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Photographic Interpretation Handbook, United States Forces: Section 05 Formulae

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SECTION 5
FORMULAE

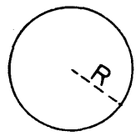
5.01 —— 5.99

FORMULAE
GEOMETRIC

CIRCUMFERENCE OF A CIRCLE = $2 \pi R$

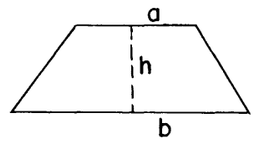
AREA OF A CIRCLE = πR^2

Where: π = 3.1416
R = radius of circle



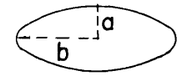
AREA OF A TRAPEZOID = $\frac{a + b}{2} h$

Where: a and b are the lengths of the parallel sides and h is the distance between.



AREA OF AN ELLIPSE = πab

Where: a and b are the semi-axes

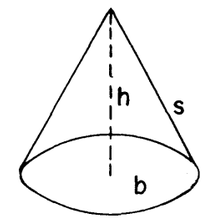


SURFACE OF A CONE = πRs

Where: R = radius of base
s = slant height

VOLUME OF A PYRAMID OR CONE = $\frac{bh}{3}$

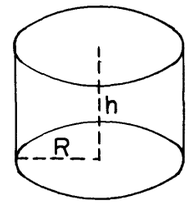
Where: b = area of base
h = height of pyramid or cone



SURFACE OF A CYLINDER = $2 \pi Rh$

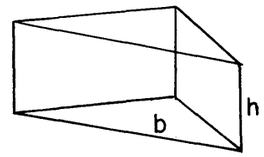
Where: R = radius
h = height

VOLUME OF A CYLINDER = $\pi R^2 h$



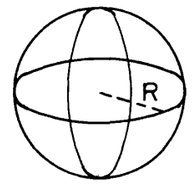
VOLUME OF A PRISM = bh

Where: b = area of base
h = height of prism



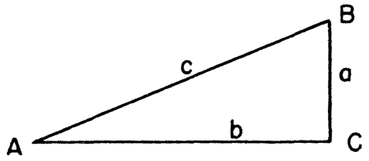
SURFACE OF A SPHERE = $4 \pi R^2$

VOLUME OF A SPHERE = $\frac{4 \pi R^3}{3}$



FORMULAE
TRIGONOMETRIC

RIGHT TRIANGLE

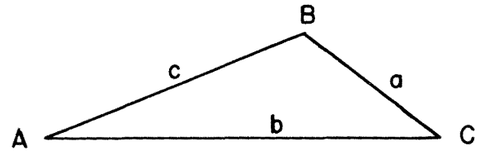


$\sin A = \frac{a}{c} = \cos B$ $\quad \tan A = \frac{a}{b} = \cot B$ $\quad \sec A = \frac{c}{b} = \operatorname{cosec} B$

$\cos A = \frac{b}{c} = \sin B$ $\quad \cot A = \frac{b}{a} = \tan B$ $\quad \operatorname{cosec} A = \frac{c}{a} = \sec B$

$A + B = 90°$ $\qquad a^2 + b^2 = c^2$

OBLIQUE TRIANGLE



If two angles and a side are known, an angle and two sides are known, or if three sides are known, the remaining angles and sides can be determined.

$$A + B + C = 180°$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{a}{\sin (B + C)} = \frac{b}{\sin (A + C)} = \frac{c}{\sin (A + B)}$$

$a^2 = b^2 + c^2 - 2bc \times \cos A$ $\qquad a = b \times \cos C + c \times \cos B$

Also, if: $s = \frac{1}{2}(a + b + c)$

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s - b)(s - c)}{bc}} \qquad \cos \tfrac{1}{2}A = \sqrt{\frac{s(s - a)}{bc}}$$

$$\sin A = 2\sqrt{\frac{s(s - a)(s - b)(s - c)}{bc}}$$

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} = \frac{a^2 \times \sin B \times \sin C}{2 \sin A} = \tfrac{1}{2}ab \times \sin C$$

GENERAL TRIGONOMETRIC RELATIONS

$$\tan A = \frac{\sin A}{\cos A} \qquad \tan A = \frac{\sin 2A}{1 + \cos 2A} \qquad \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A + \cos^2 A = 1$ $\qquad \sin 2A = 2 \sin A \times \cos A$ $\qquad \cos 2A = \cos^2 A - \sin^2 A$

$\sin A = \cos (90° - A) = \sin (180° - A)$

RESTRICTED

RESTRICTED

FORMULAE
PHOTOGRAMMETRIC

SCALE

Vertical Photographs

$$RF = \frac{f}{H} \qquad RF = \frac{f}{Hs - h} \qquad RF = \frac{d}{D} \qquad RF = \frac{d \times RF'}{d'}$$

Where:
- RF = scale or representative fraction
- f = focal length of camera lens
- H = altitude above ground
- Hs = altitude above sea level
- h = elevation of ground
- d = distance on photograph
- D = distance on ground
- d' = distance on second photograph or map
- RF' = scale of second photograph or of map

HEIGHT

$$ho = \frac{f \times P}{RF\,(W + P)} \qquad ho = \frac{H \times P}{W + P} \qquad ho' = \frac{ho \times P'}{P}$$

Where:
- ho = height of object
- ho' = height of second object
- f = focal length of camera lens
- P = differential parallax measured on stereo pair by height finder or stereocomparagraph
- P' = differential parallax measured for second object
- RF = scale of photograph
- W = photo distance between centers of stereo pair in line of flight
- H = height of camera above ground

DEPTH

$$depth = \frac{H \times P}{(W - P)f}\sqrt{w^2 (n^2 - 1) + n^2 f^2} \qquad depth = \frac{1.34 \times H \times P}{W} \text{ (approx.)}$$

Where:
- depth = depth of water
- n = index of refraction of water with respect to air
- W = photo distance between centers of stereo pairs in line of flight
- w = distance from point of depth measurement to center of photograph
- f = focal length of camera lens
- H = altitude of camera
- P = differential parallax measured between determinable point on bottom to surface of water (at shore line)

All measurements in same units!

RESTRICTED

FORMULAE

PHOTOGRAMMETRIC (CONT.)

HEIGHT FROM SHADOW

$$\sin x = \cos a \times \cos b \times \cos c \pm \sin a \times \sin b$$

$$ho = L \times \tan x$$

$$\frac{\sin c}{\sin N} = \frac{\cos x}{\cos a}$$

$$\sin x = \frac{\cos b \times \cos N \sqrt{\cos^2 a - \cos^2 b \times \sin^2 N} \pm \sin a \times \sin b}{1 - \cos^2 b \times \sin^2 N}$$

Where:
- x = angle of elevation of sun
- a = declination of latitude of sun
- b = latitude of photograph
- c = difference in longitude between sun and photograph
- ho = height of object casting shadow
- L = length of shadow on level ground
- N = angle between shadow direction and north-south direction