Two complex orthogonal space-time codes for eight transmit antennas

Le Chung Tran
University of Wollongong

J. Seberry
University of Wollongong

Yejing Wang
University of Wollongong

B. J. Wysocki
University of Wollongong

T. A. Wysocki
University of Wollongong, wysocki@uow.edu.au

See next page for additional authors

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Le Chung Tran, J. Seberry, Yejing Wang,
B.J. Wysocki, T.A. Wysocki, Tianbing Xia
and Ying Zhao

Two new constructions of complex orthogonal space-time block codes of order 8 based on the theory of amicable orthogonal designs are presented and their performance compared with that of the standard code of order 8. These new codes are suitable for multi-modulation schemes where the performance can be sacrificed for a higher throughput.

Introduction: Transmit antenna diversity can be accomplished with the use of space-time codes (STCs) [1]. Out of the STCs, space-time block codes (STBCs) developed from the amicable orthogonal designs [2] lead to the simplest receiver structures and minimum processing delays if combined with modulation schemes having complex signal constellations, such as in the case of quaternary phase shift keying (QPSK) or quadrature amplitude modulation (QAM). The simplest STBC based on amicable orthogonal designs is an Alamouti code [1] providing a transmission rate of 1 for the two transmit antenna system. STBCs based on amicable orthogonal designs, usually referred to as complex orthogonal STBCs, for a larger number of transmit antennas cannot provide the transmission rate of 1 but they are attractive nevertheless since they can provide a full diversity for the given number of transmit antennas and are usually simple to decode. There exist some complex orthogonal STBCs designs for four transmit antennas and eight transmit antennas, e.g. [3], providing the rates of 3/4 and 1/2, respectively. They are usually based on those amicable orthogonal designs where each variable is represented just once in a design. Hence, the code matrices have many zeros (50% for eight transmit antennas) resulting in many time slots when no useful information is being transmitted. In this Letter, we introduce two new complex orthogonal STBCs for eight transmit antennas having less unused time slots. In the first of these new codes, one of the variables is repeated twice and in the second code, one variable is repeated four times per every transmit antenna. These properties can be further exploited to increase the code rate over 1/2 using more sophisticated modulation schemes, where the higher number of information bits is associated with those signals that appear in more than one time slot per each antenna.

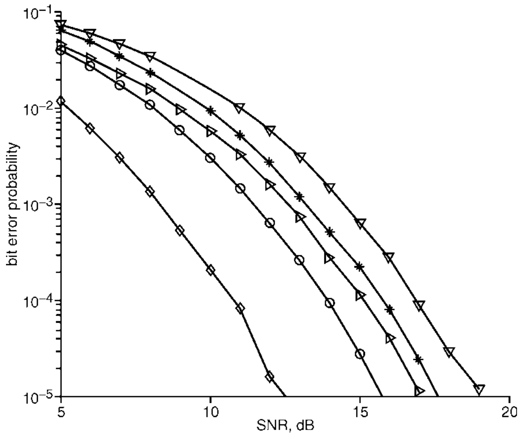

Fig. 1 *Bit error performance of S_1 in single and multi-modulation schemes*

–○– code S_1, multimod, QPSK and 8 PSK
–∗– code S_1, multimod, QPSK and 16 QAM
–◇– code S_1, single-modulation, QPSK
–▷– reference code, multimod, QPSK and 8 PSK
–▽– reference code, multimod, QPSK and 16 QAM

New codes and their performance: Orthogonal STBCs that can be used with complex signal constellations can be constructed using complex orthogonal designs (CODs) defined as follows.

Definition 1: A complex orthogonal design (COD) $\mathbf{X}$ of order n is an $n \times n$ matrix on the complex indeterminates $s_1, \ldots, s_t$, with entries chosen from $0, \pm s_1, \ldots, \pm s_t$, their complex conjugates $\pm s_1^*, \ldots, \pm s_t^*$, or their linear combinations, such that

$$\mathbf{X}^H \mathbf{X} = \left(\sum_{k=1}^{t} |s_k|^2 \right) \mathbf{I}_n \quad (1)$$

where $\mathbf{X}^H$ denotes the Hermitian transposition of $\mathbf{X}$ and $\mathbf{I}_n$ is the identity matrix of order n.

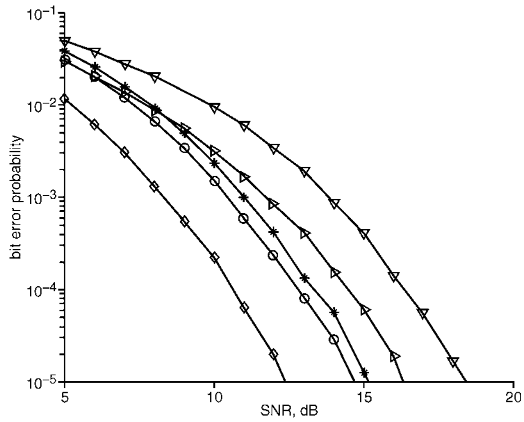

Fig. 2 *Bit error performance of S_2 in single and multi-modulation schemes*

–○– code S_2, multimod, QPSK and 8 PSK
–∗– code S_2, multimod, QPSK and 16 QAM
–◇– code S_2, single-modulation, QPSK
–▷– reference code, multimod, QPSK and 8 PSK
–▽– reference code, multimod, QPSK and 16 QAM

CODs are strongly connected to the amicable orthogonal designs (AODs), which have been explained in [4]. The detailed theory of amicable orthogonal designs (AODs), including limitations on the number of different variables for the given design order can be found in [2]. Drawing from the theory of the existence of AODs presented there, we found two new CODs of order 8. The first one, corresponding to the AOD (8; 1, 1, 2, 2; 1, 1, 2, 2), which indicates that the 8 columns of the matrix contain both real and imaginary parts of $s1$ and $s2$ once, and of $s3$ and $s4$ twice, is of the form:

$$\begin{bmatrix}
s_1 & s_2 & \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & 0 & 0 & \frac{s_4}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} \\
-s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} & 0 & 0 & \frac{s_4}{\sqrt{2}} & -\frac{s_4}{\sqrt{2}} \\
\frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & -s_1^R + is_2^I & -s_2^R + is_1^I & \frac{s_4}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} & 0 & 0 \\
\frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & s_2^R + is_1^I & -s_1^R - is_2^I & \frac{s_4}{\sqrt{2}} & -\frac{s_4}{\sqrt{2}} & 0 & 0 \\
0 & 0 & \frac{s_4^*}{\sqrt{2}} & \frac{s_4^*}{\sqrt{2}} & s_1 & s_2 & -\frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} \\
0 & 0 & \frac{s_4^*}{\sqrt{2}} & -\frac{s_4^*}{\sqrt{2}} & -s_2^* & s_1^* & -\frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} \\
\frac{s_4^*}{\sqrt{2}} & \frac{s_4^*}{\sqrt{2}} & 0 & 0 & -\frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & -s_1^R + is_2^I & -s_2^R + is_1^I \\
\frac{s_4^*}{\sqrt{2}} & -\frac{s_4^*}{\sqrt{2}} & 0 & 0 & -\frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & s_2^R + is_1^I & -s_1^R - is_2^I
\end{bmatrix} \quad (2)$$

where s_k^R, and s_k^I denote real and imaginary parts of s_k, respectively. The second design is based on AOD (8; 1, 1, 1, 4; 1, 1, 1, 4) and is of the form:

$$\begin{bmatrix} s_1 & 0 & s_3^R + is_2^I & s_2^R + is_3^I & \frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} \\ 0 & s_1 & -s_2^R + is_3^I & s_3^R - is_2^I & \frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} \\ -s_3^R + is_2^I & s_2^R + is_3^I & s_1^* & 0 & \frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} \\ -s_2^R + is_3^I & -s_3^R - is_2^I & 0 & s_1^* & \frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} \\ -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & s_1^R - is_3^I & s_2^* & s_3^R - is_1^I & 0 \\ -\frac{s_4^*}{2} & \frac{s_4^*}{2} & -\frac{s_4^*}{2} & \frac{s_4^*}{2} & -s_2 & s_1^R + is_3^I & 0 & s_3^R - is_1^I \\ -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & \frac{s_4^*}{2} & \frac{s_4^*}{2} & -s_3^R - is_1^I & 0 & s_1^R + is_3^I & -s_2^* \\ -\frac{s_4^*}{2} & \frac{s_4^*}{2} & \frac{s_4^*}{2} & -\frac{s_4^*}{2} & 0 & -s_3^R - is_1^I & s_2 & s_1^R - is_3^I \end{bmatrix} \quad (3)$$

If all the symbols of both new STBCs, S_1 and S_2 defined by the CODs given by (2) and (3), respectively, are associated with QPSK complex symbols, then the bit error rate (BER) performance in a Gaussian channel of both S_1 and S_2 is exactly the same as performance of the complex orthogonal STBC of order 8 described in [3]. The achieved code rate is also the same and equal to $1/2$. From (2) and (3) it is visible that in S_1 and S_2 some of the symbols are transmitted in more than a single time slot per given antenna. In fact, in S_1, symbols s_3 and s_4 are transmitted twice as often as s_1 or s_2. In S_2, the symbol s_4 is transmitted four times as often as s_1, s_2 or s_3. Thus, by associating s_3 and s_4 in S_1 and s_4 in S_2 with symbols from multilevel complex modulation schemes and the remaining symbols in each of S_1 and S_2 with QPSK symbols, the overall code rates can be increased. Of course, there is a trade-off between the rate increase and the BER performance. This is illustrated in Figs. 1 and 2 for S_1 and S_2, respectively. In both Figures, signal-to-noise ratio (SNR) is defined by the ratio between the total power received in each symbol time slot and the noise power at the receive antenna. Multi-modulation, using QPSK, 8 PSK and 16 QAM constellations, is utilised and the bit error performance of the STBC given in [3] is also simulated to compare with the proposed codes. In simulation, the signal power per transmission in each symbol time slot is normalised to 1. Figs. 1 and 2 show that the proposed codes associated with QPSK single-modulation provide a good bit error performance. Additionally, by sacrificing 3 and 5 dB, with respect to SNR at $\text{BER} = 10^{-5}$, in the case of S_1, and by 2.5 and 3 dB, in the case of S_2, one can increase the code rate from $1/2$ to higher code rates of $5/8$ and $3/4$, in the case of S_1, and of $9/16$ and $5/8$, in the case of S_2, respectively. Furthermore, at the same code rate, both proposed codes provide better bit error performance than the code in [3], by around 1 dB (QPSK + 8 PSK) and 1 dB (QPSK + 16 QAM), in case of S_1, and around 1.5 dB (QPSK + 8 PSK) and 2.5 dB (QPSK + 16 QAM), in case of S_2, respectively, at $\text{BER} = 10^{-5}$.

Conclusions: We have presented two new complex orthogonal STBCs for eight transmit antennas that can provide higher data rates (up to $3/4$) than other complex STBCs of the same order. This is achieved by employing multilevel modulation schemes for those code symbols that are transmitted more often than other symbols for which QPSK is used. This feature can be utilised in the design of adaptive STBC schemes, where the code rate can be traded for BER and vice versa.

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Le Chung Tran, J. Seberry, Yejing Wang, B.J. Wysocki, T.A. Wysocki, Tianbing Xia and Ying Zhao (*University of Wollongong, NSW 2522, Australia*)

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