2015

NCUWM Talk Abstracts 2015

The Seventeenth Annual Nebraska Conference for Undergraduate Women in Mathematics

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TALK ABSTRACTS

PLENARY TALKS

Dr. Maria Klawe
President
Harvey Mudd College
Twists and Turns in my Life

The talk describes how the combination of my passion for mathematics and being female at a time when women were not encouraged to be mathematicians launched me on a journey to make the culture of science and engineering inclusive and supportive of everyone independent of gender, race or other interests. The talk includes some of my experiences as an a student, industry researcher and academic, as well as the joys and challenges of raising two children while pursuing ambitious career goals. It concludes with some advice for people interested in changing the world.

Dr. Karen E. Smith
Keeler Professor of Mathematics
University of Michigan
A Personal Journey towards Mathematics

Starting with a little trick I learned in third grade to check my multiplication homework, I'll share my fascination with algebra as it grew through middle school, high school, college and eventually led to research in "characteristic p rings". Along the way, I'll point out the the importance of many mentors and teachers who led me to eventually pursue my career in mathematics.

Talks by Undergraduate Students

Katharine Ahrens, Ithaca College
Orbits of Generalized Fibonacci Sequences with Complex Coefficients

Consider the generalization of the Fibonacci sequence given by the recursion $f_n = cf_{n-1} + f_{n-2}$, where c is a complex number. We show that we can realize f_n as a complex polynomial in c. We explore some interesting cycling behavior which occurs when $c = i$. We go on to show that for any even $p \geq 6$, there is a parameter c for which the sequence $\{f_n\}$ is periodic with period p. Finally, we prove that f_n is bounded if the absolute value of c is less than 2 and unbounded otherwise.

Robin Belton, Kenyon College
Numerical Techniques of Integration for Volumes of Revolution

In Calculus one learns how to approximate the definite integral with left, right, midpoint, and trapezoid Riemann Sums. The error of these sums can be described in terms of the first and second derivatives of the integrand, f. The definite integral describes the area under the graph of f. Our project explores numerical techniques for approximating the volume obtained when f is rotated about the x axis. We define left, right, midpoint, trapezoid, and Simpson approximations for this setting. Then we examine the error for these methods and create bounds.

Candace Bethea, Washington and Lee University
Realizability of Graphs

There are standard techniques for finding vertex connectivity, edge connectivity, minimum degree, and maximum degree for a given graph of size n. We look at the reverse problem: given a prescribed vertex connectivity, edge connectivity, minimum degree, and maximum degree, what are the restrictions on the size of a graph? We examine the case when the sum of vertex connectivity and maximum degree is strictly less than the sum of edge connectivity and minimum degree. We consider the strict inequality case as well as the cases when parameters may be equal.

Jerez Chen, Furman University
Distance-2 Maximum Minimal Domination in A Grid

The discussion of domination problem can be traced back to last century. While distance -1 domination has already been discussed almost thoroughly, the focus of this research is on distance-2 domination. In particular, given a graph G with vertex set V, we say $D \subseteq V$ is *distance two dominating* if $\forall w \in V$, $\exists u \in D$, $d(u, w) \leq 2$. D is minimal when $\forall u \in D, \exists w \in V$, such that $d(u, w) \leq 2$ and $\forall v \in D$ with $v \neq u$, $d(v, w) > 2$, and we are concerned with the maximum size of minimal D for $4 \times n$ grids. for general grids. Using the idea of tiles to visualize the domination conditions, we are convinced that except for a certain kind of special boards, the cardinality of dominator sets is bounded by the ceiling nm for nm board.

Grace Chester, Missouri Western State University
Katelyn Gutteridge, Missouri Western State University
Fishing for tests: What is Fisher's Exact Test good for and how can it be made better?

Statistical tests are often used to decide if, given the input, the output is unusual. Fishers Exact test is one of those many statistical tests, and, as our research team discovered this summer, it is useful when working in the biological field of programmed evolution. It is commonly used in 2x2 contingency tables, but it can be expanded. However, current documentation does not clearly explain the full potential of this test. We will explain what makes Fisher's Exact Test unique, how we used it in biological research, and how the test can be expanded to 2x3 or larger contingency tables.

Jane Coons, State University of New York at Geneseo
Combinatorics of k-Interval Cospeciation for Cophylogeny

Cophylogenetics is the branch of phylogenetics which concerns relationships between taxonomical units that are evolving concomitantly. Our research is a response to the open problem in "First steps toward the geometry of cophylogeny" by Huggins, Owen and Yoshida, about the combinatorial properties of the cophylogenetic distance metric, k-interval cospeciation. We show that k-interval cospeciation is distinct from other metrics and accounts for global congruence between locally incongruent trees. The growth of the neighborhood of trees which satisfy the largest possible k-interval cospeciation with a given tree indicates that k-interval cospeciation is useful for analyzing simulated data.

Isabel Corona, Metropolitan State University of Denver
Carolynn Johnson, Middlebury College
A New Look at Apollonian Circle Packings]

Apollonian gaskets are circle packings that are the subject of significant current research. We first extend previous results about Ford circles to the Apollonian gasket that contains them. We then create a new labeling that provides a variety of advantages when examining the gasket and use the properties of this labeling to describe a more general circle arrangement, termed a "super-packing". We consider the sets of curvatures that can occur within our arrangement and discuss the relationship between circle inversion and our construction. We then generalize our results to the case of an Apollonian sphere packing. This research was conducted at the REU program at Central Michigan University.

Julia Dandurand, California State University, Northridge
Equal Circle Packing on a Square Flat Klein Bottle

The study of maximally dense packings of disjoint equal circles is a problem in Discrete Geometry. The optimal densities and arrangements are known for packings of small numbers of equal circles into hard boundary containers, including squares, equilateral triangles, and circles. I will present my work in proving the optimality of a particular packing of three equal circles into a boundaryless container called a Klein bottle. Using numerous figures I will introduce all the basic concepts including the notion of a Klein bottle, an optimal packing, and the graph of a packing. I will outline the process of finding and eliminating potential packing graphs, present the globally maximally dense packing, and outline the proof of its global optimality. This research was conducted as part of the 2014 REU program at Grand Valley State University.

Stephanie DeGraaf, Iowa State University
Elizabeth Doebel, Iowa State University
Knight's tours on boards with odd dimensions

A closed knight's tour of a board consists of a sequence of knight moves, where each square is visited exactly once and the sequence begins and ends with the same square. For boards of size $m \times n$ where m and n are odd, we determine which square to remove to allow for a closed knight's tour.

Elizabeth Doebel, Iowa State University
see **Stephanie DeGraaf**

Rachel Domagalski, Central Michigan University
Tight Frame Structure

In $\mathbb{R}^n$, a frame is a spanning set. A collection $F = \{f_i\}_{i=1}^k \subseteq \mathbb{R}^n$ is a λ-tight frame if there exists $\lambda > 0$ such that for every $f \in \mathbb{R}^n$, $\lambda \|f\|^2 = \sum_{i=1}^k |\langle f, f_i \rangle|^2$. A factor poset for a frame $F = \{f_i\}_{i=1}^k$ is the set $P = \{J \subseteq \{1, \ldots, k\} : \{f_j\}_{j \in J}$ is a tight frame$\}$, partially ordered by set inclusion, $\emptyset \in P$. This leads to the question: given a poset P, when is P a factor poset? This question was answered for $\mathbb{R}^2$ in 2013. In our goal of answering for $\mathbb{R}^n$, we discovered combinatorial properties of tight frames and explored constructions of frames from posets.

Natalie DuBois, State University of New York at Geneseo
Colored Unlinking

In this talk I will be discussing colored unlinking. It is natural that if we have a knot we can ask how many crossings must be changed to untie it. We can then progress to untying knots that are connected, links. We then attempt to untie these by changing crossings in the components to create the unlink. In our study we specifically focus on links of 2 knots. The colored aspect of this study is that we change crossings in either one component, the other, or a combination of the two, but never crossing between components. We can then use this data to determine upper and lower bounds to achieve the unlink.

Molly Durava, North Central College
Kelleigh Kuehl, North Central College
Piecewise-Linear Psuedodiagrams

Take a piece of string, tie a knot in it, and glue the ends together. This is a physical representation of a mathematical knot. A psuedodiagram is a shadow of a mathematical knot. From the psuedodiagram we can classify the mathematical knots that have the psuedodiagram as their shadow, called a resolution. The central question investigated here is to classify piecewise-linear psuedodiagrams that have resolutions that physically cannot exist in three dimensions. We attempt to find the reasons behind why some of these psuedodiagrams cannot be resolved, leading to a complete classification of them. This generalized the concept of weighted resolution sets to the piecewise-linear case. Through the investigation of the weighted resolution sets, many future questions regarding trends in the sets and the differences between smooth and piecewise-linear psuedodiagrams are mentioned for future investigation.

Rachel Eaton, United States Air Force Academy
The Frobenius Number of Balanced Numerical Semigroups

A numerical semigroup S is a subset of the non-negative integers which contains 0, is closed under addition, and has a finite complement in the natural numbers. The largest integer not in S is called the Frobenius number, F(S). In this talk, we will investigate F(S) for a class of numerical semigroups with minimal generating set a_1, a_2, a_3, a_4 where g.c.d.$(a_1, a_2, a_3, a_4) = 1$ and $a_1 + a_4 = a_2 + a_3$. Under these assumptions F(S) can be expressed as a quadratic in $g.c.d(a_1, a_4)$.

Paula Egging, Benedictine College
Triphos: An Alternative Coordinate System

In this presentation, we will investigate characteristics and properties of the Triphos coordinate system, an alternative, two-dimensional coordinate plane consisting of three axes evenly spaced 120 apart. The idea of this coordinate system came from students at Emporia State University. After hearing a talk on the topic by Keely Grossnickle, we were inspired to further research. We will examine similarities and differences between the Triphos system and the traditional Cartesian coordinate system, explore its algebraic properties, and discuss some advantages and applications of this unique coordinate system.

Saniita FaSenntao, Kennesaw State University
Kaleigh Mulkey, Kennesaw State University
Modeling Traffic at an Intersection

The main purpose of this project is to build a mathematical model for traffic at a busy intersection. We use elements of Queueing Theory to build our model. We collected traffic data on the number of vehicles arriving to the intersection, the duration of green and red lights, and the number of vehicles going through the intersection during a green light. We built a SAS macro code to simulate traffic based on parameters derived from the data. We describe the probability distribution of the queue length in the long run and analyze its dependence on arrival time and the durations of the green and red light. Using regression, we build a model for the dependence of the average queue length and the average service time on ? and the durations of the green and red light.Based on the regression results we propose traffic models that achieve optimal queue lengths and sojourn times.

Nicole Gallagher, Iona College
Samantha Greenidge, Iona College
Computing Čech Cohomology of S^n and a Genus-n Surface

A manifold can be thought of as a space which locally looks like $\mathbb{R}^n$. To each manifold we assign a sequence of groups, called its Čech cohomology. If two manifolds are equivalent, then their cohomology groups are the same. The cohomology groups of most household spaces have been well documented via other approaches. We would like to revisit some of those computations in the context of Čech Cohomology since the tools needed are quite elementary and there are very few places where explicit examples of such computations can be found. In this talk we will define the Čech Cover of a manifold, the nerve of such a cover, and the corresponding Čech Complex. Finally, we compute the Čech Cohomology groups of the sphere and the torus.

L. Katie Gerdts, University of Wisconsin-La Crosse
Bootstrap Methods for 3-Dimensional Rotational Data

Bootstrapping is a nonparametric statistical technique that can be used to estimate the sampling distribution of a statistic of interest. This research focuses on implementing bootstrapping in a new setting, where the data of interest are 3-dimensional rotations. Confidence regions for measures of center for the 3-dimensional rotations are proposed and a simulation study is conducted to investigate the effectiveness of the bootstrap methodology under various scenarios.

Elizabeth Greco, Kenyon College
Brownian Motion in the Complex Plane

This project explores Brownian motion, a model of random motion, in the plane. Given a domain in the complex plane and a basepoint in the domain, start a Brownian traveler at that basepoint. The h-function of the domain gives information about where the Brownian traveler is likely to first hit the boundary of the domain. I will give examples of h-functions I computed for several families of domains. Next I will describe a connection between the geometry of domains and their h-functions. Finally, I will end with results about the convergence of a sequence h-functions.

Samantha Greenidge, Iona College
see **Nicole Gallagher**

Michelle Gunzel, University of Wisconsin-Platteville
The Geometry of Cubic Polynomials

Were all familiar with how to find the critical points of a polynomial, and we have results such as Rolles Theorem and the Gauss-Lucas Theorem which show us that the critical points are located in between the roots of a polynomial. In particular, the Gauss-Lucas Theorem introduces us to a geometric relationship that exists between the roots and the critical points: if p(x) is a cubic polynomial with three distinct complex roots, the critical points of p(x) lie within the triangle formed by these roots. While this theorem allows us to approximate the locations of the critical points, we would like to be able to use this geometric relationship to find their exact locations. In order to do this, we will provide a proof of what has been called by some the most marvelous theorem in mathematics, otherwise known as Mardens Theorem.

Katelyn Gutteridge, Missouri Western State University
see **Grace Chester**

Emily Heath, Occidental College
Economical Extremal Hypergraphs for the Erdos-Selfridge Theorem

A positional game can be thought of as a generalization of Tic-Tac-Toe played on a hypergraph $(V, \mathcal{H})$. We study the Maker-Breaker game in which Maker wins if she occupies all of the vertices in an edge of $\mathcal{H}$; otherwise Breaker wins. The Erdős-Selfridge Theorem, a significant result in positional game theory, gives criteria for the existence of an explicit winning strategy for Breaker for the game played on $\mathcal{H}$. The bound in this theorem has been shown to be tight, as there are several examples of extremal hypergraphs for this theorem. We will discuss examples of n-uniform extremal hypergraphs on which Maker has an economical (n-turn) winning strategy, and ultimately provide a characterization of these economical extremal hypergraphs.

Rebecca Heinen, North Central College
Self-stabilization under Minimal Liar's Domination for Path Graphs

Self-stabilizing algorithms ensure a system that starts in an incorrect state, as defined by some condition, will eventually come to and remain in a correct state in a finite number of moves. This project focuses on creating a self-stabilizing algorithm for minimal liar's domination for any path graph. First we examined various sized stabilized path graphs in order to observe a pattern that could lead to the creation of an algorithm. We then proved that the algorithm exhibits both closure and convergence for minimal liars domination. Our main theorem states a path will exhibit self-stabilization as a minimal liar's dominating set in at most 2n moves with this algorithm, which encompasses both closure and convergence of the algorithm. Because minimal liar's domination is important for maintaining the security for a network of computers, this research can be used to ensure that an entire network of computers is secure under the conditions of minimal liar's domination at all times.

Amelia Henriksen, Brigham Young University
The 6 Degrees of Separation

We begin this presentation by demonstrating the basic algorithms used to construct a shortest path between two vertices for an undirected graph. Using Python's NetworkX package, we explain methods by which this path can be found and constructed in code. With this information, we proceed to explain the notion of the "6 Degrees of Separation", implementing the game "The Six Degrees of Kevin Bacon" in Python.

Kaylee Homolka, University of Nebraska-Lincoln
Analysis of Spectral Variation in Background Gamma Radiation at U.S. Ports of Entry

The Technical Reachback (TRB) program at Sandia National Laboratories is sponsored by the Domestic Nuclear Detection Office (DNDO) of the Department of Homeland Security. This project analyzes data and builds tools to enhance the capability for interdiction of illicit nuclear material at U.S. border crossings. This analysis was done to determine if observed changes in the background gross counts also result in changes in the spectral shape.

Melissa Jay, Colorado College
Katelyn Zumpf, North Central College
Speech Intelligibility Index Model: A Key Aspect to a Child's Development of Speech and Language

Hearing is essential in the development of speech and language in children. It has been found that children with hearing loss, who fail to seek proper help, have a delay in speech development. The overall goal of the study is to determine whether it would be more beneficial, in the long run, for children with severe hearing loss to be fitted for hearing aids or cochlear implants. To do this we look at a childs Speech Intelligibility Index (SII), the standard estimate of an individuals understanding of speech. In children with cochlear implants, there is no means of acquiring an SII, so other measures such as Better Ear Pure-Tone Average (PTA), Word Attack, and Passage scores are used. In this research, we use multiple imputation and regression to find a model that predicts functional SII values. We find that a logistic model of three explanatory variables, Word Attack, Passage Score, and Mothers Education, best predicts the SII scores for children wearing cochlear implants.

Carolynn Johnson, Middlebury College
see **Isabel Corona**

Erica Johnson, The University of the Incarnate Word
Behavior of Soliton Solutions to the Korteweg de Vries equation

The Korteweg de Vries equation is a nonlinear integrable partial differential equation that models shallow water waves. This presentation focuses on the visualization of soliton waves using a matrix-valued solution to easily identify properties of the individual wave. We consider the motion of a single wave as well as the interaction of multiple waves and discuss how the properties of each wave change in these cases. Included is a discussion of the limitations of computer algebra systems associated with the interaction of a large number of waves.

Elizabeth Kelley, Harvey Mudd College
The Total Acquisition Number of the Randomly Weighted Path

There exists a significant body of work on determining the acquisition number of various graphs when the vertices of those graphs are each initially assigned a unit weight. We determine the acquisition number of the path, star, complete, complete bipartite, cycle, and wheel graphs for variations on this initial weighting scheme, with the majority of our work focusing on the acquisition number of randomly weighted graphs. In particular, we bound the expected acquisition number of the n-path when n units of integral weight, or chips, are randomly distributed across its vertices between 0.26 and 0.37n. We then use Azumas Lemma to prove that this expected value is tightly concentrated. Additionally, we offer a non-optimal acquisition protocol algorithm for the randomly weighted path and compute the expected size of the resultant residual set.

Victoria Kelley, James Madison University
Julie Richardson, Smith College
Emma Talis, Marist College
The Effect of Prey Dispersal on a Two-Patch Predator-Prey System

We consider the effect of prey dispersal in a two-patch predator-prey model in which the two patches are qualitatively different. In particular, we assume patch two has a significantly smaller carrying capacity and a correspondingly higher predation rate. Scaling the model under these assumptions introduces a parameter of arbitrarily small order, allowing for an asymptotic analysis. We show that the predator and prey will always coexist for biologically reasonable parameter values. Furthermore, we prove the existence and uniqueness of a coexistence equilibrium and determine the stability regions in the parameter space. Using numerical simulations, we illustrate the varying effects of prey dispersal on the stability of the coexistence equilibrium and find parameter values for which a Hopf bifurcation occurs.

Mary Kemp, Occidental College
Visualizing Dessins Dénfants

In this talk, we will discuss the results of a summer research project motivated by the theory of dessins. Dessin is short for dessin dénfant which means childś drawing. Mathematically speaking, a dessin is a connected bicolored graph where the edges around every vertex are cyclically ordered. Dessins can be realized by Belyi maps which are meromorphic functions $f : X \to \mathbb{P}^1(\mathbb{C})$ such that X is a Riemann surface and f is unramified outside $\{0, 1, \infty\}$. One of the goals of this project is to determine Belyi maps that realize a given loopless, connected bipartite graph on a compact Riemann surface X. We report on our considerations of certain classes of such graphs, explorations of computational methods for finding associated Belyi maps and related applications.

Saniya Khullar, Georgetown University
Site-of-Origin Prediction for Neuroendocrine Tumors

̈There has been a five-fold increase in neuroendocrine tumors (NETs) over the past 3 decades. The two most common sites of origin for these tumors are the small bowel (SBNETs) and the pancreas (PNETs). The prognosis and proper course of treatment are very different for these two sites of origin; however, it is very difficult to distinguish SBNETs from PNETs in a standard biopsy, especially if the tumor has metastasized (say, to the liver). This project explores the prospect of developing a statistical model for predicting the probability that a tumor is a PNET or SBNET based on gene expression data, in the hopes of defining a profile that can distinguish between the two kinds of tumors.̈- The University of Iowa, Summer 2014 Project List.

Cassidy Krause, University of Wisconsin-Platteville
Analysis of the Finite Element Method and the Discrete Maximum Principle

The maximum principle is a fundamental property of solutions to Laplace's equation with Dirichlet boundary conditions. To numerically solve the equation we use the finite element method. However, for certain triangulations of the domain, the numerical solutions will not satisfy the maximum principle. The triangulation of the mesh affects the values of the global stiffness matrix, which is directly used in calculating the numerical solution to the equation. If all of the entries of the inverse of the global stiffness matrix are positive, then the maximum principle will hold, (Ciarlet, 1970). An interest that naturally arises is identifying conditions of the triangulation of the mesh to ensure a positive inverse of the global stiffness matrix. Recently Korotov et al. showed that an obtuse triangulation produces a global stiffness matrix whose inverse has negative entries. We investigate the patterns of the negative entries of the inverse of the global stiffness matrix for arbitrary obtuse triangulations.

Kelleigh Kuehl, North Central College
see **Molly Durava**

Jessica Leete, Brigham Young University
Modeling Cell Movement

The principles governing movement of Dictyostelium discoideuim, a type of cellular slime mold, are the same behind the movement of skin cells and cancer cells in the body. When the unicellular amoebae begins to starve, the cells work together to move to a new area, form a fruiting body, and start their life cycle over. I will discuss a force-based model of the slug's movement that models the attachment sites (cadherins and integrins) in each cell as springs which can extend, attach, and contract to pull the cell forward or backward. The goal is to understand how changes to these cell attachment dynamics affect the slug's movement.

Su-Jin Lim, Grinnell College
Grid Diagrams and Permutations for Knots

A knot can be represented by many different projections, and each of these projections can be put into an $n \times n$ grid diagram with one X and one O in each row and column. The $n \times n$ grid diagram can then be represented by several different pairs of permutations on n elements —X_1,X_2, …, X_n and O_1, O_2, …, O_n. As grid diagrams have been proven to be very fruitful in the field of knot theory, we have studied patterns within the permutations and analyzed how they can be used to distinguish or recognize different types of knots.

Nicole Lopez, University of St. Thomas
Madeline Shogren, University of St. Thomas
Classifying Knots in Open Chains with Random Equilateral Polygon Closures

The recent discovery of knotting behavior in proteins has stimulated discussion about classifying knots in open chains. The topology is trapped in closed curves making it possible to study them mathematically. Therefore, in order to study and better understand the knotting patterns in open chains, their endpoints must be connected to make it possible to classify them. The purpose of this project is to analyze and compare two methods used to classify knots in open chains. Method I closes knots by extending rays from the endpoints out to infinity and then connecting them. Method II is being developed to close equilateral open chains using random equilateral arcs. The two methods are compared by applying each to the same set of open chains and analyzing their classifications. This project develops efficient processes for all methods and compares them to determine the most precise method for classifying an open knot.

Marina Massaro, State University of New York at Geneseo
Optimization of Intensity-Modulated Radiation Therapy with Linear Programming Techniques

Intensity-Modulated Radiation Therapy (IMRT) is an advanced type of radiation therapy where multiple small beams of varying intensities of radiation are conformed to the shape of the tumor using a multi-leaf collimator. The complexity of IMRT planning arises from its division into three sub-problems: geometry, intensity, and realization. We employ the simplex method and interior point algorithm to solve the geometry and intensity problems, and discuss the ramifications of various solutions.

China Mauck, Grinnell College
A Model for Transport in Stereocilia

Stereocilia are projections of hair cells in the inner ear, which are vital for hearing and balance in mammals. It is not currently known how their lengths are so precisely regulated and maintained throughout an organisms lifetime. We modified a previously developed mean-field PDE model of this phenomenon to account for recent biological discoveries, and our predictions from these equations more closely approximate experimental data.

Sarah McGahan, California State University, Fresno
Alexander- and Markov-type Theorems for Virtual Singular Links

A classical braid is a set of n strings passing between two horizontal bars. These strings may interact with one another but must always travel in the downward direction. If the two horizontal bars are brought together and each pair of string ends are glued together in order, the resulting structure will be a knot or a link which we call the closure of the braid. J.W. Alexander showed that any oriented classical link can be represented as the closure of a braid. In addition, it is well-known that two braids have isotopic closures if and only if they are related by braid isotopy and a finite series of the so-called Markov's moves. These two properties of braids are known as the Alexander and Markov Theorems. Analogous theorems have been proved for the set of virtual links as well as for the set of singular links. In this talk we first introduce the virtual singular braid monoid via generators and relations. We then prove Alexander and Markov-type theorems for virtual singular links.

Melania Meyer, College of Saint Benedict
Modeling a Three-Species Predator-Prey Ecosystem

During the course of this project versions of existing three-species models were modified and analyzed. This analysis included finding the models equilibria and the corresponding eigenvalues, and performing eigenvalue analysis in order to classify the equilibria. By finding and classifying the equilibria, we were better able to understand the behavior of these models. The equilibria displayed different types of behavior and were classified as a sink, saddle or source. Further analysis showed whether the equilibrium was nodal or spiral. These classifications varied between models and these variations were examined in order to look more closely at the differences in the models and what those differences were dependent on. We used Matlab and Mathematica to create and use different visualization tools to look at the behavior of the three species near their equilibria and to also examine the populations long-term behavior, which depended on the models parameters.

Alessandra Mitchell, Slippery Rock University
Optimizing Motion Correction and Spatial Smoothing Parameters for Preprocessing fMRI Brain Images

Functional magnetic resonance imaging examines changes in brain function over time. In an fMRI experiment, statistical significance tests are used to evaluate whether or not a predetermined task makes a statistically significant change in neural activation. Before these statistical tests can be run, the data must be preprocessed in order to reduce variability due to factors such as head movement and correlated data. The purpose of this study was to use the image processing software FSL to experiment with parameters to discover the best settings for two of the preprocessing steps, motion correction, which addresses the issue of unrelated variability, and spatial smoothing, which addresses the issue of correlated data. The original images were compared with the preprocessed brain images. Establishing the best parameter settings for the fMRI images improves the statistical validity of the analysis of these images and furthers the understanding of the statistical methods behind fMRI image preprocessing.

Kaleigh Mulkey, Kennesaw State University
see **Saniita FaSenntao**

Jillian Parker, Sam Houston State University
Omega Values of the Generators of Certain Primitive Numerical Monoids

Let M be a commutative, cancellative, atomic monoid with units $M^\times$ and atoms (or irreducibles) $\mathcal{A}(M)$. For $x \in M \setminus M^\times$, we define the omega function by $\omega(x) = n$ if n is the smallest positive integer such that if $x \mid a_1 \dots a_t$ with each $a_i \in \mathcal{A}(M)$, then there is a $T \subseteq \{1, \dots, t\}$ with $|T| \leq n$ such that $x \mid \Pi_{k \in T} a_k$. Moreover, the ω-function measures how close to prime an element is. We will conjecture simple formulas for determining these omega values of a primitive numerical monoid in any embedding dimension, where the set S is generated by a generalized arithmetic sequence of the form $\langle a, ah+d, ah+2d, \dots, ah+xd \rangle$ where $a,\ d,\ h$ and, x are positive integers and $gcd(a,d) = 1$. We show by applying a theroem by Omidali and Rahmati that these results are valid and enhance the understanding of generators of certain primitive numerical monoids.

Joana Perdomo, Harvey Mudd College
Mathematical Modeling of Blood Coagulation

Blood coagulation is a series of biochemical reactions that, in part, leads to blood clot formation and the cessation of bleeding. Blood platelets are vital to clot formation; clotting factors must bind to platelet surfaces for many of the reactions to occur. Many in vitro tests of coagulation involve closed systems that use lipids in place of cell surfaces for the reactions to occur. These lipids are thought to function analogously to cellular-surface binding sites, but are typically in higher concentrations than the number of binding sites on platelets. Mathematical models that simulate these lipid-based experiments exist but fail to include cell surfaces. In this study we extend a current mathematical model of the in vitro tests to include a finite number of binding sites on platelets. The purpose is to better understand the role of platelet binding sites in coagulation and to quantify the sensitivity of the extended model to the number of surfaces.

Laura Petto, Bowdoin College
Growth Functions of Finitely Generated Algebras

We studied bounds for growth functions of finitely generated algebras of the form F/I, where F is the free algebra in two variables and I is an ideal generated by monomials of length n. The algebra can be realized as a De Bruijn graph in two variables and with words of length n. Finding a bounds for the algebras growth is equivalent to finding a Hamiltonian path through cycles in the given De Bruijn graph. We defined a new, undirected graph from the De Bruijn graph and new moves on the graph to understand properties of cycles and edges between cycles in our new graph. Continuing the work of the previous REU, we showed that standard cycles will not create the desired Hamiltonian path for even graphs and proved properties of our new graph that we hope will find a path in the odd case graphs. No prior knowledge of graph theory is assumed.

Briahna Preston, Northern Arizona University
Alyssa Whittemore, Northern Arizona University
Prime Vertex Labelings of Unicyclic Graphs

A prime vertex labeling is an injective assignment of the labels 1, 2, ..., n to a simple connected graph with n vertices such that any two adjacent vertices have relatively prime labels. We will present new results involving special cases of the conjecture that all unicyclic graphs (graphs with exactly one cycle) have prime vertex labelings. No prior knowledge of graph theory is needed.

Zoe Rehnberg, Washington University in St. Louis
Exponential Random Graph Models Under Measurement Error

Understanding social networks is increasingly important in a world dominated by social media and access to big data. When analyzing social network data, we are often interested in the underlying structures that exist in the network. Exponential random graph models (ERGMs) are frequently used by analysts to gain a better understanding of the formation and operation of these structures. Data collection, however, always entails measurement error and, therefore, introduces bias into the resulting model. In our study, we investigated the robustness of ERGMs when faced with noisy data. After exploring the effects of simulated noise on summary statistics and on ERG coefficients, we then applied methods of Bayesian inference in an attempt to recover an accurate estimate of the true model.

Julie Richardson, Smith College
see **Victoria Kelley**

Maureen Salisbury, Coe College
Environmental Remote Sensing, Flux Towers, and My Role in the EcoSpec Project

Some such methods currently used to monitor and collect data include the use of environmental remote sensing and flux towers. The EcoSpec project is attempting to combine these methods and Identify associations among hyperspectral data and meteorological and biological measurements for investigating near-surface atmosphere-biosphere interactions. I give a brief background of each of these methods and explain my role in connecting the two using information from the Moderate-Resolution Imaging Spectrometer (MODIS) and the flux tower at the Fermi Agricultural Site. I explain the possible patterns and associations I have studied and how I went about studying them; including how the method used is limited.

Spencer Saunders, Regent University
Searching for Complex Pisot Numbers and Newman Polynomial Representatives

Original research by Charles Pisot defines a Pisot number as a real algebraic integer greater than one whose Galois conjugates are all less than one in absolute value. Based on his work, this study delves into field of Complex Analysis to understand the complex form of these numbers and the minimal polynomials which represent them. Many of these polynomials have a multiple with coefficients of either 0 or 1. The occurrence of these polynomials, called Newman polynomials, are the focus of remainder of our research with Zach Blumenstein (Brown University) and Dr. Michael Mossinghoff (Davidson University).

Madeline Shogren, University of St. Thomas
see **Nicole Lopez**

Katie Sipes, James Madison University
Modeling the Semi-Permeability of the Limbs of the Kidneys

The kidneys are an organ that is found on the dorsal of the body. Responsible for filtering the blood plasma, the kidneys must have a less than a .01% error in order to produce the correct concentration of urine. The semipermeability of the descending and ascending limbs are how the kidneys are able to make a low volume of concentrated urine or a high volume of dilute urine. We can model the semipermeability of the limbs with equations that explain how the kidneys are able to produce a perfect waste product.

Megan Sorenson, Concordia University Irvine
Modeling Fluid Flow Induced by C. elegans *Swimming at Low Reynolds Number*

C. elegans have been extensively researched regarding locomotion. However, most mathematical studies have focused on body dynamics rather than the fluid. As the nematodes undulate in a sinusoidal fashion, they cause fluid movement that has been studied experimentally but not modeled computationally on this scale. Utilizing the Navier-Stokes equation, regularized stokeslets, and the method of images, we computed the dynamics of the surrounding fluid. Our results strikingly matched experimental outcomes in various ways, including the distance particles travelled in one period of undulation, as well as qualitatively and quantitatively matching velocity fields. We then implemented this method using video data of swimming *C. elegans* and successfully reproduced the fluid dynamics. This is a novel application of the method of regularized stokeslets that combines theory and experiment. We expect this approach to provide insight in generating hypotheses and informing experimental design.

Sara Stover, Mercer University
Leamer Monoids and the Huneke-Wiegand Conjecture

The Huneke-Wiegand Conjecture has earned much attention in commutative algebra; once a theorem was developed connecting the Huneke-Wiegand Conjecture to numerical monoids, the study of arithmetical sequences in numerical monoids was sparked. In our research, we restricted ourselves to characterizing numerical monoids with consecutive generators (intervals). In particular, we focus on determining when a numerical monoid $\Gamma=\langle m, m+1, \ldots m+k\rangle$ has an irreducible sequence of the form $\{x, x+s, x+2s\}$ for $s \in \mathbb{N}\backslash\Gamma$. We discovered and proved a theorem for finding an irreducible sequence of this form when Γ has two generators, thus satisfying the Huneke-Wiegand Conjecture for monomial ideals generated by two elements. This lays the groundwork for finding conditions when $\{x_0, x_0+s, x_0+2s\} \subset \Gamma$ for any number of generators.

Emma Talis, Marist College
see **Victoria Kelley**

Thoa Tran, California State University, Fresno
The Kauffman Bracket for Virtual Singular Links

Virtual knot theory, introduced by Lou Kauffman in 1996, can be regarded as a projection of classical knot theory in thickened surfaces. We take one step further by studying virtual singular links, which can be thought as immersions of disjoint unions of circles into thickened surfaces. A virtual singular link diagram contains then three types of crossings: classical, singular, and virtual crossings. Much as in the case of classical knot theory, when studying virtual singular links we are interested in knowing if two virtual singular links are equivalent or not. In this talk we construct a polynomial invariant for virtual singular links by means of extending the Kauffman bracket of knots and links to virtual singular links. Such a polynomial invariant is very useful because if we compute it from two diagrams and obtain different polynomials, then we know that the two diagrams represent distinct virtual singular links.

Alyssa Whittemore, Northern Arizona University
see **Briahna Preston**

Natalie Wiens, Tabor College
Continuity Properties of the Modulus Function of Walk Families

The modulus of a family of walks on a weighted undirected graph provides a quantitative assessment of the “richness” of the family. The modulus is computed by minimizing an energy function over a set of admissible metrics on the graph. In certain special cases, the modulus has been shown to generalize the concepts of shortest path, min cut, and effective resistance. This research explores continuity properties of the modulus and the associated extremal graph metrics. We extend these concepts to more complex graphs, including long walks and balanced trees.

Qi Yang, University of Southern California
Two Novel Approaches for Learning Near-Isometric Linear Embeddings

The large size of data processed by diverse modalities poses a challenge to information processing systems. Thus we propose two non-convex approaches for learning near-isometric linear embeddings of finite sets of data points. Given a set of training points $\mathcal{X}$, we consider the secant set $S(\mathcal{X})$ that consists of all pairwise difference vectors of $\mathcal{X}$, normalized to lie on the unit sphere. The problem can be formulated as finding a symmetric and positive semi-definite matrix that preserves the norms of all the vectors in $S(\mathcal{X})$ up to a distortion parameter δ. We reformulate it into a Frobenius norm minimization problem and develop an algorithm, *FroMax*. Our other method, *NILE-Pro*, seeks a projection matrix ψ such that the restricted isometry property constraint is minimized directly. Both methods are then demonstrated to be more computationally efficient than previous convex approaches for a number of applications in machine learning.

Katelyn Zumpf, North Central College
see **Melissa Jay**