The Polar Coordinate System

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Representing a position in a two-dimensional plane can be done several ways. It is taught early in Algebra how to represent a point in the Cartesian (or rectangular) plane. In this plane a point is represented by the coordinates (x, y), where x tells the horizontal distance from the origin and y the vertical distance. The polar coordinate system is an alternative to this rectangular system. In this system, instead of a point being represented by (x, y) coordinates, a point is represented by (r, θ) where r represents the length of a straight line from the point to the origin and θ represents the angle that straight line makes with the horizontal axis. The r component is commonly referred to as the radial coordinate and θ as the angular coordinate. Just as in the Cartesian plane, the polar plane has a horizontal axis and an origin. In the polar system the origin is called the *pole* and the horizontal axis, which is a ray that extends horizontally from the pole to the right, is called the *polar axis*. An illustration of this can be seen in the figure below:
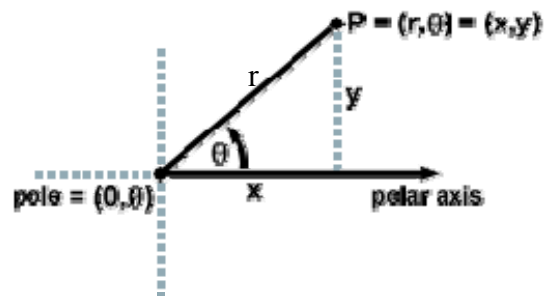
In the figure, the pole is labeled $(0, \theta)$ because the 0 indicates a distance of 0 from the pole, so $(0, \theta)$ will be exactly at the pole regardless of the angle θ. The units of θ can be given in radians or degrees, but generally is given in radians. In this paper we will use both radians and degrees. To translate between radians and degrees, we recall the conversion rules:

$$\text{To convert from radians to degrees, multiply by } \frac{180}{\pi}$$

$$\text{To convert from degrees to radians, multiply by } \frac{\pi}{180}$$

Plotting points on the polar plane and multiple representations

For any given point in the polar coordinate plane, there are multiple ways to represent that point (as opposed to Cartesian coordinates, where point representations are unique). To begin understanding this idea, one must consider the process of plotting points in the polar coordinate plane. To do this in the rectangular plane one thinks about moving horizontally and then vertically. However, in the polar coordinate plane, one uses the given distance and angle measure instead. Although the distance is given first, it is easier to use the angle measure before using the given distance.
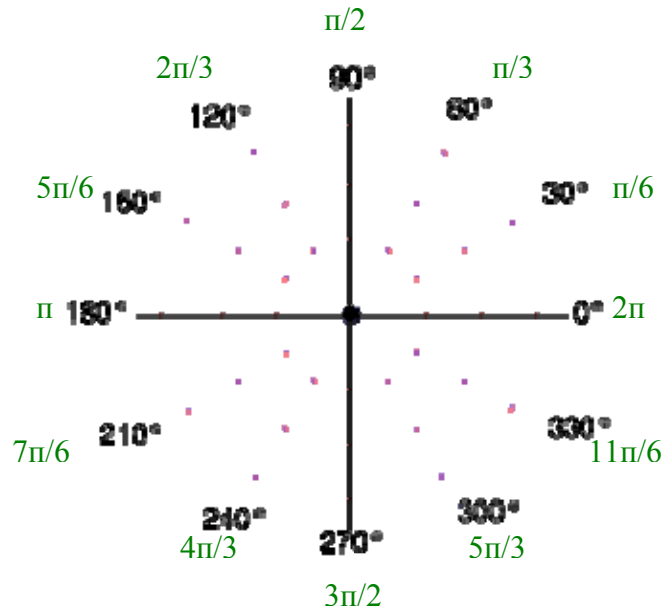
The figure above is a picture of a polar coordinate system with degrees in black and

radians in green. This is just one example of what a polar coordinate plane may look

like; other examples may have additional or fewer angle measures marked. From this

figure it is easy to see why it may be called the polar system; it resembles what one might

see looking down on the north (or south) pole with visible longitudinal and latitude lines.

To plot the polar point (2, 60°), first the positive, or counter-clockwise, angle of 60˚ from

the polar axis is located, then a distance of 2 units along that angle is determined. The

location of this point is shown in green below:
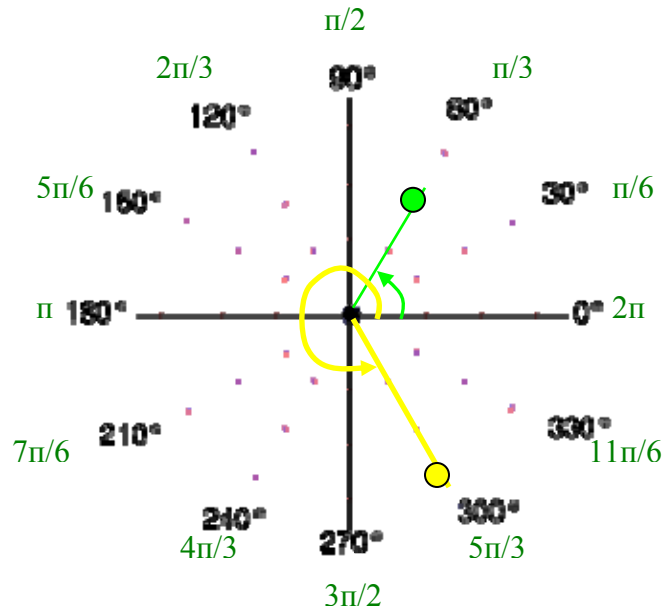
To plot (3, 5π/3), first the angle of 5π/3 is identified, then a distance of 3 along that angle from the pole is determined (seen in yellow in the figure above). To plot points with a negative *r* value such as (-2, 240°), again we first find the angle of 240° and then move a 'distance' of **-2** along that angle. Because our *r* value is negative, to plot this point we move 2 units in the opposite direction of the ray that forms the angle of 240˚. This turns out to be one of the same points as before, seen in green in the figure above.

It is easy to see now that points in the polar plane have many representations. These multiple representations happen for two reasons; (1) adding or subtracting any multiple of 360˚ or 2π (depending on whether the units of θ are in degrees or radians) will go around the circle and end in the same point (these are called *coterminal* angles) and, (2) as seen above using a –*r* value accompanied by angles that differ by a multiple of ± 180˚ (or ± π radians), will result in the same point. The formulas that allow you to

create multiple representations of a single point in a polar plane are organized in the table

below:

Original representation	New representation in degrees	New representation in radians
(r, θ)	$(r, \theta \pm 360n)$; $n = 1, 2, 3...$	$(r, \theta \pm 2n\pi)$; $n = 1, 2, 3...$
(r, θ)	$(-r, \theta \pm (2n+1)180)$; $n = 1, 2, 3...$	$(-r, (2n+1)\pi)$; $n = 1, 2, 3...$

In these formulas n can represent any integer, thus there are infinitely many

representations for any point in the polar coordinate system. Because mathematicians do

not want to worry about multiple representations, it is not unusual to impose the

following limits on r and θ to create unique representations: $r \geq 0$ and θ is in the interval

$[0, 360°)$ or $(-180°, 180°]$ or, in radian measure, $[0, 2\pi)$ or $(-\pi, \pi]$.

Converting between rectangular and polar coordinates

Because both the rectangular and the polar coordinate systems exist, it is

important to be able to convert both points and equations from one system to the other .

These conversions are important because often times certain equations are best fit or

better represented in only one of these systems. If the plane of these two systems were

superimposed on one another, such that the pole coincides with the origin and the polar

axis coincides with the positive x-axis, any point P in this new plane can be assigned

either the polar coordinates (r, θ) or the rectangular coordinates (x, y). The following

four equations show the relationships between the polar coordinates (r, θ) and the

rectangular coordinates (x, y) of any point P when the two planes are superimposed:

i) $x = r \cos \theta$ *iii)* $r^2 = x^2 + y^2$

ii) $y = r \sin \theta$ *iv)* $\tan \theta = \dfrac{y}{x}, x \neq 0$

Note: If $x = 0$ and $y \neq 0$, that would mean that $\theta = 90°$ or $-90°$, where tangent is undefined.

Understanding how the first and second equations were established requires a recollection of prior experiences with trigonometry. It is important to note here that although the following examples deal with points found in the first quadrant, these four equations can be used to convert a point found in any quadrant.
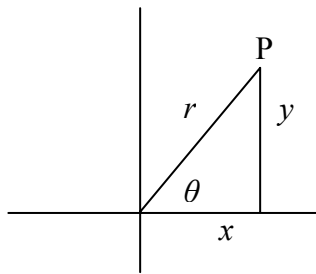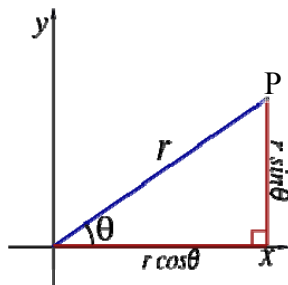

$$\sin \theta = \frac{y}{r} \qquad\qquad \text{and} \qquad\qquad \cos \theta = \frac{x}{r}$$
$$y = r\sin \theta \qquad\qquad\qquad\qquad\qquad\qquad x = r\cos \theta$$

The formula to convert from a polar coordinate to a rectangular coordinate is:

$$\mathbf{P(x, y)} = (x = r \cos \theta, y = r \sin \theta)$$

Another way to think about this conversion is to recall the unit circle. On the unit circle, any point can be represented by $(\cos \theta, \sin \theta)$. Since the r value, or radial distance, on the unit circle is 1, this is a special case of the general representation $(r \cos \theta, r \sin \theta)$, which is true for any value of r.

The following is an example of going through the process of converting a polar coordinate pair to a rectangular coordinate pair. Using the above conversion formula the polar coordinates $(8, 30°)$ are converted to rectangular coordinates as follows:

$$x = r \cos \theta \qquad\qquad y = r \sin \theta$$
$$x = 8 \cos 30 \qquad\qquad y = 8 \sin 30$$
$$x = 8 \left(\frac{\sqrt{3}}{2} \right) \qquad\qquad y = 8 \left(\frac{1}{2} \right)$$
$$x = 4\sqrt{3} \qquad\qquad y = 4$$

Thus the rectangular coordinates that correspond with the polar coordinates of $(8, 30°)$ are $(4\sqrt{3}, 4)$.

The following example will help to explain the conversion process from rectangular coordinates to polar coordinates. If the (x, y) coordinates of the point P in the figure below are (3,4) what are the polar coordinates, (r, θ), of this point? The use of the Pythagorean Theorem will help to calculate r:

P(3,4)→(r,θ)
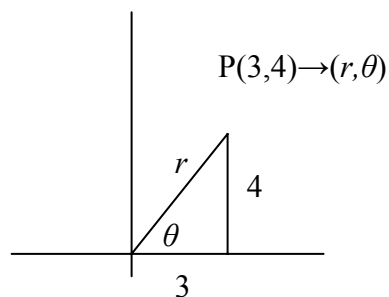
$$r^2 = 3^2 + 4^2$$
$$r^2 = 25$$
$$r = \pm 5$$

While the equation $r^2 = 25$ has two solutions, we choose $r = 5$ so that $r \geq 0$.

To help find θ, we rely on trigonometric ratios. Information regarding the sides opposite and adjacent to θ is given, so the tangent ratio can be used to help calculate θ:

$$\tan\theta = \frac{y}{x}$$
$$\tan\theta = \frac{4}{3}$$
$$\theta = \tan^{-1}\frac{4}{3}$$
$$\theta \approx 53°$$

Thus, for this problem, θ is approximately 53˚.

With the calculations above it was found that the point (3, 4), represented as a horizontal distance of 3 and vertical distance of 4, can be represented as (5, 53˚), a straight line distance of 5 units from the pole, with that straight line forming an angle of 53˚ with the polar axis. To change any point (x, y) in the first or fourth quadrant from rectangular coordinates to polar coordinates use the following formulas:

$$(r,\theta) = (\sqrt{x^2 + y^2}, \tan^{-1}\frac{y}{x})$$

Note: The range of the inverse tangent function is -90˚< θ <90˚. Thus, when converting from rectangular form to polar form, one must pay careful attention because using the inverse tangent function confines the converted angle measures to only the first or fourth quadrant.   However, we can still use the formula identified above by adding or subtracting 180º to the result. This is demonstrated in the following example in which we convert the point (-3, 5) to polar coordinates:

$$(\sqrt{x^2 + y^2}, \tan^{-1}\frac{y}{x})$$

$$(r,\theta) = \left(\sqrt{(-3)^2 + 5^2}, \tan^{-1}\frac{5}{-3}\right)$$

$$\left(\sqrt{34}, -59°\right)$$

Observe that the point (-3, 5) is in the 2nd quadrant and -59° is in the fourth quadrant. Points in the second and fourth quadrant are separated by 180°, so to find the correct θ we need to add (or subtract) 180 to -59 (-59+180 = 121). The polar coordinates that correspond with the rectangular coordinates of (-3, 5) are ($\sqrt{34}, 121°$).

Converting equations between rectangular and polar systems

Now, let us extend the idea of converting points between systems to converting equations between systems. Equation conversion between systems is done because the equations for some graphs are easier to represent in the rectangular system while the equations of other graphs are represented more simply in the polar system. To do this, we will begin by discussing some very simple rectangular equations and their corresponding polar equations:

Rectangular equation	Conversion process	Polar equation
$x = 3$	$r\cos\theta = 3 \rightarrow$	$r = \dfrac{3}{\cos\theta}$
$y = 3$	$r\sin\theta = 3 \rightarrow$	$r = \dfrac{3}{\sin\theta}$

It is easy to see from the table, that a constant function in rectangular coordinates is a more complicated function in polar coordinates. The opposite is true if we start with a constant function in polar coordinates.

Polar equation	Conversion process	Rectangular equation
$r = 3$	$r^2 = 3^2 \rightarrow$	$x^2 + y^2 = 9$
$\theta = \dfrac{\pi}{6}$	$x = r \cos \theta$ **and** $y = r \sin \theta$ $x = r \cos\left(\dfrac{\pi}{6}\right) \quad\bigg|\quad y = r \sin\left(\dfrac{\pi}{6}\right)$ $x = r\left(\dfrac{\sqrt{3}}{2}\right) \quad\bigg|\quad y = r\left(\dfrac{1}{2}\right)$ $r = \dfrac{x}{\left(\dfrac{\sqrt{3}}{2}\right)} \quad\bigg|\quad r = \dfrac{y}{\left(\dfrac{1}{2}\right)}$ $r = \dfrac{2x}{\sqrt{3}} \quad\bigg|\quad r = 2y$ <hr> $2y = \dfrac{2x}{\sqrt{3}}$	$y = \dfrac{x}{\sqrt{3}}$

From this table, it is easy to see that polar equations involving only constants aremore simply represented as polar equations than as rectangular equations.

Converting rectangular equations to polar equations is quite simple. Recall that $x = r \cos \theta$ and $y = r \sin \theta$. By using this information, the polar equation that corresponds with the rectangular equation of $3x - y + 2 = 0$ can be found. First, begin by substituting

the polar values for x and y. Second, solve for r because polar equations are often

written in function format, with r as a function of θ:

$$3x - y + 2 = 0$$
$$3(r\cos\theta) - (r\sin\theta) + 2 = 0$$
$$r(3\cos\theta - \sin\theta) = -2$$
$$r = \frac{-2}{3\cos\theta - \sin\theta}$$

A simpler rectangular function is $y = x$. Converting this rectangular function to a polar

function and then plotting it on a polar plane is shown below:

$$y = x$$
$$r\sin\theta = r\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = 1$$
$$\tan\theta = 1$$
$$\theta = 45°$$

Since the polar function is $\tan\theta = 1$, and we know that the tangent of $45°$ equals 1 (or

$\tan^{-1} 1 = 45°$),for any radial value r, the point $(r, 45°)$ will satisfy this polar function. A

graph of this equation in the polar plane is below:
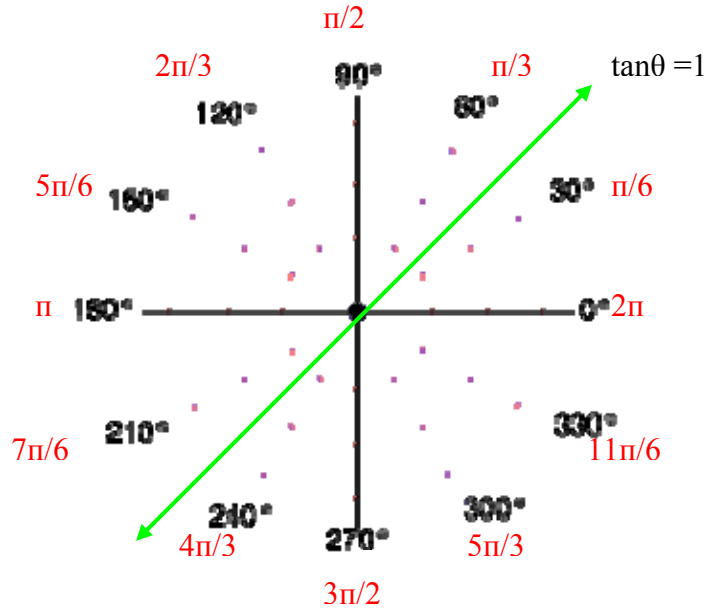
Another example of converting the common, but slightly more complex,

rectangular function $y = x^2$ to a polar equation is shown below:

$$y = x^2$$
$$r \sin \theta = (r \cos \theta)^2$$
$$r \sin \theta = r^2 \bullet \cos^2 \theta$$
$$\frac{r \sin \theta}{\cos^2 \theta} = r^2$$
$$\frac{\sin \theta}{\cos^2 \theta} = r$$

Thus the rectangular equation $y = x^2$ coincides with the polar equation $r = \dfrac{\sin \theta}{\cos^2 \theta}$.

Now that we have a polar function, we can use a table to help us plot points:

θ in radians	θ in degrees	$\sin \theta / \cos^2 \theta$ (or r)	(r, θ) θ in degrees
0	0	0	(0, 0)
п/6	30	0.6667	(.6667, 30)
п/4	45	1.4142	(1.4142, 45)
п/3	60	3.4641	(3.4641, 60)
п/2	90	undefined	undefined
2п/3	120	3.4641	(3.4641, 120)
3п/4	135	1.4142	(1.4142, 135)
5п/6	150	0.6667	(0.6667, 150)
П	180	0	(0, 0)
7п/6	210	-0.6667	(-0.6667, 210)
5п/4	225	-1.4142	(-1.4142, 225)
4п/3	240	-3.4641	(-3.4641, 240)

3π/2	270	undefined	undefined
5π/3	300	-3.4641	(-3.4641, 300)
7π/4	315	-1.4142	(-1.4142, 315)
11π/6	330	-0.6667	(-0.6667, 330)
2π	360	0	(0, 0)

*Those highlighted in red are multiple representations of points in the top part of the table.

*As shown in the table, when $\theta = 90°$ or $\theta = 270°$, this function is undefined. Thus the domain for this equation excludes those values of θ for which $\cos\theta = 0$.

The points from the table are plotted in polar plane below. They are plotted using the methods previously described. The coordinate plane with the function $y = x^2$ overlays the polar plane and it is apparent that this function connects the points from the table. The function looks the same (a parabola) on both the rectangular and polar planes:
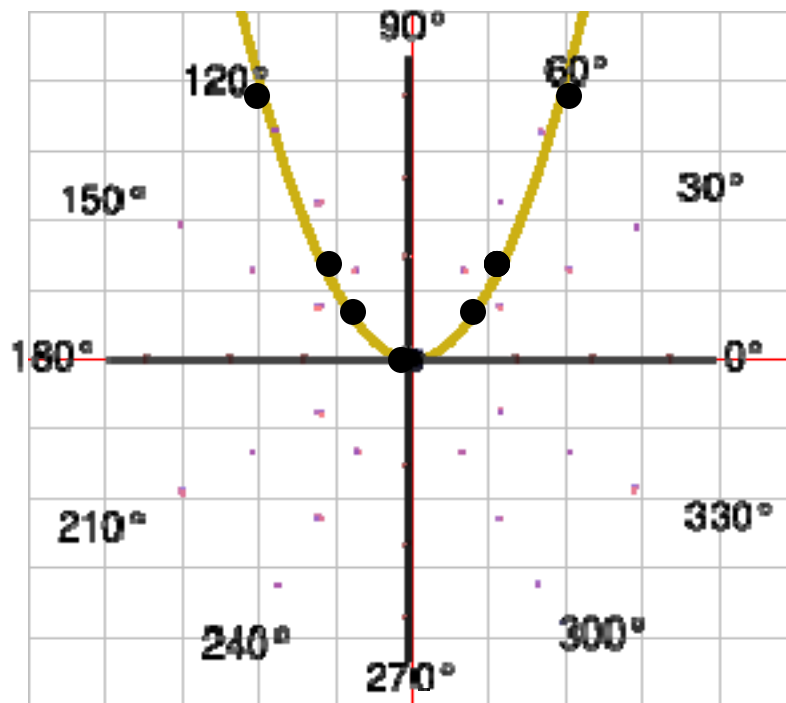
Converting from rectangular equations to polar equations, as mentioned above, is quite simple because substitutions for the polar values of x and y can be used where they are found in the rectangular equation. To graph those polar equations, a calculator or graphing program can be used, or just as in algebra, we can create a table of values to obtain the points that will be plotted.

Converting from a polar equation to a rectangular equation can be more difficult. This is because in this situation limited substitutions are available to be made (in particular, there is no explicit formula for θ in terms of x and y). Remember that $\tan\theta = \dfrac{y}{x}$ and $r^2 = x^2 + y^2$. In the polar equation an r^2 must be present to be able to replace it with an $x^2 + y^2$ and a $\tan\theta$ must be present in the polar equation to be able to substitute it with $\dfrac{y}{x}$. The substitutions used before in converting can also be used; $r\cos\theta$ in a polar equation can be substituted with x and $r\sin\theta$ can be substituted with y.

As a first example of this process we begin with the polar equation $r = 2$. This means that for any angle, the radial measure is 2. Intuitively, it is known that this will create a circle on the polar plane with a radius of 2, but how can we change the polar equation $r = 2$ into a rectangular equation? Remember the limited substitutions above; there is not a simple substitute for r, so some manipulation should be done first to obtain a value that can be substituted:

$$r = 2$$
$$r^2 = 4$$
$$x^2 + y^2 = 4$$

Now it can be seen that it is indeed an equation of a circle in the rectangular plane with a radius of 2 and center at the origin.
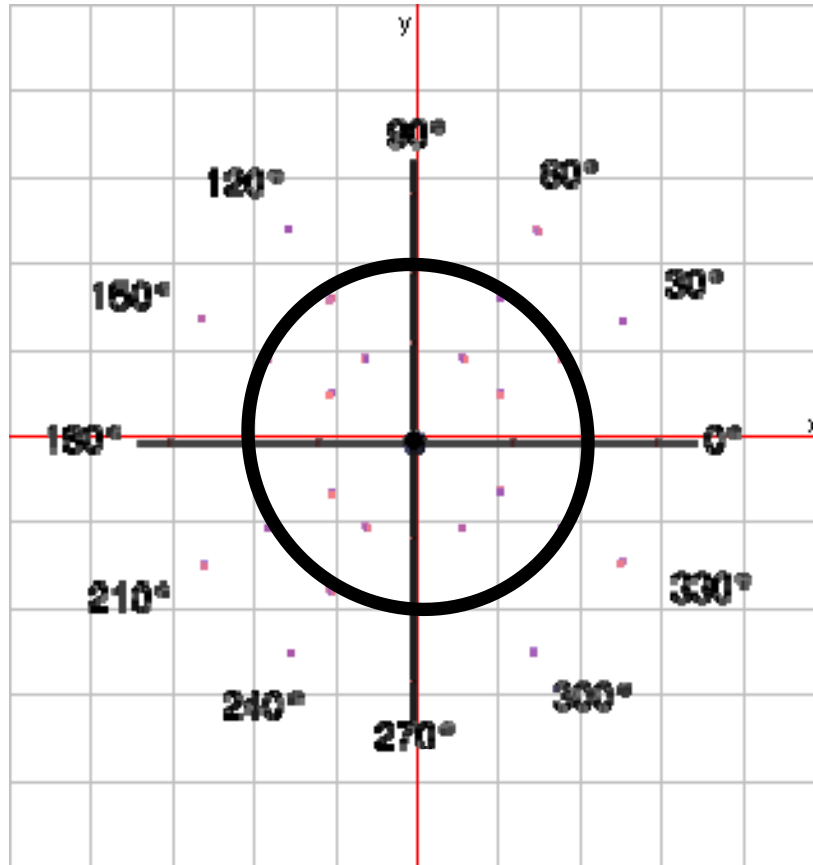
For a second example of this conversion process we again begin with a fairly simple polar equation: $\theta = 76°$. Intuitively it is known that this would be a straight line in the polar plane at the angle 76°, but what is the rectangular equation for that line? Again some manipulation will need to be done in order to find that equation:
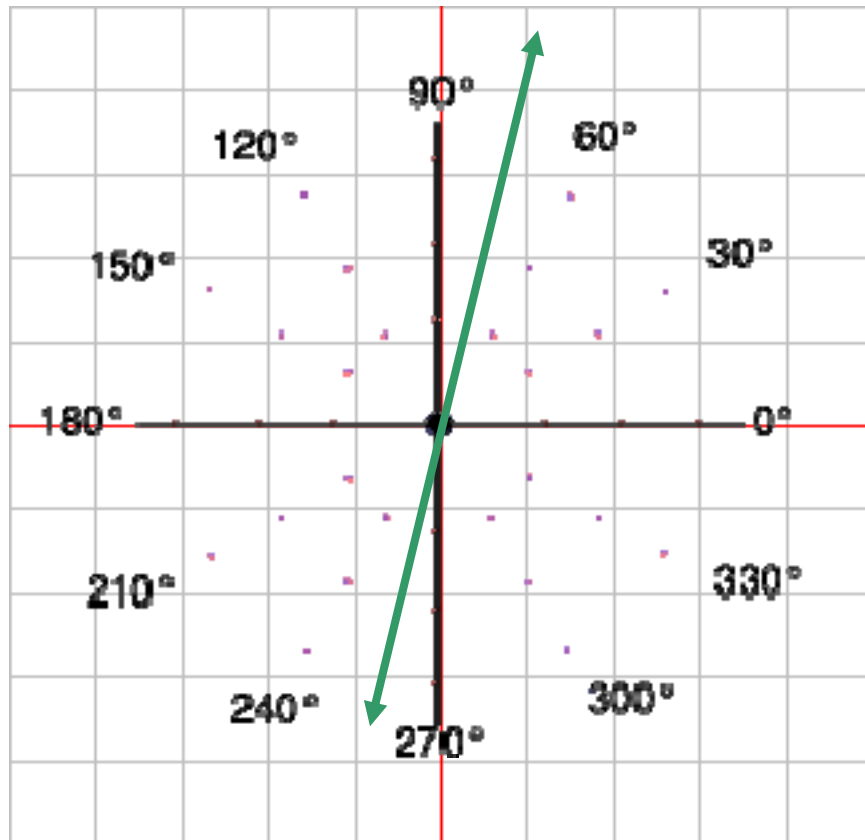
$$\theta = 76°$$

$$\tan\theta = \tan 76°.$$

$$\tan\theta = 4$$

$$\frac{y}{x} = 4$$
$$y = 4x$$

From the rectangular equation we can see that it is indeed the line that passes through the

origin with a slope of 4.

Now an example of conversion that is not quite as simple: r= 2 csc θ. Here the

manipulations are a little more complex because not only do we need to remember the

substitutions we can make, but we also must remember the relationship between

cosecant and sine:

$$r = 2\csc\theta$$

$$r = 2\left(\frac{1}{\sin\theta}\right)$$

$$r = \frac{2}{\sin\theta}$$

$$r\sin\theta = 2$$

$$y = 2$$

This is a horizontal line in both the rectangular and polar plane.

There are many polar equations where the manipulating and the algebra can become quite intensive. In these cases, it is important to stay focused and remember all the expressions that can be replaced with x's and y's, as well as all helpful trigonometric identities . Here is one such example:
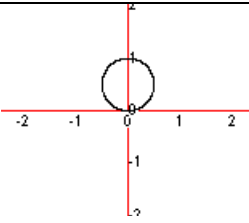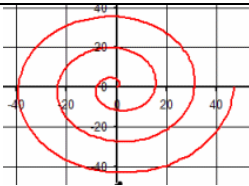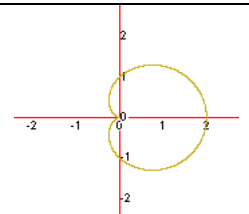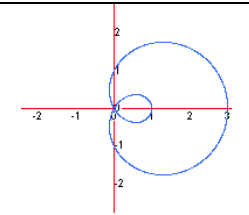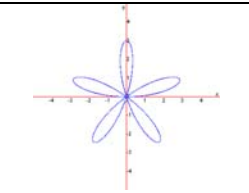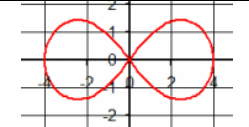
$$r = 2\sin 3\theta$$
$$r = 2\sin(2\theta + \theta)$$
$$r = 2(\sin 2\theta \bullet \cos\theta + \cos 2\theta \bullet \sin\theta)$$
$$r = 2[2\sin\theta \bullet \cos\theta \bullet \cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta]$$
$$r = 2(2\sin\theta \bullet \cos^2\theta + \sin\theta \bullet \cos^2\theta - \sin^3\theta)$$
$$r = 2(3\sin\theta \bullet \cos^2\theta - \sin^3\theta)$$
$$r = 6\sin\theta \bullet \cos^2\theta - 2\sin^3\theta$$
$$r = 6\sin\theta \bullet \cos\theta \bullet \cos\theta - 2\sin\theta \bullet \sin\theta \bullet \sin\theta \qquad \text{(multiply both sides by } r^3)$$
$$r^4 = 6r\sin\theta \bullet r\cos\theta \bullet r\cos\theta - 2r\sin\theta \bullet r\sin\theta \bullet r\sin\theta$$
$$\left(r^2\right)^2 = 6y \bullet x \bullet x - 2y \bullet y \bullet y$$
$$\left(x^2 + y^2\right)^2 = 6x^2 y - 2y^3$$

The last line above can be considered as an equation in rectangular form. Clearly it is sometimes easier to represent equations in polar form.

Common Polar Graphs

There are several examples of common graphs that are better represented in the polar plane. A few of these common graphs were briefly discussed above, more

examples in greater detail will follow. The table that follows displays some of these

examples in a very basic manner.

Basic Equation	Name of Graph	Basic shape of Graph
$r = 1\sin\theta$	Circle	
$r = a\theta$	Archimedean spiral	
$r = 1(1 \pm \cos\theta)$	Cardioid "heart"	
$r = 1 \pm 2\cos\theta$	Limaçon	
$r = 3\sin 5\theta$	Rose	
$r^2 = 4^2 \cos\theta$	Lemniscate	

Circles

I. $r = a$. This equation indicates that no matter the angle, the distance from the origin must be a. To sketch the graph, one can take $\theta \in [0,360°)$ as the domain. Recall $r = 2$ (discussed previously) was an example of a circle in this form.

II. $r = 2a\cos\theta$. This is a circle of radius $|a|$ with center at $(a, 0)$, on the x-axis. Note that a might be negative and so the absolute value bars are required on the radius. They should not be used, however, on the center. To sketch the graph, one can use the domain of $\theta \in [0,180°)$. The equation $r = -8\cos\theta$ has an a value of -4 because 2(-4)=-8, and therefore this circle would have a radius of 4 and a center at (-4,0).

III. $r = 2b\sin\theta$. This is similar to the previous example. It is a circle of radius $|b|$ and center $(0,b)$ on the y-axis.

r=2(2) cos θ

r=2(3) sin θ
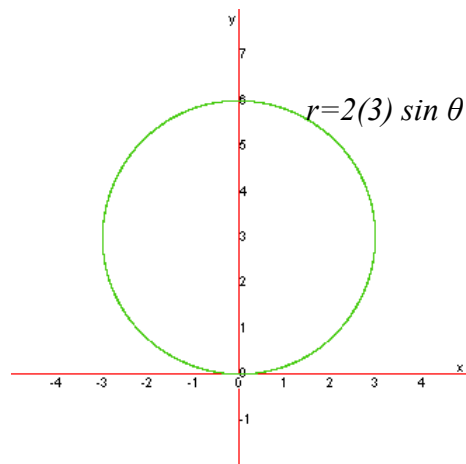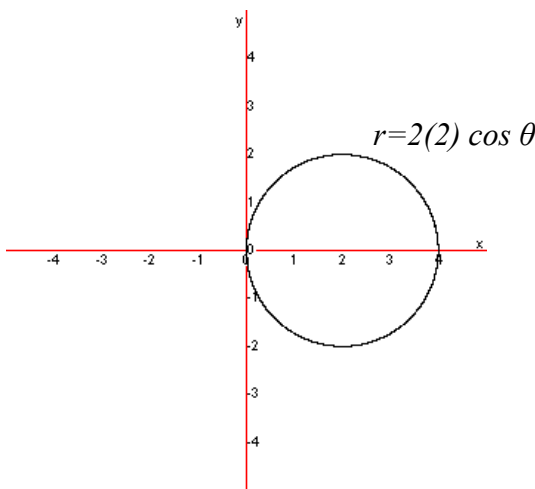
Spirals

I. Archimedean spiral, $r = a\theta$. As $|a|$ increases the spiral spreads out and as $|a|$ decreases the spiral compresses. The graph of the spiral turns about the origin, crossing the y-axis as the expression $a\theta$ changes signs.

The graph is sketched for domain $\theta \in [0, \infty)$. If negative values of θ are included in the domain, the reflection of this graph about the y-axis is also obtained.
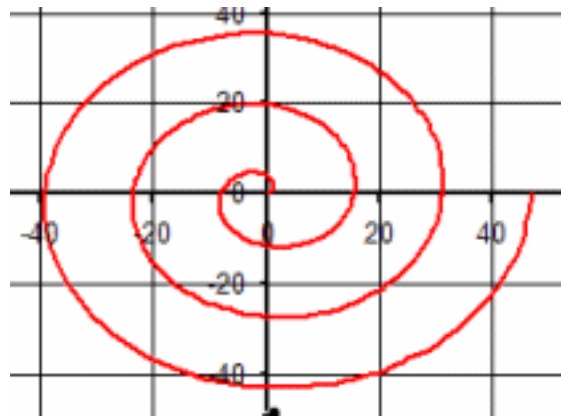

II. Equiangular spiral, $r = e^{a\theta}$. Again as $|a|$ increases or decreases the spiral expands or compresses respectively. The spiral crosses the x-axis as the sign of a changes.
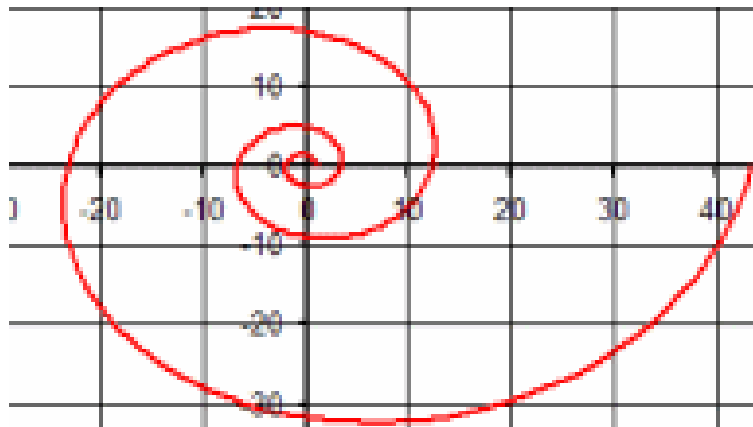
Cardioids

The domain needed to sketch these graphs is (0, 360] or (0, 2π].

The equation $r = a(1 \pm \sin\theta)$ or $r = a(1 \pm \cos\theta)$ will produce what is referred to as a cardioid. These graphs have a shape that is similar to a heart and always contain the origin. As $|a|$ increases or decreases the "heart" shape expands or compresses. A way to look at the role of *cos* and *sin* in the polar world is to think about symmetry. There are ways to test for symmetry, but those are not necessary if the following fact is known. If *cos* is used the axis of symmetry is the *x*-axis and if *sin* is used the axis of symmetry is the *y*-axis. Changes in the sign of the expression will reflect the cardioid over the axis of symmetry.

r=2(1-sin θ)

r=2.5(1+cos θ)
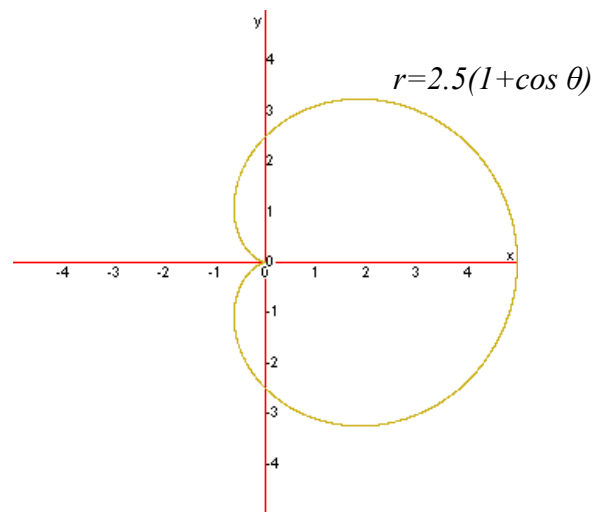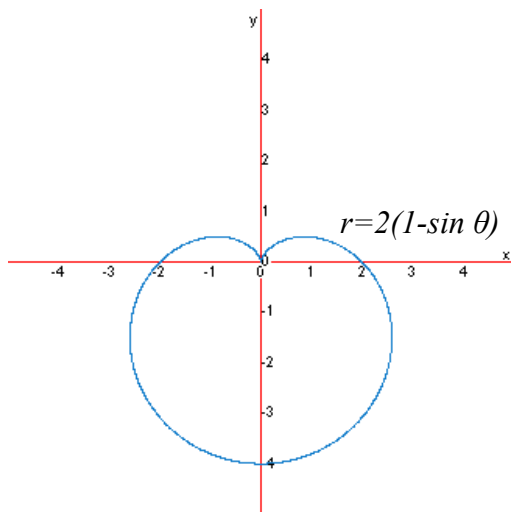
Limaçons

The domains needed to sketch these graphs is (0, 360] or (0, 2π].

I. Limaçons with an inner loop: $r = a \pm b \cos\theta$ and $r = a \pm b \sin\theta$. If $|a/b|<1$ ($|b|$ is

greater than $|a|$), these graphs will look similar to cardioids with an inner loop and will

always contain the origin. Similar to cardioids, the rules of symmetry using *cos* or *sin*

apply here. As *b* changes signs the limaçon is reflected of the *x* or *y* axis. As $|b|$ gets

larger while $|a|$ stays fixed, the size of the body expands, whereas as $|a|$ gets smaller and

$|b|$ stays fixed, the size of the inner loop increases.

r=1+2 cos θ

r=1-4sin θ
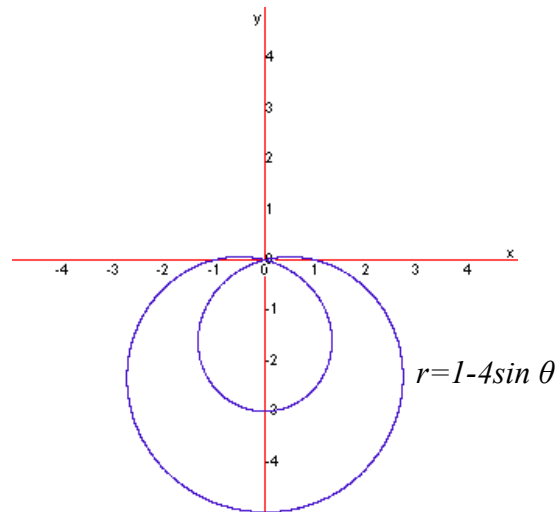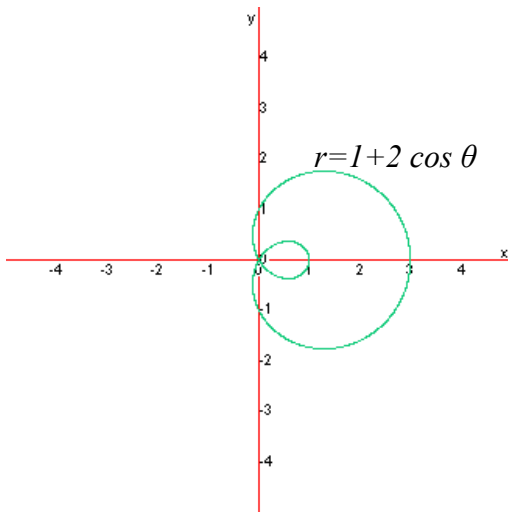
II. Limaçons with a dimple: $r = a \pm b \cos\theta$ and $r = a \pm b \sin\theta$. If $1<|a/b|<2$, these

graphs do not have an inner loop and do not contain the origin. The constant *b* plays the

same role as before, expanding the body as $|b|$ increases as well as reflecting the graph

over the axis of symmetry if the sign of *b* is changed. The constant *a* in this case

determines the size of the "dimple" rather than the inner loop as before. Again *cos* and

sin determine how the limaçon lays (which axis is the axis of symmetry). These graphs look similar to cardioids.

$r=3+2\ sin\ \theta$

$r=1.5-1.25\ cos\ \theta$
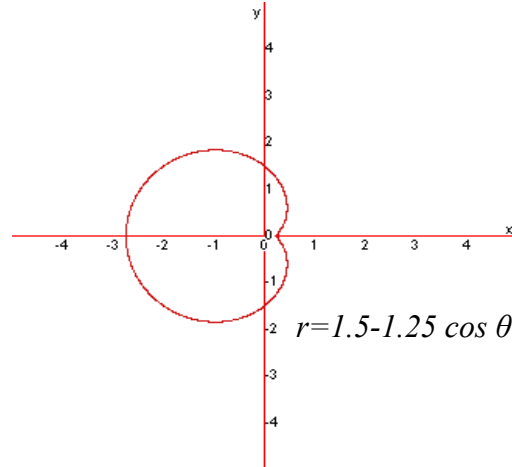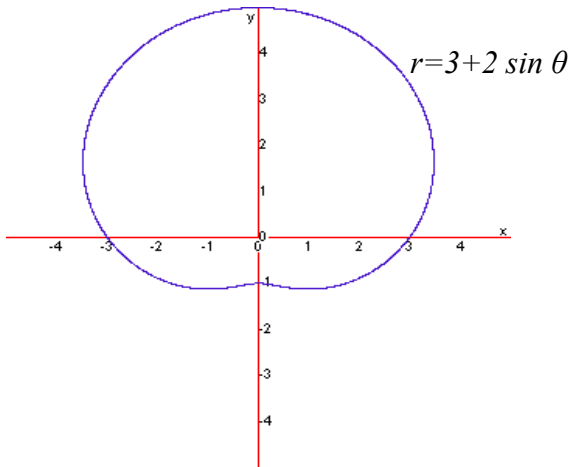
III. Convex limaçons: $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$. If $|a/b| \geq 2$, the dimple disappears completely and a convex limaçon appears and where there would have been a dimple, there is a vertical or horizontal line, as $|a/b|$ gets larger and larger this vertical or horizontal line becomes smaller and smaller. The same rules for *b* (expanding or compressing and opposite signs reflecting) and *cos/sin* (axis of symmetry) apply here as well.

$r=3-1.5\ cos\ \theta$

$r=10+3\ sin\ \theta$

Roses

$r=3sin(5\theta)$

$r=5cos(4\theta)$
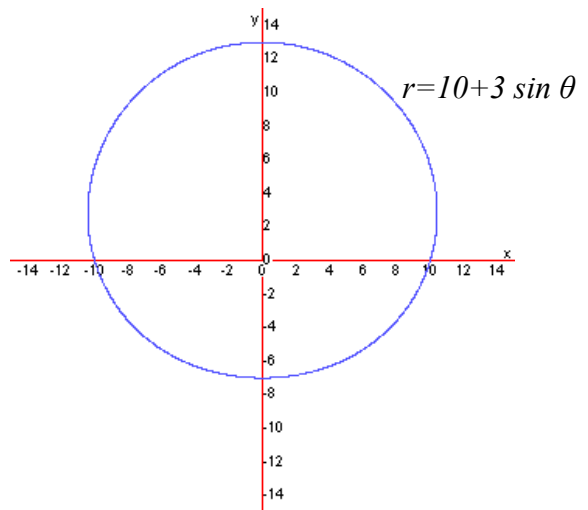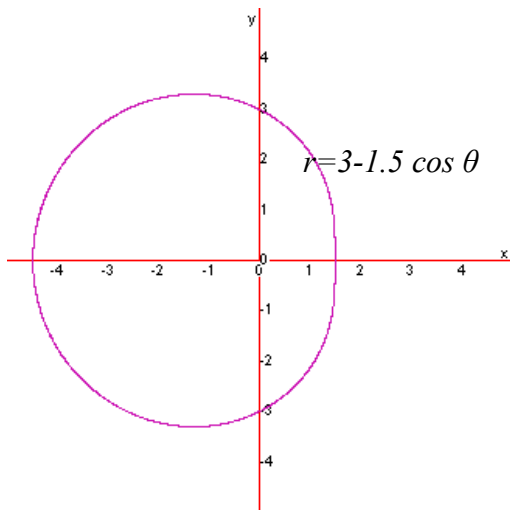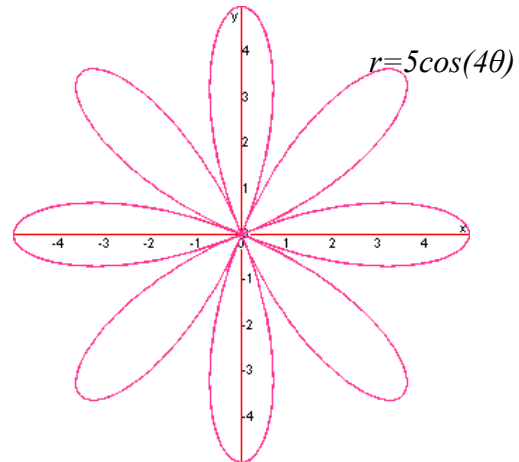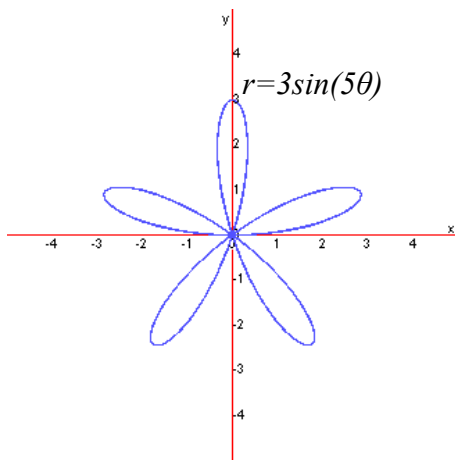
My favorite polar graphs are the rose graphs. The same rules for symmetry apply to these graphs as well. The general equation form for a rose graph is: $r = a\sin(n\theta)$ or $r = a\cos(n\theta)$, when $n \geq 2$ and an integer. If n is odd, there are n petals and the domain is $(0, \pi]$. If n is even, there are $2n$ petals and the domain is $(0, 2\pi]$. The length of each petal is $|a|$. If $n=1$, a circle is created with the center at the origin. If n is not an integer, the graph is similar to a rose but the petals overlap.

3 cos (5.5t)
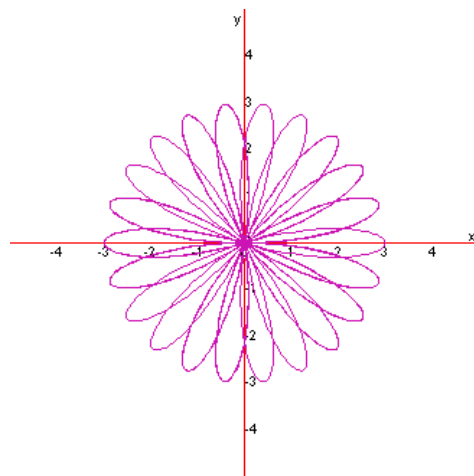
Lemniscate

The equation $r^2 = \pm a^2 \sin 2\theta$ or $r^2 = \pm a^2 \cos 2\theta$ will produce the graph of a lemniscate, a shape similar to a figure eight. The length of each "oval" (the top and bottom part of the figure eight) is determined by $|a|$. Again, *cos* and *sin* determine how the lemniscates lays on the plane.
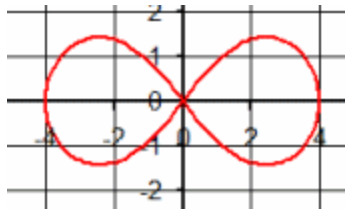
Why use the polar system?

Obviously, from what is stated above, polar coordinates are most appropriate in any context where what is being studied is tied to direction and length from a center point. There are several real-life applications that fall into this kind of context. One of the first references to polar coordinates is that of the astronomer Hipparchus who used them to establish stellar positions. This idea leads us to the use of polar coordinates and equations to help us understand the circular or orbital motion of many things in our universe.

Polar coordinates are used often in navigation as well. The destination or direction of travel can be given as an angle and a distance from the starting point. For instance, aircrafts use a slightly modified version of the polar coordinates for navigation. In this system, the one generally used for any sort of navigation, the polar axis is

generally called heading 360, and the angles continue in a clockwise direction, rather than counterclockwise, as in the mathematical system. Heading 360 corresponds to magnetic north, while headings 90, 180, and 270 correspond to magnetic east, south, and west, respectively. Thus, an aircraft traveling 5 nautical miles due east will be traveling 5 units at heading 90. It is easy to see how this system corresponds with the polar coordinate system.

Systems with radial or point forces are also good candidates for the use of the polar coordinate system. These systems include gravitational fields and radio antennas. Radially asymmetric systems may also be modeled with polar coordinates. For example, a microphone's pickup pattern illustrates its proportional response to an incoming sound from a given direction, and these patterns can be represented as polar curves. Three dimensional polar modeling of loudspeaker output patterns can be used to predict their performance.

In all of these situations where the polar system is applicable, it is easier to think about these situations in polar terms rather than with a rectangular system. This is because they lend themselves to using a distance and a direction much better than using just vertical and horizontal directions.

Other coordinate systems

During my research over the polar coordinate system, I found examples of other coordinate systems, some that I had heard of before and some that were new to me. I was very surprised to learn that NASA uses a three dimensional coordinate system for each

space shuttle. But after thinking about it, it does make sense to have such a system in place so that when astronauts are repairing the shuttle while in space, specific directions can be given from here on Earth that enable the astronaut to find whatever needs to be repaired quickly.

A more common such system is the geographic coordinate system. Generally, one learns a simplified version of this system in geography classes. A geographic coordinate system enables every location on the earth to be specified. There are three coordinates: latitude, longitude and geodesic height. Most people are familiar with both the ideas of latitude and longitude, but many are not familiar with geodesic height. Latitude and longitude enable us to specify a location on a perfect sphere. Because the Earth is not a perfect sphere, a third number is need to represent how far above or below a point is located from the given latitude and longitude. The geodesic height does just that. Generally, it represents the distance from sea-level at a given latitude and longitude.

In recent years, it seems that this geographic coordinate system has become very useful. GPS (global positioning systems) devices use ideas behind the geographic coordinate system. GPS systems are becoming more popular by the day. Many people have them in their cars to help get from place to place, and I even have a watch with a GPS that tells me how fast and far I am running.

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