309	170	-1.38
freq		1		5	-1.3000	0.3309	170	-3.93
freq		1		6	-1.6429	0.3309	170	-4.97
freq		2		3	-0.2143	0.3309	170	-0.65
freq		2		4	-1.1500	0.3309	170	-3.48
freq		2		5	-1.9929	0.3309	170	-6.02
freq		2		6	-2.3357	0.3309	170	-7.06
freq		3		4	-0.9357	0.3309	170	-2.83
freq		3		5	-1.7786	0.3309	170	-5.38
freq		3		6	-2.1214	0.3309	170	-6.41
freq		4		5	-0.8429	0.3309	170	-2.55
freq		4		6	-1.1857	0.3309	170	-3.58
freq		5		6	-0.3429	0.3309	170	-1.04

Differences of Least Squares Means

Effect	position	freq	_position	_freq	Pr > |t|	Adjustment	Adj P
position	2		4		<.0001	Scheffe	<.0001
position	3		4		<.0001	Scheffe	<.0001
freq		1		2	0.0377	Scheffe	0.4977
freq		1		3	0.1499	Scheffe	0.8355
freq		1		4	0.1689	Scheffe	0.8608
freq		1		5	0.0001	Scheffe	0.0108
freq		1		6	<.0001	Scheffe	0.0003
freq		2		3	0.5181	Scheffe	0.9947
freq		2		4	0.0006	Scheffe	0.0380
freq		2		5	<.0001	Scheffe	<.0001
freq		2		6	<.0001	Scheffe	<.0001
freq		3		4	0.0052	Scheffe	0.1627
freq		3		5	<.0001	Scheffe	<.0001
freq		3		6	<.0001	Scheffe	<.0001
freq		4		5	0.0117	Scheffe	0.2670
freq		4		6	0.0004	Scheffe	0.0287
freq		5		6	0.3015	Scheffe	0.9559

The MEANS Procedure

Variable	Mean	N	Std Dev

a1	21.3142857	35	1.7110614
a2	20.2857143	35	1.7076201
a3	19.8285714	35	2.0649130
a4	19.7142857	35	1.8875977
a5	21.9714286	35	1.3169866
a6	22.5142857	35	1.8370601
a7	14.8000000	35	1.4712939
a8	13.8857143	35	1.7784896
a9	13.8857143	35	2.1797386
a10	15.5142857	35	1.5600043
a11	16.8000000	35	2.1115927
a12	17.0571429	35	2.2873419
a13	8.6000000	35	2.0891879
a14	8.0571429	35	2.3507553
a15	8.4571429	35	1.8840329
a16	10.5142857	35	1.8688063
a17	10.5428571	35	2.0050357
a18	10.8857143	35	2.3107349
a19	3.8000000	35	1.6591990
a20	3.5142857	35	1.6155754
a21	4.4285714	35	1.3992795
a22	4.6000000	35	1.5185132
a23	4.4000000	35	1.3762695
a24	4.6285714	35	2.0448747