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A Note on the Instability of the Unprojected Individual Level Premium Cost Method

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Abstract

We compare the unit credit and the unprojected individual level premium cost methods in a continuous time environment and show that the latter may produce unstable contribution rates in a dynamic environment. Specifically, assuming there are no unfunded liabilities, we prove that the unprojected individual level premium cost method may produce non-bounded contributions if benefits change too close to the normal retirement age.

Key words and phrases: *pension funding, unit credit cost, individual level premium, unfunded liability*

1 Introduction

Pension funding methods are more than ever a key issue for actuaries, especially in the context of the so-called pay-as-you-go public pensions systems crisis. The demographic changes expected over the next few decades in developed countries represent a major challenge for public social security systems. Fortunately actuarial funding methods seem to, at least partially, offer an adequate response to these

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challenges. [For a discussion of the basics of pension methods see, for example, Trowbridge (1952), Berin (1986), and Anderson (1992).] Financial markets in recent years, however, have shown an extraordinary volatility, thereby inducing significant solvency problems for many pension funds.

Although many new pension plans are defined contribution plans (thus transferring the market risk to the plan's participants), many pension plans are still defined benefit plans. Actuaries performing valuations of defined benefit plans may have to consider several alternative funding methods and compare the evolution of the contributions under different scenarios.

In Europe, two important funding methods used by actuaries are the *unit credit* cost method and the *unprojected individual level premium* cost method. The unit credit cost method has become the *de facto* standard method used, for instance, by international standard accounting norms (FAS and IFRS), although not necessarily imposed by plan regulations. The unprojected individual level premium is often used with the aim of inducing level (constant) contributions and is often applied by European insurers in group pension contracts.[1]

To describe the fundamentals of these methods, let us consider a defined benefit pension plan operating in a simple static environment with constant benefits, no preretirement decrements, no unfunded liabilities,[2] and no actuarial gains or losses. Our objective is to look at the evolution of the contributions for a typical plan member currently age x at time 0 up to retirement age y at time T, where $T = y - x$. Assume a constant plan valuation (actuarial) force of interest r, and contributions (normal costs) are paid continuously at rate $\pi(t)$ at time t. The annual retirement benefit is B_0 paid continuously[3] from retirement age until death. Thus, the actuarial present value of the retirement benefit at retirement age y is $K_0 = B_0\overline{a}_y$.

The unit credit cost method with service proration produces an actuarial (accrued) liability at time t, $\mathrm{AL_{UC}}(t)$, of

$$\mathrm{AL_{UC}}(t) = \frac{t}{T} K_0 e^{-r(T-t)}. \tag{1}$$

As there is no interest gain or loss, the assets at time t, $F(t)$, are simply the accumulation of the past contributions, i.e.,

[1] Collinson (2001) provides an extensive discussion of the various cost methods used in Europe. The unprojected individual level premium is not used in North America.

[2] An unfunded liability occurs whenever assets are not equal to liabilities.

[3] In our analysis it does not matter how often the retirement benefits are paid per year.

$$F(t) = \int_0^t \pi(s)\mathrm{e}^{r(t-s)}\mathrm{d}s. \tag{2}$$

As there is no unfunded liability, equating assets and liabilities at t results in the following integral equation for the contribution rate:

$$\frac{t}{T}K_0\mathrm{e}^{-r(T-t)} = \int_0^t \pi(s)\mathrm{e}^{r(t-s)}\mathrm{d}s, \tag{3}$$

which yields

$$\pi_{\mathrm{UC}}(t) = \frac{1}{T}K_0\mathrm{e}^{-r(T-t)}, \tag{4}$$

i.e., bounded contributions grow exponentially.

Under the unprojected individual level premium method, we equate the assets and actuarial liability at retirement age, assuming a constant contribution rate, π_{LP}. As actuarial liability at retirement age y is K_0 and the accumulated assets is $\pi_{\mathrm{LP}}\bar{s}_{\overline{T}|r}$, we get

$$\pi_{\mathrm{LP}} = \frac{K_0\mathrm{e}^{-rT}}{\bar{s}_{\overline{T}|r}} = \frac{K_0 r\mathrm{e}^{-rT}}{(1-\mathrm{e}^{-rT})} \tag{5}$$

and the actuarial liability at t, $\mathrm{AL}_{\mathrm{LP}}(t)$, is

$$\mathrm{AL}_{\mathrm{LP}}(t) = K_0\mathrm{e}^{-r(T-t)}\frac{1-\mathrm{e}^{-rt}}{1-\mathrm{e}^{-rT}}. \tag{6}$$

At this point we introduce the notion of stability. A pension cost method is said to be *stable* if its contribution rate is bounded at *all ages prior to* the plan's normal retirement age y. Comparing these two cost methods, we see $\mathrm{AL}_{\mathrm{UC}}(t) \le \mathrm{AL}_{\mathrm{LP}}(t)$ for $0 \le t \le T$ and that $\pi_{\mathrm{UC}}(t)$ increases monotonically and eventually exceeds π_{LP} before the retirement age. Thus, both methods yield stable contribution rates in this static environment.

It turns out that the stability exhibited by the unprojected individual level premium disappears under dynamic conditions. The purpose of this paper is to analytically compare the contributions generated by these two cost methods in a continuous time deterministic dynamic environment. For simplicity, country-specific laws and regulations are not considered in this paper. Simple assumptions are used to focus on the main effect of the methods and to obtain closed forms of the contributions.

2 Instability in a Dynamic Economy

2.1 Same Rate of Return for Assets and Liabilities

As was the case of the static economy, we assume no preretirement decrements, no unfunded liabilities, no actuarial gains or losses from any source, a constant plan valuation force of interest r for both assets and liabilities, and contributions (normal costs) are paid continuously at rate $\pi(t)$ at time t. In contrast to the static economy, however, we assume here that the retirement benefit is no longer constant over time; rather it is a function of time. Let $B(t)$ denote the promised annual benefit based on the salary known at time t, so that the actuarial present value of the promised retirement benefit at age y, based on the information available at time t, is $K(t) = B(t)\bar{a}_y$.

Under the unit credit cost method with proration,[4] the actuarial liability at time t now becomes:

$$\mathrm{AL}_{\mathrm{UC}}(t) = \frac{t}{T}K(t)\mathrm{e}^{-r(T-t)}.$$

Again, as there is no interest gain or loss, the assets at time t, $F(t)$ is given in equation (2). As there is no unfunded liability, assets and liabilities must be equal at t, which results in the following integral equation for the contribution rate:

$$\frac{t}{T}K(t)\mathrm{e}^{-r(T-t)} = \int_0^t \pi_{\mathrm{UC}}(s)\mathrm{e}^{r(t-s)}\mathrm{d}s.$$

Differentiating both sides with respect to t we obtain the general solution:

$$\pi_{\mathrm{UC}}(t) = \left(\frac{1}{T}K(t) + \frac{t}{T}\frac{\partial K}{\partial t}\right)\mathrm{e}^{-r(T-t)}. \quad (7)$$

If we assume salaries increase exponentially at rate g, so that $B(t) = B_0\mathrm{e}^{gt}$ and $K(t) = K_0\mathrm{e}^{gt}$, the contribution rate becomes:

$$\pi_{\mathrm{UC}}(t) = \frac{K}{T}\mathrm{e}^{gt}(1 + gt)\mathrm{e}^{-r(T-t)}, \quad (8)$$

which is a bounded non-decreasing function of t. If the promised retirement benefits increase linearly so that $B(t) = B_0 + B_1 t$ and $K(t) = K_0 + K_1 t$, the contribution rate becomes:

[4]In the case of the projected unit credit cost method, our approach is the same as in the unit credit cost method with proration, except that we now use an estimate of the final benefit, $B(T)$, taking into account salaries projection until retirement.

$$\pi_{\mathrm{UC}}(t) = \frac{1}{T}(K_0 + 2K_1 t)\mathrm{e}^{-r(T-t)} \tag{9}$$

which is again bounded.

Under the *unprojected* individual level premium, we assume that the contribution rate calculated at time t remains constant from time t to retirement at time T, i.e., for $T - t$ years. Again, as there is no interest gain, the fund at time t is given in equation (2). The actuarial liability is defined as the prospective reserve at time t based on a constant contribution rate from time t to T:

$$\mathrm{AL}_{\mathrm{LP}}(t) = K(t)\mathrm{e}^{-r(T-t)} - \pi_{\mathrm{LP}}(t)\bar{a}_{\overline{T-t}|r}.$$

As there is no unfunded liability, the contribution rate is now the solution to the integral equation:

$$\int_0^t \pi_{\mathrm{LP}}(s)\mathrm{e}^{-rs}\mathrm{d}s = K(t)\mathrm{e}^{-rT} - \pi_{\mathrm{LP}}(t)\frac{\mathrm{e}^{-rt} - \mathrm{e}^{-rT}}{r}.$$

Differentiating both sides with respect to t and simplifying yields

$$\frac{\partial \pi}{\partial t} = \frac{r}{(\mathrm{e}^{r(T-t)} - 1)}\frac{\partial K}{\partial t}$$

with initial condition (cf. formula (5)) $\pi_{\mathrm{LP}}(0) = K_0 r\mathrm{e}^{-rT}/(1 - \mathrm{e}^{-rT})$. The solution to this differential equation is:

$$\pi_{\mathrm{LP}}(t) = \frac{rK_0}{\mathrm{e}^{rT} - 1} + \int_0^t \frac{r}{(\mathrm{e}^{r(T-s)} - 1)}\frac{\partial K}{\partial s}\mathrm{d}s. \tag{10}$$

Now, for $0 \le h \le 1$, it is well-known that $1 + (\mathrm{e}^r - 1)h \ge \mathrm{e}^{rh}$. It follows that for $0 \le \epsilon \le 1$,

$$\int_{T-\epsilon}^{T} \frac{r}{(\mathrm{e}^{r(T-s)} - 1)}\frac{\partial K}{\partial s}\mathrm{d}s \ge \frac{r}{(\mathrm{e}^r - 1)}\int_{T-\epsilon}^{T} \frac{1}{(T-s)}\frac{\partial K}{\partial s}\mathrm{d}s.$$

Thus we have established the following result:

Result 1. *When the pension plan uses a single constant valuation interest rate, a sufficient condition for the* unprojected *individual level premium method to be unstable (unbounded) in the neighborhood of the retirement age is for the condition*

$$\lim_{t\to T-} \frac{1}{T-t}\frac{\partial K}{\partial t} = \infty \tag{11}$$

to hold.

It turns out that condition (11) holds in most practical dynamic environments.

For example, if we have a case of a linear benefit growth until time T, i.e., $K(t) = K_0 + K_1 t$, then condition (11) holds and the contribution density becomes, for $0 \le t \le T$,

$$\pi_{LP}(t) = \pi_{LP}(0) + K_1(-rt + \ln(\frac{e^{rT} - 1}{e^{r(T-t)} - 1})), \tag{12}$$

which is not bounded as $t \to T$. As another example, consider an exponential growth model where salaries grow at rate $g > 0$, i.e., $K(t) = K_0 e^{gt}$. Again, condition (11) holds and the contribution density now becomes, for $0 \le t \le T$,

$$\pi_{LP}(t) = \pi_{LP}(0) + K_0 r \int_0^t \frac{g e^{gs}}{(e^{r(T-s)} - 1)} ds, \tag{13}$$

which is not bounded as $t \to T$.

2.2 Different Rates of Return for Assets and Liabilities

We will now relax the assumption that the rate of return on assets and liabilities are the same. Let r denote the actuarial force of interest used for liabilities, and let $\delta(t)$ denote the deterministic force of return at time t assumed for assets, with the conservative (safe) assumption that $0 < \delta(t) < r$. Again, we do not permit unfunded liabilities.

Under the unit credit cost method, the basic equivalence formula (3) becomes:

$$\frac{t}{T}K(t)e^{-r(T-t)} = \int_0^t \pi_{UC}(s)e^{\int_s^t \delta(u)du} ds.$$

Taking the derivative with respect to t and simplifying gives:

$$\begin{aligned}\pi_{UC}(t) = &\left(\frac{1}{T}K(t) + \frac{t}{T}\frac{\partial K}{\partial t}\right) e^{-r(T-t)} \\ &+ (r - \delta(t))\left(\frac{t}{T}K(t)e^{-r(T-t)}\right).\end{aligned} \tag{14}$$

For the unprojected individual level premium method with no unfunded liabilities, we equate assets and the actuarial liability to get the integral equation:

$$\int_0^t \pi_{\mathrm{LP}}(s)\mathrm{e}^{\int_s^t \delta(u)du} \mathrm{d}s = K(t)\mathrm{e}^{-r(T-t)} - \pi_{\mathrm{LP}}(t)\bar{a}_{\overline{T-t}|r}.$$

Taking the derivative of both sides with respect to t and simplifying, we obtain the differential equation

$$\frac{\partial \pi_{\mathrm{LP}}}{\partial t} = \pi_{\mathrm{LP}}(t)(\delta(t) - r) + r\frac{(\frac{\partial K}{\partial t} + K(t)(r - \delta(t))}{(\mathrm{e}^{r(T-t)} - 1)}$$

with initial condition $\pi_{\mathrm{LP}}(0) = K_0 r\mathrm{e}^{-rT}/(1 - \mathrm{e}^{-rT})$. The solution is

$$\begin{aligned}\pi_{\mathrm{LP}}(t) &= \pi_{\mathrm{LP}}(0)\mathrm{e}^{\int_0^t(\delta(s)-r)\mathrm{d}s} \\ &\quad + r\int_0^t \left(\frac{\frac{\partial K}{\partial s} + K(s)(r - \delta(s))}{(\mathrm{e}^{r(T-s)} - 1)}\right)\mathrm{e}^{\int_s^t(\delta(u)-r)\mathrm{d}u}\mathrm{d}s.\end{aligned} \tag{15}$$

Comparing formulas (10) and (15), we see formula (15) has an extra term, which may be an extra source of instability in the neighborhood of the normal retirement age y at T. This extra term satisfies

$$\begin{aligned}&\int_{T-\epsilon}^{T} \left(\frac{rK(s)(r - \delta(s))}{(\mathrm{e}^{r(T-s)} - 1)}\right)\mathrm{e}^{\int_s^t(\delta(u)-r)\mathrm{d}u}\mathrm{d}s \\ &\geq \frac{r}{(\mathrm{e}^{r} - 1)}\int_{T-\epsilon}^{T} \left(\frac{K(s)(r - \delta(s))}{(T - s)}\right)\mathrm{e}^{\int_s^t(\delta(u)-r)\mathrm{d}u}\mathrm{d}s.\end{aligned} \tag{16}$$

As $K(T) > 0$ and $0 < \delta(t) < r$, it follows that the right side of inequality (16) is unbounded. Thus, we have the following result:

Result 2. *If the actuarial force of interest used for liabilities is constant and always exceeds the deterministic force of return used for assets, then the contribution rate under the unprojected individual level premium will be unstable (unbounded) in the neighborhood of the normal retirement age y at T.*

3 Conclusion

Unprojected individual level premium is often used in group pension arrangements in Europe (e.g., in Belgium). A continuous time environment is used to obtain simple explicit formulas for comparing contribution rates (normal costs) under the unprojected unit credit and individual level premium cost methods. While the unit credit cost seems to be safe and coherent with respect to changes in the benefits or in the rate of return on assets, the dangers of the unprojected individual level premium method have been highlighted. We have shown that when the benefits over the career are increasing and bounded functions with bounded first derivative:

- the contribution rate under unit credit cost method is bounded and stable, while
- the contribution rate under the unprojected individual level premium is generally not bounded.

Of course, in practice periodic contributions are computed instead of densities, but the property of unbounded density leads then to huge increases in the contribution rate just before retirement. We hope this observation convinces pension managers to move away from the unprojected individual level premium method and use the unit credit cost method (as recommended by the IFRS norms) or the *projected* individual level premium cost method (as in North America).

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