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Can Contracts Replace Qualification in a Sourcing Process With Competitive Suppliers and Imperfect Information?

Yue Jin and Jennifer K. Ryan

Abstract—This paper considers a manufacturer who outsources the production of a product to multiple competing suppliers, who differ in their cost structures and in their capabilities for producing high-quality products. The manufacturer must design the sourcing process to ensure that the selected supplier has sufficient quality capability, while encouraging competition among the suppliers. We develop and analyze a mathematical model of performancebased contracting, a sourcing method that is appropriate when the manufacturer has imperfect information regarding the suppliers' costs and capabilities. We compare the performance of performance-based contracting with that of a two-stage sourcing process, an alternative sourcing method that is more commonly used in practice. The theoretical results and managerial insights derived from this research can enable manufacturing firms to improve the management of their sourcing processes. In particular, we demonstrate that performance-based contracting with a symmetric linear penalty/reward function will always outperform the two-stage sourcing process from the perspective of the buyer and that the optimal penalty/reward rate is less than or equal to the unit warranty cost. In addition, performance-based contracting generally leads to a higher quality level provided by the winning supplier. However, the winning supplier is generally better off under the two-stage sourcing process.

Index Terms—Auction mechanisms, procurement, sourcing.

I. INTRODUCTION AND MOTIVATION

R ECENT years have seen an increase in the use of outsourcing in a variety of industries, with the goal of reducing costs and obtaining operational efficiencies [1]. As a result, manufacturers must make strategic decisions regarding the design of their sourcing processes, including how to qualify potential suppliers and how to select among the set of potential suppliers. Recent years have also seen an increase in the use of procurement auctions as part of the sourcing process [2]. Auctions can induce competitive bidding, resulting in increased competition between suppliers and reduced procurement costs [3].

Sourcing process design, including the design of auctionbased mechanisms, can be complex due to a number of

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factors. First, manufacturers often care about attributes besides just price, including quality, reliability, and payment terms. A second source of complexity arises when the manufacturer has imperfect information regarding the suppliers' characteristics. Finally, sourcing process design is complex due to the fact that the nature of that process can influence the product characteristics or attributes offered by the suppliers. Suppliers choose their bidding strategies, including how they set the nonprice attributes, such as product quality, in order to maximize their chance of winning the buyer's business, while simultaneously minimizing their costs. Thus, how the buyer evaluates bids will influence the suppliers' bidding strategies and product offerings.

A. Multiattribute Procurement Mechanisms

There are a number of alternatives for selecting among suppliers when the buyer cares about multiple attributes. Engelbrecht-Wiggans et al. [4] provide the following categorization. Under request for quotation (RFQ) and request for proposal (RFP), the buyer provides detailed specifications to the suppliers. Under RFQ, the suppliers are required to meet the specifications and the buyer selects among those suppliers based on cost. Under RFP, the suppliers submit proposals which are evaluated by the buyer, with the contract awarded to the best overall supplier, as determined by the buyer, perhaps through the use of a score function. Reverse auctions are "structured" versions of the RFQ and RFP mechanisms. Price-based (PB) reverse auctions are analogous to an RFQ. They are binding, i.e., the buyer commits in selecting the supplier with the lowest bid price. Buyer-determined (BD) reverse auctions are analogous to an RFP. They are not binding, i.e., the buyer can select the winning supplier as she sees fit. Multiattribute auctions allow suppliers to bid on multiple dimensions, rather than just price. The buyer evaluates bids using a score function which converts the bid into a single number. These auctions are not widely used in practice [5], [6].

Additional mechanisms for selecting among suppliers with multiple attributes include a two-stage sourcing process (TSP) and performance-based contracting (PBC). In a TSP, the first stage is the qualification stage, in which the potential suppliers are screened for various nonprice capabilities. The second stage is the supplier selection stage, in which the qualified suppliers are invited to compete in a price-only procurement auction [7], [8]. Under PBC, the suppliers participate in a priceonly reverse auction. After the contract is awarded, the winning supplier is assessed a penalty (or reward) based on his

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realized performance. PBC is useful when the suppliers' nonprice attributes are unknown to the buyer at the time of bidding [9]. PBC is also referred to as a fixed price auction with incentives [10].

Given the growth in the use of procurement auctions, there has been significant interest in comparing the performance of these alternative approaches, both theoretically and empirically. Engelbrecht-Wiggans *et al.* [4] compare RFQ (or PB auctions) with RFP (or BD auctions) and find that the buyer's choice will depend on the number of potential suppliers, as well as the correlation between the cost and the nonprice attribute. Chang [9] uses behavioral experiments to compare PBC, PB auctions, and TSP and finds that PBC provides a higher surplus to the buyer.

B. Problem Statement and Related Literature

We study a manufacturer who outsources the production of a product to competing suppliers, who differ in their cost structures, as well as in their capabilities for producing high-quality products. The manufacturer has imperfect information regarding the suppliers' costs and quality capabilities. In contrast to much of the literature, we model the suppliers' quality endogenously, i.e., the suppliers select their quality based on the incentives induced by the sourcing process, as designed by the buyer, subject to their exogenously specified quality capabilities. We study how the manufacturer can design the sourcing process to 1) ensure that the selected supplier has sufficient quality capability to produce a high-quality product, and 2) encourage competition among the potential suppliers in order to reduce procurement costs. In addition, we consider how imperfect information regarding the suppliers' quality capabilities can be incorporated into procurement process design in practice.

We do not focus on the design of a new procurement mechanism. Instead, we compare the performance of two mechanisms that are commonly used in practice: 1) TSP; and 2) PBC. Both mechanisms consider the suppliers' quality capabilities, but still allow supplier selection to be conducted using a price-only procurement auction. In addition, both TSP and PBC are appropriate when, initially, there is uncertainty regarding the suppliers' quality capabilities. PBC assesses the penalty/reward after the bidding process, when the winning supplier's quality level is realized, while TSP involves a prebidding qualification stage in which the buyer learns about the suppliers' quality capabilities. Despite the similarities between TSP and PBC, there are some differences.

- TSP has been more widely used, including in defense procurement [11], as well as in the telecommunications [7], [8], pharmaceutical [12], and automotive [13] industries. On the other hand, PBC is commonly used in federal and state government procurement [9], [10].
- 2) Previous literature [7], [8] has demonstrated that TSP has a significant limitation: because supplier selection is based only on price, TSP provides no incentive for the suppliers to offer quality levels beyond the manufacturer's specification. In contrast, PBC offers the suppliers a penalty/reward based on their realized performance relative to a quality target. Thus, if properly designed, PBC can be used to

induce the suppliers to offer quality levels that exceed the target.

- 3) TSP has a qualification stage in which quality is assessed prior to supplier selection. Under PBC, quality is evaluated after the contract is awarded, based on realized performance. This implies that any quality problems may be discovered when it is too late to make adjustments. Thus, poor quality product may reach the consumer, damaging the firm's reputation. Although the qualification process (under TSP) and the potential for damaged reputation (under PBC) are both costly, because these costs are difficult to quantify, we do not include them in our analysis.
- 4) TSP corresponds to the traditional organizational structure of many manufacturing firms. The qualification process (involving the production or engineering departments) is decoupled from supplier selection (involving the procurement department). Thus, decisions regarding quality capabilities are separated from decisions regarding pricing. In contrast, PBC requires joint decision making, i.e., the departments must collaborate in setting an appropriate penalty/reward rate.

Our main contribution relative to the previous work on TSP [7], [8] is the introduction of a new model of PBC, for which we provide analytical results, as well as a detailed sensitivity analysis, to understand which input parameters have the most impact on the buyer's performance. In addition, we perform analytical and numerical comparisons of PBC and TSP from the perspective of the buyer and the suppliers, with the goal of understanding why TSP is more commonly used in practice.

Chang [9] also compares TSP and PBC. However, there are a number of differences between that work and this paper. First, the analysis in [9] is largely empirical. In contrast, our analysis applies analytical models, coupled with simulation. Second, [9] takes the suppliers' quality levels as exogenous. Our analysis considers endogenous quality. This distinction is critical given that PBC is a mechanism used to encourage the suppliers to offer higher levels of quality. Finally, [9] does not link the suppliers' unit costs to their quality levels. In contrast, in our analysis, we assume that the suppliers' production costs are increasing and convex in their quality levels. Gupta and Chen [10] consider the design of incentive functions, which are analogous to designing a PBC mechanism. However, most of their analysis considers exogenously specified supplier quality, they do not consider limited supplier capabilities, and they do not compare the performance of PBC and TSP.

There has been previous work comparing PB and BD mechanisms [4], [14]. Our research has some similarities to this previous work. The TSP is similar to a PB mechanism, since while PBC is similar to a BD mechanism. However, our work differs from this previous work by assuming endogenous supplier quality.

There has been some previous work on procurement auctions with endogenous quality. For example, Branco [15] and Che [16] consider multiattribute procurement auctions with endogenous quality and derive the optimal score function for the buyer. However, our model differs from this previous work in that the suppliers choose their quality levels subject to exogenously specified and heterogeneous quality capabilities, which significantly complicates the analysis.

Our work also differs from much of the previous literature by considering a more complex, but more realistic, cost structure for the suppliers. Much of the literature on auctions with quality considerations assumes the suppliers' unit costs are independent of their quality level [2], [9], [14]. Engelbrecht-Wiggans *et al.* [4] allow for a correlation between the suppliers' costs and qualities. However, the relationship between cost and quality is assumed to be linear. In contrast, our unit production cost is increasing and convex in quality.

C. Contributions and Managerial Insights

This research extends the literature on sourcing process design to compare the performance of two mechanisms that are appropriate when the buyer has uncertainty regarding the suppliers' quality capabilities: TSP and PBC. We do so using a model that captures several complexities not considered in the existing literature. In particular, we assume the suppliers endogenously determine their quality levels, in response to the sourcing process design, subject to exogenously specified heterogeneous quality capabilities, which are (initially) unknown to the buyer. In addition, we explicitly model how the suppliers' unit production costs vary as a function of their endogenous quality levels using a heterogeneous production cost function that is increasing and convex in quality. Our models allow us to compare the performance of TSP and PBC to determine conditions under which each is preferred by the buyer. While, as will be seen, PBC outperforms TSP from the perspective of the buyer, TSP is much more widely used in practice, for the reasons outlined in Section I-B. Thus, our analysis focuses on understanding the magnitude of the performance gap between TSP and PBC and identifying conditions under which using PBC provides the most value to the buyer, relative to TSP.

We find that PBC is most beneficial for buyers who faces significant uncertainty regarding the suppliers' costs, and for whom maintaining a high level of quality is critical. We also find that the final delivered quality is generally higher under PBC than under TSP and that the gap between the quality levels is largest when there is more uncertainty regarding the potential suppliers' costs and when the number of potential suppliers and the unit warranty cost are small. Given that the buyer prefers PBC to TSP, we also provide guidance regarding how the buyer should set the penalty/reward rate under PBC. We find that the optimal rate is always less than the unit warranty cost and that the optimal rate is largest when the number of potential suppliers and the unit warranty cost are large. Finally, while the buyer always prefers PBC, the winning supplier is generally better-off under TSP. The supplier's preference for TSP is strongest when the number of suppliers is large, the uncertainty in the suppliers' costs is small, and the unit warranty cost is large.

II. TSP AND PBC: MODELS AND COMPARISON

We consider a buyer (she) who sells a commodity-like item to consumers at a fixed price. The buyer can purchase the item from any of n potential suppliers (he, denoted by i = 1, ..., n), who

TABLE I LIST OF NOTATION

n	Number of potential suppliers, indexed by $i = 1, 2, \ldots, n$.
Q	Quality threshold under TSP and PBC, a decision variable for the buyer.
m(Q)	Number of qualified suppliers under TSP given threshold Q .
$C_w(q)$	Buyer's quality cost function (cost of a single unit of product with
- (1)	quality level q).
w	Unit warranty cost in a linear quality cost function, i.e.,
	$C_w(q) = w(1-q).$
$C_{p}(Q)$	Buyer's unit procurement cost under TSP given qualification threshold
-	Q.
C_B	Buyer's total cost function.
$H(q_i; Q)$	Penalty/reward function used by buyer under PBC,
	$H(q_i; Q) = h(q_i - Q).$
h	Penalty/reward rate under PBC.
p_i	Supplier i 's bidding price, a decision variable for supplier i .
q_i	Supplier i's quality level, a decision variable for supplier i, where
	$0 \le q_i \le 1.$
q_i^{\max}	Maximum achievable quality level of supplier $i, q_i^{\max} \sim$
	$\text{Uniform}(q_L, q_H), q_H \leq 1.$
$f_q\left(\cdot\right)$	Probability density function for q_i^{max} .
$\hat{q}_{i}\left(h ight)$	Supplier <i>i</i> 's optimal unconstrained quality level under PBC given h .
$q_i^*(h)$	Supplier i 's optimal constrained quality level under PBC given h ,
	$q_i^*(h) = \min\{\hat{q}_i, q_i^{\max}\}.$
$E[q^*]$	Expected delivered quality by the winning supplier under PBC.
c_i	Unit production cost for supplier <i>i</i> to produce a unit with $q_i = 1, c_i \sim$
	$Uniform(c_L, c_H).$
C(i)	<i>i</i> th smallest value in a random sample of size n from the distribution of
	c_i .
z	Constant parameter in the suppliers' production cost function, $c_i q_i^2$.
$C_{\rm Si}(q_i)$	Supplier <i>i</i> 's quality-related costs under PBC, C_{i} (c_{i}) C_{i} (c_{i})
*	$\bigcup_{i} (q_i) = c_i q_i^* - H(q_i; Q).$
π_i, π_S	Supplier <i>i</i> s profit, the winning supplier's optimal profit.
C_T	Iotal system cost, $C_T = C_B - \pi_S$.

differ in their capabilities for producing high-quality products, as well as in their production costs. The buyer has chosen to singlesource and will select a single supplier from the set of potential suppliers. We normalize the buyer's volume (which is fixed and known) to 1. The buyer cares about both procurement cost (price charged by the supplier) and the quality of the product. We let p_i and q_i denote supplier *i*'s bid price and offered quality. Both p_i and q_i are decision variables for supplier *i*.

We model quality, q_i , as the probability that a randomly selected unit of supplier i's product is not defective. Thus, $q_i \in [0, 1]$, with $q_i = 1$ representing perfect quality. The use of the defective rate (or, equivalently, the conformance rate) to represent product quality is fairly common in the literature [17], [18]. We assume that supplier *i*'s unit production cost is $c_i q_i^z$, where z > 1 is a constant, and c_i can be thought of as supplier *i*'s cost to produce a unit of perfect quality. This cost function is increasing and convex, which implies that the marginal cost of a unit of quality is increasing in the quality level, i.e., higher levels of quality are increasingly costly to obtain. A larger value of zimplies more curvature in the cost function, with z = 1 implying a linear cost in quality. A convex cost function is commonly used in the literature, e.g., the unit production cost function in [19] takes this form. The c_i are assumed to vary across the suppliers. In order to obtain closed-form results, we assume the c_i are independent draws from a uniform $[c_L, c_H]$ distribution, where $c_L > 0$. We will use $c_{(i)}$ to denote the order statistics for the c_i , so that $c_{(1)} \leq c_{(2)} \leq \cdots \leq c_{(n)}$. See Table I for a list of notation.

The suppliers have differing capabilities for producing highquality products. Specifically, let q_i^{\max} denote supplier *i*'s quality capability, i.e., supplier *i*'s maximum achievable quality level. Thus, supplier *i*'s decision problem includes the constraint $q_i \leq q_i^{\max}$. In practice, this capability will depend on various factors, including the supplier's experience, access to skilled labor and access to technology. We assume the q_i^{\max} are independent draws from a uniform $[q_L, q_H]$ distribution, where $q_H \leq 1$.

The buyer designs the sourcing process to minimize the unit cost, including the procurement cost (i.e., the winning bid price), the costs associated with poor quality and, in the case of PBC, the penalty/reward payments made to the winning supplier. While more complex models are possible [7], [8], for simplicity, we model the buyer's cost of poor quality as the expected unit warranty cost, which can be written as $C_w(q) = w(1-q)$, where 1-q is the probability a randomly selected unit is defective, and w is the warranty cost associated with a single defective unit. Finally, we assume that each supplier will choose the quality level and bid price in order to maximize his expected unit profit, which includes the unit production cost, revenue from sales to the buyer (winning bid price), and, in the case of PBC, the penalty/reward payments made to the buyer.

A. TSP

We first consider a TSP, commonly used in industrial and government procurement. In the qualification stage, potential suppliers are screened for a variety of capabilities, including quality and reliability. In the supplier selection stage, the qualified suppliers are invited to compete for the buyer's business by participating in a sealed-bid (closed) price-only procurement auction. Thus, each stage has a fundamentally different objective. The qualification stage ensures that the buyer sources from only the most capable suppliers, while the supplier selection stage induces competition between the potential suppliers. Jin et al. [8] present a model of a TSP in which the buyer exerts effort in the qualification stage in order to learn about the suppliers' quality capabilities. We present only the simplest model in their paper, in which the qualification process is costless and provides the buyer with perfect information on the suppliers' capabilities.¹ This simplest model is the most favorable version of TSP, and thus offers the most favorable comparison with PBC.

In the qualification stage, the buyer specifies the qualification threshold, Q, which suppliers must meet in order to participate in the supplier selection stage, i.e., only suppliers with $q_i^{\max} \ge Q$ are invited to participate in price-only bidding. After the qualification stage, some number, $m(Q) \le n$, of the potential suppliers will be qualified. Note that m(Q) is a random variable which depends on Q and on the distribution of the q_i^{\max} . These m(Q) qualified suppliers will submit sealed price-only bids to the buyer, who will select among them on the basis of price. Using standard results from auction theory [20], the expected winning bid price, which is the buyer's expected unit procurement cost, can be written as $C_p(Q) = Q^z E[c_H \times \frac{2}{m(Q)+1} + c_L \times \frac{m(Q)-1}{m(Q)+1}]$. This result uses the fact that all of the qualified suppliers will set their quality just equal to the qualification threshold, Q, since they have no incentive to offer a higher level of quality. Thus, the buyer's problem is to choose Q to minimize her total cost, $C_B(Q) = C_p(Q) + C_w(Q)$. Jin *et al.* [7], [8] show that the optimal threshold satisfies $\frac{d^-C_p}{d^-Q} = \frac{d^-C_w}{d^-Q} = w$, i.e., at the optimal Q, the marginal increase in procurement costs due to an increase in Q just equals the marginal savings in warranty costs due to an increase in Q.

Although Jin *et al.* [7], [8] assume a closed auction, i.e., sealed bids, their results also hold under an open auction, due to revenue equivalence (see, for example, [20]). We can thus rewrite the results for the special case in which the suppliers have unlimited quality capabilities, and z = 2, as follows: the qualification threshold is $Q^* = \frac{w}{2E[c_{(2)}]}$, the buyer's cost is $C_B^* = w - \frac{w}{2}Q^* = w - \frac{w^2}{4}\frac{1}{E[c_{(2)}]}$, and the winning supplier earns $\pi_S^* = (Q^*)^2(E[c_{(2)}] - E[c_{(1)}])$.

B. PBC

Under the TSP, the qualified suppliers compete only on the basis of price. Thus, they have no incentive to offer a quality level greater than the qualification threshold, Q. In constrast, under PBC, the buyer offers the suppliers a penalty/reward based on the winning supplier's realized quality, relative to the target quality, Q. Thus, a properly designed penalty/reward function may encourage the suppliers to offer quality levels greater than Q. Another benefit of PBC is the potential for increased competition, relative to TSP. Under TSP only the $m(Q) \leq n$ qualified suppliers may participate in the supplier selection stage. In contrast, under PBC, all n potential suppliers are allowed to bid.²

Under PBC, the buyer declares that the winning supplier's payment will include a penalty/reward, denoted by $H(q_i; Q)$, where Q is the buyer's target quality level. If $q_i > Q$, then $H(q_i; Q) > 0$ represents a reward paid to the supplier; if $q_i < Q$, then $H(q_i; Q) < 0$ represents a penalty charged to the supplier. Thus, supplier *i*'s profit, given that he wins the buyer's business, is $\pi_i = p_i - c_i q_i^z + H(q_i; Q)$. The buyer will select among the n potential suppliers by running a price-only procurement auction. We assume no cost for bidding and thus all n potential suppliers participate in the auction. Supplier i will bid in order to maximize his expected unit profit, subject to $q_i \leq q_i^{\max}$ and $p_i \ge c_i q_i^z - H(q_i; Q)$. Since we assume price-only bidding, the probability that supplier i wins is decreasing as p_i increases. Thus, the best q_i will be the value that minimizes supplier *i*'s total unit quality-related costs, denoted by $C_{\rm Si}$, where $C_{\rm Si}(q_i) = c_i q_i^z - H(q_i; Q)$. We let \hat{q}_i denote that value of q_i that minimizes $C_{Si}(q_i)$. However, supplier *i* is constrained by his maximum achievable quality level, q_i^{max} . Thus, supplier iwill set his quality level equal to $q_i^* = \min\{\hat{q}_i, q_i^{\max}\}.$

¹Here, we do not include a detailed analysis of the TSP model with non-fully capable suppliers because that analysis has been published in previous papers [7], [8].

²Of course, there are some settings in which it would not be feasible to allow all n suppliers to bid, regardless of their quality capabilities, e.g., settings in which there is a hard minimum on the quality level that must be achieved. In such a setting, a pure PBC approach would not be appropriate.

While alternative forms for the penalty/reward function are possible, we will focus on a symmetric and linear function, $H(q_i; Q) = h(q_i - Q)$, where h is the penalty/reward rate, which is a decision variable for the buyer. In this case, we can solve $\frac{\partial C_{\text{Si}}}{\partial q_i} = 0$ to find $\hat{q}_i = (\frac{h}{zc_i})^{\frac{1}{z-1}}$. When z = 2, $\hat{q}_i = \frac{h}{2c_i}$. As noted in [10], linear incentive functions are commonly used in practice. In Section V, we will demonstrate that the buyer does not see a significant increase in expected cost if she uses a symmetric function.

Before analyzing PBC in detail, we argue that revenue equivalence applies, and thus the buyer's results for open and closed bidding will be identical. The costs c_i for producing one unit of perfect quality and the quality capabilities q_i^{\max} are independent across all suppliers. The optimal quality level for supplier i, q_i^* , is equal to $\min\{(\frac{h}{zc_i})^{\frac{1}{z-1}}, q_i^{\max}\}$. The suppliers' optimal costs, i.e., $C_{\rm Si}(q_i^*) = c_i (q_i^*)^z - h(q_i^* - Q)$ for i = 1, ..., n, are thus functions of c_i and q_i^{max} . For a given h, these optimal costs are independent across the suppliers. In addition, the costs follow identical distributions since c_i and q_i^{max} are drawn from identical distributions. Thus, the auction has a set of bidders with independent and identically distributed (i.i.d.) costs. Following the revenue equivalence result for forward auctions [20], for reverse auctions, all standard auction forms, including open auctions and first-price closed auctions, with i.i.d. costs, will yield the same expected cost for the buyer.

C. Comparison of PBC and TSP

The goal of this work is to compare the performance of the TSP and PBC mechanisms. We would like to do so from three perspectives: those of the buyer, the suppliers, and the system as a whole. It is straightforward to argue that PBC with a general penalty/reward function always outperforms TSP. To do so, one can argue that the penalty/reward function under PBC can always be designed to achieve the results of TSP with a specified threshold, Q. Specifically, the buyer can design the penalty/reward function such that if a supplier's quality capability is at least equal to Q, then there is no penalty; otherwise, an infinite penalty is assigned. Given this penalty/reward function, only those suppliers with quality capability above the threshold Q would participate in the auction, and the winning supplier would always set his quality equal to Q, thus replicating the TSP outcome.

While PBC with a general penalty/reward function, such as an asymmetric linear function, will always perform at least as well as the TSP, from the buyer's perspective, it is not clear how the more practical and easy-to-implement symmetric linear penalty/reward function will perform relative to the TSP since the above argument does not apply for the symmetric case. Thus, we next prove that PBC, with a symmetric penalty/reward rate, h, selected to minimize the buyer's expected cost, always outperforms the TSP, with qualification threshold, Q, selected to minimize the buyer's expected cost, from the perspective of the buyer. The proofs of all theorems can be found in the technical supplement.

Theorem 1: Consider a setting in which the potential suppliers have limited quality capabilities, i.e., supplier i must

satisfy $q_i \leq q_i^{\max}$, where $q_i^{\max} \leq 1$ for all *i*. The buyer's expected cost under PBC, with a symmetric penalty/reward function, $H(q_i; Q) = h(q_i - Q)$, in which the penalty/reward rate, *h*, is set optimally, is no greater than the buyer's expected cost under the TSP, in which the qualification threshold, *Q*, is set optimally.

From the buyer's perspective, PBC will always outperform TSP, i.e., PBC will provide a lower expected cost. From the proof of Theorem 1, there are two factors contributing to this result. First, the suppliers' quality levels are endogenous and PBC provides a greater degree of flexibility to the suppliers in choosing their quality levels. Under TSP, the suppliers must set their quality levels at least equal to the qualification threshold Q. Since the suppliers' expected costs are increasing in Q under TSP, all qualified suppliers will choose to set their quality levels equal to exactly Q. Under PBC, although the penalty/reward function specifies a target quality level Q the suppliers are able to choose their optimal quality level, i.e., the quality level that minimizes their costs, subject to their quality capabilities. This flexibility implies that each supplier may offer a different quality level, i.e., the suppliers will offer differentiated quality levels to the buyer. Second, PBC allows all n potential suppliers to bid in the auction. On the other hand, under TSP, only the $m(Q) \leq n$ qualified suppliers, i.e., the suppliers with $q_i^{\max} \ge Q$, are allowed to bid in the auction stage. Thus, there is less competition in the bidding stage under TSP than under PBC, leading to a higher winning bid price for the buyer.

III. PBC: DETAILED ANALYSIS

We next perform a more detailed analysis of the buyer's decisions under PBC. Supplier *i* will set his optimal quality level, q_i^* , to minimize $C_{\text{Si}}(q_i)$, subject to $q_i \leq q_i^{\text{max}}$. One of the main goals of this research is to study a setting in which the suppliers set their quality levels endogenously, based on the incentives provided by the buyer, but subject to heterogeneous quality capabilities. Thus, in Section III-A, we consider a setting in which the suppliers do not have unlimited quality capabilities, which implies $q_i^* = \min{\{\hat{q}_i, q_i^{\max}\}}$ for all *i*. Then, to derive additional results, in Section III-B, we consider the case in which all suppliers have unlimited quality capability and thus $q_i^* = \hat{q}_i$, for all *i*.

A. Suppliers are Not Fully Capable

To model the suppliers' heterogeneous quality capabilities, we assume the q_i^{\max} are independent draws from a uniform $[q_L, q_H]$ distribution for i = 1, ..., n, where $q_H \le 1$ (and thus $q_i^{\max} \le 1$ for all *i*). In this case, assuming a symmetric and linear penalty/reward function, $H(q_i; Q) = h(q_i - Q)$, with penalty/reward rate *h*, and z > 1, supplier *i*'s optimal quality level is $q_i^*(h) = \min\{\hat{q}_i(h), q_i^{\max}\}$, where $\hat{q}_i(h) = (\frac{h}{zc_i})^{\frac{1}{z-1}}$. When bidding in a price-only auction, supplier *i* will bid based on his optimal cost

$$C_{\rm Si}^*(h) = c_i (q_i^*(h))^z - H(q_i^*(h);Q)$$

= $c_i (q_i^*(h))^z - h(q_i^*(h) - Q).$ (1)

For this model, determining the winning supplier and the expected winning bid price, and characterizing the buyer's optimal penalty/reward rate, are challenging. As will be seen, in the case of fully capable suppliers (with $q_i^*(h) = \hat{q}_i(h)$ for all *i*), supplier *i*'s total cost, $C_{\text{Si}}(\hat{q}_i(h))$, is decreasing in c_i , and thus the supplier with the lowest unit cost wins the bidding. However, this result does not hold when the suppliers are not fully capable. Instead, the winning supplier will be the one with the lowest $C_{\text{Si}}^*(h)$, which depends in a complex manner on both c_i and q_i^{\max} , as well as *h*.

To characterize the suppliers' bid prices, we need to find the probability distribution of $C_{\rm Si}^*(h)$, noting that since the c_i and $q_i^{\rm max}$ are i.i.d., the $C_{\rm Si}^*(h)$ will also be i.i.d. Let $F_C(\cdot)$ denote the cdf $C_{\rm Si}^*(h)$. Furthermore, to characterize the buyer's expected cost, we must characterize the order statistics of $C_{\rm Si}^*(h)$. Under open bidding, the buyer will choose the supplier with the lowest unit cost, i.e., the lowest $C_{\rm Si}^*(h)$, and will pay a bid price equal to the second lowest value of $C_{\rm Si}^*(h)$. Consider a setting with just two suppliers, labeled as i and j. Suppose that supplier i wins the bidding. This implies that $C_{\rm Si}^*(h) \leq C_{Sj}^*(h)$. The buyer's total cost is

$$C_B(Q,h) = C^*_{Sj}(h) + w(1 - q^*_i(h)) + h(q^*_i(h) - Q)$$

= $c_j q^*_j(h)^z - h(q^*_j(h) - q^*_i(h)) + w(1 - q^*_i(h)).$ (2)

Note that the Q cancels out of the buyer's cost function. Hence, for the remainder of this section, we will write the buyer's cost as $C_B(h)$. We can now state the following result.

Theorem 2: Consider PBC with a symmetric penalty/reward function, $H(q_i; Q) = h(q_i - Q)$, when the suppliers are not all fully capable, i.e., when $q_i^{\max} \leq 1$ for all *i*, with z > 1.

1) Given $C_{Si}^{*}(h) \leq C_{Sj}^{*}(h)$, then $E[q_{i}^{*}(h)] \geq E[q_{j}^{*}(h)]$, $E[q_{i}^{\max}] \geq E[q_{j}^{\max}]$ and $E[c_{i}] \leq E[c_{j}]$.

This theorem holds for any pair of suppliers i and j not just those with the lowest and second lowest total unit cost. The theorem states that if supplier i provides the buyer with a lower total unit cost than supplier j ($C_{Si}^*(h) \leq C_{Sj}^*(h)$), then the expected quality from supplier i must be no less than that from supplier j ($E[q_i^*(h)] \geq E[q_j^*(h)]$). Thus, the theorem implies that, for any h, the supplier who wins the bidding based on the costs $C_{Si}^*(h)$ for i = 1, ..., n, will also provide the highest expected level of delivered quality. The final two results of the theorem imply that, in expectation, the auction process favors suppliers with higher quality capability and lower unit costs.

1) Optimal Penalty/Reward Rate: We would like to characterize the optimal penalty/reward rate, h. In order to understand the impact of h on which supplier wins the bidding, it is useful to consider two extreme cases.

When h→∞, q_i^{*}(h) = min{q̂_i, q_i^{max}} = q_i^{max} and thus C_{Si}^{*} = c_i(q_i^{max})^z - h(q_i^{max} - Q). In this case, C_{Si}^{*} is dominated by the term -h(q_i^{max} - Q). The impact of c_i is negligible. Therefore, the supplier with the highest q_i^{max} will win the auction. Thus, the expected delivered quality is equal to the expected highest q_i^{max} among n random draws, which is 1/(n+1)q_L + n/(n+1)q_H. Since the impact of c_i is negligible, and since c_i is independent of q_i^{max}, a supplier with any c_i has an equal chance of winning the auction.

Thus, the expected unit cost of the winning supplier is $\frac{c_L + c_H}{2}$.

2) When $h \to 0$, $q_i^*(h) = \min\{\hat{q}_i, q_i^{\max}\} = \hat{q}_i = (\frac{h}{zc_i})^{\frac{1}{z-1}}$ and $C_{\text{Si}}^* = hQ - (1 - \frac{1}{z})(h)^{\frac{1}{z-1}}(\frac{1}{zc_i})^{\frac{1}{z-1}}$. In this case, C_{Si}^* is an increasing function of c_i . Therefore, the supplier with the lowest c_i will win the auction and the expected unit cost of the winning supplier will be $\frac{n}{n+1}c_L + \frac{1}{n+1}c_H$. Because c_i is independent of q_i^{\max} , a supplier with any q_i^{\max} has an equal chance of winning the auction. Thus, the expected delivered quality level is $\frac{q_L+q_H}{2}$.

Intuitively, when h = 0, the lowest unit cost supplier wins. As h increases away from 0, suppliers with higher unit cost starts to have a chance of winning the auction, if they happen to have a high-quality capability. Finally, when h becomes sufficiently large, the most capable supplier wins.

We are now ready to characterize the optimal penalty/reward rate for the buyer.

Theorem 3: Under PBC, with a symmetric penalty/reward function, $H(q_i; Q) = h(q_i - Q)$, when the suppliers are not all fully capable, i.e., when $q_i^{\max} \leq 1$ for all *i*, with z > 1, the optimal penalty/reward rate h^* satisfies $h^* \leq w$, where *w* is the unit warranty cost.

The theorem implies that the buyer does not share all of her warranty costs with the winning supplier. Instead, it is optimal for the buyer to absorb some of the warranty costs. To understand this result, note that Theorem 2 indicates that the auction process will favor suppliers who have lower unit costs and those who can provide a higher level of delivered quality. Thus, although a high penalty/reward rate h can be used to encourage the supplier's to offer a high level of quality, it is not the only mechanism for doing so. Finally, we note that this result is consistent with the findings in [10], although the assumptions and model settings have a number of differences.

2) Evaluating the Buyer's Cost: While Theorem 3 presents general results regarding the magnitude of the optimal penalty/reward rate h providing an exact expression is more challenging. In fact, even deriving a useful closed-form expression for the buyer's expected cost is difficult. From (2), we can write the buyer's expected cost as $C_B(Q, h) =$ $E[C_{S[2]}^*(h)] + w + (h - w)E[q_{[1]}^*(h)] - hQ$, where we use the subscript [i] to denote the order statistics for the $C_{Si}^*(h)$, so that $C_{S[1]}^*(h) \leq C_{S[2]}^*(h) \leq \cdots \leq C_{S[n]}^*(h)$, and $C_{S[2]}^*(h) =$ $c_{[2]}(q_{[2]}^*)^2 - hq_{[2]}^* + hQ$. The expected profit for the winning supplier, i.e., supplier [1], will then be

$$\pi_S(h) = E[C^*_{S[2]}(h)] - E[C^*_{S[1]}(h)].$$
(3)

To evaluate the buyer's expected cost, we need to evaluate $E[q_{[1]}^*(h)]$ and $E[C_{S[2]}^*(h)]$. To do so, for simplicity, we will focus on the case in which $q_L \leq \hat{q}_i(h) \leq q_H$ for all i, where $\hat{q}_i(h) = (\frac{h}{zc_i})^{\frac{1}{z-1}}$. For a given c_i , using the fact that $q_i^*(h) = \min\{\hat{q}_i(h), q_i^{\max}\}$, we have

$$E_{q_{i}^{\max}}[q_{i}^{*}(h)|c_{i}] = \int_{q_{L}}^{q_{i}(h)} q_{i}^{\max} f_{q}(q_{i}^{\max}) dq_{i}^{\max} + \int_{\hat{q}_{i}(h)}^{q_{H}} \hat{q}_{i}(h) f_{q}(q_{i}^{\max}) dq_{i}^{\max}$$
(4)

where $f_q(q_i^{\max})$ denotes the density function for q_i^{\max} , defined on $[q_L, q_H]$. From (1), using $q_i^*(h) = \min\{\hat{q}_i(h), q_i^{\max}\}$, we have

$$E_{q_{i}^{\max}}[C_{\mathrm{Si}}^{*}(h)|c_{i}] = \int_{q_{L}}^{q_{i}(h)} (c_{i}(q_{i}^{\max})^{2} - hq_{i}^{\max} + hQ)f_{q}(q_{i}^{\max})dq_{i}^{\max} + \int_{\hat{q}_{i}(h)}^{q_{H}} \left(hQ - \left(1 - \frac{1}{z}\right)(h)^{\frac{z}{z-1}} \left(\frac{1}{zc_{i}}\right)^{\frac{1}{z-1}}\right)f_{q}(q_{i}^{\max})dq_{i}^{\max}.$$
(5)

We can now write the buyer's expected cost as

C

$$B(Q,h) = E_{c_{[2]}} \{ E_{q_i^{\max}}[C_{\mathrm{Si}}^*(h)|c_{[2]}] \} + w$$

+ $(h-w)E_{c_{[1]}} \{ E_{q_i^{\max}}[q_i^*|c_{[1]}] \} - hQ$ (6)

where $E_{q_i^{\max}}[C_{Si}^*(h)|c_{[2]}]$ and $E_{q_i^{\max}}[q_i^*|c_{[1]}]$ are computed using (5) and (4), respectively.

We next present the results of an extensive numerical study to demonstrate the behavior of the optimal penalty/reward rate, as well as the buyer's expected cost. In Section III-B, we consider the case in which all suppliers are fully capable, which allows for closed-form results and additional insights.

3) Numerical Results: We fix z = 2, so that the unit production cost is quadratic in quality. We have six problem parameters to consider $(c_L, c_H, q_L, q_H, w, \text{ and } n)$. We set $q_H = 1$ and the average unit cost to $\frac{c_L+c_H}{2} = 2$. We write $c_L = 2 - \delta$ and $c_H = 2 + \delta$. Thus, we have four parameters to vary $(q_L, \delta, w, \text{ and } n)$. For each, we consider four values: $q_L \in \{0.1, 0.3, 0.5, 0.7\}, \delta \in \{0.5, 1, 1.5, 2\}, w \in \{0.25, 1, 2, 3\}$, and $n \in \{5, 10, 15, 20\}$, resulting in a total of 256 experiments. To set the range for the unit warranty cost is equal to 2. Thus, we include cases in which the unit warranty cost is substantially less than the average unit cost, as well as cases in which the unit warranty cost exceeds the average unit cost. In the latter case, the unit warranty cost includes a goodwill cost, i.e., a cost of customer dissatisfaction due to the low quality of the product.

We next describe how we evaluate the buyer's cost for a given penalty/reward rate h and search for the optimal h. We first generate two sets of one million random numbers from a uniform[0, 1] distribution. For each combination of the parameters $(c_L, c_H, q_L, q_H, w, n)$, we use the first set to generate a set of one million $c_i \in [c_L, c_H]$ and the second set to generate a set of one million $q_i^{\max} \in [q_L, q_H]$. For a given h, we find the optimal quality $q_i^* = \min\{\hat{q}_i(h), q_i^{\max}\}$ for each i and compute the overall cost $C_{\text{Si}}^*(h) = c_i(q_i^*)^2 - h(q_i^* - Q)$ for each i. We then repeat the following process 5000 times.

- Generate a set of indexes (integers) from a uniform distribution between 0 and 1 million. The size of the set is the number of suppliers, n.
- 2) Use the indexes to extract the corresponding $C_{Si}^{*}(h)$ and rank these costs in ascending order.
- 3) Find the winning supplier (the supplier with the lowest $C_{\text{Si}}^*(h)$) and the winning bid price (the second lowest $C_{\text{Si}}^*(h)$).
- Record the buyer's total cost, the delivered quality and the winning bid price.

After 5000 repetitions, we compute the average of the winning supplier's total cost, the delivered quality and the winning bid

TABLE II PBC Solution for Various Values of n

	C_B^*	$E[q^*]$	h^*	C_T	π_S^*	(w-h)/h
n = 5	1.004	0.442	1.096	0.915	0.088	46.1%
n = 10	0.900	0.510	1.130	0.823	0.077	42.5%
n = 15	0.845	0.546	1.149	0.777	0.068	39.7%
n = 20	0.810	0.568	1.157	0.749	0.062	39.4%

TABLE III PBC SOLUTION FOR VARIOUS VALUES OF w

	C_B^*	$E\left[q^*\right]$	h^*	C_T	π^*_S	(w-h)/h
w = 0.25	0.227	0.166	0.197	0.221	0.005	35.0%
w = 1	0.737	0.452	0.776	0.696	0.041	37.5%
w = 2	1.164	0.671	1.476	1.065	0.098	43.9%
w = 3	1.432	0.778	2.083	1.282	0.150	51.3%

TABLE IV PBC Solution for Various Values of q_{L}

	C_B^*	$E[q^*]$	h^*	C_T	π_S^*	(w-h)/h
$q_L = 0.1$	0.943	0.477	1.114	0.869	0.074	43.0%
$q_L = 0.3$	0.908	0.501	1.127	0.836	0.072	42.9%
$q_L = 0.5$	0.872	0.529	1.141	0.800	0.072	41.3%
$q_L = 0.7$	0.836	0.560	1.151	0.760	0.076	40.4%

TABLE V PBC Solution for Various Values of $\delta = c_H - c_L$

	C_B^*	$E[q^*]$	h^*	C_T	π_S^*	(w-h)/h
$\delta = 1$	1.057	0.431	1.438	1.030	0.027	7.3%
$\delta = 2$	0.963	0.482	1.267	0.908	0.055	19.1%
$\delta = 3$	0.843	0.540	1.050	0.756	0.088	40.3%
$\delta = 4$	0.696	0.614	0.777	0.570	0.126	100.9%

price. From these values, we compute the buyer's cost and the total system cost. Finally, we need to find the h to minimize the buyer's cost. According to Theorem 3, $h^* \leq w$. Thus, we search for the optimal h over the range [0.1, 3] with an increment of 0.01.

Tables II–V show the impact of the parameters on the performance measures, including the buyer's optimal cost C_B^* , the winning supplier's expected profit π_S^* , the expected total system cost, denoted by $C_T = C_B^* - \pi_S^*$, the expected quality level provided by the winning supplier (referred to as the "expected delivered quality level") $E[q^*]$, and the optimal penalty/reward rate h^* . Although the results in the tables are obtained using simulation, we expect them to be quite accurate. Specifically, for each of the 256 experiments, for each performance measure, we computed the ratio of the standard deviation to the mean (across the 5000 replications). For the buyer's expected cost (system cost), the average value of this ratio across the 256 experiments was 0.0017 (0.0024), while the maximum value across the 256 experiments was 0.0055 (0.0081). Tables II–V show the impact of one parameter. Each row shows the average value of the performance measures across the 64 experiments in which the parameter took the specified value.

Table II shows the impact of the number of potential suppliers on the results. These results reflect the benefit of increased competition (larger n) for the buyer. With more potential suppliers, the competition is more intense, resulting the lower cost for the buyer and lower profit for the winning supplier. The buyer is also able to set a higher penalty/reward rate, effectively passing on more of the warranty cost to the winning supplier, and resulting in a higher expected delivered quality.

Table III shows the impact of the unit warranty cost. As wincreases, C_B^* increases, $E[q^*]$ increases, C_T increases, π_S^* increases, h^* increases, and (w - h)/h increases. The impact of the unit warranty cost on the buyer's cost, the penalty/reward rate and the delivered quality are intuitive. With a higher unit warranty cost, the buyer must set a higher penalty/reward rate in order to induce a higher delivered quality, leading to a higher expected cost for the buyer. A higher unit warranty cost is beneficial for the winning supplier, i.e., results in a higher expected profit. The winning supplier's profit, as shown in (3), is equal to the difference between the costs of the second lowest and lowest cost suppliers. This difference is increasing in h, i.e., a larger h magnifies any differences between the suppliers' c_i and q_i^{max} . Since the buyer sets a higher penalty/reward rate as the warranty cost increases, the difference between the second lowest and lowest suppliers' costs increases as the warranty cost increases, implying that the winning supplier's profit increases. Thus, the information rent earned by the winning supplier increases with the warranty cost, reflecting the increased power of the suppliers.

Table IV shows the impact of the lower bound on the suppliers' quality capabilities on the results. As q_L increases, C_B^* decreases, $E[q^*]$ increases, the expected system cost decreases, π^*_S increases, h^* increases, and (w-h)/h decreases. In our numerical examples, we fix the upper bound on the suppliers' quality capabilities to $q_H = 1$. Thus, an increase in q_L implies an increase in the average quality level across the suppliers. This increase leads to a higher expected delivered quality for the buyer and thus reduces the buyer's expected cost, as well as the expected system cost. However, the impact of a larger q_L on the winning supplier's expected profit is mixed. On one hand, an increase in the average quality level across the suppliers leads to an increase in the expected quality capability of the winning supplier, which should benefit the winning supplier. On the other hand, a larger q_L implies that the range $(q_H - q_L)$ of quality levels across the set of suppliers is smaller, i.e., there is less differentiation between the potential suppliers, while the number of suppliers competing for the buyer's business is fixed at n. Thus, as q_L increases, the difference between $C^*_{S[2]}(h)$ and $C^*_{S[1]}(h)$ decreases, implying that the winning supplier's expected profit, as given in (3), decreases. In other words, less differentiation (tighter competition) between the suppliers results in a lower expected profit for the winning supplier. Overall, considering both factors, q_L has little impact on the winning supplier's expected profit.

Table V shows the impact of the spread in the suppliers' unit costs ($\delta = c_H - c_L$) on the results. As $c_H - c_L$ increases, C_B^*

decreases, $E[q^*]$ increases, the expected system cost decreases, π_S^* increases, h^* decreases, and (w-h)/h increases. As we vary the spread $(c_H - c_L)$, the average cost $(\frac{c_H + c_L}{2})$ is kept constant. Thus, when the spread is increased, the suppliers' costs can take lower (and higher) values. Due to the competition induced by the auction, as well as the fact that the auction process favors lower cost suppliers, the buyer is able to take advantage of the lower costs, without feeling the impact of the higher costs, resulting in a lower overall cost for the buyer and allowing the buyer to achieve a higher expected delivered quality level. The larger spread $(c_H - c_L)$ also benefits the winning supplier. As noted above, the winning supplier's expected profit is equal to $C^*_{S[2]}(h) - C^*_{S[1]}(h)$, which should be increasing in $c_H - c_L$. In other words, greater differentiation between the suppliers (larger $c_H - c_L$) implies less competition and higher profits for the winning supplier.

While Tables II–V show the impact of one parameter, Fig. 1 shows the joint impact of the unit warranty cost w and the spread in the suppliers' unit costs, δ . Fig. 1(a) confirms the results shown in the tables, i.e., the buyer's optimal cost is decreasing in δ and increasing in w. However, Fig. 1(b) demonstrates an interaction not seen in the tables. Specifically, when the unit warranty cost is small, the expected delivered quality is much more sensitive to the spread in the suppliers' costs than when the unit warranty cost is large. As explained in the previous paragraph, larger δ will generally lead to larger $E[q^*]$. However, when the unit warranty cost is large, this effect is minimal, since the expected delivered quality will already be quite large due to the large penalty/reward rate induced by the high warranty cost. Finally, while not shown here, we also created additional two-way graphs, for different combinations of input parameters. All of these graphs are similar to Fig. 1(a), i.e., they confirm the results in Tables II-V without demonstrating any interactions between the parameters.

B. All Suppliers are Fully Capable

To obtain further analytical results, we consider the case in which all suppliers are fully capable, i.e., $q_i^{\max} = 1$ for all i, with z = 2. When the suppliers are all fully capable, given the penalty/reward rate equal to h, supplier i will set his quality level equal to $q_i^*(h) = \hat{q}_i(h) = \frac{h}{2c_i}$. Depending on the value of h, we may have $\hat{q}_i(h) > 1$. Given our interpretation of quality (i.e., the probability that a randomly selected unit is not defective), we should define $q_i^*(h) = \min\{\hat{q}_i(h), 1\}$. However, the goal of this section is to enable analytical results for PBC. Thus, while we will discuss the case in which $q_i^*(h) = \min\{\hat{q}_i(h), 1\}$ at the end of this section, for simplicity, we will write $q_i^*(h) = \hat{q}_i(h) = \frac{h}{2c_i}$. Given $q_i^*(h)$, supplier i's optimal cost is $C_{\text{Si}}^*(h) = \hat{C}_{\text{Si}}(q_i^*(h)) = C_{si}(\hat{q}_i(h)) = hQ - (\frac{h^2}{4c_i})$. When bidding in the price-only auction, supplier i will based on $C_{\text{Si}}^*(h)$. Since the c_i are i.i.d., the $C_{\text{Si}}^*(h)$ will also be i.i.d.

The suppliers participate in an open auction. Recall that the buyer will select the supplier with the lowest $C_{Si}^*(h)$ and will pay a bid price equal to the second lowest $C_{Si}^*(h)$. From the expression for $C_{Si}^*(h)$, it is clear that the supplier with the lowest c_i will also be the supplier with the lowest $C_{Si}^*(h)$. Thus, the



Fig. 1. Impact of δ and w on buyer's expected cost and expected delivered quality. (a) The buyer's expected cost. (b) The expected delivered quality level.

lowest cost supplier, i.e., the supplier whose unit cost is $c_{(1)}$, will win with a bid price equal to $p_{(1)}^*(h) = C_{S(2)}^*(h) = hQ - \frac{h^2}{4c_{(2)}}$ and the quality level equal to $q_{(1)}^*(h) = \frac{h}{2c_{(1)}}$. The buyer's total expected unit cost $C_B(Q, h)$ consists of the expected unit procurement cost (or expected winning bid price), the expected unit warranty cost and the expected penalty/reward payment

$$C_B(Q,h) = E[p_{(1)}^*(h)] + w(1 - E[q_{(1)}^*]) + h(E[q_{(1)}^*(h)] - Q$$

= $hQ - E\left[\frac{h^2}{4c_{(2)}}\right] + w\left(1 - E\left[\frac{h}{2c_{(1)}}\right]\right)$
+ $h\left(E\left[\frac{h}{2c_{(1)}}\right] - Q\right)$
= $w - \frac{h^2}{4}E\left[\frac{1}{c_{(2)}}\right] + \frac{h}{2}E\left[\frac{1}{c_{(1)}}\right](h - w).$

Notice that the quality threshold Q does not affect the buyer's cost. Thus, the buyer's problem is to select h to minimize $C_B(h)$. We are now ready to characterize the buyer's optimal solution.

Theorem 4: Under PBC, with a symmetric penalty/reward function, $H(q_i; Q) = h(q_i - Q)$, when all suppliers are fully capable, with z = 2, the optimal penalty/reward rate is $h^* = w(\frac{1}{2-\gamma})$, where $\gamma = \frac{E[\frac{1}{c(1)}]}{E[\frac{1}{c(1)}]} \leq 1$ and thus $0 \leq h^* \leq w$.

The optimal delivered quality level, i.e., the quality level provided by the winning supplier, is $E[q_{(1)}^*] = \frac{\hbar^*}{2}E[\frac{1}{c_{(1)}}] = \frac{w}{2}(\frac{1}{2-\gamma})E[\frac{1}{c_{(1)}}]$. The buyer's optimal expected cost is $C_B^* = w - \frac{w}{2}E[q_{(1)}^*]$.

The winning supplier earns expected profit equal to $\pi_S^* = \frac{(h^*)^2}{4} E[\frac{1}{c_{(1)}} - \frac{1}{c_{(2)}}]$. The total supply chain expected cost is $C_T = C_B^* - \pi_S^* = E[c_{(1)}(q_{(1)}^*)^2 + w(1 - q_{(1)}^*)].$

The buyer's optimal expected cost satisfies $\frac{\partial C_B^*}{\partial w} = 1 - E[q_{(1)}^*]$. Thus, when $E[q_{(1)}^*] < 1$, the buyer's optimal expected cost is increasing in the unit warranty cost w, while the winning supplier's expected profit is increasing in the unit warranty

cost, w. The optimal expected delivered quality level $E[q^*_{(1)}]$ is increasing in the unit warranty cost w.

This theorem provides the following insights.

- When designing PBC, the buyer must set the quality target Q and the penalty/reward rate h. Q can take any value. However, the optimal h is always less than the unit warranty cost w.
- For c_L > 0, as the number of potential suppliers n increases, it will generally hold that E[1/(c(2))] → E[1/(c(1))].
 Thus, γ = E[1/(c(2))]/E[1/(c(1))] → 1, which implies h* → w, as n → ∞. In other words, the optimal penalty/reward rate h* approaches the warranty cost w as the number of suppliers increases.
- 3) The winning supplier will be the supplier with the lowest unit cost $c_{(1)}$. That supplier will provide quality level $q_{(1)}^* = \frac{h^*}{2c_{(1)}}$. Applying the same argument as in the previous bullet, $E[q_{(1)}^*]$ will generally be increasing in *n*. Thus, more potential suppliers (more competition) is beneficial to the buyer, leading to a higher expected delivered quality level and lower expected cost.
- 4) The expression for the winning supplier's expected profit π_S^* indicates that this profit should generally be decreasing as the number of suppliers increases. To see this, note that h^* is approximately constant (equal to w) as $n \to \infty$, while $E[\frac{1}{f_{(x)}} \frac{1}{f_{(x)}}] \to 0$.
- while E[1/c(1) 1/c(2)] → 0.
 5) When E[q^{*}(1)] < 1, a larger warranty cost w leads to higher expected cost and a higher expected delivered quality level for the buyer. However, a larger warranty cost leads to higher expected profit for the winning supplier. This implies that the suppliers gain power over the buyer when the warranty cost is large, allowing the winning supplier to extract more profit from the buyer.
- 6) The expected delivered quality level under PBC, as specified in Theorem 4, is lower than in the system optimal solution, which can be obtained by setting h = w.

TABLE VI PBC VERSUS TSP: IMPACT OF NUMBER OF POTENTIAL SUPPLIERS

	$\% C_B$	$\% E[q^*]$	$\% \pi_S^*$	$\% C_T$	$\substack{q^{\mathrm{TSP}} \leq \\ q^{\mathrm{PBC}}}$	$\begin{array}{c} C_T^{\mathrm{TSP}} > \\ C_T^{\mathrm{PBC}} \end{array}$	$\begin{array}{c} \pi^{\mathrm{TSP}}_{S} > \\ \pi^{\mathrm{PBC}}_{S} \end{array}$
n = 5	7.55%	24.75%	1.41%	8.34%	92.19%	100.00%	57.81%
n = 10	9.29%	17.60%	5.75%	10.39%	93.75%	100.00%	65.63%
n = 15	10.36%	13.93%	9.26%	11.60%	87.50%	93.75%	84.38%
n = 20	11.07%	11.49%	12.41%	12.21%	87.50%	93.75%	89.06%

 TABLE VII

 PBC VERSUS TSP: IMPACT OF UNIT WARRANTY COST

	% C _B	$\% E[q^*]$	$\% \pi^*_S$	$\% C_T$	$q_{q^{\mathrm{PBC}}}^{\mathrm{TSP}} \leq$	$\begin{array}{c} C_T^{\mathrm{TSP}} > \\ C_T^{\mathrm{PBC}} \end{array}$	$\begin{array}{l} \pi^{\mathrm{TSP}}_{S} > \\ \pi^{\mathrm{PBC}}_{S} \end{array}$
w = 0.25	4.00%	28.41%	-0.56%	4.96%	76.56%	87.50%	57.81%
w = 1	7.37%	14.23%	6.75%	8.19%	89.06%	100.00%	75.00%
w = 2	11.52%	13.19%	8.07%	12.93%	95.31%	100.00%	75.00%
w = 3	15.38%	11.93%	14.58%	16.45%	100.00%	100.00%	89.06%

TABLE VIII PBC VERSUS TSP: IMPACT OF LOWER BOUND ON SUPPLIERS' QUALITY CAPABILITIES

	$\% C_B$	$\% E[q^*]$	$\% \pi^*_S$	$\% C_T$	$q_{q^{\mathrm{TSP}}\leq}^{\mathrm{TSP}}\leq$	$\begin{array}{c} C_T^{\mathrm{TSP}} > \\ C_T^{\mathrm{PBC}} \end{array}$	$\begin{array}{c} \pi^{\mathrm{TSP}}_{S} > \\ \pi^{\mathrm{PBC}}_{S} \end{array}$
$q_L = 0.1$ $q_L = 0.3$	9.21%	18.89%	7.78%	9.91%	93.75%	96.88%	82.81%
	9.84%	17.33%	8.48%	10.67%	92.19%	96.88%	78.13%
$\begin{array}{l} q_L = 0.5 \\ q_L = 0.7 \end{array}$	10.05%	16.74%	6.88%	11.31%	90.63%	96.88%	73.44%
	9.17%	14.80%	5.70%	10.65%	84.38%	96.88%	62.50%

In summary, the optimal penalty/reward rate h^* is proportional to the unit warranty cost w. Therefore, PBC with a linear penalty/reward function is a form of shared warranty costs, where the portion of the warranty cost paid by the selected supplier $(\frac{1}{2-\gamma})$ is increasing in the number of potential suppliers. Thus, when there is more competition, the winning supplier is forced to bear a greater portion of the buyer's warranty costs. In addition, the optimal penalty/reward rate h^* will generally approach w as the number of potential suppliers increases. Thus, as the number of suppliers increases, the optimal winning quality level increases towards the system optimal quality level.

As noted above, when $q_i^{\max} = 1$ for all *i*, the optimal quality level should be written as $q_i^*(h) = \min\{\hat{q}_i(h), 1\}$, rather than $q_i^*(h) = \hat{q}_i(h)$. In this case, while the analysis is more complex than for Theorem 4, we can also demonstrate that a unique optimal *h* exists and satisfies $h^* \leq w$.

IV. COMPARISON OF PBC AND TSP

In Section II-C, we demonstrated analytically that PBC always outperforms the TSP. However, TSP is more widely used in practice. When quality is of importance to their competitive advantage, e.g., when reputation is critical, buyers prefer to contract with suppliers who are known to be capable of meeting the buyer's standards (e.g., through the qualification stage of TSP), rather than contracting with suppliers of unknown quality

TABLE IX PBC VERSUS TSP: IMPACT OF SPREAD IN SUPPLIERS' UNIT COSTS $(\delta = c_H - c_L)$

	$\% C_B$	$\% E[q^*]$	$\% \pi^*_S$	$\% C_T$	$q_{q^{\rm PBC}}^{\rm TSP} <$	$\begin{array}{c} C_T^{\mathrm{TSP}} > \\ C_T^{\mathrm{PBC}} \end{array}$	$\begin{array}{c} \pi^{\mathrm{TSP}}_{S} > \\ \pi^{\mathrm{PBC}}_{S} \end{array}$
$\delta = 1$ $\delta = 2$ $\delta = 3$ $\delta = 4$	1.10% 3.86% 9.57% 23.73%	3.73% 9.67% 17.95% 36.42%	10.87% 11.02% 8.02% -1.07%	0.92% 3.66% 9.99% 27.96%	68.75% 95.31% 100.00% 96.88%	87.50% 100.00% 100.00% 100.00%	89.06% 84.38% 68.75% 54.69%

and then penalizing for poor performance (e.g., through PBC). Therefore, it is useful to understand the magnitude of the performance gap between TSP and PBC and to identify conditions under which buyers do not lose much by using TSP.

We consider the case in which the suppliers are not all fully capable, i.e., the case in which $q_i^{\text{max}} \leq 1$ for all *i*. Thus, we assume the q_i^{max} follow a uniform $[q_L, q_H]$ distribution, with $q_H \leq 1$, across the set of potential suppliers. As discussed in Section III-A, analytical results are not possible for PBC for this setting. Therefore, we will use numerical experiments to compare PBC and TSP. Our numerical experiments follow the experimental design outlined in Section III-A3. For PBC, we use the simulation approach outlined in Section III-A3. For TSP, we follow the approach outlined in [7].

Tables VI-IX follow the same format as Tables II-V. In the tables, $\% C_B$ represents the amount by which the buyer's expected cost under TSP exceeds the buyer's expected cost under PBC. Similarly, % $E[q^*]$ represents the amount by which the expected delivered quality under PBC exceeds the expected delivered quality under TSP, while % π^*_S represents the amount by which the winning supplier's expected profit under TSP exceeds the winning supplier's expected profit under PBC. Finally, $\% C_T$ represents the amount by which the expected total system cost under TSP exceeds the expected system cost under PBC. In these tables, the columns labeled $q^{\text{TSP}} < q^{\text{PBC}}, C_T^{\text{TSP}} > C_T^{\text{PBC}}$ and $\pi_S^{\text{TSP}} > \pi_S^{\text{PBC}}$ show the percent of cases in which the expected delivered quality, system cost, and winning supplier's expected profit, have the relationship shown. Recall that the buyer's expected cost is always lower under PBC than under TSP. The results in Tables VI–IX indicate that PBC generally outperforms TSP from the perspective of the buyer's expected cost, the system expected cost and the expected delivered quality level. However, TSP generally outperforms PBC from the perspective of the winning supplier's expected profit. We next consider some more detailed comparisons.

Table VI shows the impact of the number of potential suppliers on the performance of PBC relative to TSP. As *n* increases, the performance of PBC relative to TSP improves, in terms of both the buyer's expected cost and the system expected cost. The opposite is true for the winning supplier's expected profit, i.e., as *n* increases, the supplier's profit under TSP improves relative to under PBC. To understand these results, note that under PBC all *n* suppliers compete in the auction process. Under TSP, only the subset of qualified suppliers is allowed to compete in the auction. Thus, under TSP, the value of having more potential suppliers, in terms of increased competition in the auction stage, is tempered by the qualification stage. Thus, the higher degree of competition under PBC benefits the buyer and the system, while the lower degree of competition under TSP benefits the winning supplier.

Table VI also indicates that, as n increases, the difference between the expected delivered quality under PBC and TSP decreases. As discussed in [7], under the TSP there is a tradeoff between the optimal quality level (threshold) and the number of suppliers. Specifically, when the number of potential suppliers is small, the buyer will set a low quality level (threshold) in order to ensure that enough suppliers are qualified to maintain competition in the auction stage. However, when n gets larger, the buyer is able to be more stringent in the qualification stage, setting a higher quality level (threshold), while still maintaining competition in the auction stage. Thus, having a larger number of suppliers will reduce the quality gap between PBC and TSP.

Table VII shows the impact of the unit warranty cost on the performance of PBC relative to TSP. As w increases, the performance of PBC relative to TSP improves in terms of both the buyer's expected cost and the expected system cost. However, the opposite is true for the winning supplier's expected profit, i.e., as w increases, the winning supplier's expected profit under TSP improves relative to under PBC. The explanation here is similar to that provided for Table VI. Under PBC, the number of suppliers competing in the auction, i.e., the level of competition, is fixed. Under TSP, the buyer controls the level of competition in the auction stage through the selection of the quality level (threshold). When the warranty cost is low, the buyer is willing to sacrifice quality for increased competition by setting a low quality level (threshold). However, when the warranty cost increases, the buyer must focus more on the costs associated with poor quality and thus will set a higher quality level (threshold), which leads to fewer qualified suppliers. With fewer suppliers competing under TSP compared to PBC, the winning supplier is able to extract a higher profit.

Table VII also indicates that as w increases, the difference between expected delivered quality levels under PBC and TSP decreases. This is most likely due to the fact that when w increases the expected delivered quality levels are increased under both PBC and TSP, along with the fact that both expected delivered quality levels are constrained by 1 (the maximum quality capability). However, as w increases, the percentage of cases in which PBC has a larger expected delivered quality level than TSP increases.

Table VIII shows the impact of the lower bound on the suppliers' quality capabilities on the performance of PBC relative to TSP. While q_L does not have a consistent impact on the buyer's expected cost, the expected system cost or the winning supplier's expected profit, a larger q_L implies that the supplier is less likely to have higher profit under TSP than under PBC. Also, a larger q_L causes a decrease in the difference in the expected delivered quality levels under PBC and TSP. This result is likely due to the fact that a larger q_L implies less spread in the quality levels across suppliers.

Table IX shows the impact of the spread in the suppliers' unit costs ($\delta = c_H - c_L$) on the performance of PBC relative to TSP. As δ increases, the performance of PBC relative to TSP

improves in terms of both the buyer's expected cost and the expected system cost. In addition, as δ increases, TSP becomes less preferred by the supplier. Overall, as δ increases, all parties become more likely to prefer PBC. Finally, as δ increases, the expected delivered quality also improves for PBC relative to TSP.

As noted in the discussion of Table V, under PBC, a larger $\delta = c_H - c_L$ leads to lower expected cost for the buyer and higher expected profit for the winning supplier. The same result holds under TSP. However, the benefits to the buyer of a larger δ are more substantial under PBC than under TSP. The larger number of suppliers competing in the auction stage under PBC implies that the buyer can take more advantage of the potential for low unit costs for the suppliers when δ is larger than she can under TSP, when fewer suppliers compete in the auction stage. Thus, as δ increases, the performance of PBC relative to TSP, from the perspective of the buyer's cost, improves.

Similarly, as $\delta = c_H - c_L$ increases, the performance of PBC relative to TSP, from the perspective of the winning supplier's profit, improves. To understand this, note that, from the perspective of the suppliers, TSP is less flexible than PBC. In other words, under TSP, every qualified supplier sets their quality level equal to the threshold Q. As a result, as δ increases, the expected delivered quality increases at a slower rate under TSP than under PBC. However, under PBC, the suppliers can adjust their delivered quality with greater flexibility, i.e., the q_i^* will vary across the suppliers, which leads to a greater differentiation between the suppliers and a larger value of $C_{S[2]}^*(h) - C_{S[1]}^*(h)$. Thus, as δ increases, the winning supplier's profit increases more quickly under PBC than under TSP.

In summary, a larger $\delta = c_H - c_L$ implies a greater degree of differentiation between potential suppliers. Our results thus indicate that PBC increasingly outperforms TSP, on all performance measures, as this level of differentiation increases. This is an important insight for buyer's when designing their sourcing processes. While in some cases, for practical reasons, the buyer may prefer TSP over PBC despite the lower cost associated with PBC, the buyer must be careful when selecting TSP if there is a high level of variation in the potential suppliers' unit costs.

V. PBC: ASYMMETRIC PENALTY/REWARD FUNCTION

The above analysis assumes a symmetric and linear penalty/reward function, i.e., $H(q_i; Q) = h(q_i - Q)$, where h is the penalty/reward rate. We next consider anasymmetric penalty/reward function of the form: $H(q_i; Q) = h_1(q_i - Q)$ if $q_i \ge Q$ and $H(q_i; Q) = h_2(q_i - Q)$ if $q_i < Q$, where $h_2 > h_1$. Thus, if $q_i > Q$, then $H(q_i; Q) > 0$ represents a reward paid to the supplier. If $q_i < Q$, then $H(q_i; Q) < 0$ represents a penalty charged to the supplier. The buyer must set h_1 and h_2 , as well as Q.

For this penalty/reward function, the optimal quality level and the cost for supplier i are presented in the technical appendix. Characterizing the winning supplier, and determining the optimal penalty and reward rates for the buyer, are challenging. In addition, unlike for the symmetric penalty/reward function case, the quality threshold Q does not cancel out of the buyer's

TABLE X COMPARISON OF ASYMMETRIC AND SYMMETRIC PENALTY/REWARD FUNCTIONS

	$\% C_B$	$\% C_T$	$\% E[q^*]$	% h
Average	0.24%	0.37%	2.11%	-1.39%
Minimum	0.00%	0.00%	0.00%	-20.00%
Maximum	2.06%	3.79%	11.82%	10.00%

cost function, and thus the buyer has a three-dimensional (3-D) strategy space (h_1, h_2, Q) . Thus, analytical results are not possible. We therefore conducted a set of numerical experiments, following the methodology and experimental design described in Section III-A3, in order to understand how the results for the asymmetric penalty/reward function will differ from those with the symmetric penalty/reward function, as presented in Section III. To determine the optimal h_1 , h_2 , and Q to minimize the buyer's expected cost $C_B(h_1, h_2, Q)$, we performed an exhaustive search. Specifically, we considered $h_1 \in [0, w]$, in increments of 0.05, $h_2 \in [h_1, w]$, in increments of 0.05, and 20 values of Q, equally spaced between q_L and q_H .

The results are shown in Table X. In the table, $\% C_B$ represents the percentage difference between the buyer's expected cost under the symmetric and asymmetric cases, i.e., $\% C_B =$ $\left|\frac{C_B(h^*)-C_B(h_1^*,h_2^*,Q^*)}{C_B(h^*,h^*,Q^*)}\right|$. Since the symmetric penalty/reward function is a special case of the asymmetric penalty/reward function, the buyer's optimal cost will always be higher for the symmetric case. However, due to the use of simulation to estimate the cost functions, as well as the limitations of the 3-D search process described above, in some cases we have that $C_B(h^*)$ is slightly (no more than 2.06%) less than $C_B(h_1^*, h_2^*, Q^*)$. Thus, we report the absolute value of the percent difference. The columns labelled % C_T and % $E[q^*]$ consider the difference in the expected total system cost and the expected delivered quality, and are calculated analogously to $\% C_B$. The final column compares the penalty/reward rates. Specifically, we first compute % $h_1 = \frac{h^* - h_1^*}{h_1^*}$ and % $h_2 = \frac{h^* - h_2^*}{h_2^*}$. We then let % h = $\% h_1$ if $E[q^*] > Q^*$ and $\% h = \% h_2$ if $E[q^*] \le Q^*$. Thus, we compare the symmetric penalty/reward rate to the relevant rate for the asymmetric case, i.e., to the rate that is applied in the optimal solution for the asymmetric case. When $E[q^*] > Q^*$ $(E[q^*] < Q^*)$, the reward (penalty) will be applied.

Table X indicates that, although the buyer's expected cost will be lower under the asymmetric penalty/reward function, the buyer will not see a significant loss if he chooses to implement the simpler symmetric function. Our numerical results indicate that the buyer's expected cost and realized quality, as well as the expected system cost, do not significantly differ between the two types of function. Finally, in the technical appendix, we provide some intuition for the observation that the performance of the symmetric and asymmetric penalty/reward functions are quite similar.

VI. CONCLUSION AND MANAGERIAL INSIGHTS

In this paper, we compare the performance of two mechanisms that are appropriate, and commonly used in practice, when the buyer has uncertainty regarding the potential suppliers' costs and quality capabilities: the TSP and PBC. We do so using a model setting that captures several complexities not generally considered in the existing literature. Specifically, our model captures the fact that suppliers will generally set their quality levels endogenously, in response to the incentives provided by the sourcing process design. In addition, our model captures the reality that potential suppliers will often have differing quality capabilities, which constrain the quality levels they can offer to the buyer. Finally, our model uses a heterogeneous production cost function to capture how the suppliers' production costs vary with quality.

We find that PBC always outperforms TSP from the perspective of the buyer. This result is due, in part, to the fact that the suppliers' quality levels are endogenous, set optimally in response to the buyer's sourcing process design. PBC provides the suppliers with the flexibility to choose the quality level to minimize their own costs, given their quality capability and the penalty/reward rate assessed by the suppliers. TSP, on the other hand, imposes a minimum quality level across all of the potential suppliers. Thus, TSP does not provide the same level of flexibility for the suppliers to adjust their quality levels to their own cost structure. In addition, PBC allows all potential suppliers to compete in the bidding, while TSP allows only the subset of qualified suppliers to bid. This increased competition in the bidding stage also favors PBC from the perspective of the buyer.

While PBC can be shown to outperform TSP, in practice TSP is more widely used, for a number of practical, but hard to quantify, reasons. Thus, it is useful to understand the magnitude of the performance gap between TSP and PBC in order to identify conditions under which buyers do not lose much by choosing TSP. We find that PBC is most beneficial to the buyer when the spread in the potential suppliers' costs is large and when the unit warranty cost is large. Thus, a buyer who faces significant uncertainty regarding the suppliers' costs, and for whom maintaining a high level of quality is critical, should give extra consideration in implementing PBC, despite the practical benefits of TSP.

We also studied how the expected delivered quality differs under TSP and PBC. We find that the delivered quality is generally higher under PBC than under TSP and that the gap between the quality levels is largest when the spread in the potential suppliers' costs is large and when the number of potential suppliers and unit warranty cost are small. We find that the expected delivered quality will increase towards the system optimal quality level as the number of potential suppliers increases. Finally, while the buyer always prefers to source through PBC, we find that the winning supplier is generally better off under TSP. The supplier's preference for TSP is strongest when the number of suppliers is large, the spread in the suppliers' costs is small, and the unit warranty cost is large.

Given that the buyer prefers PBC to TSP, we also consider how the optimal penalty/reward rate varies with the problem parameters. We find that the optimal rate is always less than the unit warranty cost. In addition, when the suppliers are fully capable, the optimal penalty/reward rate is proportional to the unit warranty cost. Thus, PBC with a symmetric and linear penalty/reward function is a form of warranty cost sharing. The optimal penalty/reward rate is largest when the number of potential suppliers is large and the unit warranty cost is large. The former point indicates that more competition between the suppliers enables the buyer to pass more of the warranty costs onto the winning supplier. The latter point implies that when the warranty cost is large, the buyer must provide more incentive to the suppliers to provide high-quality products. These results provide guidance regarding how to set the penalty/reward rate for firms that choose to source using the PBC approach.

Finally, we note some future research directions. It would be useful to consider a setting in which the suppliers have limited capacity and thus the buyer may need to contract with multiple suppliers. In addition, there may be some settings in which it would not be feasible to allow all suppliers to bid, regardless of their quality capabilities, e.g., there may be a hard minimum on the quality level that must be achieved by the suppliers. In such settings, a pure PBC approach would not be appropriate. However, a hybrid approach, combining first a qualification process, perhaps with the quality threshold set to the minimum acceptable level, followed by a PBC mechanism, may be a better approach. The design and performance of such hybrid mechanisms is an important topic for future research.

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