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# **Shadow Price of Environmental *Bads*: Weak vs. Strong Disposability<sup>1</sup>**

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## ***Abstract***

This paper addresses the issue of the shadow price of environmental *bads* treated as an undesirable output (normal input) with weak (strong) disposability in a two-stage estimation. Nebraska agriculture sector time series data is spread over 1936-94. Results indicate the difference in the price due to the disposability property.

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## **Shadow Price of Environmental *Bads*: Weak vs. Strong Disposability**

**Saleem Shaik and Glenn Helmers**

There has been considerable theoretical development and application of programming methods directed to environmental analysis due to its advantages in accounting for undesirable outcomes (environmental *bads*). Taking advantage of nonparametric methods to accommodate weak (undesirable output) and strong (normal input) disposability, environmentally adjusted productivity (EAP) measures have been computed (Shaik, 1998) for the Nebraska agriculture sector. However, these measures might be misrepresented due to the inability of the nonparametric approach in multiple technology to attach appropriate weights (representing their marginal product i.e., prices) in productivity measurement (Shaik, 1997). Using an aggregate output-input technology the ratio of the slopes (equivalent to the marginal rate of transformation) of the parametric distance function allows the retrieval of shadow prices of environmental *bads*. Due to the inherent difficulty and interpretation of assuming an efficient distance function (dependent variable is one), the computed nonparametric distance measures are used as a dependent variable in the parametric distance function.

Environmental *bads*, given their public good nature are valued equally by consumers and producers only under restrictive assumptions and conditions. Recently successful attempts have been made to value environmental *bads* through contingent valuation methods from the consumer perspective. An alternative is to compare the value of environmental *bads* from the producer side. But how would the environmental policies or producers actions change if we valued environmental *bads*? It would be expected that producers' actions would change if environmental *bads* are internalized in their production choices. Currently environmental *bads* are largely disassociated from production decisions leading to increased use of chemicals and

other practices above that which would occur were environmental *bads* included in producers decisions. Contribution to this phenomenon is the uncertainty and lack of knowledge of exact losses (shadow price of environmental *bads*) experienced by individual producers. Because the prices of environmental *bads* from the producer perspective are seldom available, duality theory between the output (input) distance function and the revenue (cost) function is exploited in this paper in retrieving shadow prices. Since environmental *bads* can theoretically be treated either as an undesirable output (hyperbolic output oriented measure as in Fare et al, 1993) with weak disposability or as an input (radial input oriented measure as in Pittman, 1981; Reinhard et al, 1997, who indicated the *reason for using environmental bads as input is largely pragmatic*) with strong disposability, computation of both shadow price estimates provides alternative sets of values for comparison. This equivalency treating environmental *bads* as an undesirable output or an input can be developed given an implicit function.

Let a zero profit maximising firm accounting for environmental *bads* be represented by an implicit production function  $F(y_g, y_b, x) = 0$ . The other assumptions include joint production and separability of output ( $y_g$ ) and environmental *bads* ( $y_b$ ). The first order conditions of the implicit function with respect to its elements is positive. However given the weak disposability assumption of environmental *bads* (undesirable output) the firm conceptually would maximise with negative<sup>1</sup> (positive) marginal rates of transformation for *bads* (output). This negative marginal rate of transformation is equivalent to positive marginal product of the *bads* (when treated as conventional input) since  $\partial y_g / \partial y_b \equiv \partial y_g / \partial x |_{y_b = x}$ .

This paper contributes to the existing literature, by estimating the shadow price of environmental *bads* treated as an undesirable output (conventional input) with weak (strong)

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<sup>1</sup> The negative marginal rate of transformation reflects the inward bending of the transformation curve or backward bending of the input requirement set.

disposability in a two-stage estimation. A two-stage estimation procedure is applied to exploit the nonparametric and parametric methods in computing shadow prices for agriculture damage in Nebraska. The second section presents the duality theory models of the output (input) distance function and the revenue (cost) function used in retrieving the shadow prices. The construction of Nebraska agriculture sector data of inputs, outputs and environmental *bads* is explained in the third section. The empirical application and results are presented in the fourth section followed by conclusions in the last section.

## Nonparametric Output and Input Distance Functions

The two-stage, nonparametric approach followed by the parametric approach is effectively adapted in the estimation of distance measures. In the first stage, the distance function is computed for each observation using a piecewise linear DEA model due to its ability to impose weak (undesirable output) and strong (conventional input) disposability assumptions. Second the distance measures so obtained are used as dependent variables in the parametric distance function to account for the parametric nature allowing the retrieval of shadow prices.

Following Fare, et al (1989 pp. 92-93), the weak disposal reference set satisfying constant returns to scale, strong disposability of desirable outputs, and weak disposability of environmental externalities can be defined as:

$$(1) \quad \begin{aligned} P_w^T(x) &= \{ y : x \text{ can produce } (y_g, y_b) \text{ in year } T; \\ 0 \leq \theta \leq 1 \text{ implies } \theta(y_g, y_b) \in P_w^T(x) \quad y_g^1 < y_g \Rightarrow \theta(y_g, y_b) \in P_w^T(x) \} \end{aligned}$$

The distance function and linear programming problem<sup>2</sup> used to calculate this hyperbolic output measure can be evaluated for each year  $t$  as:

$$(2) \quad D_o^T(x^t, y_g^t, y_b^t)^{-1} = \max_{\theta, z} \{ \theta : (\theta y_g^t, \theta^{-1} y_b^t) \in P_w^T(x^t) \}$$

or

$$\max_{\theta, z} \theta \quad \text{s.t.} \quad \theta y_g^t \leq Y_g z$$

$$(2 - \theta) y_b^t = Y_b z$$

$$x^t \geq X z$$

$$z \geq 0$$

where  $Y_g = (y_g^1, y_g^2, \dots, y_g^T)$

Following Shaik (1998), the input reference set satisfying constant returns to scale and strong disposability of the inputs as well as environmental *bads* treated as an input can be defined as:

$$(3) \quad L^T(y_g) = \{ (x, y_b) : y_g \text{ produced by } (x, y_b) \text{ in year } T; \}$$

This concept can be represented by an input distance function evaluated for each year  $t$  using a reference production possibilities set  $T$ , as:

$$(4) \quad D_i^T(y_g^t, x^t, y_b^t)^{-1} = \min_{\lambda, z} \{ \lambda : (\lambda x, \lambda y_b^t) \in L^T(y_b^t) \}$$

or

$$\min_{\lambda, z} \lambda \quad \text{s.t.} \quad y_g^t \leq Y_g z$$

$$\lambda y_b^t \geq Y_b z$$

$$\lambda x^t \geq X z$$

$$z \geq 0$$

where  $X = (x^1, x^2, \dots, x^T)$

<sup>2</sup> This result uses the Fare, Grosskopf, Lovell and Pasurka linearization of the  $\theta^{-1}$  nonlinear constraint. Using a first order Talyor series expansion,  $f(X) = f(X_0) + f'(X_0)(X - X_0)$ , let  $f(\theta) = \theta^{-1}$  and if  $\theta$  is approximated around 1 then  $Y_b z \geq \theta^{-1} y_b$  would be  $Y_b z \geq (2 - \theta) y_b$

## Duality between Output (Input) Distance Function and Revenue (Cost)

Shephard (1970) and Fare and Primont (1995) have established the duality between the output (input) distance function and revenue (cost). We extended it to include environmental *bads* treated as an undesirable output and as conventional input. Shephard's lemma duality is exploited in computing the shadow prices of environmental *bads* treated as an undesirable output (conventional input) with the assumption that the revenue (cost) and output (input) distance functions are differentiable. We then calculate the shadow price of environmental *bads* treated as an undesirable output (conventional input), given the price of output (input) as the ratio of distance function derivatives with respect to desirable output (input) and environmental *bads*.

Following Fare and Primont (1995 pp. 49-51) the duality between the output distance function and the revenue function can be defined as:

$$(5) \quad \begin{aligned} R(x, p_g, p_b) &= \max_{y_g, y_b} \{(p_g y_g - p_b y_b) : D_o(x, p_g, p_b) \leq 1\} \\ D_o(x, p_g, p_b) &= \sup_{p_g, p_b} \{(p_g y_g - p_b y_b) : R(x, p_g, p_b) \leq 1\} \end{aligned}$$

Consequently they also derived the absolute output shadow prices of each observation of environmental *bads* as the ratio of the derivatives of the distance functions i.e.,

$$(6) \quad \frac{P_b}{P_g} = \frac{\partial D_o(x, y_g, y_b) / \partial y_b}{\partial D_o(x, y_g, y_b) / \partial y_g}$$

In our special case since we assume a revenue maximizing firm with zero profits the observed revenue  $[R(x, p_g, p_b)]$  equals the actual cost  $[C(y_g, y_b, w)]$ . Hence the shadow prices of environmental *bads* is computed from the distance function parameter estimates as:

$$(7) \quad P_b = C(y_g, y_b, w) * \frac{\partial D_o(x, y_g, y_b) / \partial y_b}{\partial D_o(x, y_g, y_b) / \partial y_g}$$

Following Fare and Primont (1995 pp. 44-48) the duality between input distance function and cost function can be defined as:

$$(8) \quad \begin{aligned} C(y_g, w, p_b) &= \min_{x, y_b} \{(wx + p_b y_b) : D_i(y_g, x, y_b) \geq 1\} \\ D_i(y_g, x, y_b) &= \inf_{w, p_b} \{(wx + p_b y_b) : C(y_g, w, p_b) \leq 1\} \end{aligned}$$

Thus the absolute input shadow prices of each observation of environmental *bads* is the ratio of the derivatives of the distance functions i.e.,

$$(9) \quad \frac{P_b}{w} = \frac{\partial D_i(y_g, x, y_b) / \partial y_b}{\partial D_i(y_g, x, y_b) / \partial x}$$

Similarly in a competitive environment, the firm would be market constrained at least in the long run, so the observed cost  $[C(y_g, w, p_b)]$  equals the actual revenue  $[R(x, p_g)]$ . This approach of calculating shadow prices is consistent with the shares obtained in the Theil-Tornquist index. So the input shadow prices of environmental *bads* is computed from the distance function parameter estimates as:

$$(10) \quad P_b = R(x, p_g) * \frac{\partial D_i(y_g, x, y_b) / \partial y_b}{\partial D_i(y_g, x, y_b) / \partial x}$$

In order to calculate the shadow prices from equations [7 and 10] we use the parameter estimates from the parametric distance functions. The use of a Cobb-Douglas function in the estimation would be consistent with the use of the nonparametric method in the first stage. The output and input distance functions are defined as:

$$\begin{aligned} (11a) \quad D_o &= \alpha_0 + \alpha_g Y_g + \alpha_b Y_b + \alpha_x X + \varepsilon_t \\ (11b) \quad D_i &= \alpha_0 + \alpha_g Y_g + \alpha_x X + \alpha_b Y_b + \varepsilon_t \end{aligned}$$

where,  $D_o$  and  $D_i$  is the output and input distance function respectively.



## Nebraska Output, Input and Environmental *Bads* Data

Aggregate input, aggregate output and three environmental *bads* data spread over 1936-94 time period is used in the analysis. The single output quantity index was aggregated from food grains, feed crops, vegetable and oil crops, meat animals, poultry and other livestock including milk, honey and wool production. Annual data on crop production [yield per acre times total harvested acres for each crop], livestock quantity estimates [pounds of meat produced] multiplied by prices received by farmers were used in the construction of an output Theil-Tornquist quantity index.

An aggregate input quantity index was constructed by aggregating farm equipment, breeding livestock, farm real estate, farm labor and intermediate inputs accounting for quantity and quality changes (Shaik, 1998). The capital stock (1982 Mil \$), number of breeding livestock (January 1), three types of land (non-irrigated, irrigated and pasture) and value of buildings and structures (1982 Mil \$), implicit intermediate quantity index (logarithmic difference between the rate of change in expenditures and price index) and total hours worked was used as input quantity changes. The rental value reflected their marginal products in case of capital, and breeding livestock; cash rents in case of land, expenditures in case of intermediate inputs and wage compensation in the case of farm labor was used as shares in the aggregation of input quantity index.

Excess nitrogen from agriculture is calculated as the difference between nitrogen inputs [commercial fertilizer, animal manure and legume fixation] and nitrogen removed by harvested crops. Evidence [Exner and Spalding, 1990; Muller et al, 1995] based on sampling of wells in Nebraska indicates a positive correlation between high levels of nitrate contamination in irrigation wells and fertilizer and animal manure application. This offers some support for using

nitrogen surplus as a proxy for environmental externalities produced due to agriculture.

Information on the extent of pesticide use in pounds is available only for survey years. Utilizing these point data, a time series data on quantity of an active pesticide ingredient was generated based on the rate of change of implicit pesticide quantity index for Nebraska. A pesticide leaching loss potential [PLLP] index is computed by using pounds of pesticide as shares for each pesticide. A time series PLLP index was computed by interpolation between the survey years. Deflating the pesticide use by the PLLP index gives an implicit damage quantity index.

Wetland loss is computed as the difference in the wetland inventory. A wetland inventory is computed based on unpublished wetland data [Ralph Heimlich, 1997] for Nebraska, Gersib et al [1992] data for rainwater basin and Natural Resource Commission [1993] for Sandhills. Utilizing these data, a time series is constructed by adding acreage drained for farming.

## **Empirical Application and Results**

The environmentally adjusted productivity (EAP) estimates accommodating weak (undesirable output) and strong (normal input) disposability of environmental bad for Nebraska agriculture sector has been computed using the nonparametric approach. The results of the slopes of the output and input parametric distance functions (equation 12a, 12b) estimated using SHAZAM are presented in Table 1. The stochastic distance function is also estimated with an error term, which has two components, one to account for random effects and another to account for technical inefficiency. The residuals from these estimates seem to have the wrong skewness and the likelihood values is less than that obtained using OLS leading to non-convergence of the

maximum likelihood estimation. Apparently OLS estimation seems to be a better estimator of the fit for the data. We corrected for autocorrelation using an alternative maximum likelihood procedure which incorporates the first observation and the stationarity condition of the error process following Beach and McKinnon, 1978.

The absolute shadow price for each observation of environmental *bads* can be computed as the ratio of the derivatives of the parametric distance function [equivalent to the marginal rate of transformation] reflecting the opportunity cost of agricultural production and environmental *bads* in the Nebraska agriculture sector. Alternatively the dual measures from the nonparametric piecewise linear program can be used as the slopes. For our purpose the estimates from a Cobb-Douglas function are substituted in equations (7 & 10) to compute the shadow value (shadow price of environmental *bads* at the aggregate level) of environmental *bads*.

Table 2 presents the shadow price and the value of potential nitrate pollution, pesticide contamination and wetland losses treated as an undesirable output and input. With the exception of pesticide contamination when used as an input, the signs are consistent. Results indicate higher shadow prices when the environmental bad is treated as an input compared to an undesirable output. The shadow price computed from the output distance function represents the opportunity cost to the producer to reduce pollution along with increasing agriculture production given the level of inputs. In contrast higher shadow price estimates from the input distance function reflects the value of production forgone if nitrate and pesticide are not applied or agriculture land is lost to wetlands conservation. The cost of environmental damage in 1971-80 and 1981-94 was comparatively higher than the average cost for the whole period of the analysis for potential nitrate pollution, pesticide contamination and wetland losses. The negative cost in the case of nitrate pollution prior to 1960 indicates excess nitrate was good for agriculture

production. Based on the environmentally adjusted productivity measures and the shadow prices, the results confirm that TFP measures overestimate productivity growth if environmental costs are unaccounted for and underestimate them if environmental benefits due to environmental friendly technologies are ignored. The difference in the two estimates of shadow price can be largely attributed to the disposability assumption. In the output case, the shadow price represents the cost of reducing environmental damage to produce the same amount of output using the level of inputs available in that year (hyperbolic output distance function). Treating environmental damage as an input represent the cost of reduced use of this particular input to produce given output using the level of conventional inputs available in that year (input distance function).

## **Conclusions**

The results demonstrate difference in the shadow price estimates when environmental damage is treated as an undesirable output and as an input due to the disposability assumption. A two-stage estimate accommodates weak disposability in the deterministic framework yielding parametric estimates of the slopes. This study can be a corner stone for further research, particularly related to 1) use of shadow price to adjust the productivity measures and 2) estimation of the demand for environmental *bads* in a system of equations.

## Reference:

- Beach, C., and MacKinnon, J., “ A Maximum Likelihood Procedure for Regression with Autocorrelated Errors.” *Econometrica*. 46(Jan, 1978): 51-58.
- Exner, Mary E., and R. F. Spalding. “Occurrence of pesticides and nitrate in Nebraska's ground water.” Nebraska Water Resources Center, Lincoln: Water Center, Institute of Agriculture and Natural Resources, The University of Nebraska, 1990.
- Fare, R., S. Grosskopf and C. A. K. Lovell. *Production Frontiers*. Cambridge: Cambridge University Press, 1994.
- Fare, R., S. Grosskopf, C. A. K. Lovell and C. Pasurka. “ Multilateral Productivity Comparisons When Some Outputs are Undesirable: A Nonparametric Approach” *Review of Economics and Statistics*. 71(1989): 90-98.
- Fare, R., S. Grosskopf, C.A.K.Lovell and S. Yaisawarng. “Derivation of Shadow Prices for Undesirable Outputs: A Distance function Approach.” *Review of Economics and Statistics*. 75(1993): 374-80
- Fare, R and D. Primont. *Multi-Output Production and Duality: Theory and Applications*. Boston: Kluwer Academic Publishers.
- Gersib, R.A., K. F. Dinan, J. D. Kauffeld, M. D. Onnen, P. J. Gabig, J. E. Cornely, G. E. Jasmer, J. M. Hyland, K. J. Strom. *Rainwater Basin Joint Venture Implementation Plan*, Nebraska Game and Parks Commission, Lincoln, NE. 1993: 56pp.
- Muller, D. K., Hamilton, P.A., Helsel, D.R., Hitt,K.J., and Ruddy, B.C., “*Nutrients in Ground Water and Surface Water of the United States- An analysis of data through 1992.*” U.S. Geological Survey Water-Resources Investigations (WRI) Report 95-4031, 1995.
- Natural Resources Commission. *Report on the Sandhills Area Study*, Nat. Resour. Comm., Lincoln, Nebraska. 1993: 56 pp.
- Pittman, R.W. “Issues in Pollution Control: Interplant Cost Differences and Economies of Scale.” *Land Economics*. 57(1981): 1-17.
- Ralph Heimlich. Unpublished data on Wetlands In Nebraska, 1997.
- Reinhard, Stijn., Geert Thijssen and C. A. Knox. Lovell. “Econometric Measurement of Technical and Environmental Efficiency of Dutch Dairy Farms.” *Working paper*, Agricultural Economics Research Institute (LEI-DLO), Netherlands, 1997.

Shaik Saleem. “*Environmentally Adjusted Productivity [EAP] Measures for Nebraska Agriculture Sector*” *Dissertation*, Dept. of Agriculture Economics, University of Nebraska-Lincoln, May 1998.

Shaik Saleem. “*Comparison of Index, Nonparametric and Parametric Productivity Measures: Nebraska Agriculture Sector.*” Working paper. Department of Agriculture Economics, University of Nebraska-Lincoln, 1997.

SHAZAM *User’s Reference Manual Version 7.0*, McGraw-Hill, 1993.

Shephard, R.W. “*Theory of Cost and Production,*” Princeton University Press, 1970.

**Table 1. Average Slope of the Output and Input Distance Functions.**

<i>Variables</i>	$\partial D / \partial Y_g$	$\partial D / \partial Y_b$	<i>Ratio</i>
Output Distance Function			
Nitrate Pollution	0.1203	0.0005	0.0038
Pesticide Contamination	-0.0109	-0.0001	0.0054
Wetland Losses	0.1720	0.1636	0.9512
Input Distance Function			
Nitrate Pollution	-0.4829	0.0196	-(-0.0406)
Pesticide Contamination	-0.0753	-0.0001	-(-0.0014)
Wetland Losses	0.2132	-0.0712	-(-0.3340)

Where D is an output or an input distance function,  $Y_g$  is a desirable output &  $Y_b$  is environmental *bads*

**Table 2. Shadow Price and Value of Environmental Bads Treated as Undesirable Output and Input.**

	<i>Potential Nitrate Pollution</i>		<i>Pesticide Contamination</i>		<i>Wetland Losses</i>	
	<i>Shadow Price</i>	<i>Shadow Value</i>	<i>Shadow Price</i>	<i>Shadow Value</i>	<i>Shadow Price</i>	<i>Shadow Value</i>
1936-50	0.015	-2.67	0.021	0.00	3.76	0.15
1951-60	0.024	-3.25	0.035	0.04	6.13	0.12
1961-70	0.028	7.44	0.040	0.10	7.00	0.09
1971-80	0.051	34.02	0.074	0.94	12.95	0.10
1936-80	0.028	7.60	0.040	0.24	7.05	0.12
1981-94	0.069	31.71	0.100	1.94	17.50	0.07
1936-94	0.038	13.32	0.054	0.64	9.53	0.11
			Input Distance Function			
1936-50	0.228	-46.17	-0.006	-0.001	1.32	0.053
1951-60	0.360	-54.32	-0.009	-0.010	2.15	0.042
1961-70	0.383	105.90	-0.011	-0.028	2.46	0.032
1971-80	0.759	493.65	-0.020	-0.248	4.55	0.037
1936-80	0.410	105.77	-0.011	-0.064	2.47	0.042
1981-94	1.210	545.20	-0.026	-0.512	6.15	0.025
1936-94	0.600	210.04	-0.014	-0.170	3.35	0.038

Where the units of shadow prices are \$/ lb Nitrogen, \$/ lb pesticide and \$ / acre of wetland and numbers in the parenthesis represent the shadow value in Mil \$.



**Figure 1. Nonparametric Output and Input EAP and TFP measures  
Nebraska Agriculture Sector**

